Circuits and Systems Analysis

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Grading

1 short exam	**23 February 2015**	**%10**
2 midterm exams	23 March 2015	%20
	20 April 2015	%20
1 coursework	to be announced in Ninova	%10
Final exam	**A minimum of 20/60 points from the above is required!**	**%40**

Frequently Asked Questions

- Is attendence compulsory?
  - No!
- Which topics are covered by an exam?
  - The answer is always the same: «everthing we have seen until now» .
- How can I sign up for Ninova?
  - This should be automatic, you should receive e-mails from Ninova to your ITU account. If you are not signed up or if you think there is a problem about Ninova, just send me an e-mail (karabacak@itu.edu.tr).
- Can you give us more questions to practice?
  - No, you can find many questions in the reference books.

References:

L.O. Chua, C.A. Desoer, S.E. Kuh. “Linear and Nonlinear Circuits”, Mc.Graw Hill, 1987, New York (Sections: 9,10,11,13).

B.D.O. Anderson, S. Vongpanitlerd, “Network Analysis and Synthesis”, Prentice-Hall, 1973, New Jersey.

Yılmaz Tokad, “ Devre Analizi Dersleri” Kısım II, İ.T.Ü. Yayınları, 1977.

Yılmaz Tokad, “ Devre Analizi Dersleri” Kısım IV, Çağlayan Kitabevi, 1987.

Cevdet Acar, “Elektrik Devrelerinin Analizi” İ.T.Ü. Yayınları, 1995.

M. Jamshidi, M. Tarokh, B. Shafai. “Computer-Aided Analysis and Design of Linear Control Systems”, Prentice Hall, 1992 (Sections: 2,3).

What have you learned in «Basics of Electrical Circuits»?

Goal: To predict the $\underbrace{\text{electrical behaviour}}_{\text{current and voltage}}$ of *physical circuits.*

Undefined quantities in circuit theory: Current and voltage.

Axioms in circuit theory: KCL, KVL and lumped circuit axiom.

Circuit elements: Linear and nonlinear resistors, capacitors, inductors.

Methods to analyze circuits: Node analysis, mesh analysis.

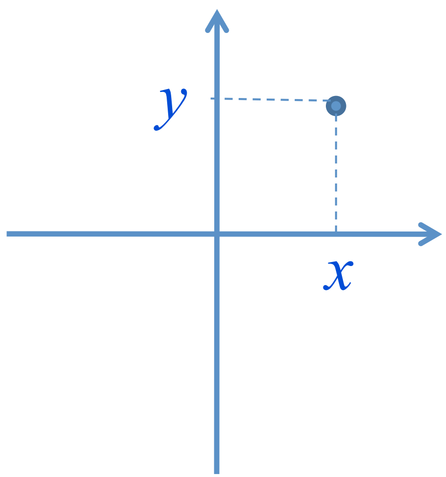
Some theorems: Tellegen's theorem, Additivity, Multiplicativity, Thevenin and Norton theorems.

Analysis of Dynamical Circuits: ....

Background: Complex numbers

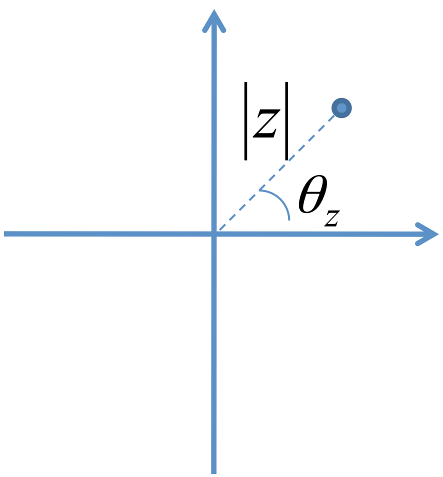
Cartesian Coordinates

$$z = x + jy$$



Polar Coordinates

$$z = |z| e^{j\theta_z}$$



$$\mathrm{Re}\{z\} = x \quad \mathrm{Im}\{z\} = y$$

$$|z| = \sqrt{x^2 + y^2} \quad \theta_z = \arctan\left(y/x\right)$$

$$z_1 = x_1 + jy_1 \qquad z_2 = x_2 + jy_2$$

$$z_1 = |z_1| e^{j\theta_{z1}} \qquad z_2 = |z_2| e^{j\theta_{z2}}$$

$$z_1 + z_2 = (x_1 + jy_1) + (x_2 + jy_2) = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 . z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$$

$$z_1 . z_2 = |z_1||z_2| e^{j(\theta_{z1} + \theta_{z2})}$$

$$\frac{z_1}{z_2} = \frac{(x_1 + jy_1)}{(x_2 + jy_2)} = \frac{(x_1 + jy_1)(x_2 - jy_2)}{(x_2 + jy_2)(x_2 - jy_2)}$$

$$= \frac{(x_1 x_2 + y_1 y_2) + j(-x_1 y_2 + x_2 y_1)}{(x_2^2 + y_2^2)}$$

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} e^{j(\theta_{z1} - \theta_{z2})}$$

$$z + \bar{z} = 2x$$

$$z - \bar{z} = 2jy$$

$$z.\bar{z} = |z|^2 = x^2 + y^2$$

Background: Solutions of dynamical circuits

State Transition Matrix

$$\boxed{x(t) = \Phi(t)x(t_0) + x_{par}(t) - \Phi(t)x_{par}(t_0)}$$

$$x(t) = \underbrace{\Phi(t)x(t_0)}_{\text{zero-input solution}} + \underbrace{x_{par}(t) - \Phi(t)x_{par}(t_0)}_{\text{zero-state solution}}$$

$$x(t) = \underbrace{e^{At}x(t_0)}_{\text{zero-input solution}} + \underbrace{\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau}_{\text{zero-state solution}}$$

Zero-input solution:

eigenvalues

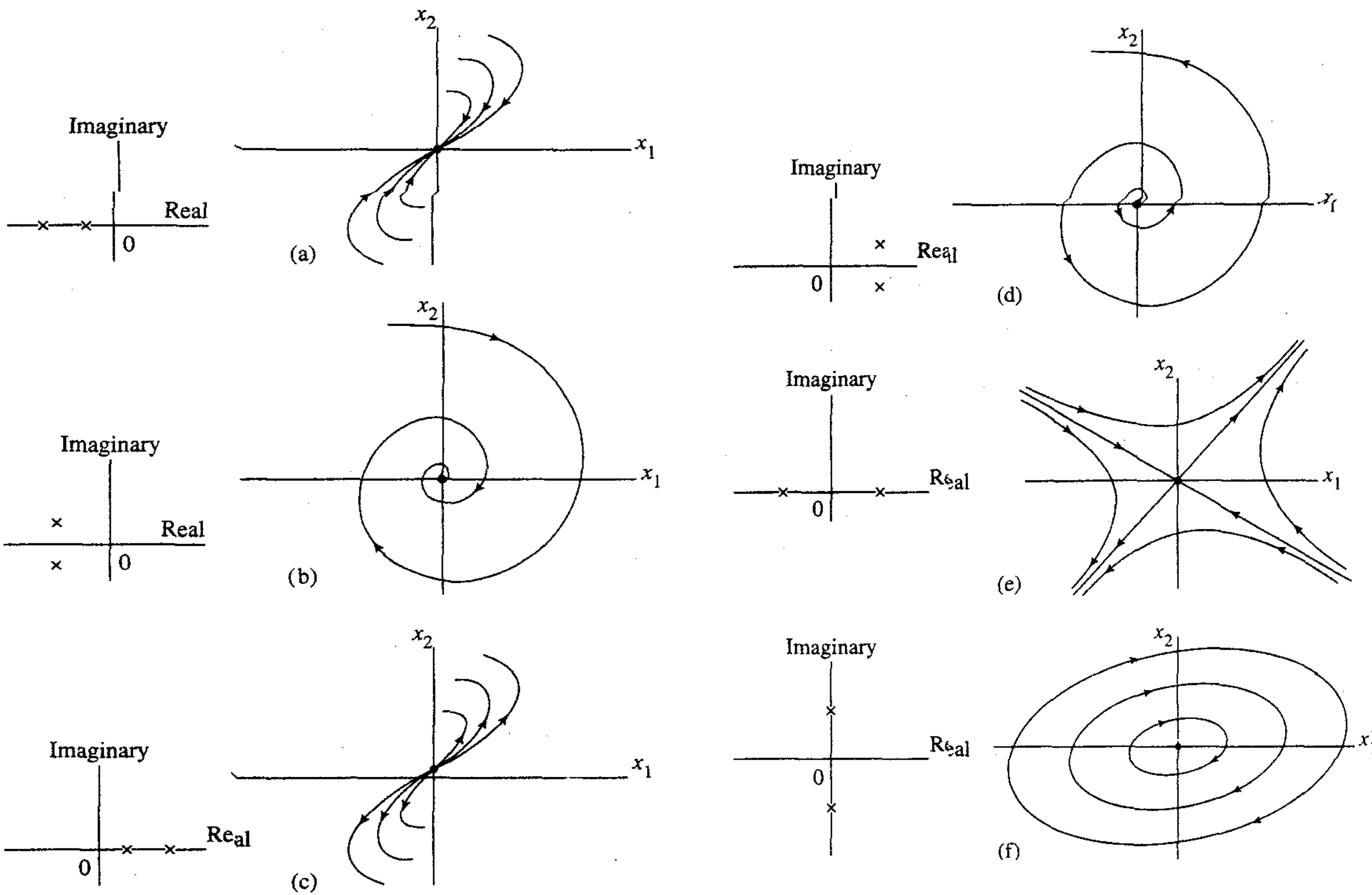
$$x_{zi}(t) = e^{\lambda_1 (t-t_o)} S_1 c_1 + e^{\lambda_2 (t-t_o)} S_2 c_2 + \ldots\ldots e^{\lambda_n (t-t_o)} S_n c_n$$

eigenvectors

How do eigenvalues and eigenvectors affect the solution?

....

Solutions of Linear Systems



S. Haykin, "Neural Networks- A Comprehensive Foundation" 2nd Edition, Prentice Hall, 1999, New Jersey.

What is common in all these systems?

A special solution of a dynamical system: Equilibrium

$$\dot{x}(t) = Ax(t) \quad \rightarrow \quad 0 = Ax_d$$

$$\dot{x}(t) = f(x(t)) \quad \rightarrow \quad 0 = f(x_d)$$

How many equilibria are there?

What happens near the equlibrium?

<u>Definition:</u> Lyapunov stability

Let x_d be an equilibrium of the system given by $\dot{x}(t) = f(x(t))$.
The equilibrium is Lyapunov stable if for every $\varepsilon > 0$ there exists a $\delta(\varepsilon) > 0$ such that

$$\|x(0) - x_d\| < \delta(\varepsilon) \quad \Rightarrow \quad \|x(t) - x_d\| < \varepsilon \quad \forall t > 0.$$

A Lyapunov stable equilibrium x_d is asymptotically stable if there exists a $\delta > 0$ such that $\|x(0) - x_d\| < \delta \quad \Rightarrow \quad \lim_{t \to \infty} \|x(t) - x_d\| = 0$.

Sinusoidal Steady-State Analysis

Goal: To find zero-state solution.

Why "sinusoidal and steady"?

steady → We are interested in the steady solution (kalıcı çözüm). The zero-input solution is assumed to converge to zero.

sinusoidal → Source that drive the circuit are assumed to be sinusoidal. Hence, the solution is also sinusoidal.

The method is not limited to circuit theory; it can also be applied to control theory, quantum theory and electromagnetic theory.

Tool: Phasor

Sinusoidal

$$\boxed{x(t) = A_m \cos(wt + \varphi)}$$

amplitude frequency phase

$$x(t) = A_m \cos(wt + \varphi)$$

$$A_m > 0 \qquad w : [rad/sn], \;\; T \hat{=} \frac{2\pi}{w}, \;\; w = 2\pi f$$

$$f : [Hz]$$

Phasor

$$\boxed{A \hat{=} A_m e^{j\varphi}}$$

If the phasor is given, how can we find the sinusoidal signal?

If frequency w and the phasor A is known, then

$$\mathrm{Re}[Ae^{jwt}] = \mathrm{Re}[A_m e^{j(wt+\varphi)}]$$
$$= A_m \cos(wt + \varphi)$$

Sinusoidal

Phasor

$$x(t) = A_m \cos(wt + \varphi) \Longleftrightarrow A = A_m e^{j\varphi}$$

$$= A_m \cos(\varphi)\cos(wt) + (-A_m)\sin(\varphi)\sin(wt)$$

$$= A_m \cos\varphi + jA_m \sin\varphi$$

$$\rightarrow A_m \cos(wt + \varphi) = \mathrm{Re}(A)\cos wt - \mathrm{Im}(A)\sin wt$$