Istanbul Technical University Faculty of Mechanical Engineering
MAK422E Engineering Design and CAD Midterm Exam,
October 28, 2014, Instructor: Hikmet Kocabas Time: 90 minutes

1. (10) Write the file transfer standards between computer programs.

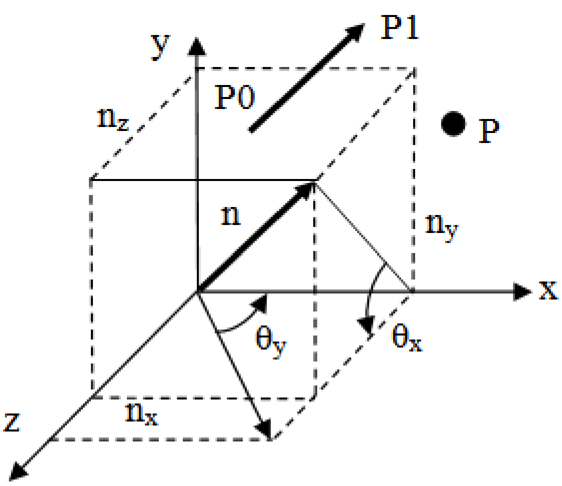
CAD data transfer formats : IGES, PDES, STEP, DXF, DWG, CADL, Parasolid, SAT, ACIS

2. (15) How to calculate the transformation matrix of a rotation about an arbitrary 3D space vector that is not passing through the origin?

$$Rx(\theta) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Ry(\theta) := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T0 = \begin{pmatrix} 1 & 0 & 0 & P0_x \\ 0 & 1 & 0 & P0_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T1 = \begin{pmatrix} 1 & 0 & 0 & -P0_x \\ 0 & 1 & 0 & -P0_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = T0\, R_x(-\theta_x)\, R_y(-\theta_y)\, R_x(\theta)\, R_y(\theta_y)\, R_x(\theta_x)\, T1 \qquad P' = T \cdot P$$



$$n_1 = \sqrt{n_y^{\,2} + n_z^{\,2}}$$

$$\cos(\theta_x) = \frac{n_z}{n_1} \qquad \sin(\theta_x) = \frac{n_y}{n_1}$$

$$\cos(\theta_y) = \frac{n_x}{n} \qquad \sin(\theta_y) = \frac{n_1}{n}$$

$$\theta_x = \mathrm{acos}\left(\frac{n_z}{n_1}\right) \qquad \theta_y = \mathrm{acos}\left(\frac{n_x}{n}\right)$$

3. (25) Extend L_1 to L_2 by using intersection point P_5 of L_1 and L_2.

Find the u parameter corresponding to this extention point P_5.

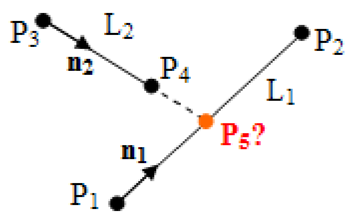
P_1 (3,1) , P_2 (7,6) , P_3 (1,7) , P_4 (3, 4).

Use vector algebra.

$n_1 = (P_2\ \ P_1) / |P_2\ \ P_1|$ $\qquad L_1 = P(u) = P_1 + u\,(P_2\ \ P_1)$

$L_1 = P(u) = P_1 + u\,(P_2\ \ P_1) = L_2 = P_3 + v\,(P_4\ \ P_3) \quad \cdot\, n_5$

$$P_1 := \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \quad P_2 := \begin{pmatrix} 7 \\ 6 \\ 0 \end{pmatrix} \quad P_3 := \begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix} \quad P_4 := \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$



$$n_1 := \frac{P_2 - P_1}{|P_2 - P_1|} \qquad n_1 = \begin{pmatrix} 0.625 \\ 0.781 \\ 0 \end{pmatrix}$$

$$n_2 := \frac{P_4 - P_3}{|P_4 - P_3|} \qquad n_2 = \begin{pmatrix} 0.555 \\ -0.832 \\ 0 \end{pmatrix} \qquad n_3 := (n_1 \times n_2) \qquad n_3 = \begin{pmatrix} 0 \\ 0 \\ -0.953 \end{pmatrix}$$

$$n_4 := (n_3 \times n_1) \qquad n_4 = \begin{pmatrix} 0.744 \\ -0.595 \\ 0 \end{pmatrix} \qquad n_5 := (n_2 \times n_3) \qquad n_5 = \begin{pmatrix} 0.793 \\ 0.529 \\ 0 \end{pmatrix}$$

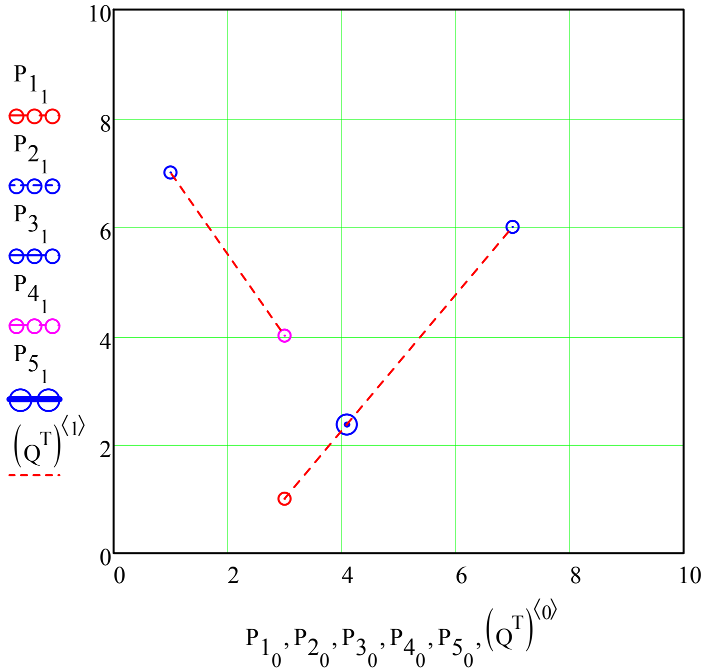
$$P_1 + u\cdot(P_2 - P_1) = P_3 + v\cdot(P_4 - P_3) \qquad \mathbf{.\,n_5}$$

$$u := \frac{(P_3 - P_1)\cdot n_5}{(P_2 - P_1)\cdot n_5} \qquad u = 0.273 \qquad P_5 := P_1 + u\cdot(P_2 - P_1) \qquad P_5 = \begin{pmatrix} 4.091 \\ 2.364 \\ 0 \end{pmatrix}$$

$$P_1 + u\cdot(P_2 - P_1) = P_3 + v\cdot(P_4 - P_3) \qquad \mathbf{.\,n_4} \qquad v := \frac{(P_1 - P_3)\cdot n_4}{(P_4 - P_3)\cdot n_4} \qquad v = 1.545$$

$$P_5 := P_3 + v\cdot(P_4 - P_3) \qquad P_5 = \begin{pmatrix} 4.091 \\ 2.364 \\ 0 \end{pmatrix}$$

for drawing purpose far point $\quad F := \begin{pmatrix} 10^{10} \\ 10^{10} \\ 0 \end{pmatrix} \qquad Q := \text{augment}(P_1, P_2, F, P_3, P_4, F, P_5)$



4. (25) Scale by 3 the triangle formed by end points P1 (2, 3), P2 (1, 2), P3 (3, 1) in Fig.2. Sketch the results. Transformation will keep the corner P2 of triangle at the same coordinate.

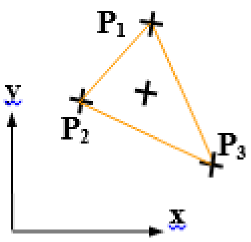
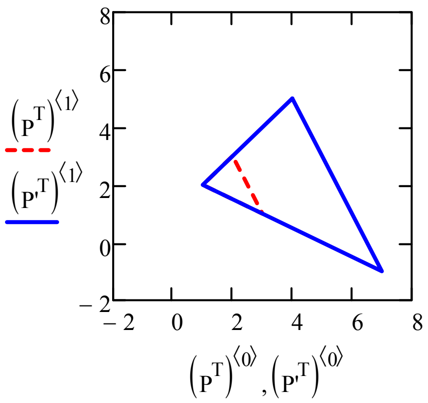


Fig.2.

$$P := \begin{pmatrix} 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad S := \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T := \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P' := T \cdot S \cdot T1 \cdot P \qquad P' = \begin{pmatrix} 4 & 1 & 7 & 4 \\ 5 & 2 & -1 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



$$T \cdot S \cdot T1 = \begin{pmatrix} 3 & 0 & -2 \\ 0 & 3 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

5. (25) Consider the planar cubic Hermite curve in Fig.3 defined by four points $P_0=(0,6)$, $P_1=(3,6)$, $P_2=(6,3)$, $P_3=(3,0)$, Compute the point p(0.3) of the curve equation.

$$p(u) = \left(2\cdot u^3 - 3\cdot u^2 + 1\right)\cdot p_0 + \left(-2u^3 + 3\cdot u^2\right)\cdot p_1 + \left(u^3 - 2\cdot u^2 + u\right)\cdot p'_0 + \left(u^3 - u^2\right)\cdot p'_1$$

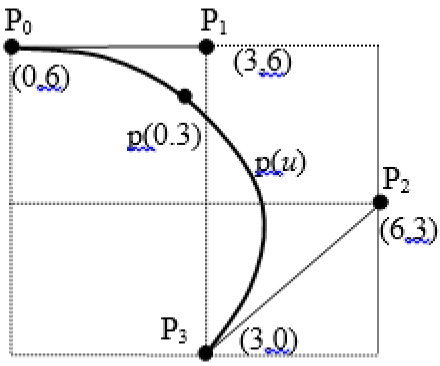
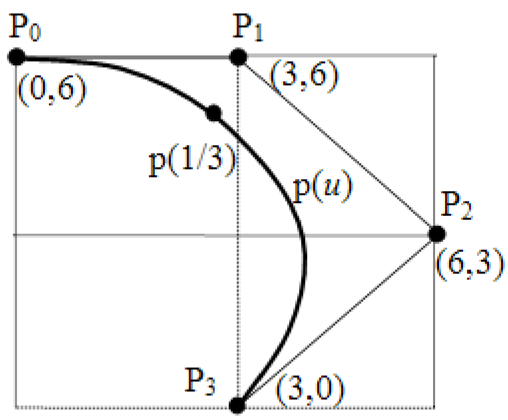


Fig.3.

$u := uu$

$n := 10 \qquad i := 0..n \qquad u_i := \frac{i}{n}$



cubic Hermite curve

Control Points :

$p_0 := \begin{pmatrix} 0 \\ 6 \end{pmatrix} \qquad p_1 := \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad p_2 := \begin{pmatrix} 6 \\ 3 \end{pmatrix} \qquad p_3 := \begin{pmatrix} 3 \\ 0 \end{pmatrix} \qquad P := \text{stack}\left(p_0^T, p_1^T, p_2^T, p_3^T\right)$

$p'_0 := \left(p_1 - p_0\right) \qquad p'_3 := \left(p_3 - p_2\right) \qquad P = \begin{pmatrix} 0 & 6 \\ 3 & 6 \\ 6 & 3 \\ 3 & 0 \end{pmatrix}$

Cubic Hermite Curve Equation:

$$p(u) := \left(2\cdot u^3 - 3\cdot u^2 + 1\right)\cdot p_0 + \left(-2u^3 + 3\cdot u^2\right)\cdot p_3 + \left(u^3 - 2\cdot u^2 + u\right)\cdot p'_0 + \left(u^3 - u^2\right)\cdot p'_3$$

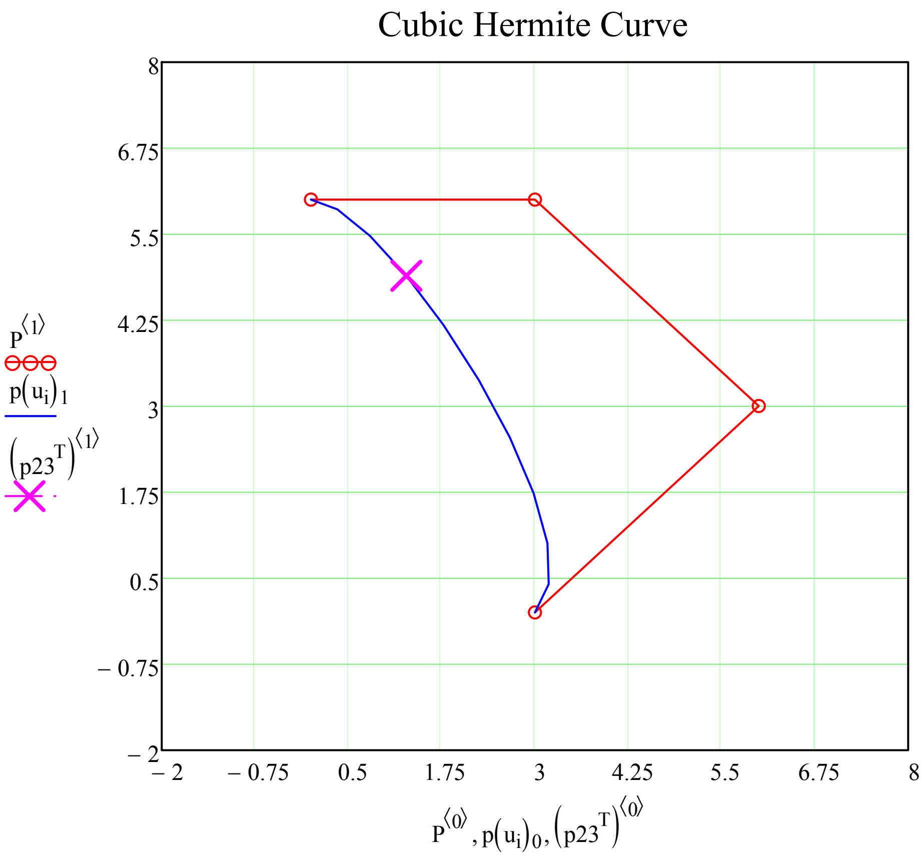
$F0(u) := \left(2\cdot u^3 - 3\cdot u^2 + 1\right)$ $F0(0.3) = 0.784$

$F1(u) := \left(-2u^3 + 3\cdot u^2\right)$ $F1(0.3) = 0.216$

$F2(u) := \left(u^3 - 2\cdot u^2 + u\right)$ $F2(0.3) = 0.147$

$F3(u) := \left(u^3 - u^2\right)$ $F3(0.3) = -0.063$

$v := 0.3$ $p23 := p(0.3)$ $p23 = \begin{pmatrix} 1.278 \\ 4.893 \end{pmatrix}$



$$p_0 := \begin{pmatrix} 0 \\ 6 \end{pmatrix} \qquad p_1 := \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad p_2 := \begin{pmatrix} 6 \\ 3 \end{pmatrix} \qquad p_3 := \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$p'_0 := \frac{(p_1 - p_0)}{|(p_1 - p_0)|} \qquad p'_3 := \frac{(p_3 - p_2)}{|p_3 - p_2|} \qquad P = \begin{pmatrix} 0 & 6 \\ 3 & 6 \\ 6 & 3 \\ 3 & 0 \end{pmatrix}$$

Cubic Hermite Curve Equation:

$$p(u) := \left(2\cdot u^3 - 3\cdot u^2 + 1\right)\cdot p_0 + \left(-2u^3 + 3\cdot u^2\right)\cdot p_3 + \left(u^3 - 2\cdot u^2 + u\right)\cdot p'_0 + \left(u^3 - u^2\right)\cdot p'_3$$

$$v := 0.3 \qquad p23 := p(0.3) \qquad p23 = \begin{pmatrix} 0.84 \\ 4.749 \end{pmatrix}$$