ITU MAK422E-4025E Engg. Design and CAD CRN: 13989
Final Exam, Jan. 9, 2016 Time: 90 minutes, Instructor: Hikmet KOCABAS

1. (20) Explain which transformation matrices have to be applied in which order to rotate the point (2, 2) on the xy plane by 30 degrees in the counter clockwise direction with the point (3, 4) as the center of rotation. Calculate the coordinates of the rotated point by applying the transformation matrices derived above to the point (2, 2).

The rotation axis must pass through the origin while the same relative position is maintained between the object and the rotation axis. Thus the object is translated by (-3,-4,0) together with the rotation axis so that the rotation axis coincides with the z axis of the world coordinate system. Then the object is rotated about z axis by 30 degrees CCW. Now the object is translated again, by (3,4,0), to return to the original position. These operations can be expressed as

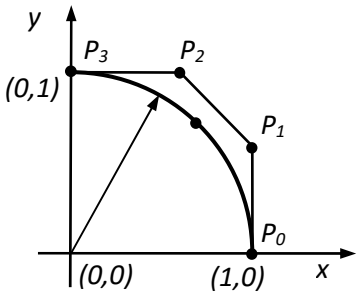
$$[X_w \quad Y_w \quad Z_w \quad 1]^T = Trans(3,4,0)\cdot Rot(z,30^\circ)\cdot Trans(-3,-4,0)\cdot[2 \quad 2 \quad 0 \quad 1]^T$$

Expanding above equation gives

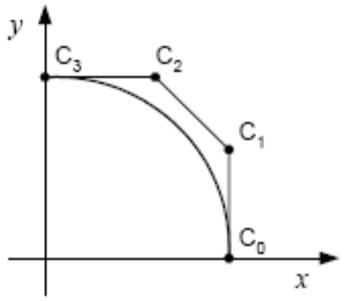
$$[X_w \quad Y_w \quad Z_w \quad 1]^T = \begin{bmatrix}1&0&0&3\\0&1&0&4\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos 30^\circ & -\sin 30^\circ & 0 & 0\\ \sin 30^\circ & \cos 30^\circ & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1&0&0&-3\\0&1&0&-4\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}2\\2\\0\\1\end{bmatrix}$$

$$= [3.134 \quad 1.768 \quad 0 \quad 1]^T$$

2. (20) Determine a Bezier curve of degree 3 which approximates a quarter circle centered at (0, 0). The end points of the quarter circle are (1, 0) and (0, 1). Calculate the coordinates of the middle point of this Bezier curve and compare them with those of the middle point of the quarter circle.



$$P(u) = \sum_{i=0}^{n} C_i^n \, u^i (1-u)^{n-i} \, P_i \qquad C_i^n = \frac{n!}{i\,!\,(n-i)\,!}$$



Parametric expression of unit quarter circle: $\mathbf{P}(u)$

$$\mathbf{P}(u)=\begin{pmatrix}\cos\frac{\pi}{2}u\\ \sin\frac{\pi}{2}u\\ 0\end{pmatrix} \qquad \mathbf{P}'(u)=\begin{pmatrix}-\frac{\pi}{2}\sin\frac{\pi}{2}u\\ \frac{\pi}{2}\cos\frac{\pi}{2}u\\ 0\end{pmatrix}$$

If $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, *and* $\mathbf{C}_3$ *are the control points of Bezier curve which represents the unit quarter circle,*

$$\mathbf{P}(0)=(1,0,0)=\mathbf{C}_0 \qquad \mathbf{P}(1)=(0,1,0)=\mathbf{C}_3$$
$$\mathbf{P}'(0)=(0,\pi/2,0) \qquad \mathbf{P}'(1)=(-\pi/2,0,0)$$

Bezier curve form expression of the unit quarter circle,

First derivative of $\mathbf{P}(u)$*:*

$$\frac{d\mathbf{P}(u)}{du}=3\cdot\sum_{i=0}^{2}\binom{2}{i}u^i(1-u)^{2-i}(\mathbf{P}_{i+1}-\mathbf{P}_i)$$

$$\left[\frac{d\mathbf{P}(u)}{du}\right]_{u=0}=3\cdot(\mathbf{P}_1-\mathbf{P}_0)=\mathbf{P}'(0) \qquad \therefore \mathbf{P}_1=\mathbf{P}(0)+\frac{1}{3}\mathbf{P}'(0)=\begin{pmatrix}1\\ \pi/6\\ 0\end{pmatrix}$$

$$\left[\frac{d\mathbf{P}(u)}{du}\right]_{u=1}=3\cdot(\mathbf{P}_3-\mathbf{P}_2)=\mathbf{P}'(1) \qquad \therefore \mathbf{P}_1=\mathbf{P}(1)-\frac{1}{3}\mathbf{P}'(1)=\begin{pmatrix}\pi/6\\ 1\\ 0\end{pmatrix}$$

Bezier curve is expanded as follows.

$$\mathbf{P}(u)=\sum_{i=0}^{3}\binom{3}{i}u^i(1-u)^{3-i}\mathbf{P}_i$$
$$=\binom{3}{0}(1-u)^3\begin{pmatrix}1\\0\\0\end{pmatrix}+\binom{3}{1}(1-u)^2u\begin{pmatrix}1\\ \pi/6\\ 0\end{pmatrix}+\binom{3}{2}(1-u)u^2\begin{pmatrix}\pi/6\\1\\0\end{pmatrix}+\binom{3}{3}u^3\begin{pmatrix}0\\1\\0\end{pmatrix}$$
$$=\begin{pmatrix}(1-u)^3+3(1-u)^2u+\pi/2\cdot(1-u)u^2\\ \pi/2\cdot(1-u)^2u+3(1-u)u^2+u^3\\ 0\end{pmatrix}$$

$$\therefore \mathbf{P}(\frac{1}{2})=\begin{pmatrix}0.6963\\0.6963\\0\end{pmatrix} \quad : mid\ \ point\ in\ Bezier\ expression$$

The mid point of the quarter circle is obtained.

$$\mathbf{P}\left(\frac{1}{2}\right) = \begin{pmatrix} \cos\left(\frac{\pi}{2}\cdot\frac{1}{2}\right) \\ \sin\left(\frac{\pi}{2}\cdot\frac{1}{2}\right) \\ 0 \end{pmatrix} = \begin{pmatrix} 0.707 \\ 0.707 \\ 0 \end{pmatrix}$$

It can be said that the curve approximated by Bezier curve is flatter than the exact circular arc.

3. (10) Explain the solid modeling methods in CAD?

There are many ways to model a part,
the major categories are,

Surfaces (such as polygons used in ES 206)

Elemental (using lines and points like drafting)

Boundary Representation (B-rep)

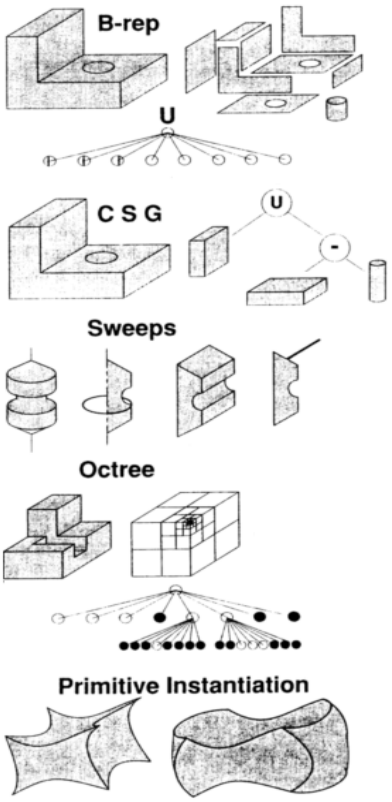
Constructive Solid Geometry (CSG)

Sweep

Hybrid (feature based modellers)

Tessellated Models, Octree Model

Primitive Instantiation, Analitical Solid Modeling (ASM)



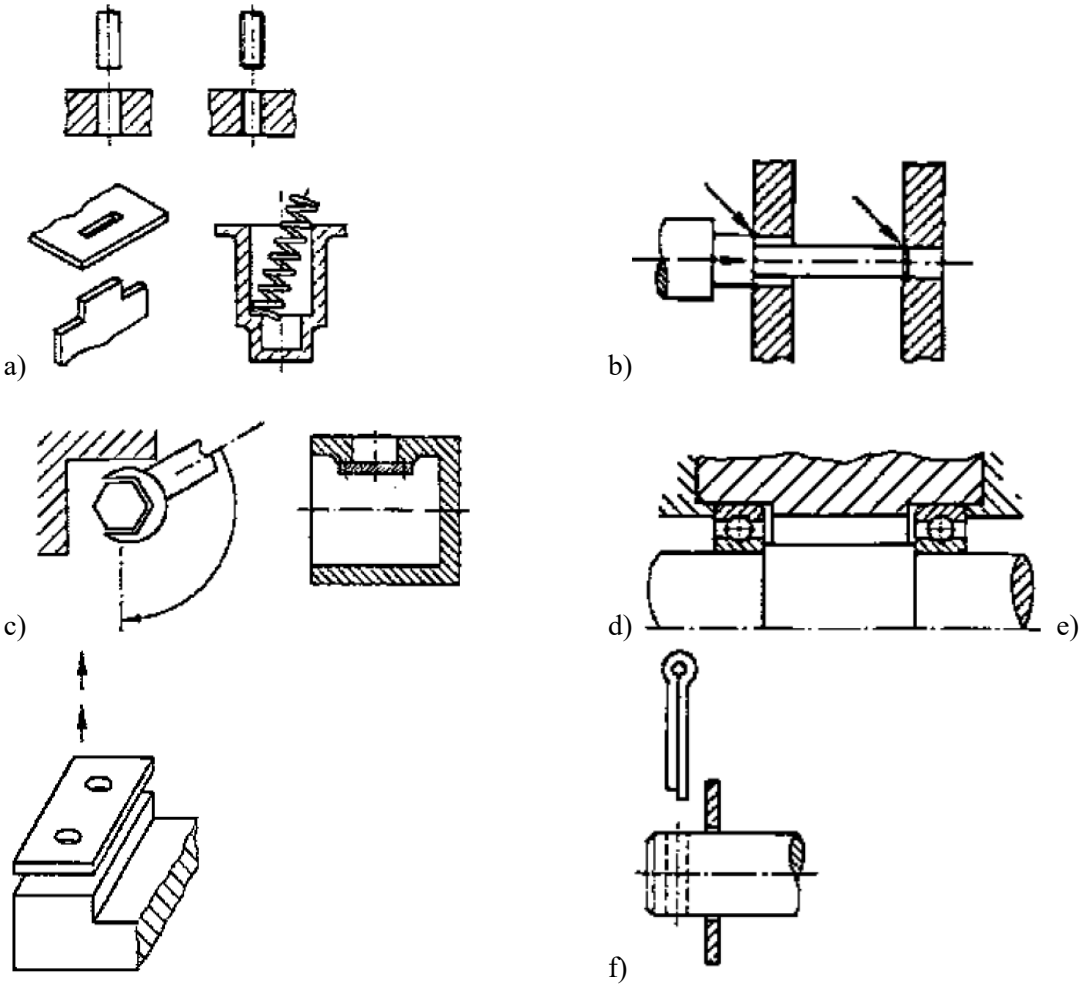
4. (15) What are the advantages of engineering design techniques?

the advantages of engineering design techniques:
Advantages of these techniques are,
- shorter production times
- fewer production steps
- smaller parts inventory
- more standardized parts
- simpler designs that are more likely to be robust
- they can help when expertise is not available, or as a way to reexamine traditional designs
- proven to be very successful over decades of application

5. **(15)** Write down the main principles of Design for Manufacture.

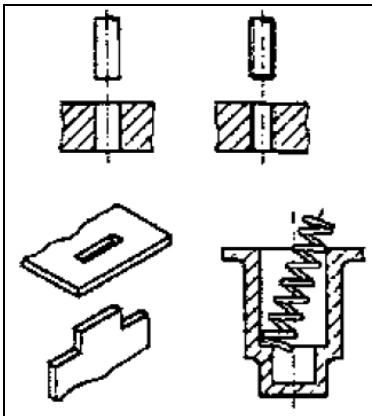
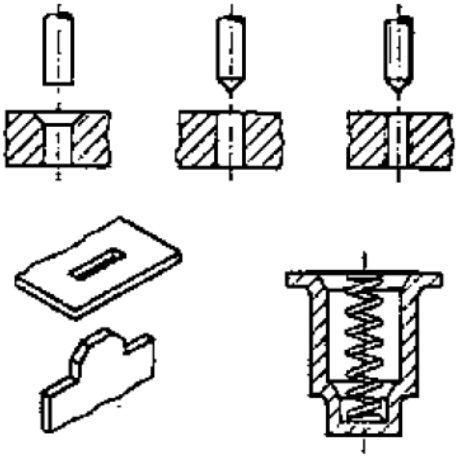
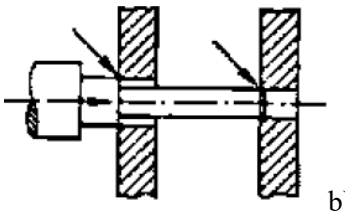
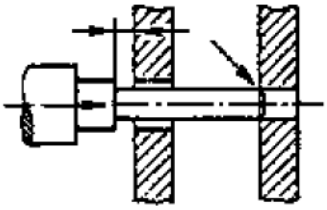
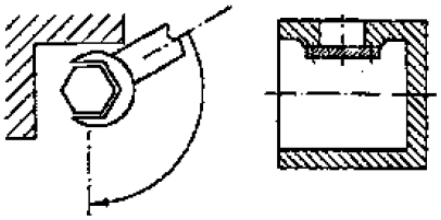
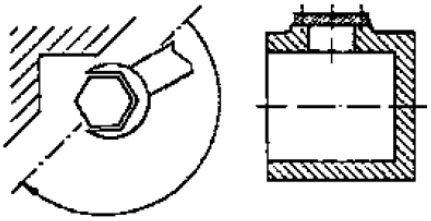
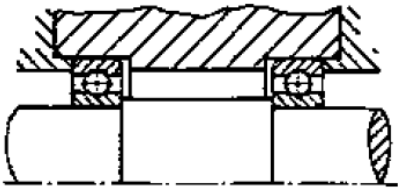
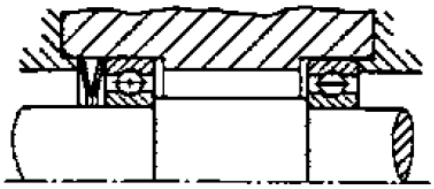
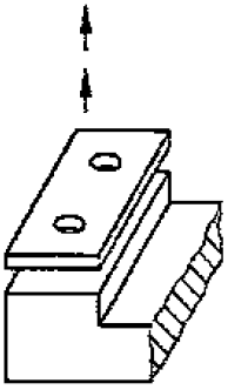
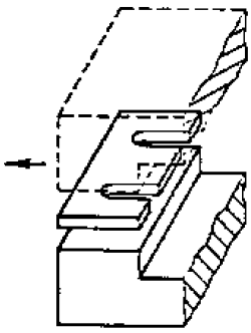
PRINCIPLES OF DFM
1. Reduce the total number of parts
2. Develop modular designs
3. Use standard components
4. Parts should be multi-functional
5. Parts should be multi-use
6. Parts should be designed for ease of fabrication
7. Avoid separate fasteners
8. Minimize the number of assembly directions.
10. Minimize handling

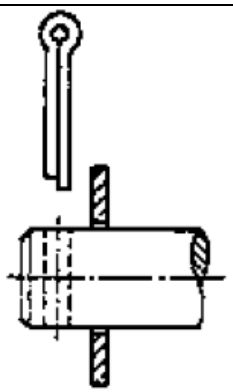
6. **(20)** How can you improve following designs in terms of DFX (Assembly)? Draw sketches and explain.



7. **(20)** How can you improve following designs in terms of DFX (Assembly)? Draw sketches and explain.

Text book: Engineering Design, Springer (2007)Pahl_Beitz.pdf, pg.382, Fig.7.125, Embodiment Design

 a) Provide tapering to ease joining	
 b) Avoid simultaneous operations that influence each other.	
 c) Provide access for assembly tools	
 d) Allow for large tolerances through assembly parts that are flexible	
 e) Adapt using standardised matching parts without disassembling	

 f) Apply locking elements that are easy to assemble