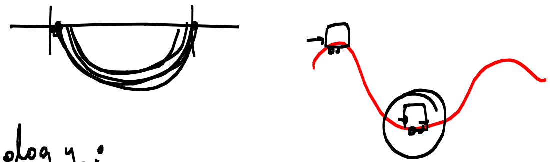
Chp 14: Periodic Motion ~ oscillations

Seasons:



Some terminology:

Period: The time to complete one full cycle

Amplitude: max. mag. of displacement from equil. position.

0-point ⇒ stable eq. position

Frequency: # of cycles per unit time ⇒ $f = \frac{1}{T} = s^{-1}$ ⇒ Hz

Hz ⇒ Heinrich Hertz (1850~)

Ang. Freq: $\frac{2\pi}{T} = \omega$

Simple Harmonic Motion:

$$\boxed{F = (\text{const})\, x} \qquad \boxed{F = -k x} \quad \text{constant}$$

$$F_x = m a_x = \boxed{\frac{m}{m}\frac{d^2x}{dt^2} = \frac{-kx}{m}}$$

$$\frac{d^2x}{dt^2} = -\overbrace{\frac{k}{m}}^{\omega} x$$

$$\frac{d^2\theta}{dt^2} = -\overbrace{\frac{g}{l}}^{\omega} \theta$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

$A\cos(x) + B\sin x$

$\cos(at)$

$-a\sin(at)$

$-a^2\cos(at)$

$A\cos\left(\sqrt{\frac{k}{m}}\, t\right)$

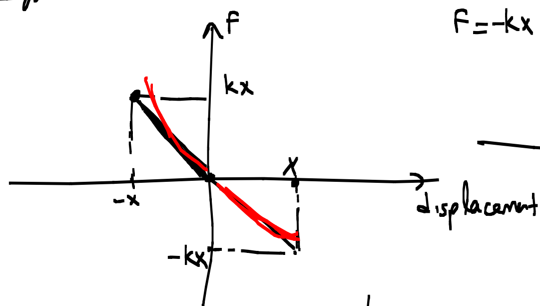
$e^{ix} \Rightarrow \cos x + i\sin x$

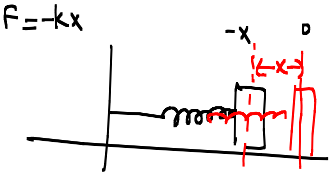
$$\frac{dx}{dt} = 3 \Rightarrow \int dx = \int 3\,dt$$

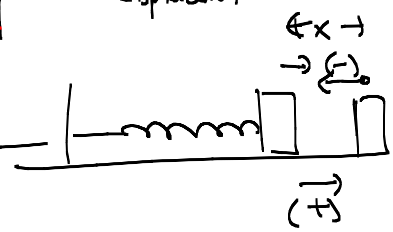
$$\boxed{x = 3t + c}$$

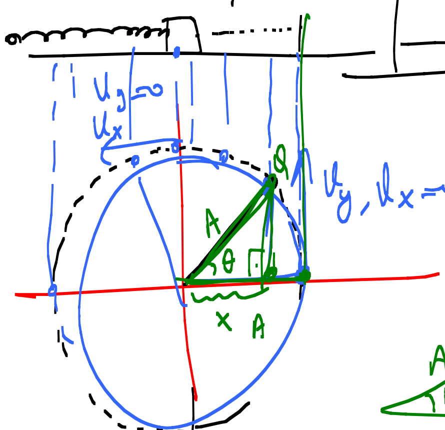
Analogy with Circular Motion

$F = -kx$

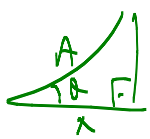








$$\boxed{x = A\cos\theta}$$



$$f = \frac{mv^2}{R} = m\omega^2 R = ma$$

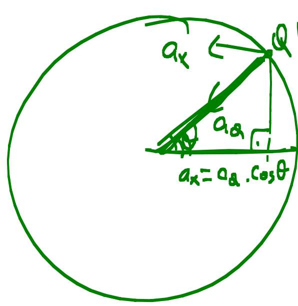
$$\boxed{a_Q = \omega^2 A}$$

$a_Q \rightarrow a_x$

$$a_x = -a_Q \cos\theta$$

$$\boxed{a_x = -\omega^2 A\cos\theta}$$

$$\boxed{a_x = -\omega^2 x}$$



$$\boxed{\begin{aligned} F &= -kx \\ F &= ma_x \end{aligned}}$$

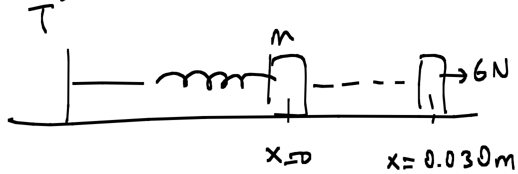
$$f = -kx = ma_x = m(-\omega^2 x)$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$\omega = 2\pi f \Rightarrow f = \frac{1}{2\pi}\omega = \boxed{\frac{1}{2\pi}\sqrt{\frac{k}{m}} = f}$

$f = \frac{1}{T}$ &, $\omega = \frac{2\pi}{T}$ $\boxed{R}$

Example



$F = -kx$

a) $k = \frac{6}{0.03} = 200\ N/m$ $m = 0.50\ kg$

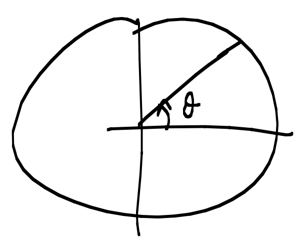
b) $\omega = ?$, $f = ?$, $T = ?$ $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.5}} = \sqrt{400} = 20$

$\omega = 20\ rad/s$

$f = \frac{\omega}{2\pi} = \frac{20\ rad/s}{2\pi\ rad/cycle} = \frac{20}{2\pi}\ \underbrace{cyc/sec}_{Hz} = 3.2\ Hz$

$T = \frac{1}{f} = \frac{1}{3.2\ cy/sec} = 0.31\ s$

$x = A\cos\theta$



$\boxed{\theta(t) = \omega t + \theta_0}$

$x(t) = v t + x_0$

$\boxed{x(t) = A\cos[\omega t + \phi]}$

$\boxed{\omega = \sqrt{\frac{k}{m}}}$

$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$

$$-\frac{k}{m}x + \frac{k}{m}x = 0$$

$$v(t) = \frac{dx}{dt}$$

$$v = \frac{d}{dt}\left(A\cos(\omega t+\phi)\right) = -A*\omega \sin(\omega t+\phi)$$

$$\boxed{v = -A\omega \sin(\omega t+\phi)}$$ vel in. S.H.M

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \Rightarrow \frac{d}{dt}\left[-A\omega \sin(\omega t+\phi)\right]$$

$$a_x = -A\omega * \omega * \cos(\omega t+\phi)$$

$$\boxed{x = A\cos(\omega t+\phi)}$$

$$\boxed{a_x = -A\omega^2 \cos(\omega t+\phi)}$$ acc. in SHM

$$\boxed{a = -\omega^2 x = -\omega^2 A\cos(\omega t+\phi)}$$

$$\boxed{\frac{d^2x}{dt^2} = \frac{k}{m}x}$$

$$\boxed{v_{0x},\ x_0}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\boxed{v(t) = -A\omega \sin(\omega t+\phi)} \qquad \boxed{x(t) = A\cos(\omega t+\phi)}$$

$$v(t=0) = v_0 = -A\omega \sin\phi \qquad x(t=0) = x_0 = A\cos\phi$$

$$\frac{v_0}{x_0} = \frac{-A\omega\sin\phi}{A\cos\phi} = -\omega\tan\phi = \frac{v_0}{x_0}$$

$$\boxed{\phi = \arctan\left(-\frac{v_0}{\omega x_0}\right)} \qquad \tan\phi = -\frac{v_0}{\omega x_0}$$

$$v_0^2 = A^2\omega^2 \sin^2\phi$$

$$+\quad \omega^2 x_0^2 = A^2\omega^2\cos^2\phi$$

$$v_0^2+\omega^2x_0^2 = A^2\omega^2(\cancel{\sin^2\phi+\cos^2\phi})$$

$$A=\sqrt{\frac{v_0^2+\omega^2x_0^2}{\omega^2}}$$

$$-\int f\cdot dx = U(x) \qquad \boxed{-\frac{dU}{dx}=f}$$

$$+kx\cdot dx$$

En. in SHM'S

$$E=\frac{1}{2} m v_x^2+\frac{1}{2} kx^2 = \text{constant} = \frac{1}{2} kA^2$$

$$v_x=0 \Rightarrow x=A \Rightarrow \quad E=\frac{1}{2} kA^2=\text{constant}$$

Putting $v_x, x \Rightarrow$

$$E=\frac{1}{2} m\left[-A\omega\sin(\omega t+\phi)\right]^2+\frac{1}{2} k\left[A\cos(\omega t+\phi)\right]^2$$

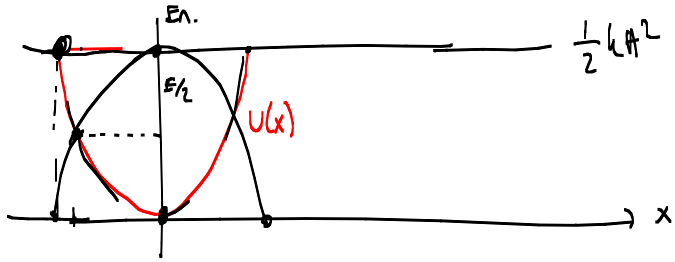
$$=\frac{1}{2} \cancel{m}A^2\underbrace{\omega^2}_{\frac{k}{\cancel{m}}}\sin^2(\omega t+\phi)+\frac{1}{2} kA^2\cos^2(\omega t+\phi)$$

$$=\frac{1}{2} kA^2\sin^2(\omega t+\phi)+\frac{1}{2} kA^2\cos^2(\omega t+\phi)=\frac{1}{2} kA^2$$

$$\cancel{\frac{1}{2}}kx^2+\cancel{\frac{1}{2}}mv^2=\cancel{\frac{1}{2}}kA^2$$

$$\frac{\cancel{m}v^2}{\cancel{m}}=kA^2-kx^2=\frac{k}{m}(A^2-x^2)$$

$$v^2=\frac{k}{m}(A^2-x^2) \Rightarrow \boxed{v=\sqrt{\frac{k}{m}(A^2-x^2)}}$$

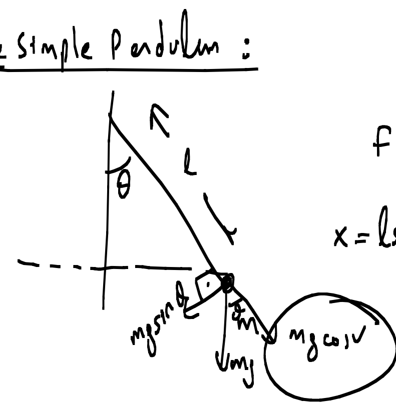


$\frac{1}{2}mv^2 = KE$

$\frac{1}{2}kx^2 = U(x)$

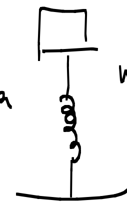
$KE + U(x) = \text{const}$

The Simple Pendulum :



$F_\theta = -mg\sin\theta = ma$

$x = l\sin\theta$



$\omega = \sqrt{\frac{k}{m}}$

$a = \frac{d^2x}{dt^2}; \quad \alpha = \frac{d^2\theta}{dt^2}$

$$-\cancel{m}g\sin\theta = m\alpha l = \cancel{m}l\frac{d^2\theta}{dt^2}$$

$$-\frac{g}{l}\sin\theta = \frac{l}{l}\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\underbrace{\frac{g}{l}}_{\omega}\theta$$

$\sin x = x - \cancel{\frac{x^3}{3!}} + \cancel{\frac{x^5}{5!}}$

$$\frac{d^2x}{dt^2} = -\overbrace{\frac{k}{m}}^{\omega}x$$

$$\omega = \sqrt{\frac{g}{L}} \Rightarrow f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$