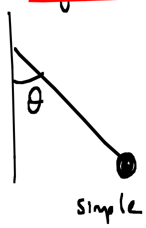
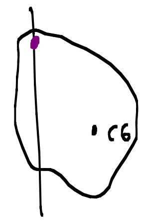
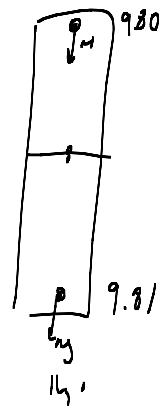
Physical Pendulum





solid object



$v = v_0 + at$

$\omega = \omega_0 + \alpha t$

$\Delta x = v_0 t + \frac{1}{2} a t^2$

$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$

$F = ma$ Linear

$\boxed{\tau = I\alpha \quad \text{Rot}}$

$\tau = F d$ $m a r$

$I = mr^2$ $\frac{a}{r}$

$P = mv$

$L = I\omega$

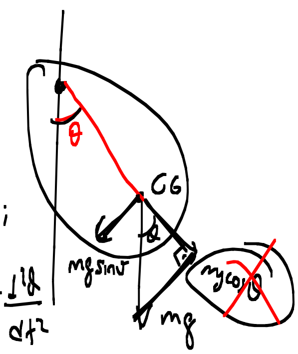
$F = ma$

$\tau = I\alpha$

$a = \frac{d^2x}{dt^2}$;

$\tau = F_\perp \cdot d$

$\alpha = \frac{d^2\theta}{dt^2}$



$\boxed{\tau = -mg\sin\theta \cdot d = I\alpha}$

$\boxed{\frac{-mg\sin\theta d}{I} = \frac{\cancel{I}}{\cancel{I}} \frac{d^2\theta}{dt^2}}$

$\frac{-g\sin\theta}{l} = \frac{l}{l}\frac{d^2\theta}{dt^2}$

$\boxed{\frac{d^2\theta}{dt^2} = -\frac{mgd}{I} \frac{\sin\theta}{\theta}}$

$\sin\theta \simeq \theta$

S.H.O

$\boxed{\frac{d^2x}{dt^2} + \underbrace{\frac{k}{m}}_{\omega^2} x = 0}$

$\cancel{x}(t) = A\cos(\omega t + \phi)$

$\theta(t) = A\cos(\omega t + \phi)$

$\sqrt{\frac{\cancel{d}}{t^2 \cancel{d}}} = \frac{1}{t}$

$\frac{d^2\theta}{dt^2} + \underbrace{\frac{mgd}{I}}_{\omega^2} \theta = 0$

$\boxed{\omega = \sqrt{\frac{mgd}{I}}}$ $\sqrt{\frac{mgd}{md^2}}$

$$\frac{\omega}{2\pi} = f \Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}} \qquad T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{mgd}}$$

SHO

$x(t)$

$$\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}}$$

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad f = \frac{1}{T} \qquad \frac{2\pi}{T} = \omega$$

$$\boxed{2\pi f = \omega}$$

Damped Oscillations:

In reality we have friction (like) forces.

friction force

$$\text{S.H.O.}: \; F = -kx - b v_x = m a_x = m\frac{d^2x}{dt^2}$$

Extra

$$-kx \boxed{- b\frac{dx}{dt}} = m\frac{d^2x}{dt^2} \Rightarrow \qquad \frac{1}{t^2} \qquad t^2$$

$$\boxed{x(t) = A e^{-\left(\frac{b}{2m}\right)t}\cos(\omega' t + \phi)}$$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

critical damping $\Rightarrow \omega' = 0 \Rightarrow \frac{k}{\cancel{m}} = \frac{b^2}{4m^{\cancel{2}}}$

$$\boxed{b_c = 2\sqrt{km}}$$

$b < b_c$

$b > b_c \Rightarrow$ overdamping

$$x(t) = A\, \underbrace{e^{-\left(\frac{b}{2m}\right)t}}\, \underbrace{\cos(\omega' t + \phi)}$$

$b > b_c \Rightarrow \omega' = \sqrt{}$

$\omega' \rightarrow$ complex $\qquad \cos(i\omega t)$

$$\cos(ix) = \frac{e^{iix} + e^{-iix}}{2} \qquad \frac{e^{-x} + e^{+x}}{2} = \cos ix$$

$$e^{ix} = \cos x + i\cancel{\sin x}$$
$$e^{-ix} = \cos x - i\cancel{\sin x}$$
$$+ \quad \frac{2\cos x}{2} =$$

overdamping soln.

$$\boxed{x(t) = C_1 e^{-a_1 t} + C_2 e^{-a_2 t}}$$

Energy in Damped Osc:

$$E = \underbrace{\frac{1}{2} m v_x^2} + \underbrace{\frac{1}{2} k x^2}$$

$$\frac{dE}{dt} = \frac{1}{\cancel{2}} m \underbrace{\frac{d}{dt}(v_x^2)}_{\cancel{2} v_x a_x} + \frac{1}{2} k \frac{d}{dt}\overbrace{(x^2)}^{2x v_x}$$

$$\frac{d}{dt} v_x^2 = v_x \frac{dv_x}{dt} + \frac{dv_x}{dt} v_x = 2 v_x \underbrace{\frac{dv_x}{dt}}_{a_x}$$

$v_x \; v_x$ chain rule

$$\frac{dE}{dt} = m v_x a_x + k x v_x = v_x \underbrace{[m a_x + k x]}_{-b v_x}$$

$$\Sigma F = -b v_x = m a_x + kx$$

$$\frac{dE}{dt} = v_x \cdot -b v_x = \boxed{-b v_x^2 = \frac{dE}{dt}}$$

2nd way:

$$\int F_{ex} \cdot dx = W$$

$$\int -b v_x \, dt \, \frac{dx}{dt} = \int -b v_x^2 \, dt = \int dE$$

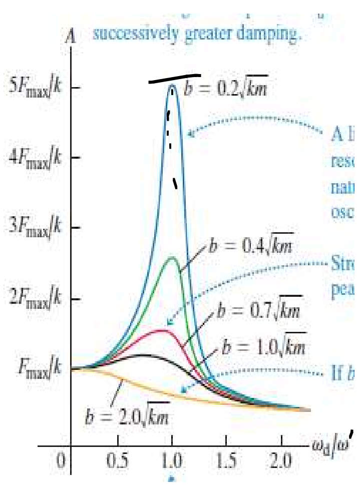
$$\boxed{\frac{dE}{dt} = -b v_x^2} \Leftarrow dE = -b v_x^2 \, dt$$

Forced Oscillation and Resonance

Driving force is periodic; ω_d = ang. freq.

$\omega_d \simeq \omega'$ ← natural ang. freq

$$A = \frac{F_{max}}{\sqrt{(k - \omega_d^2)^2 + b^2 \omega_d^2}}$$



$$\boxed{k - m\omega_d^2 = 0}$$ $A \to max$

HW — Young Freedman — libgen.info

Chp 14 → Sec 14.2, 3, 5, 6

Sears Univ. Phys 13th ed

1 • ⇒ type

1 •• ⇒ type

14.7, 14.8

1 prob 1 prob

10 ⇒