$\boxed{9.6}$ $\theta(t) = 250t - 20t^2 - 1.5t^3$

a) $\omega(t) = 0 \Rightarrow t_0 = ?$

$\rightarrow \omega = \frac{d\theta}{dt} = 250 - 40t - 4.5t^2 = 0$

$\therefore t_0 = 4.23\,s$

b) $\alpha(t) = ?$ at $t = t_0$

$\alpha = \frac{d^2\theta}{dt^2} = -40 - 9t = -78\ rad/s^2$

c) $\theta(t_0) = ?$

$\theta = 250 \cdot 4.23 - 20(4.23)^2 - 1.5(4.23)^2 = 586\ rad \xrightarrow[\frac{1}{2\pi}]{} 933\ rev$

d) $w(t=0) = ?$ $250\ rad/s$

e) $w_{av}(t=0 \rightarrow t = t_0) \rightarrow$ $\frac{\Delta\theta}{\Delta t} = \frac{586}{4.23}\frac{rad}{s} = 139\ rad/s$

$\boxed{9.7} \rightarrow$ similar

$\boxed{9.26}$ $r = \frac{0.75m}{2}$ $w_0 = 0.25\frac{rev}{s}$ $\alpha = 0.9\frac{rev}{s^2}$

a) $w(t=0.2) = ?$ $w = w_0 + \alpha t = 0.25 + 0.9 \times 0.2 = 0.43\ rev/s$

b) $\Delta\theta = ?$ $w_0 t + \frac{1}{2}\alpha t^2 = 0.25 \times 0.2 + \frac{1}{2} \times 0.9 \times (0.2)^2$

$= 0.05 + 0.018 = 0.068\ rev$

c) $v_t = ?$ at $t = 0.2$ $v = w \cdot r = 0.43 \times 0.375\ \frac{m \cdot rev}{s}$

$1\ rev = 2\pi\ rad \Rightarrow v = 0.43 \times 0.375 \times 2\pi \simeq 1\ m/s$

d) $a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + (\alpha \cdot r)^2} = \sqrt{\left(\frac{1}{0.375}\right)^2 + (0.9 \times 2\pi \times 0.375)^2} \simeq 3.5\ \frac{m}{s^2}$

$\boxed{9.67}$ $r = 25\text{ cm}$, $a(t) = At$ $v_0 = 0$ $a(3) = 1.8\ m/s^2$

a) $A = ? \Rightarrow \frac{a}{t} = A = \frac{1.8}{3} = 0.6\ m/s^3$

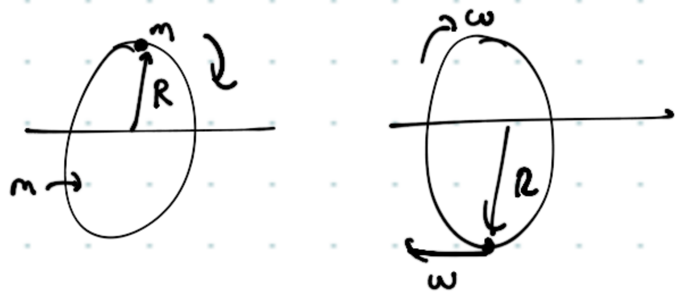
b) $\alpha = ?$ $\alpha = \frac{a}{r} = \frac{0.6t}{0.25} = 2.4t$

c) $w(t) = 15\ rad/s \Rightarrow t = ?$

$$w(t) = \int_0^t \alpha(t')\,dt' = \int_0^t (2.4t')\,dt' = 15 \Rightarrow 2.4\frac{t^2}{2} = 15 \Rightarrow t = \sqrt{\frac{30}{2.4}} \cong 3.5s.$$

d) $\theta(3.5) = ?$ $$\theta = \int_0^t w(t')\,dt' = \frac{1.2t^3}{3} = 0.4t^3 = 0.4(3.5)^3 \sim 18\text{ rad}$$

$\boxed{9.80}$



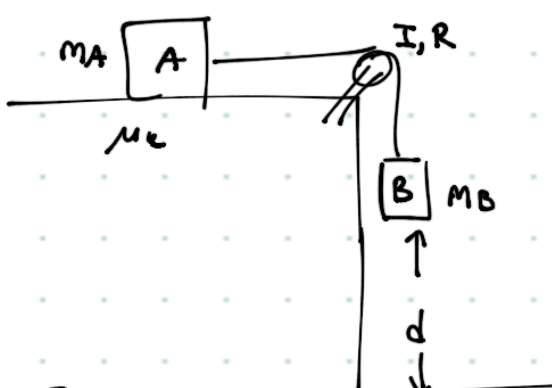
$I_1 = I_{disc} = \frac{1}{2}mR^2$

$I_2 = I_{obj} = mR^2$

$$E_i = E_f \Rightarrow mg2R = \frac{1}{2}I_1w^2 + \frac{1}{2}I_2w^2 = \frac{3}{4}mR^2w^2$$

$\therefore \boxed{w^2 = \frac{8g}{3R}}$

$\boxed{9.85}$



$v = ?$

$E_i = E_f \Rightarrow$

$$m_Bgd - \mu_k(m_Ag)\cdot d = \frac{1}{2}(m_A+m_B)v^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

$$\therefore\ gd\left[m_B - \mu_k m_A\right] = \frac{v^2}{2}\left[m_A+m_B+\frac{I}{R^2}\right] \Rightarrow v = \sqrt{\frac{2gd(m_B-\mu_k m_A)}{m_A+m_B+\frac{I}{R^2}}}$$

10.8 $M = 8.4$ kg $\quad r = 25$ cm $\quad$ 4 small $m = 2$ kg masses

$\tau_{Friction} = ?$ $\quad \omega_i = 75$ rpm $\quad \omega_f = 50$ rpm $\quad \Delta t = 30$s

$\tau = I\alpha$ $\quad$ Calculate $I, \alpha \rightarrow$ get τ !

$\uparrow$const $\rightarrow \uparrow$const

0.6

$\cancel{1.2}$

$\cancel{4.8}$

$$I = I_{sphere} + I_{balls} = \frac{2}{3}MR^2 + 2mR^2 = 2R^2\left[\frac{M}{3} + m\right] = \cancel{2}(0.25)^2\left[\frac{8.4}{3} + 2\right]$$

$$\frac{\Delta\omega}{\Delta t} = \alpha = \frac{50-75}{30}\ \frac{\text{rev}}{\text{min}} \cdot \frac{1}{s} \Rightarrow -\frac{25}{30} \cdot \frac{2\pi}{60}\ \frac{\text{rad}}{s^2} \cong -0{,}09$$

$$\rightarrow \tau = 0.6 \times -0.09 \sim -0.54 \text{ N.m}$$

10.29 $m = 1.5$ kg, $\quad r = 0.1$ m,

a) $\tau = ?$ $\quad \omega_0 = 0 \quad \omega_f = 1200 \frac{\text{rev}}{\text{min}} \quad \Delta t = 2.5$s

$\tau \rightarrow$ const, $\alpha \rightarrow$ const

$$\therefore \omega_f = \cancel{\omega_0}^{\,0} + \alpha\Delta t \Rightarrow \qquad \alpha = \frac{\frac{\cancel{1200}\cdot 2\pi}{\cancel{60}}}{\cancel{2.5}} \cong 50 \text{ rad/s}^2$$

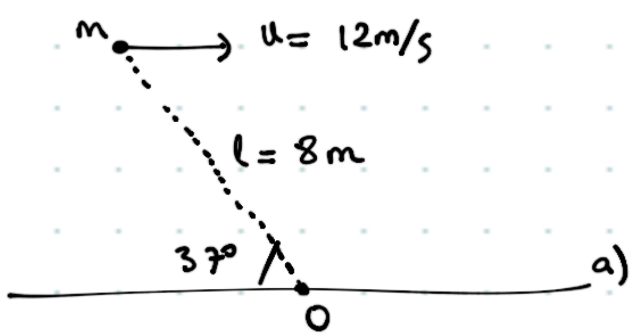
$$\tau = I\alpha = \frac{1}{2}\cdot 1.5 \cdot (0.1)^2 \cdot 50 \sim \frac{1.5}{4} \sim 0.375 \text{ N.m}$$

b) $\Delta\theta = ?$, $\quad \theta = \cancel{\omega_0 t}^{\,0} + \frac{1}{2}\alpha t^2 = \frac{1}{2}\cdot 50 \cdot (2.5)^2 = \frac{\cancel{50}\cdot 25}{\cancel{8}4} = \frac{625}{4} \cong 156$ rad

c) $W = \vec{F}\cdot\Delta\vec{x} \Rightarrow \tau\cdot\Delta\theta = 0.375 \times 156 \simeq 59$ Joule $\leftarrow$ same

d) $K = \frac{1}{2}I\omega^2 = \frac{1}{\cancel{2}} \times \frac{1}{\cancel{2}} \times 1.5 \times (0.1)^2 \times \left(\frac{\cancel{1200}\cdot 2\pi}{\cancel{60}}\right)^2 = 1.5 \times 4\pi^2 \sim 60$ Joul

10.35



$m = 2$kg ; $\vec{L}_O = ?$

$$\vec{r} \times \vec{P} = 8 \times 2 \times 12 \times \frac{3}{5}$$

a) $L = \frac{48 \times 12}{5} \simeq \boxed{115}$ $\otimes$ into the page

b) $\frac{d\vec{L}}{dt} = \vec{\tau}$ along $m \rightarrow O$

$\odot$ Torque out of the page

$$mg \cdot l\cos\theta = \cancel{20}^{\,4} \cdot 8 \cdot \frac{4}{\cancel{5}} \simeq \boxed{128}$$

$\boxed{10.36}$ $m = 12\,kg$ $r = 24\ cm$ $\theta(t) = At^2 + Bt^4$ $A = 1.5,\ B = 1.1$

a) units of A, B b) $L(3) = ?$, $\tau(3) = ?$

$L = I\omega$ for rotation : $\frac{dL}{dt} = \tau = I\alpha$

$\therefore\ \omega(t) = \frac{d}{dt}\{At^2 + Bt^4\} = 2At + 4Bt^3$

$\therefore\ L = I\{2At + 4Bt^3\}$. $\tau = \frac{d}{dt}L = I\{2A + 12Bt^2\}$

$\boxed{10.40}$ $m = 0.025\,kg$, $r_0 = 0.3m$, $w_0 = 1.75\ rad/s$

$\rightarrow r_f = 0.15\ m$

a) $\vec{\tau} = \vec{r} \times \vec{T} = 0$

$\therefore\ \frac{d\vec{L}}{dt} = 0$

b) $L_i = L_f$

$\rightarrow\ \cancel{m} v_i r_i = \cancel{m} v_f r_f$

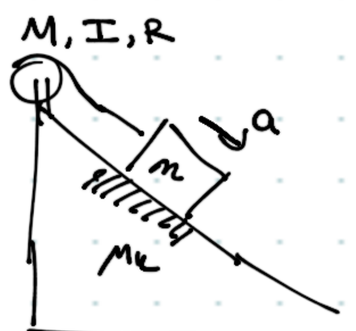
$\therefore\ w_f = 7\ rad/s$ $\Rightarrow\ 1.75 \times \underbrace{\cancel{0.3}}_{4} \times \cancel{0.3} = w_f \cdot \cancel{(0.15)^2}$

c) $\Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m \left\{[7 \times (0.15)]^2 - [1.75 \times 0.3]^2\right\}$

$= \frac{1}{2} \times 0.025 \times (1.75 \times 0.3)^2 \left[4 - 1\right] = \frac{1}{2} \times \underbrace{25}_{10} \times 10^{-3} \times \underbrace{\underbrace{(1.75)^2 \cdot 3}_{\sim 3}\ \underbrace{(0.3)^2}_{0.1}}_{1} \sim 0.01$

d) $W = \Delta KE$

$\boxed{10.64}$



$I = 0.5\ kg\,m^2$

$M = 25\,kg$ $R = 0.2m$

$\mu_k = 0,25$, $m = 5\,kg$

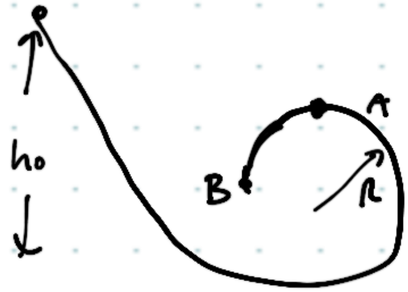
$mg\sin\theta - \cancel{T} - \mu_k mg\cos\theta = ma$

$+ \quad \frac{\cancel{T}\cancel{R}}{\cancel{R}} = I\alpha = \frac{I}{R}\frac{a}{R}$

$mg(\sin\theta - \mu_k \cos\theta) = a\left[m + \frac{I}{R^2}\right]$

$\therefore\ \boxed{a = \frac{mg(\sin\theta - \mu_k \cos\theta)}{m + \frac{I}{R^2}}}$

$\boxed{10.70}$



$$mg + N = m\frac{v^2}{R}$$

$$v_{min} \to N = 0$$

$$\therefore \cancel{m}g = \frac{\cancel{m}v^2}{R} \Rightarrow v^2 = gR$$

$$\omega^2 = \frac{v^2}{R^2}$$

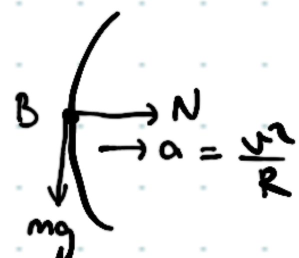
$$I_{hollow} = \frac{2}{3}MR^2$$

a) $E_i = E_f \Rightarrow mg(h_0 - 2R) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$\therefore mg(h_0 - 2R) = \frac{1}{2}mv^2 + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}mv^2 = \frac{5}{6}mv^2 = \frac{5}{6}mgR$$

$$\cancel{mg}(h_0 - 2R) = \frac{5}{6}\cancel{m}gR \Rightarrow h_0 = \left(2 + \frac{5}{6}\right)R = \boxed{\frac{17}{6}R}$$

b) N (at B) = ?



$$N = m\frac{v^2}{R}$$

$$\cancel{m}g(h_0 - R) = \frac{5}{6}\cancel{m}v^2 \Rightarrow g\left(\frac{17}{6}R - R\right) = \frac{5}{6}v^2$$

$$g\frac{11}{\cancel{6}}R = \frac{5}{\cancel{6}}v^2$$

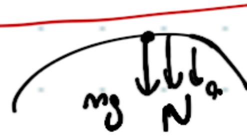
$$\therefore v^2 = \frac{11}{5}gR$$

$$N = \frac{mv^2}{R} = \boxed{\frac{11}{5}mg}$$

c) of course d)

$$\cancel{m}g(h - 2R) = \frac{1}{2}\cancel{m}v^2 \Rightarrow g\left(\frac{17}{6} - 2\right)R = \frac{5gR}{6} = \frac{1}{2}v^2$$

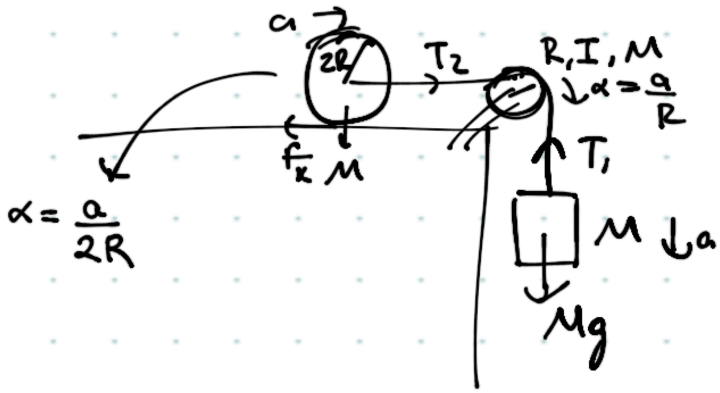
$$\therefore v^2 = \frac{5gR}{3}$$



$$\Rightarrow mg + N = m\frac{v^2}{R} = \frac{5mg}{3}$$

$$\therefore \boxed{N = \frac{2mg}{3}}$$

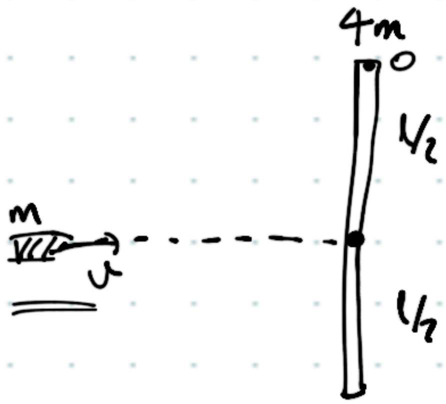
10.83



$$Mg - \cancel{T_1} = Ma$$
$$(\cancel{T_1} - \cancel{T_2})\cancel{R} = \frac{1}{2} M \cancel{R}^{\cancel{2}} \frac{a}{\cancel{R}}$$
$$\cancel{T_2} - \cancel{f_k} = Ma$$
$$(\cancel{f_k}) \cdot \cancel{2R} = \frac{1}{2} M (2\cancel{R})^{\cancel{2}} \cdot \frac{a}{\cancel{2R}}$$
$$+$$

$$\cancel{M}g = \cancel{M}a\left[1 + \frac{1}{2} + 1 + \frac{1}{2}\right] \Rightarrow \boxed{a = \frac{g}{3}}$$

10.87



a) $\omega = ?$ · $\vec{r} \times \vec{p}$

$$L_i = L_f$$
$$\frac{mvl}{2} = I_T\,\omega \rightarrow (A)$$
$$\frac{\cancel{m}vl}{2} = \left(\cancel{m}\left(\frac{l}{2}\right)^2 + \frac{4\cancel{m}}{3} l^2\right)\omega$$

$$\frac{v\cancel{l}}{\cancel{2}} = \frac{19}{\cancel{12}6} l^{\cancel{2}}\,\omega \Rightarrow \boxed{\omega = \frac{6v}{19L}}$$

b) $$\frac{\frac{1}{\cancel{2}} m v^2}{\frac{1}{\cancel{2}} I_T \omega^2} = \frac{mv^2}{I_T \omega \cdot \omega} \Rightarrow (A)\ \frac{\cancel{m}\cancel{v}\cancel{v}}{\frac{\cancel{m}\cancel{v}\cancel{l}}{\cancel{2}} \cdot \frac{3\cancel{6}\cancel{v}}{19\cancel{l}}} = \boxed{\frac{19}{3}}$$

10.91

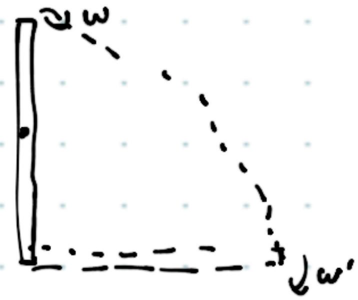
$$r \times p = I\omega$$
$$L_i = L_f$$

a) $$\cancel{m} v \cdot \frac{2\cancel{l}}{3} = \frac{(\cancel{3}\cancel{m})}{\cancel{3}} l^{\cancel{2}}\,\omega$$

$$\boxed{\omega = \frac{2v}{3l}} = 2$$

$$\frac{2 \times 2.\cancel{25}}{\cancel{3} \times 0.\cancel{75}}$$

b)



$$E_i = E_f$$

$$\cancel{\frac{1}{2}}\,\frac{I\omega^2}{\cancel{I}} + \frac{(3m)g}{I}\cdot\frac{l}{2} = \cancel{\frac{1}{2}}\,\frac{\cancel{I}\,\omega'^2}{\cancel{I}}$$

$$\omega'^2 = \omega^2 + \frac{3mgl}{I}$$

$$\omega' = \sqrt{\omega^2 + \frac{3\cancel{m}g\cancel{l}}{\frac{3\cancel{m}}{3}\,l^{\cancel{2}}}} = \sqrt{\omega^2 + \frac{3g}{l}} = \sqrt{4+40} \sim 2\sqrt{11}$$

$\boxed{10.100}$

$$\frac{2\pi\sqrt{\left(R^2 - \frac{d^2}{4}\right)}}{2\pi R}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi R}{T} \quad \times \qquad \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$v_{cm} = \omega\sqrt{R^2 - \frac{d^2}{4}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\,\omega^2\left[R^2 - \frac{d^2}{4}\right] + \frac{1}{2}\,\overbrace{\frac{2}{5}MR^2}^{I}\,\omega^2$$

$$\cancel{m}gh = \frac{1}{2}mv^2 + \frac{1}{2}I\,\frac{v^2}{\left(R^2 - \frac{d^2}{4}\right)} = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\,\frac{2}{5}\cancel{m}R^2\,\frac{v^2}{R^2 - \frac{d^2}{4}}$$

$$5 \times 2gh = \cancel{2}\,\frac{v^2}{\cancel{2}}\,5\left[1 + \frac{2}{\cancel{5}}\,\frac{R^2}{R^2 - \frac{d^2}{4}}\right] \Rightarrow 10gh = v^2\left[5 + \frac{2R^2}{R^2 - \frac{d^2}{4}}\right]$$

$$v^2 = \frac{10gh}{5 + \frac{2\cancel{R^2}/\cancel{R^2}}{\frac{\cancel{R^2}}{\cancel{R^2}} - \frac{d^2}{4R^2}}} \Rightarrow v = \sqrt{\frac{10gh}{5 + \frac{2}{1 - \frac{d^2}{4R^2}}}}$$

$\boxed{13.16}$ $m = 0.4\ kg$, $a_x = -2.7\ m/s^2$, $x = 0.3m$, $T = ?$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad F = -kx = ma$$

$$-k \cdot 0.3 = 0.4 \cdot (-2.7)$$

$\boxed{13.19}$ $m = 1.5\ kg$ $\quad x(t) = 7.4cm\ \cos[(4.16s^{-1})t - 2.42]$

$$\underline{x(t) = A\cos[\omega t + \phi]}$$

a) $T = ?$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{4.16}$$

b) $k = ?$ $\quad \omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2 = 1.5 \times (4.16)^2$

c) $\underline{v(t) = -A\omega\ \sin[\omega t + \phi]} \xrightarrow[max]{} A\omega = \frac{7.4}{100} \cdot 4.16\ m/s$

d) $F_{max} = m a_{max} = m \cdot A\omega^2 = 1.5 \times \frac{7.4}{100} \cdot (4.16)^2$

$$a(t) = \underline{-A\omega^2\ \cos[\omega t + \phi]}$$

e) $t = 1s$, a, v, x, F

$\boxed{13.22}$ ω, A

a) $\underline{U = KE} \Rightarrow \cancel{\frac{1}{2}}kx^2 = \cancel{\frac{1}{2}}mv^2$

$$\frac{k}{m}x^2 = v^2 \Rightarrow \omega^2x^2 = v^2$$

$$\cancel{\omega^2}\cancel{A^2}\cos^2[\ ] = \cancel{\omega^2}\cancel{A^2}\sin^2[\ ]$$

$$\Rightarrow \sin\underset{\omega t}{[\ ]} = \cos[\ ] = [\ ] = \pi/4$$

$$x = A\cos[\ ] = \boxed{\frac{A\sqrt{2}}{2}} \quad ; \quad v = |-A\omega\ \sin[\ ]| = \boxed{\frac{A\omega\sqrt{2}}{2}}$$

b) 4 times $\Rightarrow \omega t = \frac{\pi}{4} + n\frac{\pi}{2} \Rightarrow \omega\,\Delta t = \frac{\pi}{2} \Rightarrow \boxed{\Delta t = \frac{\pi}{2\omega}}$

d) $x = \frac{A}{2}$ KE, PE % ?

75% 25%

$x = A\cos\underbrace{[\;]}_{60°}$ $v = -\omega A \underbrace{\sin[\;]}_{\sqrt{3}/2} \Rightarrow -\frac{\omega A\sqrt{3}}{2}$

$\underline{PE}$ $\underline{KE}$

$\frac{1}{2}kx^2$ $\frac{1}{2}mv^2 = \frac{1}{2}m\left[\frac{-\omega A\sqrt{3}}{2}\right]^2 = \frac{1}{2}\cancel{m}\frac{\overbrace{\omega^2}^{k/\cancel{m}}A^2\,3}{4}$

$\frac{1}{2}k\left(\frac{A}{2}\right)^2 = \boxed{\frac{1}{8}kA^2}$ $= \boxed{\frac{3}{8}kA^2}$

$\boxed{13.23}$ $m = 0.5\,kg$; $k = 450\,N/m$; $A = 0.04\,m$

a) $v_{max} = \omega A$; $\omega = \sqrt{\frac{k}{m}}$

b) $v = ?$ $x = -0.015\,m$

$v = \mp\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2} = \sqrt{\frac{450}{0.5}}\sqrt{(0.04)^2 - (-0.015)^2}$

$\frac{30}{100}\sqrt{4^2 - 1.5^2} \sim \frac{30\times 3.5}{100} \sim 1\,m/s$

c) $a_{max} = A\omega^2 = A\frac{k}{m} = 0.04 \times \frac{450}{\underbrace{0.5}_{900}} = 36\,m/s^2$

$F = ma = -kx \Rightarrow a = -\frac{k}{m}x$

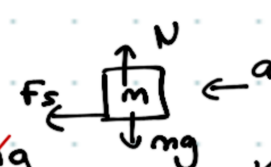
d) $x = -0.015\,m \Rightarrow$ $a = -\omega^2\underbrace{A\cos[\;]}_{x} =$ $a = -\underbrace{\omega^2}_{900}\underbrace{x}_{-0.015} = -\frac{k}{m}x$

$a = 13.5\,m/s^2$

e) $\frac{1}{2}kA^2$

$\boxed{13.68}$

$F = \cancel{m}a = \mu_s \cancel{m}g$

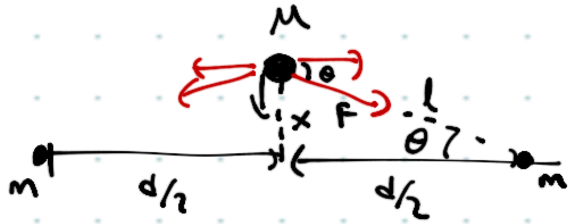


$F_s = ma$
$= \mu_s N$
$f_s = \mu_s mg$

$\omega = \sqrt{\frac{k}{m+M}}$ $a_{max} = \omega^2 \underline{\underline{A}} = \mu_s g$

$$A = \frac{\mu_s g}{\omega^2} = \frac{\mu_s g\,(m+M)}{k}$$

13.83



$l \cong \frac{d}{2}$

$x = l\sin\theta \sim \frac{d}{2}\theta$

$\theta = \frac{2x}{d}$

$$F = ma$$

$$-2F\theta = m\frac{d^2x}{dt}$$

$$-\frac{4Fx}{d} = m\frac{d^2x}{dt^2}$$

$$F_{net} = 2F\sin\theta \simeq 2F\theta$$

$$\therefore \frac{d^2x}{dt^2} \doteq -\underbrace{\frac{4F}{md}}_{\omega^2}x$$

$$\omega = \sqrt{\frac{4F}{md}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md}{4F}} = 2\pi\sqrt{\frac{\cancel{m}d}{16G\frac{M\cancel{m}}{d^2}}}$$

$$F = G\frac{Mm}{l^2} \cong G\frac{Mm}{(d/2)^2} = 4G\frac{Mm}{d^2}$$

$$\therefore T = \frac{\pi}{2}\sqrt{\frac{d^3}{GM}} = \boxed{\frac{\pi d}{2}\sqrt{\frac{d}{GM}}}$$

13.84 $F_x = -cx^3$; $U(x=0) = 0$ $v(x) = ?$

a) $$U = -\int F_x\,dx = -\int -cx^3\,dx = \frac{cx^4}{4} + d\,\Big|_{x=0} = 0 \therefore d = 0$$

$$U(x) = \frac{c}{4}x^4$$

b) KE + U =

$$\cancel{2}\frac{1}{\cancel{2}}\cancel{m}v^2 + \cancel{2}\frac{c}{\cancel{4}_{m}}x^4 = \cancel{2}\frac{c}{4_m}A^4$$

$$v^2 = \frac{c}{2m}(A^4 - x^4) \Rightarrow v = \sqrt{\frac{c}{2m}(A^4 - x^4)}$$

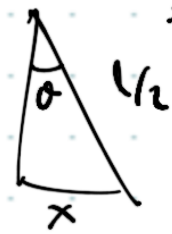
$$\frac{dx}{dt} = \sqrt{\frac{c}{2m}\left(A^4 - x^4\right)} \Rightarrow \int_0^T dt = 4\int_0^A dx \sqrt{\frac{2m}{c\left(A^4 - x^4\right)}}$$

$\boxed{13.91}$

$$\tau = I\alpha$$

$$-kx \cdot \underbrace{\cos\theta}_{1} \frac{l}{2} = \frac{1}{12} M l^2 \alpha$$

$$x = \frac{l}{2}\sin\theta = \frac{l}{2}\theta$$



$$-k \cdot \frac{\cancel{l}}{\cancel{2}}\theta \cdot \frac{\cancel{l}}{\cancel{2}} = \frac{Ml^2}{12}\alpha = \frac{M\cancel{l^2}}{\underset{3}{\cancel{12}}} \frac{d^2\theta}{dt^2}$$

$$-\underbrace{\frac{3k}{M}}_{\omega^2}\theta = \frac{d^2\theta}{dt^2} \qquad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\omega = \sqrt{\frac{3k}{M}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{M}{3k}}}$$

L, m

$\frac{M}{L}$

$dm = \lambda \cdot ds = \lambda R \, d\theta$

$\int dF = \int \frac{G M \, dm}{R^2} \cos\theta$

$\lambda R \int d\theta \cos\theta$

$\frac{GM}{R^2} \int dm$

M