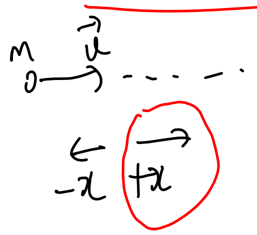
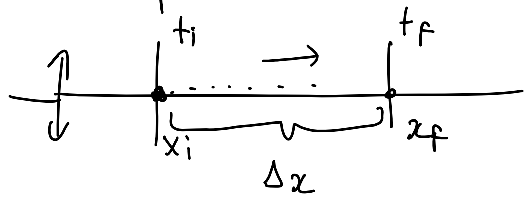
Note Title 2/15/2012

Motion in 1-D

Displacement = $\Delta x = x_f - x_i$



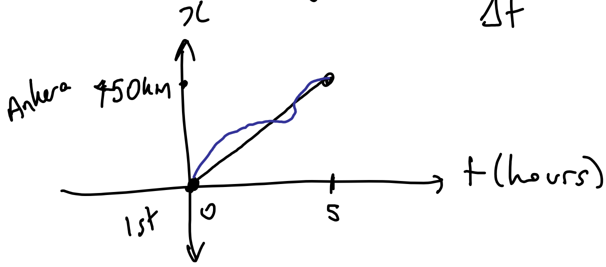


1st → Ankara

$t_i = 8\,am$ ↓ 1 pm

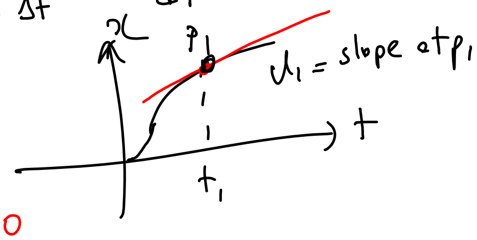
5 hours

Average velocity ⇒ $\vec{v}_{av} = \dfrac{\Delta \vec{x}}{\Delta t} = \dfrac{450}{5} = 90\ km/h$ $x_i = 0$ $x_f = 450\,km$

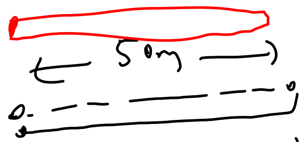


- Instanteneous Velocity ⇒ $\lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{x}}{\Delta t} = \dfrac{d\vec{x}}{dt} = \vec{v}_{inst}$

$\dfrac{Dx}{Dt}$ $\dfrac{\Delta x}{\Delta t}$



- Popov



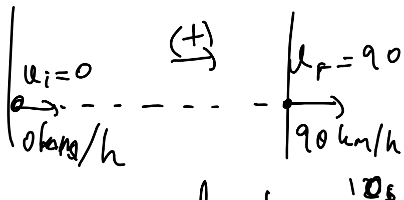
$\dfrac{\Delta x}{\Delta t} = 0$

46 s ⇒

$v_{inst} \neq 0$

$v_{av} = 0$

- Acceleration ⇒ $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$ ⇒ $\dfrac{m/s}{s}$ ⇒ $\dfrac{m}{s^2}$

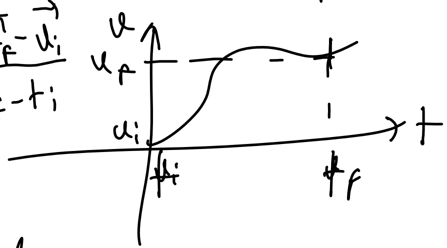


Change of velocity wrt time ⇒ acceleration → $2.5\ m/s^2$

with respect to

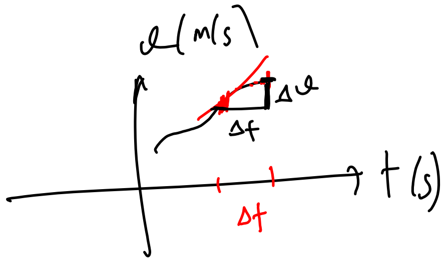
Average acceleration:

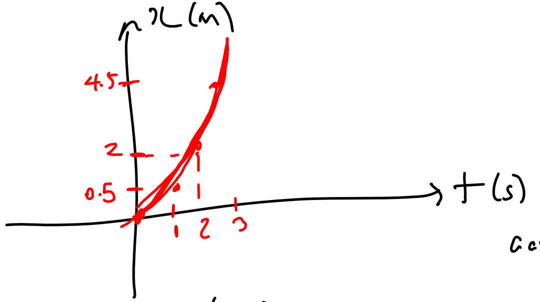
$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$



Instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$



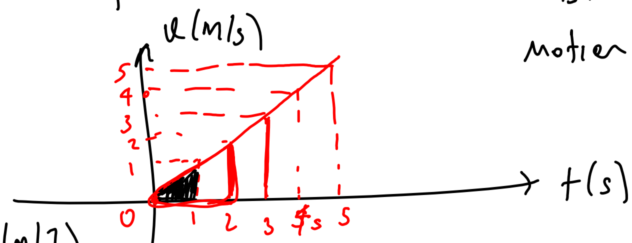


$x_i = 0$ $\quad$ $x_f = ?$

$v_i = 0$ $\quad$ $t = 5\,s$ $\quad$ $v_f = 5\,m/s$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{5-0}{5-0} = 1\,m/s^2 \quad \frac{1\,m/s}{1\,s}$$

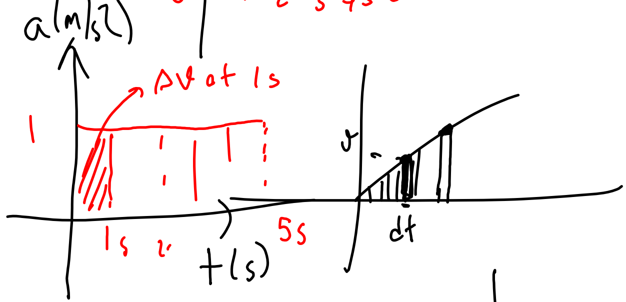
Motion with constant acceleration

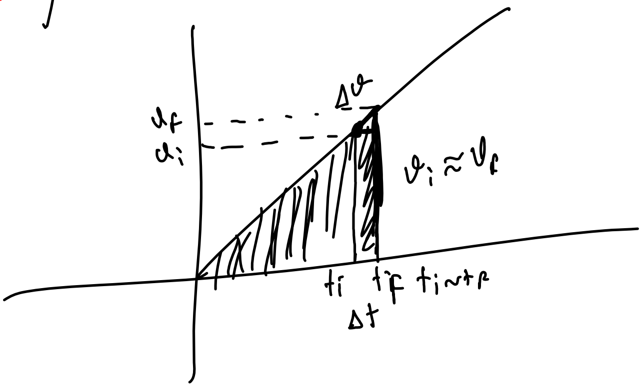


• Δx = Area under $v-t$ graph

$$\frac{vt = x}{\Delta v} = \frac{dx}{dt}$$

$$v\,dt = dx$$





$$v = \frac{dx}{dt} \quad ; \quad a = \frac{dv}{dt} = \text{const}$$

$$\Rightarrow \int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt$$

$$\int_{v_i}^{v_f} dv = a \int_{t_i}^{t_f} dt$$

$$v\,dt = dx$$

↓ take $\int$ of both

$$(v_f - v_i) = a\,(t_f - \cancel{t_i})$$

$$\Delta v = a\,\Delta t \to at$$

$$\int_{\cancel{t_i}\,0}^{t_f} v\,dt = \int_{x_i}^{x_f} dx$$

$$\boxed{v_f = v_i + a\,\Delta t}$$

$$v(t) = v_i + a\,\Delta t$$

↓ $\Delta t \to t$, $t_i \neq 0$

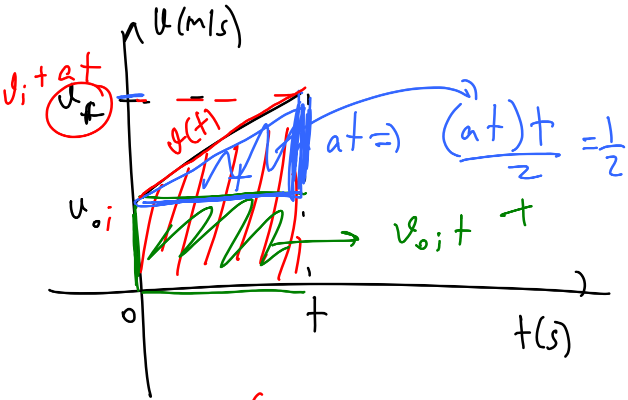
$$① \int_{\cancel{t_i}\,0}^{t_f} (at)\,dt = x_f - x_i \Rightarrow \frac{d^2x}{dt^2} = a$$

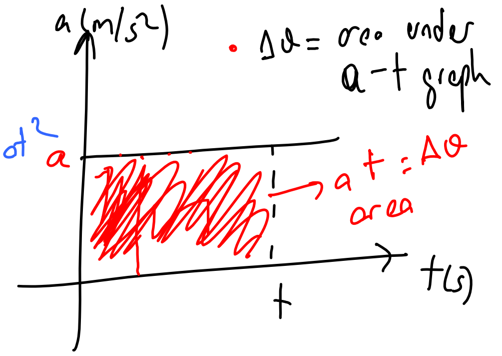
$$\frac{a(\Delta t)^2}{2} \qquad a\,\frac{t^2}{2}\Big|_{\cancel{t_i}\,0}^{t_f} = \frac{a(\Delta t)^2}{2}$$

$$\Delta x = v_i t + \frac{1}{2} a t^2$$

$$② \int_{t_i}^{t_f} v_i\,dt = v_i\,\Delta t \Rightarrow x_f - x_i =$$

$$\boxed{\Delta x = v_i\,\Delta t + \frac{1}{2} a\,\Delta t^2}$$





$\int dx = \int v\,dt$

$\int dv = \int a\,dt$

$\boxed{\Delta v = at}$

$v_f - v_i = at$

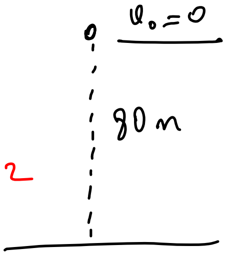
$v_f = v_i + at$

Area under a-t $\Rightarrow$ Δv

Area under v-t $\Rightarrow$ Δx

$\Delta x = v_0 t + \frac{1}{2}at^2$

• free fall-motion



$g = 9.81\ m/s^2$

$g \sim 10\ m/s^2$

$80 = \Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$

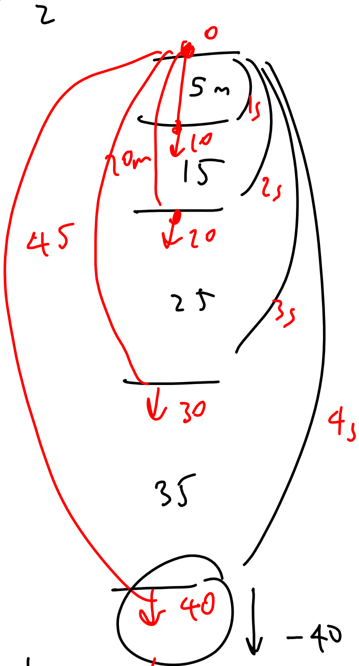
0 10

$\cancel{80} = \frac{1}{2} \times \cancel{10} \times t^2 = \quad t^2 = 16 \Rightarrow t = 4s$

$\frac{1}{2}at^2 = 5t^2$

$5 \times (2)^2 = 20$

$5 \times 3^2 = 45$



↓ (+)/(−)

↓(−) $g = -10\ m/s^2$

$v_f = \cancel{v_0}^{0} + at$

$v = -gt \Rightarrow$

$v_f = -10 \times 4 = -40$

$\Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$

$\boxed{\Delta x = -\frac{1}{2}gt^2}$ → $-\frac{1}{2} \times 10 \times 4^2 = -80m$

Time dependent acceleration

Example

$$\boxed{a(t) = t^2 + 3t}$$

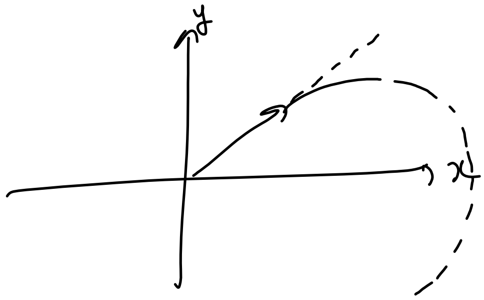
$$v(t) = v_0 + \int a(t)\,dt$$

$$x_f = x_0 + \int v(t)\,dt$$

$$\int \frac{e^{-t^2}}{t}\,dt$$

Another example

Chp-3 3D- Motion (2D)



$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} \Rightarrow$$

$$\vec{v}_{inst} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dt}$$

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt} \; ; \quad \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

Vectors : $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$\frac{d\vec{r}}{dt} = \vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

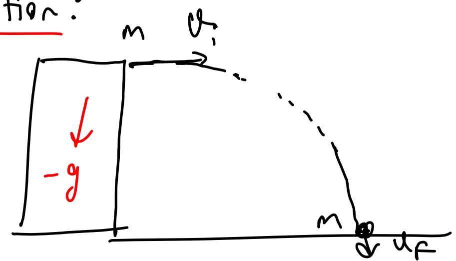
$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$= \underbrace{\frac{d^2x}{dt^2}}_{\frac{dv_x}{dt}}\hat{i} + \underbrace{\frac{d^2y}{dt^2}}_{\frac{dv_y}{dt}}\hat{j} + \underbrace{\frac{d^2z}{dt^2}}_{\frac{dv_z}{dt}}\hat{k}$$

Speed is a number $|\vec{v}| = \frac{|d\vec{r}|}{dt} =$

$$\sqrt{v_x^2 + v_y^2 + v_z^2} = |\vec{v}|$$

Projectile Motion :

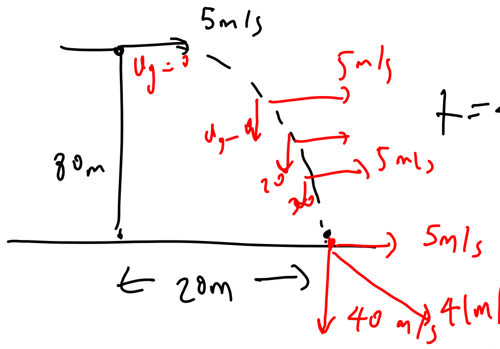


It is nothing but a const motn along x-direction and a free fall motion along y-direction.

$v_x \to$ const since Force along $x \Rightarrow \phi$

Along $x \Rightarrow \quad \Delta x = v_{0x} t$

Along $y \Rightarrow \quad \boxed{\Delta y = \frac{1}{2}(-g)t^2} \Rightarrow$ free fall



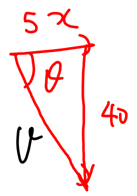
$t = 4s$ since it's free fall

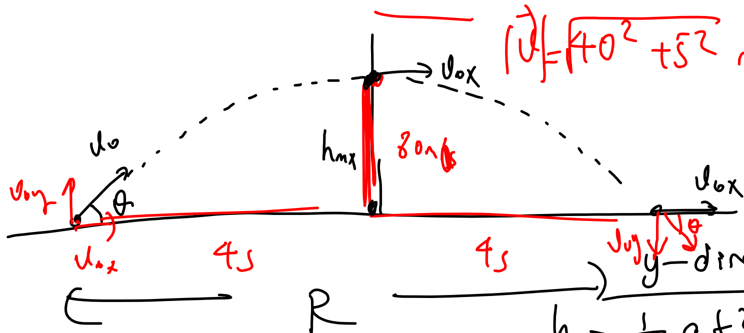
$-80 = -\frac{1}{2} 10 \times t^2$

$16 = t^2 \Rightarrow t = 4$

$41 m/s = |\vec{v}|$

$|\vec{v}| = \sqrt{40^2 + 5^2} \sim 41 m/s$





y-direction

$h = \frac{1}{2} g t^2 = \frac{1}{2} \frac{g^2 t^2}{g} = \frac{v_y^2}{2g}$

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

$v_{f_y} = \cancel{v_0} + at \Rightarrow v_y = gt$

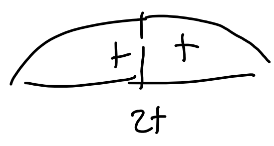
$t = \frac{v_y}{g}$

$$\boxed{h_{mx} = \frac{v_0^2 \sin^2\theta}{2g}}$$

$R = \text{Range} = v_{0x} \cdot t_{range} = v_0 \cos\theta * 2\frac{v_0 \sin\theta}{g}$

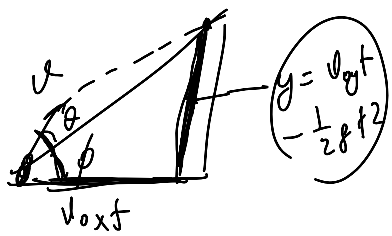
$t_{range} = \frac{2 v_y}{g}$

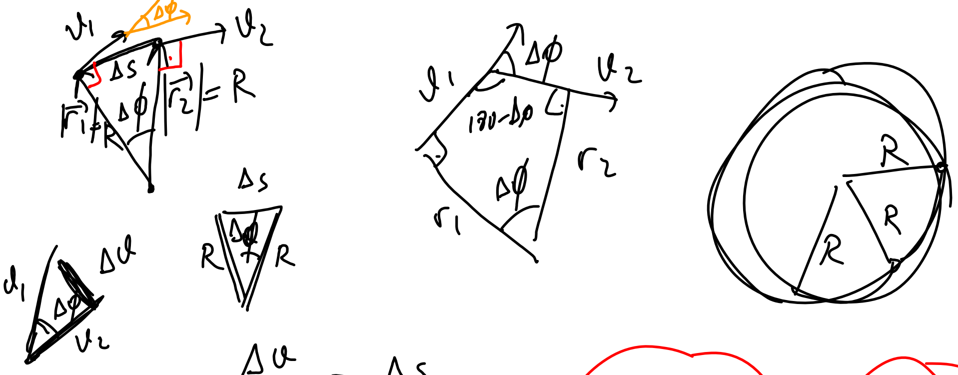
$$\boxed{R = \frac{v_0^2 \sin 2\theta}{g}}$$



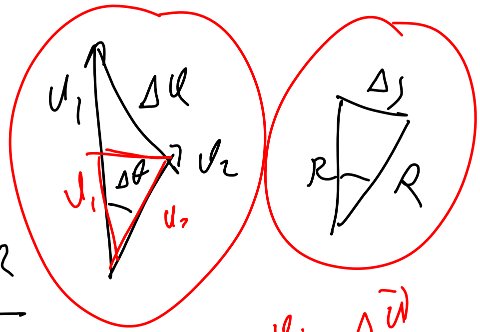
<u>Circular Motion:</u>

$$\boxed{\vec{v} = \frac{d\vec{x}}{dt}}$$





$$\frac{\Delta v}{v_1} = \frac{\Delta s}{R}$$



$$\Delta v = \frac{v_1 \Delta s}{R}$$

$$a_{av} = \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v_1 \Delta s}{R \Delta t} = \frac{v_1^2}{R}$$

$$\boxed{a_{rad} = \frac{v^2}{R}}$$

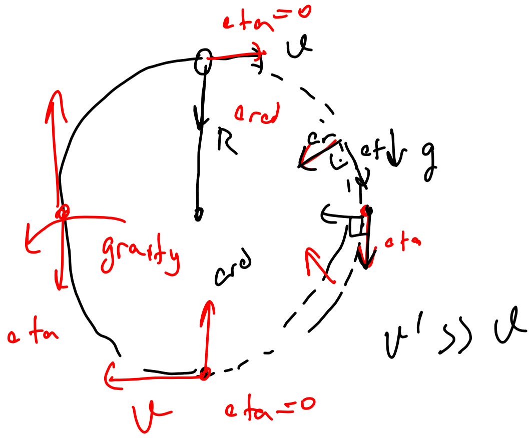
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Period : The total time takes for the particle to complete a full circle.

$$T = \frac{\Delta D}{v_{av}} = \frac{2\pi R}{v} = \text{Period}$$

$$v = \frac{2\pi R}{T}$$

$$a_{rad} = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \boxed{\frac{4\pi^2 R}{T^2} = a_{rad}}$$



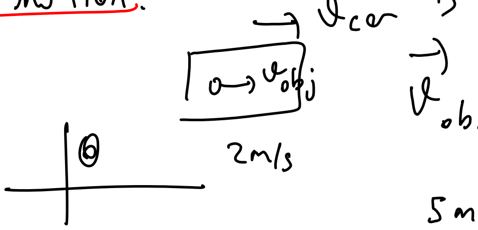
a_{rad} on top < a_{rad} on bottom

$a_{tan} \perp a_{rad}$

$$|\vec{a}_{tot}| = \sqrt{a_t^2 + a_{rad}^2}$$

$$\vec{a}_{tot} = \vec{a}_{rad} + \vec{a}_{tan}$$

Relative Motion:



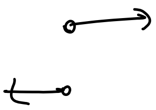
$$\vec{v}_{observed} = \vec{v}_{obj} + \vec{v}_{car}$$

5 m/s

$v_{car} = 3\,m/s$

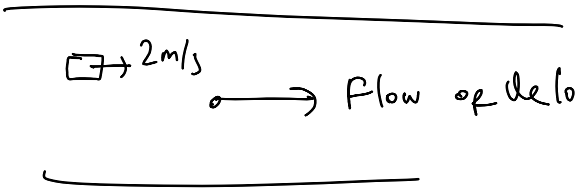
$\Rightarrow 3 - 1 = 2\,m/s \Rightarrow$ day ex

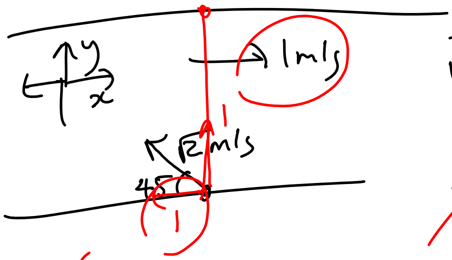
$v_{obj} = -1\,m/s$



River problem

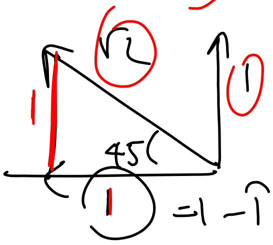
$\Rightarrow v_{observed} = 2 + 1$





$\vec{v}_{flow} + \vec{v}_{obj} = v_{observed}$

$\cancel{1\hat{i}} + [\cancel{-\hat{i}} + \hat{j}] = \hat{j}$



$= 1 - \hat{i}$

$\sqrt{2} = 1 \Rightarrow -\hat{i} + \hat{j}$