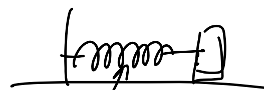
Note Title 2/29/2012

Chp 5 - App. of Newton's Laws

Types of forces: $F \propto \frac{1}{r^2}$

1) • Grav. force (universal) valid on all scales (galaxies, earth, small particles electrons)

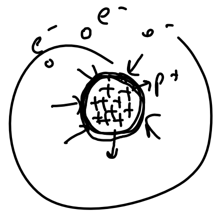


• Force friction (μN), fluid resistance (kv, Dv^2), spring force (kx), are complicated in nature.

2) • Electromagnetic force $\propto \frac{1}{r^2}$

3) • Weak force (nuclear decays) ; 4) Strong force (force which holds atoms)

rubber band

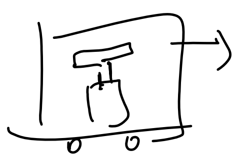


I) Equilibrium (of a system)

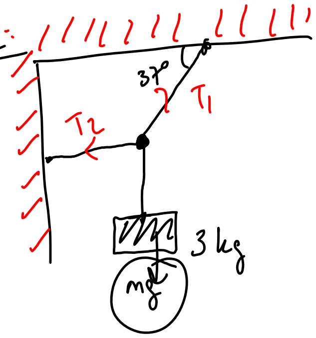
The system has total $\vec{a} = 0$. (not necessarily $\vec{v} = 0$)

$\Downarrow$

$\sum \vec{F} = 0$



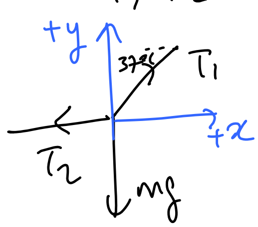
Example:



FBD (free body diagram)

⇒ system is in equilibrium

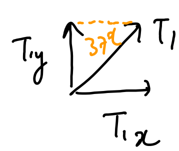
T_1, T_2 ?



$\sum \vec{F} = 0$

Equil. cond:

$$\left.\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned}\right\}$$



$T_{1y} = T_1 . \sin 37°$

$T_{1x} = T_1 \cos 37°$

$\sum F_x = 0 \Rightarrow T_1 \cos 37° - T_2 = 0$ (1) $\sum F_y = 0 \Rightarrow T_1 \sin 37° - mg = 0$ (2)

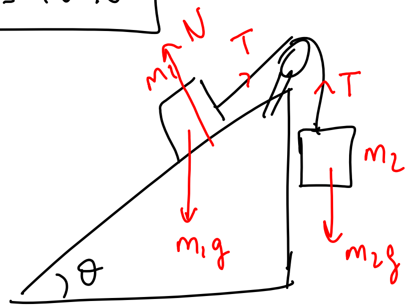
(1) $T_1 \frac{4}{5} = T_2$ $\Rightarrow$ (2) $\Rightarrow$ $T_1 \frac{\cancel{3}}{5} = \cancel{3} \times 10$ $\quad T_1 \sin 37^\circ = mg$

$$\boxed{T_1 = 50\ N}$$

$\Rightarrow \cancel{5}0 \times \frac{4}{\cancel{5}} = T_2$

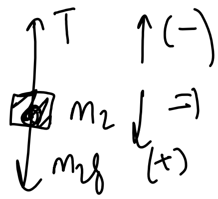
$$\boxed{T_2 = 40\ N}$$

Example



If the sys. is in equil. then what is $\frac{m_1}{m_2} = ?$

FBD (m_2) $\Rightarrow$

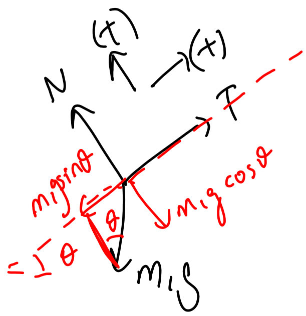


$\sum \vec{F} = 0$

$m_2 g - T = 0$

$$T = m_2 g$$

FBD (m_1)



$\sum \vec{F} = 0$

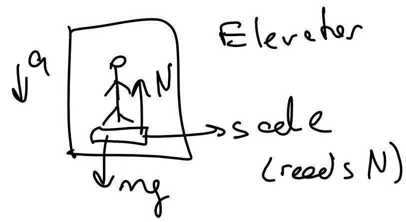
$\sum F_\perp = N - m_1 g \cos\theta = 0$

$\sum F_\parallel = T - m_1 g \sin\theta = 0$

$m_2 \cancel{g} - m_1 \cancel{g} \sin\theta = 0 \Rightarrow \boxed{\frac{m_1}{m_2} = \frac{1}{\sin\theta}}$

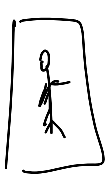
II) Dynamics (a non-zero $\vec{a}$)

$$\sum \vec{F} = \sum m \vec{a}$$

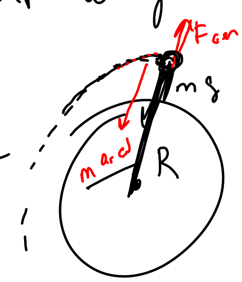


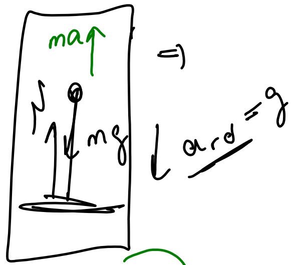
$mg - N = ma$

$N = mg - ma \Rightarrow$ Apparent weight



Astronauts on a satellite they feel weightless





$\Rightarrow \quad mg - ma = 0$

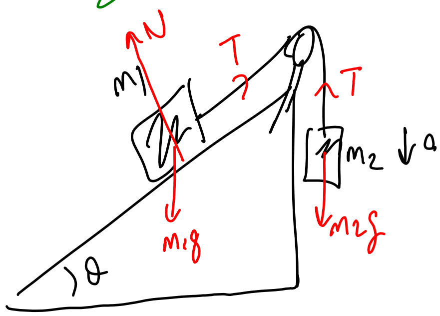
$$\boxed{\cancel{mg} - N = \cancel{ma}}$$

$N = 0$

$\sum F = \sum ma$

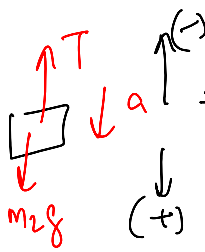
$\sum (F - ma) \Rightarrow mg - N - ma = 0$

• Example



$\frac{m_1}{m_2} = ?$

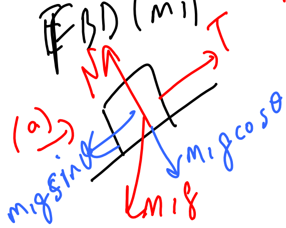
FBD (m_2)



$\Rightarrow \sum F = \sum ma$

$m_2 g - T = m_2 a \quad (1)$

FBD (m_1)



$\sum \vec{F} = ma$

$\sum F_\perp = 0 \quad ; \quad \sum F_\parallel = m_1 a$

$N - m_1 g\cos\theta = 0 \quad ; \quad T - m_1 g\sin\theta = m_1 a$

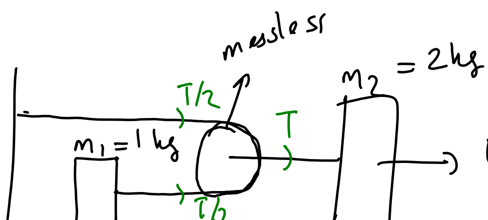
$T = m_1(a + g\sin\theta)$

$m_2 g - m_1(a + g\sin\theta) = m_2 a$

$m_2(g - a) = m_1(a + g\sin\theta)$

$$\boxed{\frac{m_1}{m_2} = \frac{g - a}{a + g\sin\theta}}$$

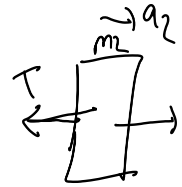
Example



$F = 40N$

$\rightarrow a(m_1) = ?$

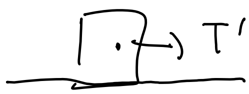
FBD(m_2)



$\Rightarrow$ $\Sigma F = m_2 a$

$$\boxed{F - T = m_2 a_2}$$

FBD(m_1)

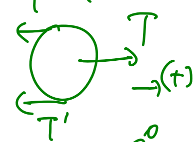


$\Rightarrow$ $\Sigma F = m_1 a_1$

$T' = m_1 a_1$

$\Downarrow$

$\frac{T}{2} = m_1 a_1 = m_1 2a_2$



$\Sigma F = \Sigma \cancel{m}a^{0}$

$\Sigma F = 0$

$T - T' - T' = 0$

$T = 2T'$

$T' = T/2$

$a_1 = 1 a_2$

$a_1 = 2a_2$

$F - \cancel{T} = m_2 a_2$

$\frac{T}{2} = 2m_1 a_2 \Rightarrow \cancel{T} = 4 m_1 a_2$ } $m_2 a_2$ $4 m_1 a_2$

$F - 4m_1 a_2 = m_2 a_2 \Rightarrow F = a_2 [m_2 + 4m_1]$

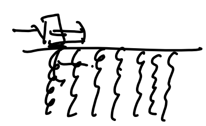
$a_2 = \frac{F}{m_2 + 4m_1} = \frac{40}{2 + 4} = \frac{40}{6} = \frac{20}{3} \cong 6.7$

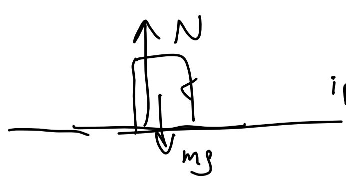
$a_1 \cong 13.4 \ m/s^2 = 2a_2$

Friction

Kinetic Friction : $F_k = \mu_k N$

Static Friction $\Rightarrow$ $F_s \leq \mu_s N$



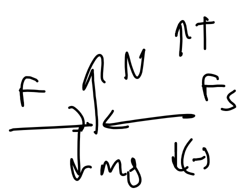


if it is not move

not pushing it $F_s = 0$

defined as

$\mu_s \Rightarrow$ max amount of F so that m will not move

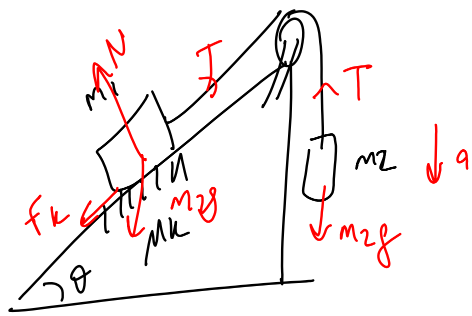


$\Rightarrow$ $N - mg = 0$ $\Rightarrow$ $N = mg$

$F - F_s = 0$

$F = F_s = \mu_s N = \mu_s mg$

$$\boxed{\mu_s = \frac{F}{mg}}$$



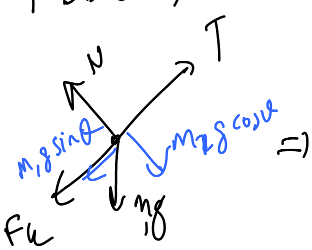
$$\frac{m_1}{m_2} = ?$$

★ The direction of friction is always opposite to the direction of motion!

FBD (m_2)

$\Rightarrow -T + m_2 g = m_2 a \quad (1)$

FBD (m_1)



$\Rightarrow$

$$N - m_1 g \cos\theta = 0 \quad (2)$$

$$T - m_1 g \sin\theta - f_k = 0\ m_1 a \quad (3)$$

$$F_k = \mu_k N \quad (4) \qquad \frac{m_1}{m_2} = ?$$

$$N = m_1 g \cos\theta \Rightarrow f_k = \mu_k m_1 g \cos\theta$$

$$T = f_k + m_1 g \sin\theta + m_1 a = (\mu_k g \cos\theta + g \sin\theta + a)\, m_1$$

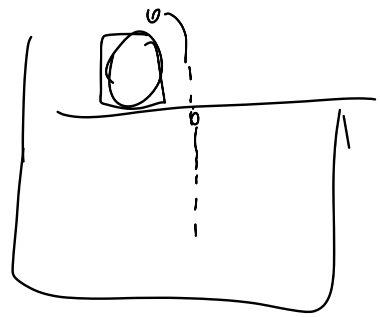
$$m_2 g - T = m_2 a$$

$$m_2 (g - a) = T = m_1 (\mu_k g \cos\theta + g \sin\theta + a)$$

$$\frac{m_1}{m_2} = \frac{g - a}{\mu_k g \cos\theta + g \sin\theta + a}$$

$$\boxed{\frac{m_1}{m_2} = \frac{g - a}{a + g \sin\theta}} \rightarrow \text{without friction}$$

Fluid resistance



$F_f = kv$ (small velocity)

$F_f' = Dv^2$ (large velocity)

stone in a water pool

kv

mg a

$\sum F = \sum ma$

$mg - kv = ma \Rightarrow mg - k\frac{dx}{dt} = m\frac{d^2x}{dt^2}$

(Terminal speed $\Rightarrow a = 0$) $v_t = v_{terminal}$

$mg - kv_t = 0 \Rightarrow v_t = \frac{mg}{k}$

$mg = kv_t$

$mg - kv = ma \Rightarrow mg - kv = m\frac{dv}{dt}$

$kv_t - kv = m\frac{dv}{dt} \Rightarrow k(v_t - v)\,dt = m\,dv$

$\Downarrow$

$-\frac{k}{m}dt = \frac{dv}{v - v_t}$

$\int_0^v \frac{dv'}{v' - v_t} = -\frac{k}{m}\int_0^t dt'$

$x = (v' - v_t)$

$dx = dv'$

$\int \frac{dx}{x} = \ln x$

$= \ln(v' - v_t)$

$\ln|v - v_t| = -\frac{k}{m}t \Rightarrow \ln\left(\frac{-v + v_t}{v_t}\right) = -\frac{k}{m}t$

$\ln a - \ln b = \ln\frac{a}{b}$

$\frac{-v + v_t}{v_t} = e^{-\frac{k}{m}t}$

$-\frac{v}{v_t} + 1 = e^{-\frac{k}{m}t}$

$v(t) = v_t\left(1 - e^{-\left(\frac{k}{m}\right)t}\right)$

e^{-100}

$t \gg 1$ $\frac{1}{e^{100}} \sim 0$

$v(t) = v_t$

$t = 0 \Rightarrow v = 0$

$$a(t) = \frac{dv}{dt} = \frac{k}{m} v_t \, e^{-\frac{k}{m}t} = g \, e^{-\frac{k}{m}t}$$

$t = 0 \rightarrow g$

$t \gg 1 \Rightarrow a = 0$

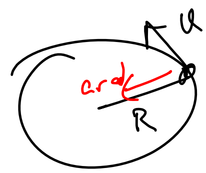
$$x(t) = \int v(t)\, dt$$

$$x(t) = v_t \left[t - \frac{m}{k}\left(1 - e^{-\left(\frac{k}{m}\right)t}\right) \right]$$

Air $\Rightarrow$ $v_t = \sqrt{\frac{mg}{D}}$ $\Rightarrow$ $mg \downarrow$ $\uparrow Dv^2$ $\Rightarrow$ terminal

$mg - Dv^2 = 0$

Circular Motion

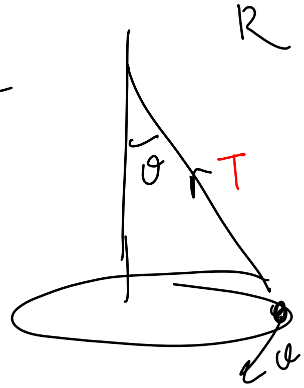


$$a_{rad} = \frac{v^2}{R}$$

$$T = \frac{2\pi R}{v} \Rightarrow a_{rad} = \frac{4\pi^2 R}{T^2}$$

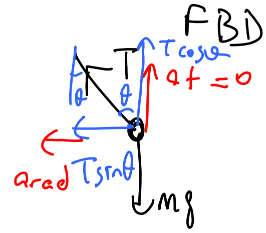
$$F = m\, a_{rad} = \frac{m v^2}{R}$$

Example



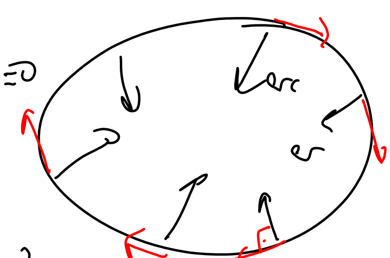
$mg, v \quad \theta = ?$

FBD



$$\sum F_{tang} = 0 = T\cos\theta - mg = 0$$

$$\sum F_{rad} = m\, a_{rad}$$



$$T\sin\theta = m\, a_{rad} = m\frac{v^2}{R}$$

$$T\cos\theta = mg$$

$$\frac{\cancel{m}g \sin\theta}{\cos\theta} = \frac{\cancel{m} v^2}{R} \Rightarrow \tan\theta = \frac{v^2}{gR}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$