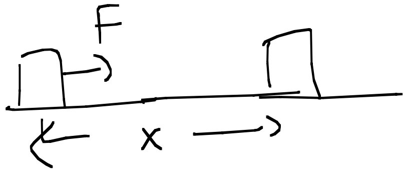
Work and Energy



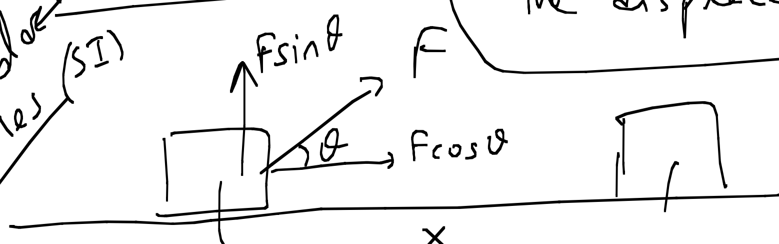
$Fx = \text{Work}$

$$\boxed{W = \vec{F} \cdot \Delta\vec{x}}$$

scalar

Joules (SI)

Force along the direction of motion times the displacement

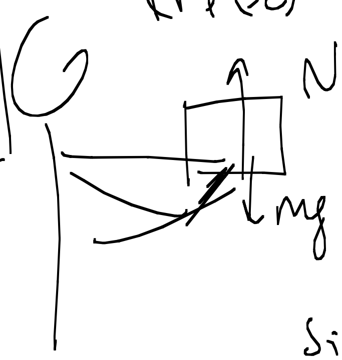


$N * m$

$kg \frac{m}{s^2} * m$

$$\boxed{kg \frac{m^2}{s^2} = \text{Joule}}$$

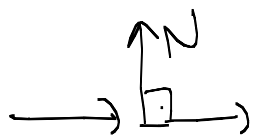
$$W = \underbrace{|F|\cos\theta\,|x|}_{(\vec{F})\cdot(\vec{x})}$$



hold it

No work

Sine $\Delta x = 0$



$\vec{F} \cdot \Delta\vec{x}$

$W = \vec{N} \cdot \Delta\vec{x} = 0$

Work-Energy Theorem

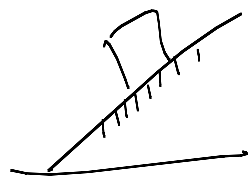
$$\boxed{W = \Delta K.E.}$$ $W = KE_f - KE_i$

* Total energy of the universe constant.

No energy can be created or destroyed.

(Is true for a closed system)

Inclined plane with no friction



$$W = \Delta K = KE_f - KE_i$$

$$\underline{KE = \frac{1}{2} m v^2}$$

$KE_{in} = 0$

$$W = F\Delta x = m a \Delta x = \underline{\Delta KE}$$

$W = max = m$

$v_f^2 = v_0^2 + 2a\Delta x$

$a = \dfrac{v_f^2 - v_0^2}{2\Delta x}$

$$m\left[\frac{v_f^2 - v_0^2}{2\cancel{\Delta x}}\right]\cancel{\Delta x} = \Delta KE$$

$KE_f - KE_i$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = KE_f - KE_i$$

$$W = \Delta K = KE_f - KE_i$$

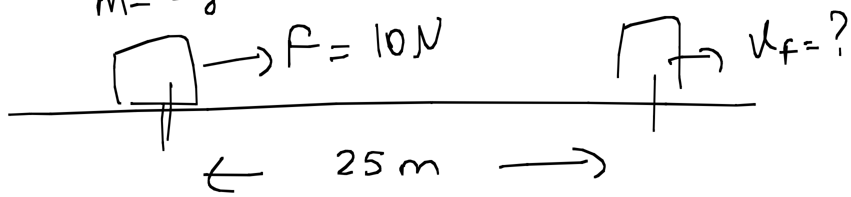
$$\boxed{\vec{F} \cdot \vec{\Delta x} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2}$$

$$\vec{F} \cdot \vec{x} = |\vec{F}|\,|\vec{x}| \cos\theta = W$$

$-1 \leq \cos\theta \leq 1 \Rightarrow$ W can be both $(+, -)$

Ex:

$M = 2kg$



$v_i = 0 \quad ; \quad v_f = ?$

1st way: $W = \Delta KE$ or $\sum \vec{F} = m\vec{a}$

$\vec{F} \cdot \vec{x} = KE_f - \cancel{KE_i}^{\,0}$

$10 * 25 * \underbrace{\cos 0}_{1} = \frac{1}{\cancel{2}} \cancel{m}^{\,2} v_f^2$

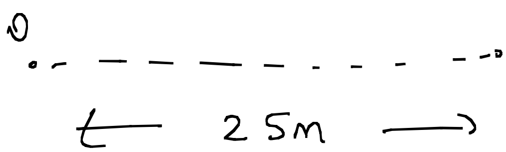
$v_f^2 = 250 \Rightarrow v_f = \sqrt{250}$

$v_f = 5\sqrt{10} \cong 16.5 \frac{m}{s}$

2nd way

$\sum F = ma = \quad F = 10N = 2 * a$

$a = 5 m/s^2$



$\Delta x = \frac{1}{2} a t^2 \Rightarrow \quad \overset{5}{\cancel{25}} = \frac{1}{2} \cancel{5} * t^2$

$t^2 = 10$

$t = \sqrt{10}$

$v_f = \cancel{v_0}^{\,0} + at$

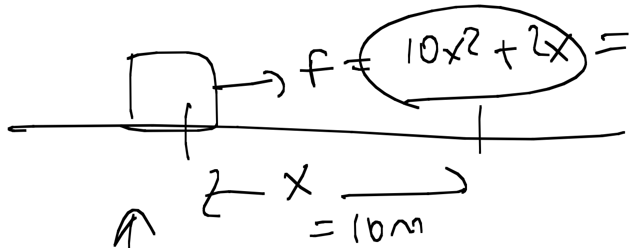
$v_f = at$

$v_f = 5 * \sqrt{10}$

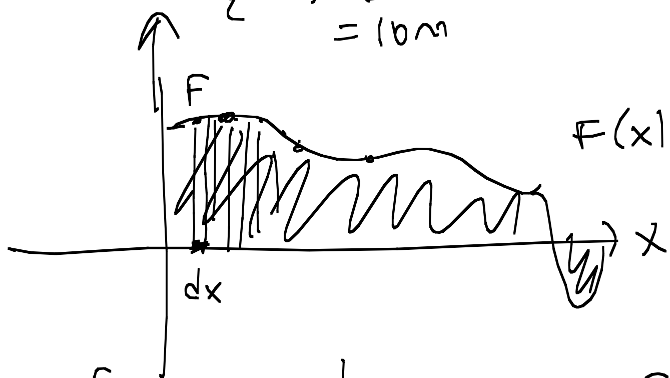
$\underline{v_f^2 = \cancel{v_0^2}^{\,0} + 2a\Delta x}$

Non-constant Forces

$W = \int \vec{F} \cdot d\vec{x} = \Delta KE \qquad w = \vec{F} \cdot \vec{x}$



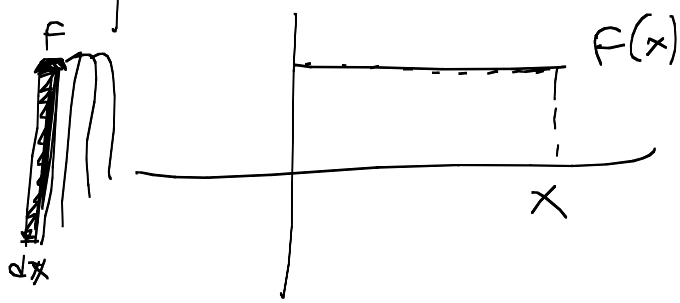
work is the area under F-x graph



line-integral

$F(x) \Rightarrow \int \vec{F} \cdot d\vec{x} = W$

$\int$ntegral

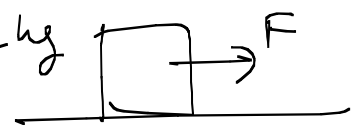


Ex =

$F(x) = 10x^2 + 2x$

$v_0 = 0$ $\quad \Delta x = 10m$

$v_f = ?$ $\quad m = 2kg$



$\int \vec{F} \cdot d\vec{r} = \Delta KE$ $\quad F \cdot dx \underbrace{\cos\theta}_{1}$ $\quad \vec{\Delta x}$

$W = \Delta KE = KE_f - \underbrace{\cancel{KE_i}}_{0}$ $\quad v_i = 0$

$$\int_0^{10} (10x^2 + 2x)dx = \frac{1}{2} m v_f^2$$

$$\frac{10}{3}x^3 + x^2 \Big|_0^{10} = \frac{1}{\cancel{2}} * \cancel{2}\, v_f^2$$

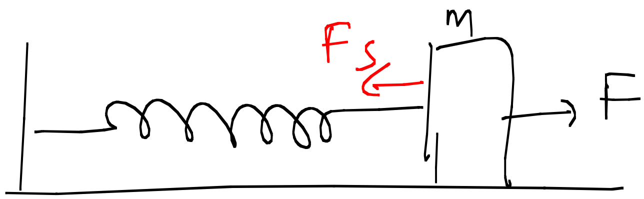
$$\frac{10000}{3} + 100 = v_f^2$$

$$v_f = \sqrt{3433} \cong 60 \text{ m/s} \ (58.6)$$

Spring force (F is not const)

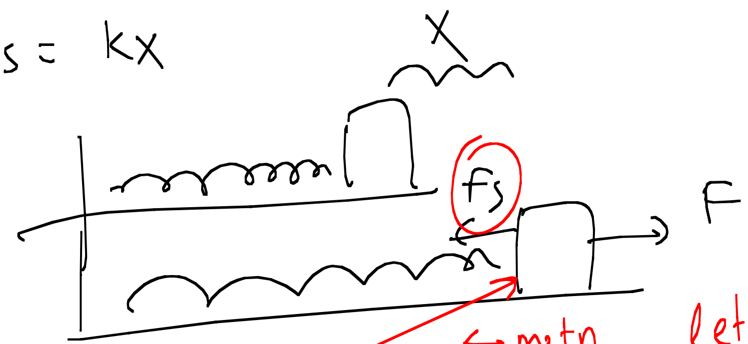
change from eq. pos.

$F_s = k\Delta x$ (Hooke's Law)



$F = kx$

$F_s = kx$



let it go

$F = 0$

$$W = \int \vec{F}_s \cdot d\vec{x} = \int_0^X (kx)\,dx \rightarrow k\int_0^X x\,dx$$

$\vec{F}_s \cdot d\vec{x} = +F\,dx$

cos 0

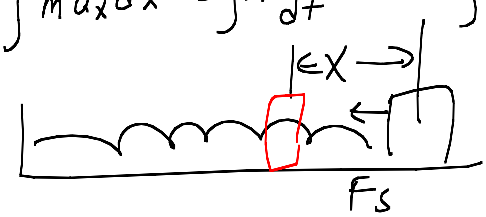
$$\frac{kx^2}{2}\Big|_0^X = \boxed{\frac{1}{2}kx^2}$$

v_x

$$W = \int \vec{F} \cdot d\vec{x} = \int m\vec{a}_x d\vec{x} = \int m\frac{dv_x}{dt} \cdot d\vec{x} = \int m\frac{dv_x}{\cancel{dx}}\frac{dx}{dt}\cancel{dx}$$

$$W = \int m v_x\, dv_x$$

$$= m\int_{v_1}^{v_2} v\,dv$$



High school

$$\frac{mv^2}{2}\Big|_{v_1}^{v_2}$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W = \int \vec{F}_s \cdot d\vec{x} = \Delta KE$$

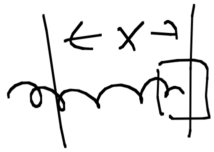
$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$E_{in} = E_f$$

$$PE_i + \cancel{KE_i}$$

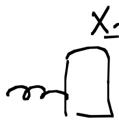
$$= \cancel{PE_f} + KE_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$



$u_i = 0 \Rightarrow PE_i = \frac{1}{2}kx^2$

$u = 0 \Rightarrow KE_i = 0$ } Initial



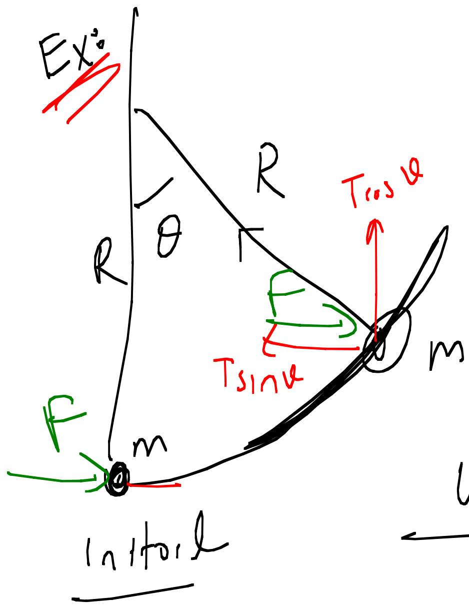
$u = u_f$

$PE_f = 0$ (not stretched)

$KE_f = \frac{1}{2}mu_f^2$

$$\frac{1}{2}kx^2 = \frac{1}{2}mu^2$$

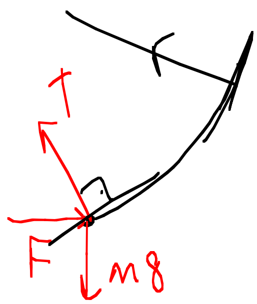
Ex:



Find $(\theta = \theta_0)$

Work ? (net force) $\Rightarrow 0$

since it is in eq

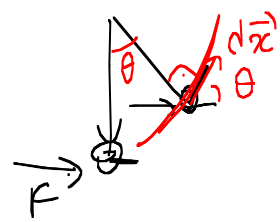


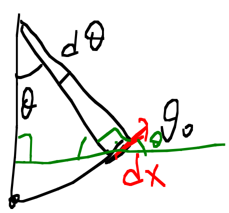
$W(T) = 0$ T ⊥ motion

$\sum Work = 0 = \sum W_T + W_{mg} + W_F$

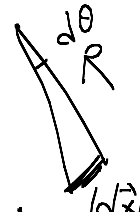
$W = \vec{F}\cdot\vec{x}$

$\int \vec{F}\cdot d\vec{x}$

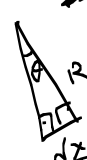




$\int \vec{F}\cdot d\vec{x}$



$\int |\vec{F}||d\vec{x}| \cos\theta$

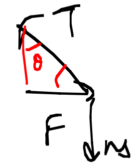


$\int F R\, d\theta \cos\theta$

$W_F = \int mg \underbrace{\tan\theta \cos\theta}_{\sin\theta} R\, d\theta$

$dx = R \tan d\theta$ dθ

In equil $\quad \sum F_x = 0 \quad \sum F_y = 0$



$F - T\sin\theta = 0$

$-mg + T\cos\theta = 0$

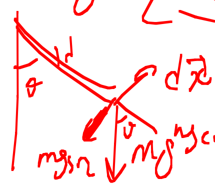
$T = \frac{mg}{\cos\theta}$

$F = T\sin\theta = mg\tan\theta$

$$W_F = \int mg\sin\theta\, R\, d\theta = mgR\int \sin\theta\, d\theta$$

$$= mgR(-\cos\theta)\Big|_0^{\theta_0} = mgR[-\cos\theta_0 + \underbrace{\cos 0}_{1}]$$

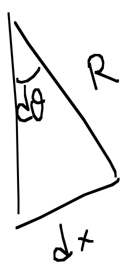
$$W_F = mgR(1 - \cos\theta_0)$$



$$\int m\vec{g}\cdot d\vec{x}$$

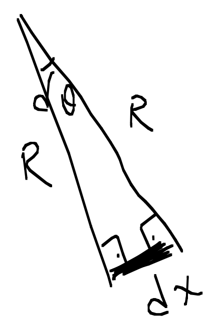
$$W_{mg} = mgR(\cos\theta_0 - 1)$$

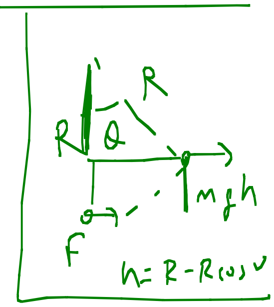
$$\int -mg\sin\theta R\, d\theta = mgR\cos\theta\Big|_0^{\theta_0} = mgR[\cos\theta_0 - 1]$$



$d\theta \ll 1$

$dx = R\, d\theta$





$mg \quad R(1-\cos\theta)$

$R\sin d\theta = dx$

$R\, d\theta = dx$

$(\sin x = x)$?

small angles

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$$

$$e^{ix} = \cos x + i\sin x$$

// $\cos x = 1 - \frac{x^2}{2}$

$\boxed{\sin x = x} - \frac{x^3}{3!}$

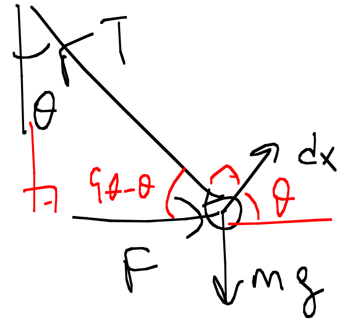
$$\sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = 1 + (ix) + \frac{(ix)^2}{2} + \frac{(ix)^3}{3!} + \dots$$

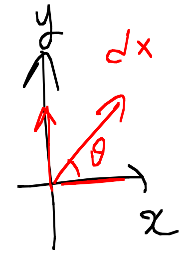
$x \ll 1$

0.01

$$1 + ix - \frac{x^2}{2} - i\frac{x^3}{3!} = \cos x + i\sin x$$

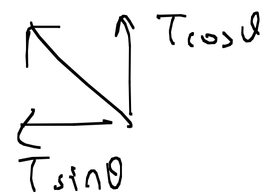
$$\int \vec{F} \cdot d\vec{x}$$





$$\vec{F} = |\vec{F}|\,\hat{i}$$

$$\vec{T} = -|\vec{T}|\sin\theta\,\hat{i} + |\vec{T}|\cos\theta\,\hat{j}$$



$$m\vec{g} = -|m\vec{g}|\,\hat{j}$$

$$d\vec{x} = |d\vec{x}|\cos\theta\,\hat{i} + |d\vec{x}|\sin\theta\,\hat{j}$$

$$W = \int \vec{F} \cdot d\vec{x} \qquad W_F = \int \left[(|\vec{F}|\,\hat{i}) \cdot \left(|d\vec{x}|\cos\theta\,\hat{i} + |d\vec{x}|\sin\theta\,\hat{j} \right) \right]$$

$$W_F = \int |\vec{F}|\,|d\vec{x}|\cos\theta$$

$$W_T = \int T \cdot d\vec{x} = \int \left(-|\vec{T}|\sin\theta\,\hat{i} + \vec{T}\cos\theta\,\hat{j} \right) \cdot \left(|d\vec{x}|\cos\theta\,\hat{i} + |d\vec{x}|\sin\theta\,\hat{j} \right)$$

$$= \int \left[-|\vec{T}|\sin\theta\,\cancel{\cos\theta\,|d\vec{x}|} + |\vec{T}|\cancel{\cos\theta\,|d\vec{x}|}\sin\theta \right] = 0$$

$$W_{mg} = \int \left[(-mg\,\hat{j}) \cdot \left(|d\vec{x}|\cos\theta\,\hat{i} + |d\vec{x}|\sin\theta\,\hat{j} \right) \right]$$

$$W_{mg} = -\int mg\,|d\vec{x}|\sin\theta$$

$$|d\vec{x}| = R\,d\theta$$

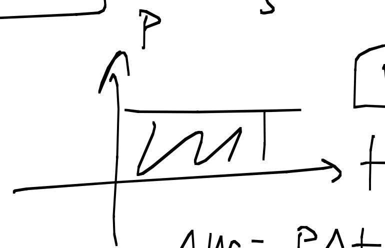
From eq $\Rightarrow$ $|\vec{F}| = mg\tan\theta$

Power

$$\boxed{1\,hp = 746\,W}$$

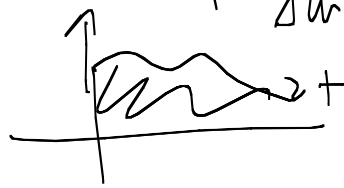
$$\boxed{P_{av} = \frac{\Delta W}{\Delta t}} = \frac{kg\,m^2/s^2}{s} = \frac{kg\,m^2}{s^3}$$

$\boxed{\text{Watt}}$



$$\Delta W = P\Delta t$$

$$\boxed{P = \frac{dW}{dt}}$$



$$\Delta W = \int P\,dt$$

$$\vec{P} = \vec{F}\cdot\vec{v}$$

$$P = \frac{dW}{dt} = \frac{d}{dt}\left[\int \vec{F}\cdot d\vec{x}\right]$$

$$\vec{P} = \vec{F}\cdot\frac{d\vec{x}}{dt} = \boxed{\vec{F}\cdot\vec{v}}$$