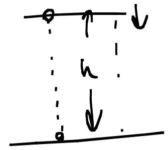
Note Title 3/21/2012

Work ⇒ Potential Energy and Conserv. of Energy

$W = \vec{F} \cdot \Delta \vec{x}$ ⇒ grav. is an external force

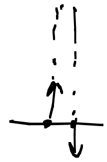
$-mg$

$W = \Delta K$



Potential energy ⇒ $W = -\Delta U = U_i - U_f$

Two kinds of forces in nature:



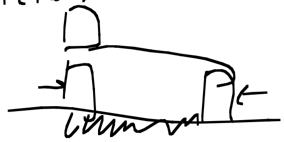
1) Conservative Forces:

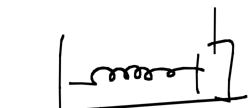
$W_F = 0$

if $\Delta x = 0$

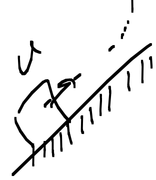
2) Non-conservative forces:

friction, Air resistance





Spring



$\boxed{W_{NF} = \Delta E}$

$\boxed{W_{NF} = E_f - E_i}$

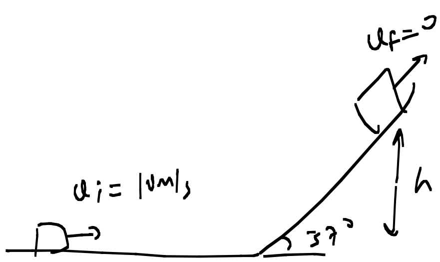
$E_f - E_i = 0$ ⇒

$E_i = E_f$

if $W_{CF} = \boxed{\Delta E = 0}$

$PE_i + KE_i$

Energy conservation $= PE_f + KE_f$



Gravity = Conservative

Ex:

What is the max Δx for the spring?

$W = \Delta E = 0$ (since no nonconservative force exists)

$E_f - E_i = 0$ ⇒ $\underline{E_f = E_i}$

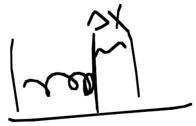
$KE_f + PE_f = KE_i + PE_i$

0

$\frac{1}{2} m v^2$

$PE_i = PE_S + PE_G$

$PE_{Si} = 0 \qquad PE_{iG} = mgh$



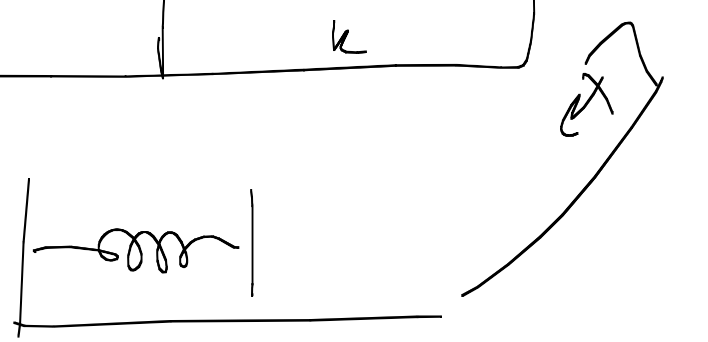
$\Rightarrow PE_f = PE_{fS} + \underbrace{PE_{fG}}_{0}$

$PE_{fS} = \frac{1}{2}k\Delta x^2$

$E_f = E_i \Rightarrow \frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2 + mgh$

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2 + mgh$$

$$\boxed{\Delta x = \sqrt{\frac{m(v^2+2gh)}{k}}}$$



$$W = 0 = \int \vec{F}_f \cdot d\vec{x} = E_f - E_i$$

$\downarrow$

k V

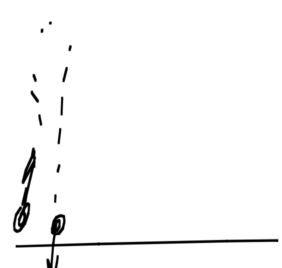
<u>Prop. of Cons. Forces</u>

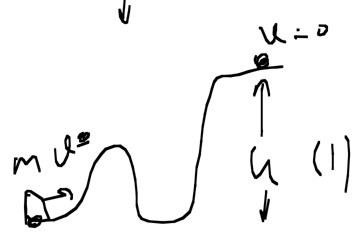
1) $W = 0$ for $\Delta x = 0$

2) Reversible

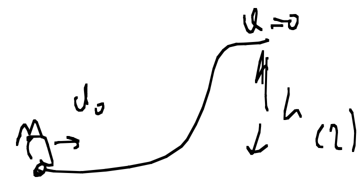
3) W = indep. of the path

just depends on $\vec{x}_i$, $\vec{x}_f$

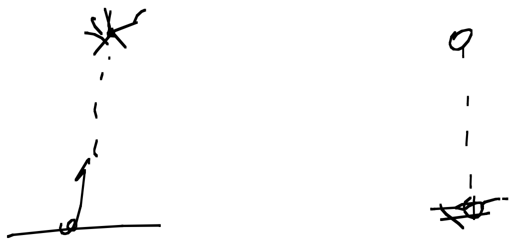




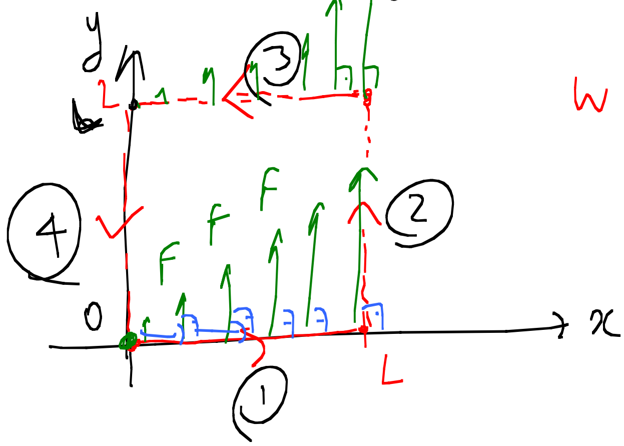
$\boxed{\Delta \vec{x}}$



4) $W = -\Delta U$



Ex: $\vec{F} = cx\,\hat{j}$



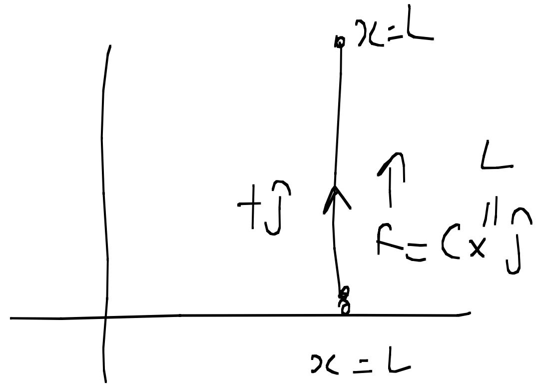
$W_F = ?$

$W = \int \vec{F}\cdot d\vec{l} \Rightarrow 0$

if F is cons.

Thm: $\oint_{\text{consv}} \vec{A}\cdot d\vec{x} = 0$

$$\int \vec{F}\cdot d\vec{l} = \cancel{\int_{(1)} \vec{F}\cdot d\vec{l}}^{\;\perp\;0} + \int_{(2)} \vec{F}\cdot d\vec{l} + \cancel{\int_{(3)} \vec{F}\cdot d\vec{l}}^{\;\perp} + \cancel{\int_{(4)} \vec{F}\cdot d\vec{l}}^{\;F=0}$$



$\int \vec{F}\cdot d\vec{l}$ $\qquad$ $d\vec{l} = |d\vec{l}|\,\hat{j}$

$\vec{F} = CL\,\hat{j}$

$$\vec{F}\cdot d\vec{x} = CL|d\vec{x}|\underbrace{\hat{j}\cdot\hat{j}}_{1} = CL\,dx$$

$$\int \vec{F}\cdot d\vec{x} = \int CL\,dl = CL\int_0^L dx$$

L

$\int dl$

$\boxed{CL^2}$

$\boxed{W_F = CL^2}$

$$\boxed{W = -\Delta U} = \int \vec{F}\cdot d\vec{x}$$

Infinitesimal work = $F\cdot\Delta x$ where Δx is small

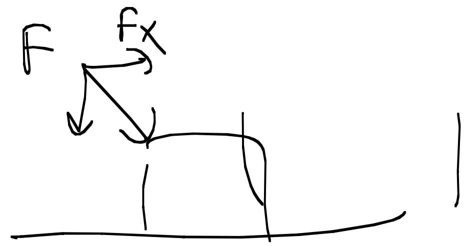
$$dW = \vec{F}\cdot d\vec{x} = -\Delta U = -dU$$

$$\therefore \boxed{F_x = -\frac{dU}{dx}} \Rightarrow \frac{\partial}{\partial y}\left[mgy\right]$$

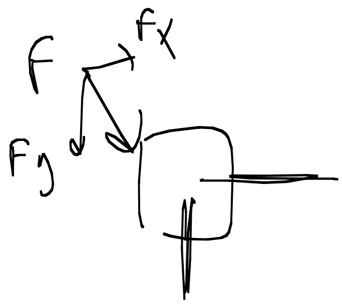
$$mg\left(\frac{d}{dy}y\right) = mg$$

$$F_x = -\frac{dU}{dx}$$

$$F_y = -\frac{dU}{dy} \;; \quad F_z = -\frac{dU}{dz}$$



$$W_x = F_x (\Delta x)$$



Gradient

$$mgy$$

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

$$F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

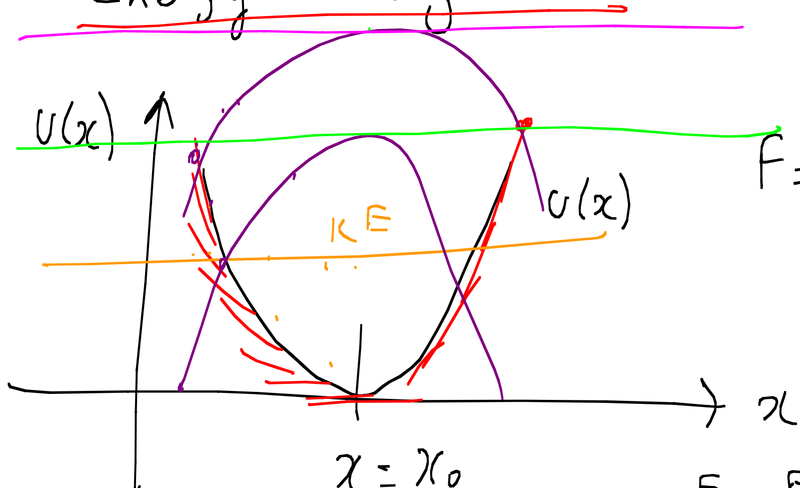
$$\boxed{\vec{F} = -\vec{\nabla} U}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Gradient

$$\nabla U$$

Energy Diagrams

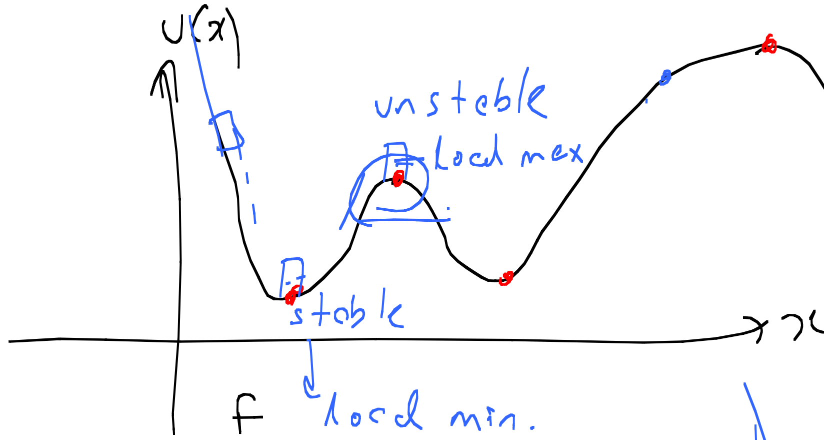


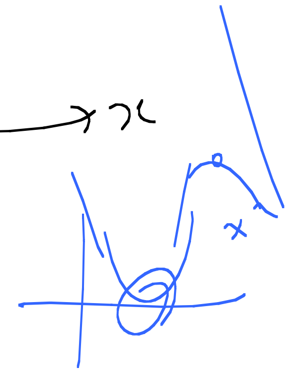
$$F = -\frac{dU}{dx}$$

$$E_i = E_f$$

$$W = \Delta E = 0$$

cons for PE+KE=





$$F = -\frac{dU}{dx}$$