Note Title 3/28/2012

Momentum, Impulse, Collisions

En. conservation is less general than momentum conservation.

$v \gg 1$ mom. is still conserved. $kg \frac{m}{s}$

Momentum $\propto$ mass
$\propto$ velocity } $\vec{p} = m\vec{v}$

$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$

$\vec{p} = m v \Rightarrow \frac{d\vec{p}}{dt} = \frac{d}{dt}(m v) = m\frac{d\vec{v}}{dt} = \vec{F}$ (1) $\frac{d\vec{p}}{dt} = \vec{F}$

$\vec{p} = m\vec{v}$: $\frac{d\vec{p}}{dt} = \vec{F} \Rightarrow d\vec{p} = \vec{F}\,dt \Rightarrow \int_{P_1}^{P_2} d\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt$

$\therefore \Delta\vec{P} = \vec{P}_2 - \vec{P}_1 = \int_{t_1}^{t_2} \vec{F}\,dt = \vec{J}$: Impulse

If $\vec{F} =$ const $\Rightarrow \int F\,dt = F\int_{t_1}^{t_2} dt = F(t_2 - t_1)$

$\boxed{\Delta\vec{P} = \vec{F}\,\Delta t} = \vec{J}$

Momentum Conservation

$\boxed{\vec{P}_i = \vec{P}_f}$ If there is no external force (closed)

$\Rightarrow$ Momentum will be constant

$$\boxed{\sum_{i=1}^{N} \vec{P}_{0i} = \sum_{i=1}^{N} \vec{P}_{fi}} \Rightarrow \vec{P}_{01} + \vec{P}_{02} + \vec{P}_{03} + \cdots$$
$$= \vec{P}_{f1} + \vec{P}_{f2} + \vec{P}_{f3} + \cdots$$

Collisions

1) Elastic Collision (Billiard Balls) [En is also conserved]

2) Inelastic Collision (Rest ...) [En. is not conserved]
(Explosion)

Ex: A rifle : $m_R = 3\,kg$; $m_b = 5\,gr$

Initial final

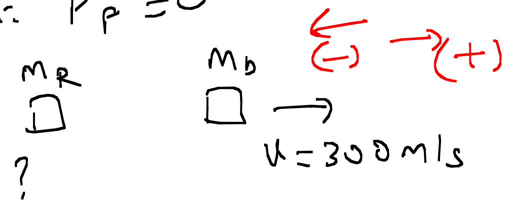
$\rightarrow 300\ m/s$

Rest $\leftarrow v_R = ?$

$v = 0$

$\vec{P}_i = \vec{P}_f$

$$\vec{p}_i = 0 \therefore \vec{p}_f = 0$$

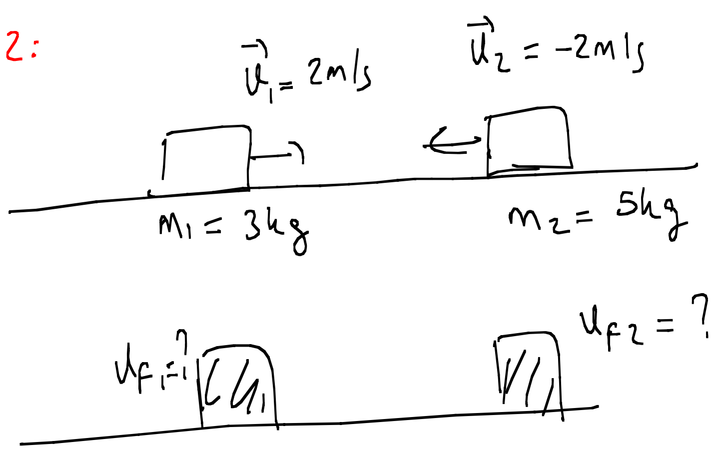


$$\vec{p}_f = 0 = \vec{p}_b + \vec{p}_R = m_b \vec{v}_b + m_R \vec{v}_R$$

$$0 = +5\times10^{-3} * 300 + 3\, v_R$$

$$-3 v_R = 1.5 \Rightarrow v_R = -\frac{1.5}{3} = -0.5\,m/s$$

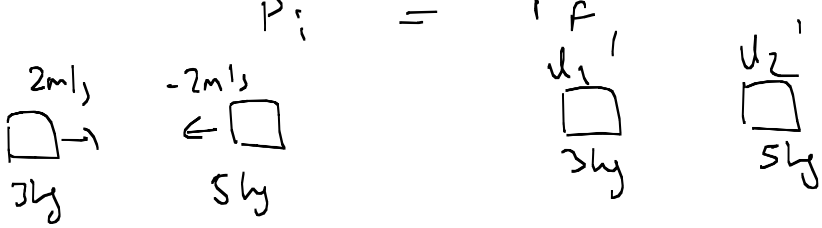
Ex 2:



Mom. is conserved (elastic)

En. is also conserved

$$\vec{P}_i = \vec{P}_f$$



$$\underbrace{\vec{P}_1 + \vec{P}_2}_{m_1\vec{v}_1 + m_2\vec{v}_2} = \underbrace{3*2 - 5*2}_{-4} = \underbrace{3v_1' + 5v_2'}_{m_1 v_1' + m_2 v_2'}$$

$$-4 = 3v_1' + 5v_2' \quad (1)$$

En. conservation: $E_i = E_f$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

$$\cancel{\tfrac{1}{2}} * 3 * 2^2 + \cancel{\tfrac{1}{2}}\, 5(-2)^2 = \cancel{\tfrac{1}{2}} * 3\, u_1'^2 + \cancel{\tfrac{1}{2}} * 5\, u_2'^2$$

$$12 + 20 = 3u_1'^2 + 5u_2'^2 \quad (2)$$

from eqn (1) ⇒ $\boxed{u_2' = \dfrac{-4 - 3u_1'}{5}}$

$$32 = 3u_1'^2 + \frac{\cancel{5}}{\cancel{25}\ 5}\left[9u_1'^2 + 24u_1' + 16\right]$$

$$160 = 15u_1'^2 + 9u_1'^2 + 24u_1' + 16$$

$$24u_1'^2 + 24u_1' - 144 = 0$$

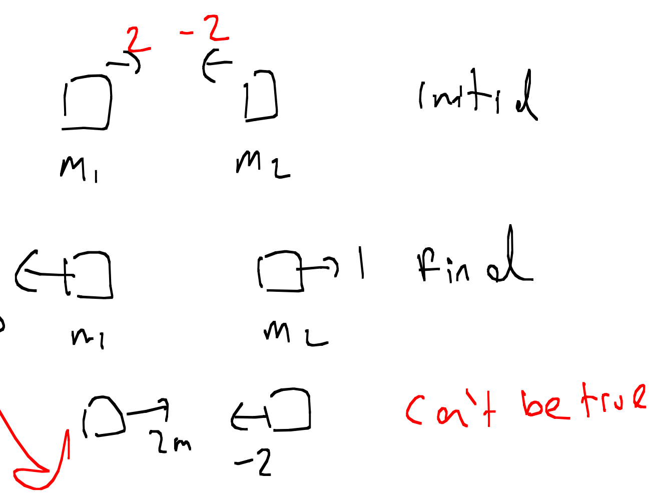
$$u_1'^2 + u_1' - 6 = 0 = (u_1' + 3)(u_1' - 2) = 0$$

u_1' $+3$

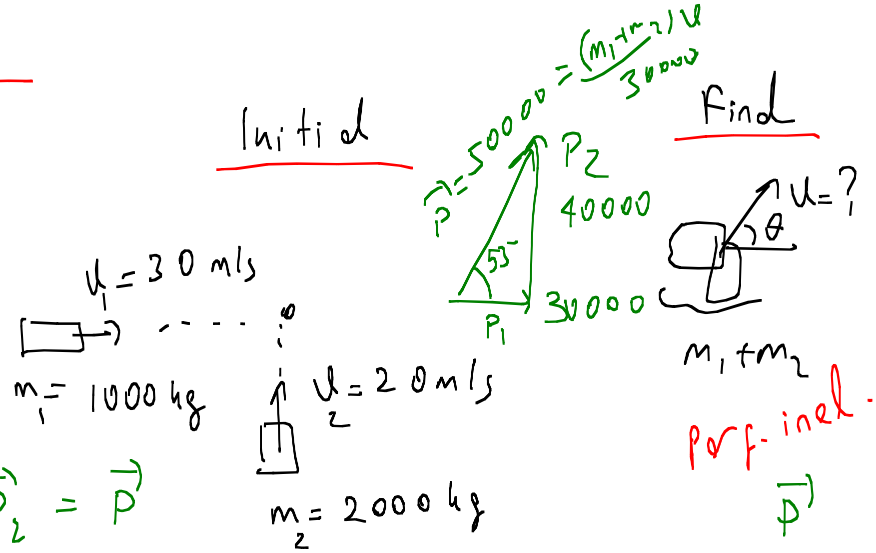
u_1' -2

$\boxed{u_1' = -3 \;\Downarrow\; u_2' = 1}$, $u_1' = 2$ ⇓ X $\boxed{u_2' = \dfrac{-4 - 3u_1'}{5}}$

$u_2' - 2$



Ex:



$$\vec{P}_1 + \vec{P}_2 = \vec{P}$$

$$\vec{P}_i = \vec{P}_f$$

$$(1) \quad P_{ix} = P_{fx}$$

$$(2) \quad P_{iy} = P_{fy}$$

$$(1) \Rightarrow \quad m_1 * u_1 = (m_1 + m_2)\, u \cos\theta$$

$$(2) \Rightarrow \quad m_2\, u_2 = (m_1 + m_2)\, u \sin\theta$$

$$(1)^2 + (2)^2 \Rightarrow \quad m_1^2 u_1^2 + m_2^2 u_2^2 = (m_1 + m_2)^2 u^2 \underbrace{[\cos^2\theta + \sin^2\theta]}_{1}$$

$$m_1^2 u_1^2 + m_2^2 u_2^2 = (m_1 + m_2)^2 u^2 \Rightarrow u = \frac{\sqrt{m_1^2 u_1^2 + m_2^2 u_2^2}}{m_1 + m_2}$$

$$(3) \quad 30 = 3u\cos\theta \qquad \cancel{1000} * 30 = \cancel{3000} * u\cos\theta$$

$$(4) \quad 40 = 3u\sin\theta \qquad \cancel{2000} * 20 = \cancel{3000}\, u\sin\theta$$

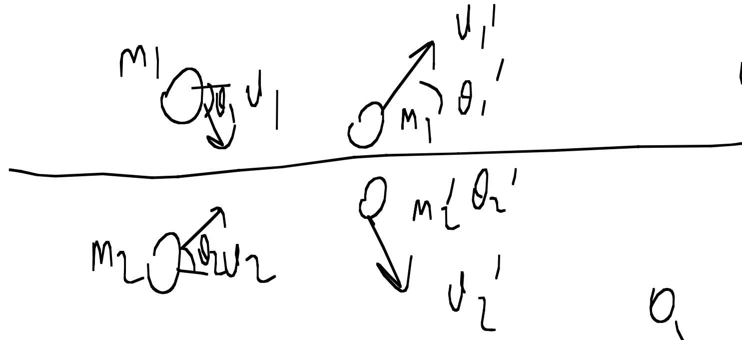
$$(3)^2 + (4)^2 = 900 + 1600 = (3u)^2[\cos^2\theta + \sin^2\theta]$$

$$2500 = (3u)^2$$

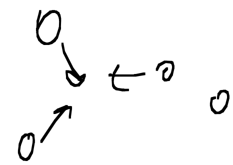
$$3u = 50 \quad \boxed{u = 16.7 \text{ m/s}}$$

$$(4) \Rightarrow \quad 40 = 3u\sin\theta \Rightarrow \sin\theta = \frac{40}{3 * 16.7}$$

$$\theta = \sin^{-1}\left[\frac{40}{50}\right] \Rightarrow \boxed{53°}$$



$\theta_1' = ?$

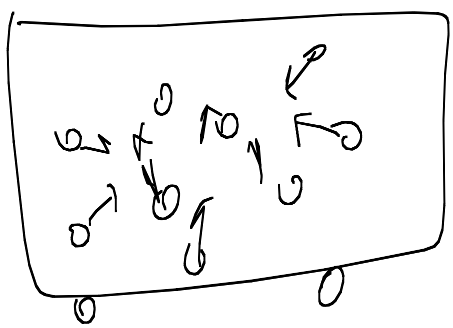


$v_1, v_2, \theta_1, \theta_2$

$E_i = E_f$

$$\boxed{\begin{aligned} P_{ix} &= P_{fx} \\ P_{iy} &= P_{fy} \end{aligned}}$$

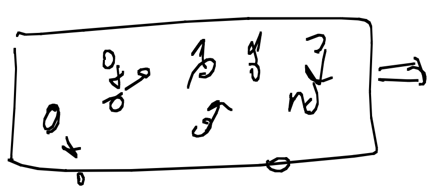
Center of Mass



$\rightarrow v$

$\sum P_i$

$\Downarrow$

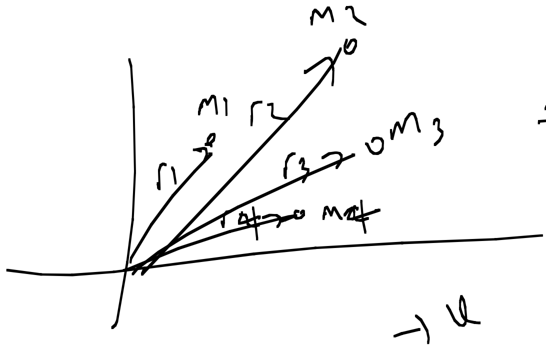


$\Rightarrow v' \; \sum P_f$

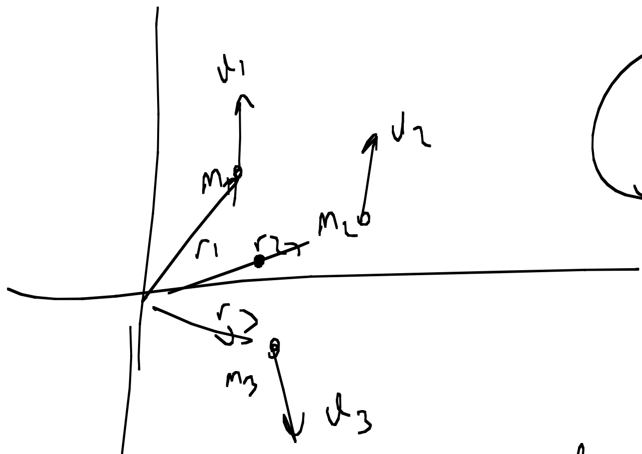
$$\sum \vec{P_i} = \sum P_f$$

[Center of Mass] $v_{system} = P_{Total}$

$$\frac{\sum m_i \vec{r_i}}{\sum m_i} = \vec{r}_{cm}$$

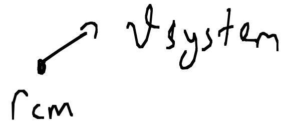


$$= \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4 r_4}{m_1 + m_2 + m_3 + m_4}$$



$\vec{v}_{system}$

$$\frac{\vec{p}_1 + \vec{p}_2 + \vec{p}_3}{m_1 + m_2 + m_3} = \vec{v}_{sy}$$



$\Delta \vec{p} = 0$ (Mom- conservation)

Extend force applied

$$\Delta \vec{P} = \vec{P}_f - \vec{P}_i = \int_{t_1}^{t_2} f(t)\,dt$$