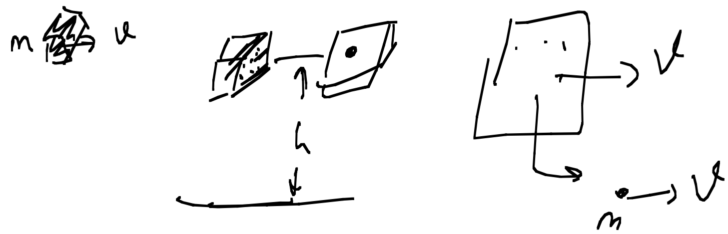
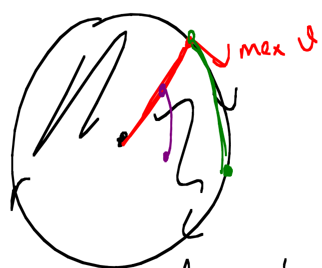
Note Title 4/4/2012

Rotation of Rigid Bodies

Angular velocity and ang. acceleration, Rigid bodies

Point particles $\Rightarrow$ $m\vec{a} = \vec{F}$ point

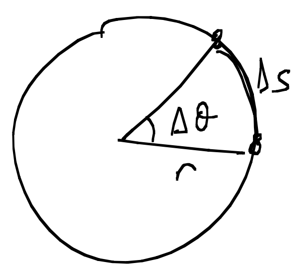




. Note that each single part of the disk has a diff. tangential velocity. We need to define a velocity that has an invariant value.

$\Rightarrow$ Angular velocity: the amount of angle that changes with unit time.

$$\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \frac{rad}{s}$$



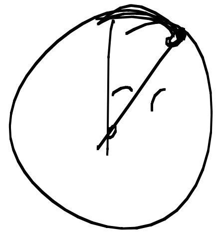
$$\Delta\theta = \frac{\Delta s}{r} \rightarrow \text{radians} \Rightarrow \frac{2\pi r}{r} = 2\pi_{\text{radians}}$$

2π radians $= 360^\circ \Rightarrow$

$$1\,rad = \frac{360^\circ}{2\pi} = 57.3^\circ$$

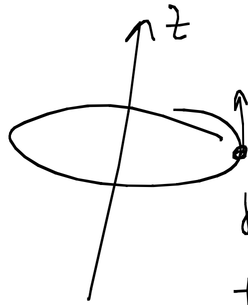
$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{r\,\Delta t} = \frac{1}{r}\underbrace{\boxed{\frac{\Delta s}{\Delta t}}}_{v}$$

$$\omega = \frac{v}{r} \Rightarrow \boxed{v = \omega r}$$



$$\vec{\omega}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

$$\vec{v}_{ins} = \frac{d\vec{x}}{dt}$$



four fingers along tangential

thumb $\Rightarrow \vec{\omega}$

$+\omega_z \Rightarrow \boxed{|\vec{\omega}| + \hat{k}}$

$$2\,\frac{rad}{s} \Rightarrow +2\hat{k}\,\frac{rad}{s}$$

Angular acceleration

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \Rightarrow \frac{rad}{s^2} \quad \alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left[\frac{d\theta}{dt}\right] = \frac{d^2\theta}{dt^2}$$

$$a_{lin} = \frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{d}{dt}(\omega r)$$

$$a_{lin} = r\frac{d\omega}{dt} = r\alpha \Rightarrow \boxed{a_{lin} = r\alpha}$$

lin	rot
$v_f = v_0 + at$	$\omega_f = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v_f^2 = v_0^2 + 2a\Delta x$	$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$
$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$

Moment of Inertia

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \dots$$

$$\left.\begin{aligned} v_1 &= \omega_1 r_1 \\ v_2 &= \omega_2 r_2 \\ v_3 &= \omega_3 r_3 \end{aligned}\right\}$$

Rigid bodies

$$\omega_1 = \omega_2 = \omega_3 = \dots = \omega$$

$$\frac{1}{2}\omega^2(m_1 r_1^2) + \frac{1}{2}\omega^2(m_2 r_2^2) + \frac{1}{2}\omega^2(m_3 r_3^2) + \dots$$

$$\text{Rotational } KE = \frac{1}{2}\omega^2 \underbrace{\left[m_1 r_1^2 + m_2 r_2^2 + \dots\right]}_{\text{Moment of inertia}}$$

$$\boxed{I = mr^2}$$

Mass $\Rightarrow$ is the resistance of an object to its acceleration $\Rightarrow F = ma \Rightarrow a = \frac{F}{m}$

Moment of inertia I : resistance of an object towards rotational motion. $\tau = I\alpha$

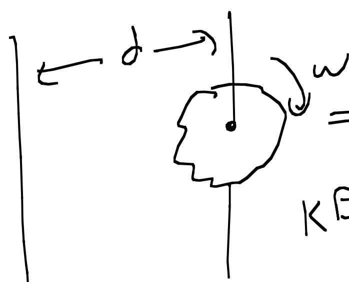
$\Downarrow$

$I = \frac{\tau}{\alpha}$ $\tau \rightarrow$ rot force, $\alpha \rightarrow$ rot acc.

$\alpha = \frac{\tau}{I}$

$KE_R = \frac{1}{2} I \omega^2$

Parallel axis theorem



$\Rightarrow I = I_{object}$

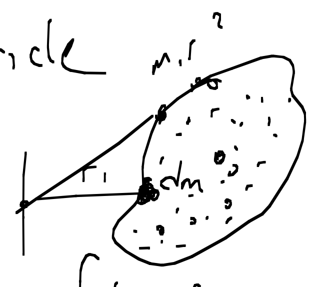
$KE = \frac{1}{2} I \omega^2$

$$\boxed{I' = I_{object} + Md^2}$$

Calculating Moment of Inertia

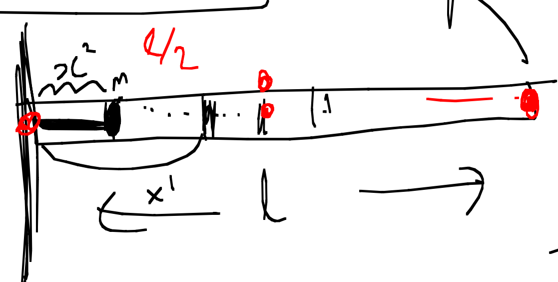
$I = mr^2 \rightarrow$ point particle

$$\boxed{I = \int r^2 \, dm}$$



$\int dm \, r^2$

Ex:

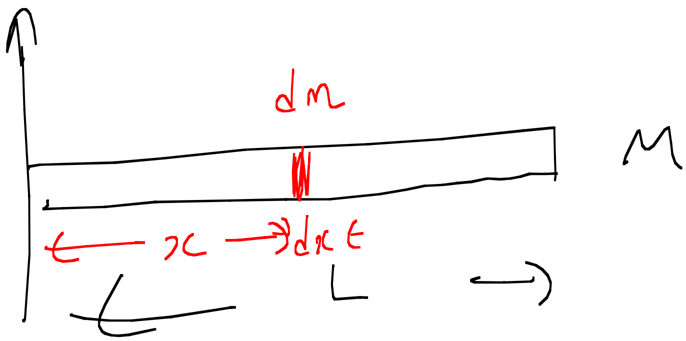


M

rod, metal

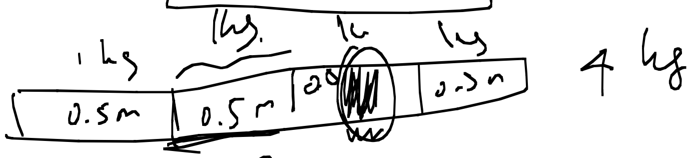
$I = mr^2 \Rightarrow \frac{1}{4} M L^2$

$I = mr^2 \Rightarrow ML^2 \qquad \frac{ML^2}{2}$



$\int r^2 dm \quad \Big| \quad \int x^2 dm$

mass density $\Rightarrow \boxed{\lambda = \frac{M}{L}} \Rightarrow \lambda = \frac{4}{2} = 2$



$\lambda = \frac{4}{2} = 2 \frac{kg}{m}$

$0.5\,m * \lambda = 1kg$ (2)

$\lambda = \frac{4}{2} = 2\frac{kg}{m} * 0.1m = 1kg \quad 0.2kg$

dist

$0.5m * \boxed{2\frac{kg}{m}} = 1kg$

$dm = \lambda dx$

$I = \int x^2 dm = \int x^2 \lambda dx = \lambda \int_0^L x^2 dx$

$\lambda \frac{x^3}{3}\Big|_0^L = \lambda \frac{L^3}{3} \Rightarrow \frac{M}{L} * \frac{L^3}{3} = \frac{1}{3} ML^2$

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