Note Title 5/2/2012

Law of Gravitation - Chp 12

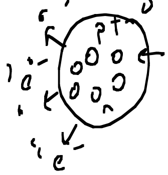
4 - fund. forces in universe

1) Electro Mag. Force ⇒ charges, computers, walking .. $\alpha = \frac{1}{137}$

2) Weak force ⇒ Nuclear Bomb related ⇒ radioactive reactions

Plutonium → lighter ⇒ (decays) $\alpha \sim 0.1$

3) Strong force ⇒ core of the atom

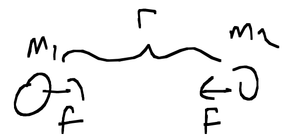


repel each other p $\alpha \sim 1$

Gluons

Glue ⇒ spring

4) Gravitational force: ⇒



$\alpha \sim 10^{-12}$

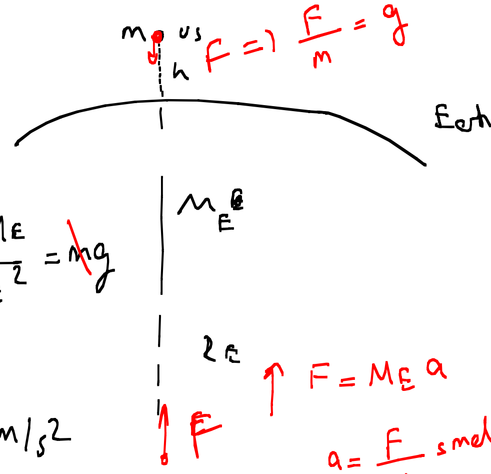
Apple → earth

Galaxy rotation, Earth rotating around Sun.

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

F_G ⇒ is responsible weight of objects in Earth.

$$F_g = G\frac{m M_E}{R_E^2} \Rightarrow$$



$F \Rightarrow \frac{F}{m} = g$

$$F_g = ma = G\frac{m M_E}{(R_E+h)^2} \sim G\frac{\cancel{m} M_E}{R_E^2} = \cancel{m}g$$

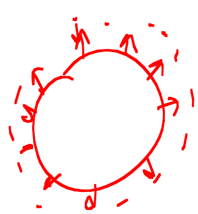
$h \ll R_E$ 10^8 km

$F = M_E a$

$$g = \frac{G M_E}{R_E^2} = \text{const} \sim 9.81 \ m/s^2$$

$a = \frac{F}{M_E}$ small

$\frac{a_E}{g} \sim 10^{-23}$

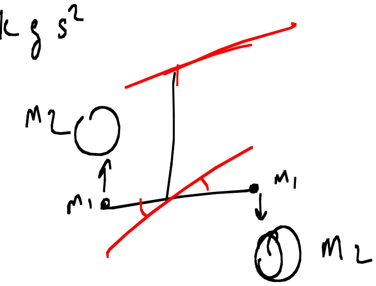


$$G = \boxed{6.67 \times 10^{-11}} \ \frac{m^3}{kg\ s^2}$$

$$F = G\frac{m^2}{r^2} \Rightarrow G = \frac{Fr^2}{m^2} = \frac{kg\frac{m}{s^2} * m^2}{kg^2}$$

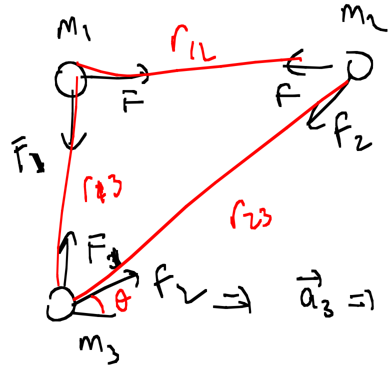
$$\Rightarrow G \Rightarrow \frac{kg\ m^3}{s^2\ kg^2} = \frac{m^3}{kg\ s^2}$$

Cavendish Experiment

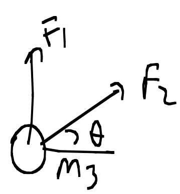


$\vec{F} \Rightarrow$ attractive

Ex:



$$\Rightarrow \vec{a}_3 \Rightarrow \frac{\vec{F}_1 + \vec{F}_2}{m_3} = \frac{\Sigma \vec{F}}{m}$$



$$\Rightarrow a_{3x} = \frac{\cancel{F_{1x}}^{0} + F_{2x}}{m_3} = \frac{F_2 \cos\theta}{m_3}$$

$$a_{3y} = \frac{F_{1y} + F_{2y}}{m_3} = \frac{F_1 + F_2 \sin\theta}{m_3}$$

$$a_{3x} = \frac{G\, m_2 \cancel{m_3}}{r_{23}^2} * \frac{\cos\theta}{\cancel{m_3}} = \frac{G\, m_2 \cos\theta}{r_{23}^2}$$

$$a_{3y} = \frac{G}{\cancel{m_3}}\left[\frac{\cancel{m_3} m_1}{r_{13}^2} + \frac{\cancel{m_3} m_2}{r_{23}^2}\sin\theta\right] = G\left[\frac{m_1}{r_{13}^2} + \frac{m_2}{r_{23}^2}\sin\theta\right]$$

$$|\vec{a}_3| = \sqrt{a_{3x}^2 + a_{3y}^2}$$

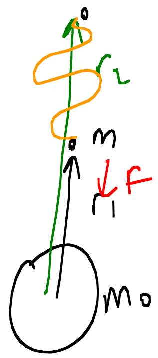
Ex:

after seconds $\Rightarrow$

Force is not constant

$W = \int F \cdot dr$ F is a function of r

Gravitation Potential Energy



$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

F and r ⇒ opposite
$\cos 180^\circ = -1$

$$W = -G m_0 m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$\int \frac{1}{r^2} = -\frac{1}{r}$$

$$W = G m_0 m \left(\frac{1}{r} \Big|_{r_1}^{r_2} \right)$$

$$\therefore W = \frac{G m_0 m}{r_2} - \frac{G m_0 m}{r_1}$$ ⇒ Conservative force

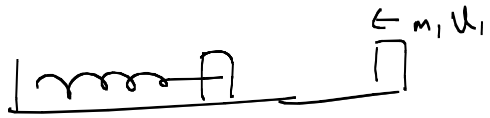
$$W = -\Delta U = U_1 - U_2 = \frac{G m_0 m}{r_2} - \frac{G m_0 m}{r_1}$$

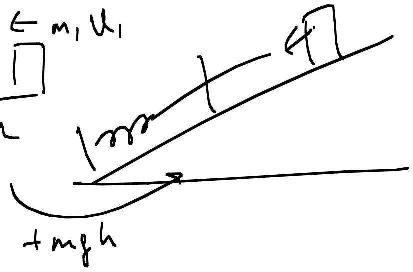
$$\Rightarrow \boxed{U_G = -\frac{G M_1 M_2}{r}}$$

Total Energy is conserved if we assume that Gravity as a potential energy.

$F = kx$ ⇒   E is const but

spring energy ⇒ $\frac{1}{2} kx^2$

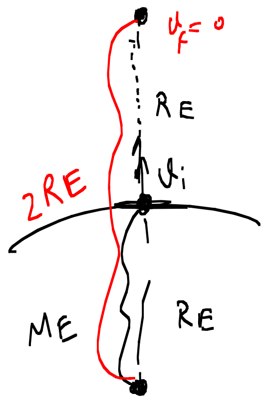




$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

<u>Ex:</u>

a) Find ~~the~~ min initial vertical velocity of a bullet to reach to a distance of R_E after it is fired upwards.



$$E_i = E_f$$

$$KE_i + PE_i = \cancel{KE_f} + PE_f$$

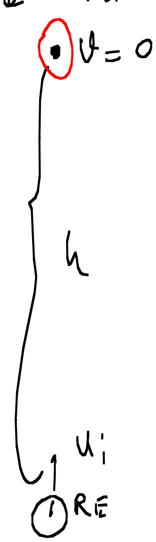
$$\frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = -\frac{GmM_E}{2R_E}$$

$$\frac{1}{2}mv_i^2 = \frac{GmM_E}{R_E}\left(1 - \frac{1}{2}\right)$$

$$\cancel{\frac{1}{2}}\cancel{m}v_i^2 = \frac{G\cancel{m}M_E}{R_E} \times \cancel{\frac{1}{2}}$$

$$v_i = \sqrt{\frac{GM_E}{R_E}}$$

b) What is the min. velocity to escape from Earth



$$h \gg R_E$$

$$KE_i + PE_i = \cancel{KE_f}^{\,0} + \cancel{PE_f}^{\,0}$$

$$\frac{1}{2}mv^2 - \frac{GmM_E}{R_E} = -\frac{GmM_E}{h+R_E}$$

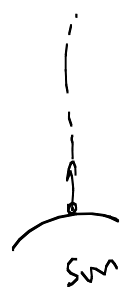
$$\frac{1}{2}mv^2 = \frac{GmM_E}{R_E} - \cancel{\frac{GmM_E}{(h+R_E)}} \quad ; \quad h \gg R_E$$

Escape speed cond

$$\frac{1}{2}mv^2 = GmM_E\left(\underbrace{\frac{1}{R_E}}_{} - \cancel{\frac{1}{h}}\right) \qquad \frac{h - \cancel{R_E}}{R_E * h}$$

$$\frac{1}{2}mv^2 - \frac{GmM_E}{R_E} = 0 \qquad \cancel{\frac{h}{R_E * h}}$$

$$\frac{1}{2}\cancel{m}v_e^2 = \frac{G\cancel{m}M_E}{R_E} \Rightarrow v_e = \sqrt{\frac{2GM_E}{R_E}}$$



for ~~bound~~ bounded particles

Circular orbits

$F = ma$

$$a = \frac{v^2}{R}$$

$$F_G = \frac{G\cancel{m}M_E}{r^2} = \cancel{m}a$$

$$\therefore \frac{GM_E}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}} \Rightarrow v_0 = \sqrt{\frac{GM_E}{r}}$$

v orbital speed

T = period $\Rightarrow$ $2\pi r \Rightarrow$ distance

$$v_{orbit} = \sqrt{\frac{GM_E}{r}}$$

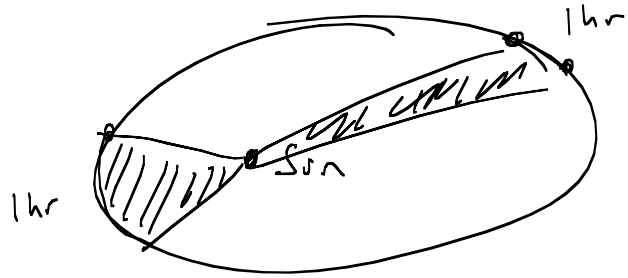
$$\Delta t = \frac{\Delta x}{v} \Rightarrow v = \frac{\Delta x}{\Delta t}$$

$$T = \frac{2\pi r}{\sqrt{\frac{GM_E}{r}}} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

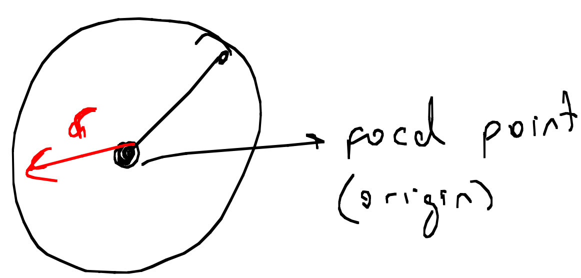
Kepler's Law

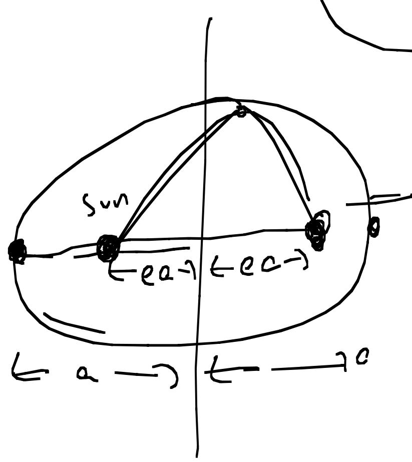
i) The planets move on an elliptic path.

ii) All planets move on a path such that the total area covered by the planet is the same per the same time.

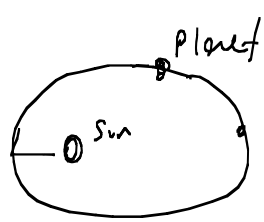


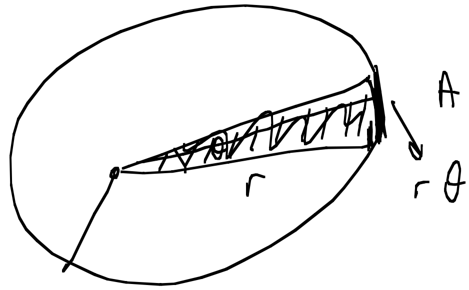
iii) $T \propto \frac{2\pi a^{3/2}}{\sqrt{Gm}}$



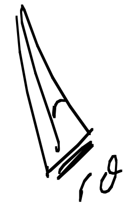


two focal points





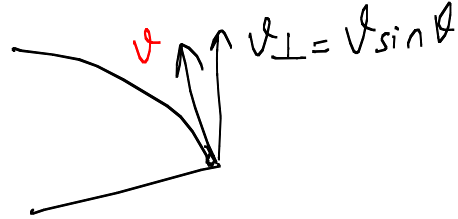
$A = \frac{1}{2} r r \theta$



small $\Rightarrow d\theta$

$$dA = \frac{1}{2} r^2 \, d\theta$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \underline{\text{const}}$$



$$r \, r \frac{d\theta}{dt}$$

$$v_{\perp} = r\omega$$

$$v_{\perp} = r \frac{d\theta}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2} r v_{\perp} = \frac{1}{2} r v \sin\theta = \text{const}$$

multiply by m $\quad m\frac{dA}{dt} = \frac{1}{2} r \underbrace{m v \sin\theta}_{\vec{p}}$

$$m\frac{dA}{dt} = \vec{r} \times \vec{p} = \frac{L}{2} = \text{const}$$

$$\frac{dL}{dt} = 0 = \text{Torque} \Rightarrow \underline{\vec{r} \times \vec{F} = 0}$$

$$\frac{d\vec{p}}{dt} = \text{Force} \quad ; \quad \frac{d\vec{L}}{dt} = \text{Torque}$$