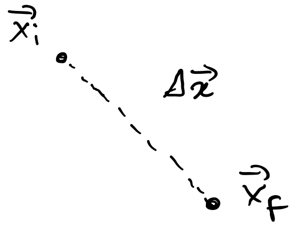
Motion

$x_0, \to x_f$

Displacement $\Delta \vec{x} = \vec{x}_f - \vec{x}_i$

Difference



$\Delta \to$ Diffrene

$\delta \to$ diffence

Greek | Latin

Variational dif.

$\frac{\delta f}{\delta x} \to \frac{df}{dx}$

Average velocity:

$\boxed{V_{av} = \frac{\Delta \vec{x}}{\Delta t}}$

Avg. of any fnc ⇒ f over variable x on an interval $0 \to L$

$$\boxed{\langle f(x)\rangle = \frac{1}{L}\int_0^L f(x)\,dx}$$

$$\langle f(t)\rangle = \frac{1}{T}\int_0^T f(t)\,dt$$

0, 1, 2, 3, 4

$$\frac{1}{5}\sum_{i=1}^{5} x_i$$

$$\frac{0+1+2+3+4}{5} \Rightarrow$$

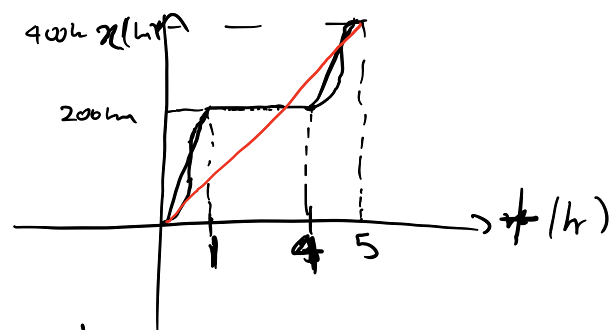
~~Instantaneous~~ Velocity:

$$\boxed{\vec{v}(t) = \frac{d\vec{x}}{dt}} = \lim_{\Delta t \to 0}\frac{\Delta \vec{x}}{\Delta t}$$

Avg of $\vec{v}(t)$ ⇒ $\frac{1}{\Delta t}\int_0^{\Delta t} \vec{v}(t)\,dt \to \frac{\Delta x}{\Delta t}$

$$\vec{v}(t) = \frac{d\vec{x}}{dt} \Rightarrow \overbrace{\int d\vec{x}}^{\Delta\vec{x}} = \int_0^{\Delta t} \vec{v}(t)\,dt = \Delta \vec{x}$$

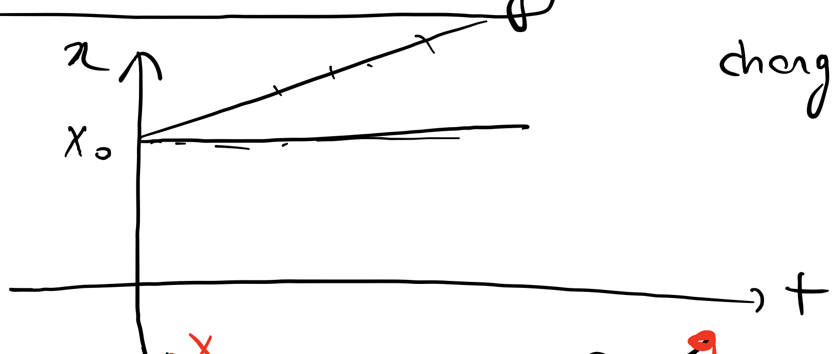
$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{400\text{ km}}{5\text{ hr}} = 80\ \frac{\text{km}}{\text{h}}$$



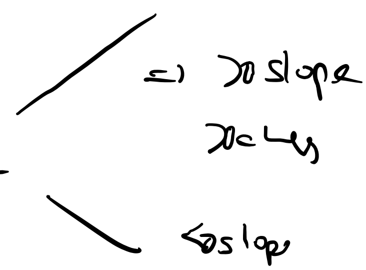
Newton => Calculus! $\lim \Delta t \to 0 \Rightarrow dt$

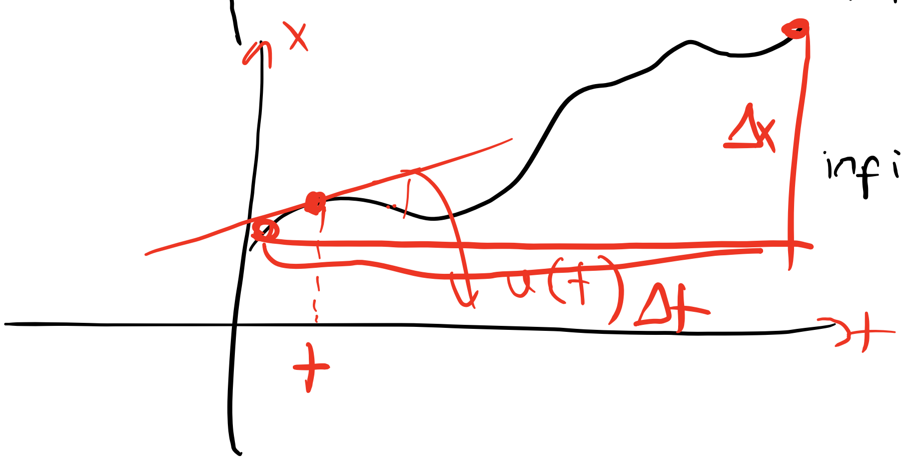
infinitesimally small time intervals (dt)

$\boxed{\text{Avg} \to \text{overall change} \quad \text{inst} \Rightarrow \text{sudden change}}$ of position



change ↔ slope





infin. change

sudden change

Change of velocity $\Rightarrow$ acceleration

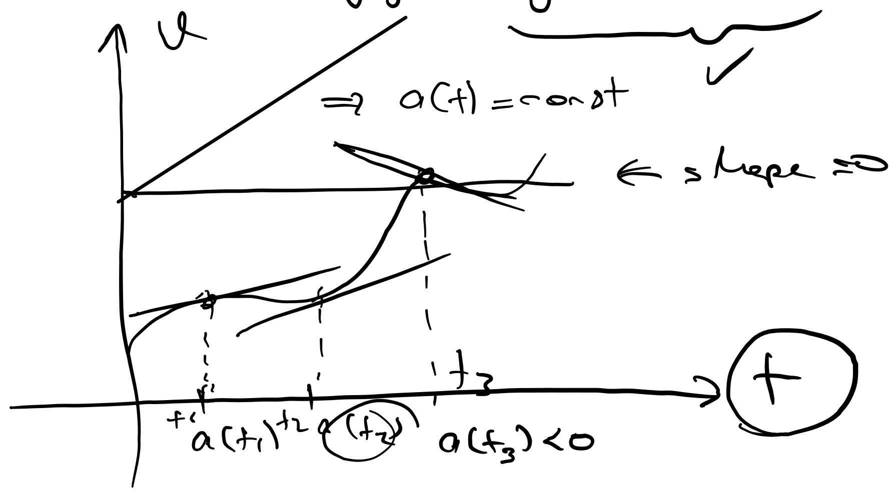
Avg $\rightarrow$ overall change of velocity

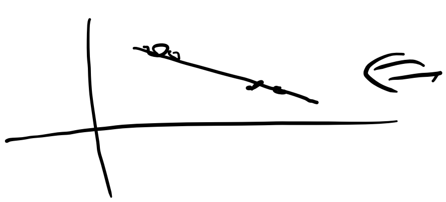
Inst $\rightarrow$ sudden change

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$a_{av}(t) = \frac{1}{\Delta t} \int_0^{\Delta t} a(t)\, dt = \frac{\Delta v}{\Delta t}$$

$$a(t) = \frac{dv}{dt} \Rightarrow \Delta v = \int_{v_0}^{v_f} dv = \int_0^{\Delta t} a(t)\, dt$$





$$v(t) = \frac{dx}{dt} \quad \Rightarrow \quad \int_0^{x(t)} dx = \int v(t)\, dt \quad (\cancel{A})$$

assume $\boxed{a_0 = \text{const}}$

$$a(t) = \frac{dv}{dt} \rightarrow \underbrace{\int_{v_0}^{v(t)} dv}_{\Delta v} = \int a(t)\, dt$$

$$v(t) - v_0 \qquad a_0 \underbrace{\int_0^t dt}_{t} = a_0 t$$

$$\boxed{v(t) = v_0 + a_0 t}$$

$$v = v_0 + at$$

$$\int_{x_0}^{x(t)} dx = x \Big|_{x_0}^{x(t)} = x(t) - x_0 = \int_0^t \underline{v(t)}\, dt$$

$$x(t) - x_0 = \int_0^t [\underset{①}{v_0} + \underset{②}{a_0 t}]\, dt \qquad (\cancel{A})$$

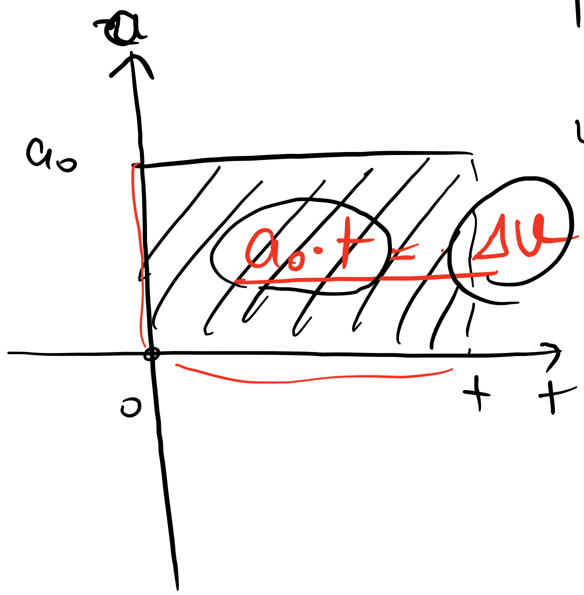
① $\int_0^t \underline{v_0}\, dt = v_0 \int_0^t dt = \underline{\underline{v_0 t}}$

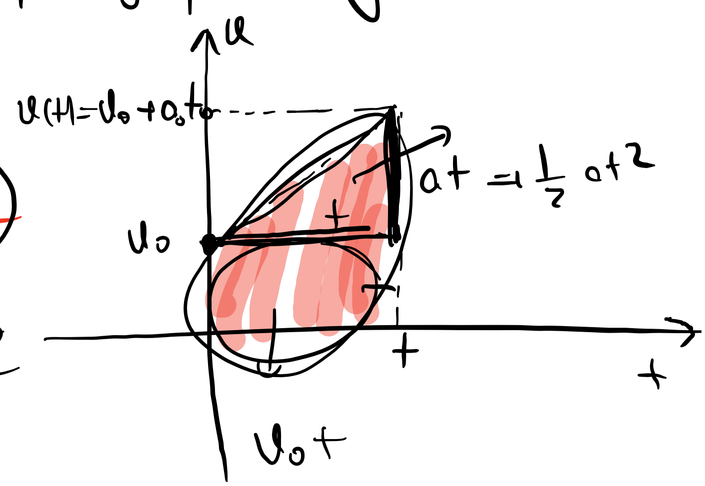
② $\int_0^t (a_0 t)\, dt = a_0 \underbrace{\int_0^t t\, dt}_{t^2/2} = \underline{\frac{1}{2} a_0 t^2}$

(A) $\Rightarrow$ $x(t) - x_0 = v_0 t + \frac{1}{2} a_0 t^2$

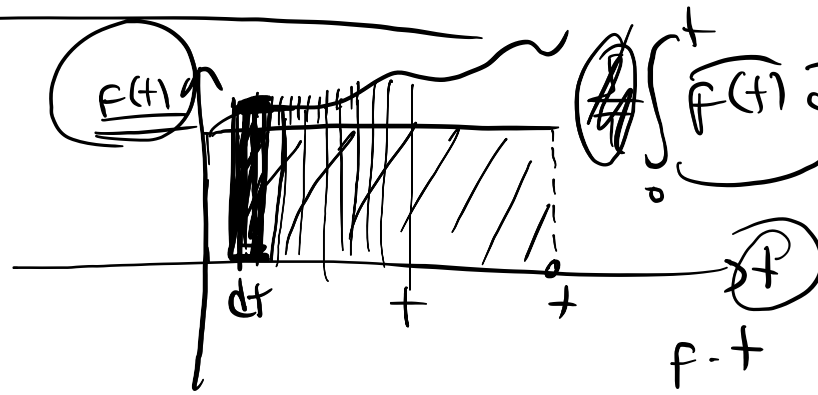
$\therefore$ $x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$ HSE

Prove this eqn. graphically!





change $\Rightarrow$ derrivetive



$\int_0^t f(t)\, dt = Q$

$f \cdot t$

$$F \int dt = F \cdot t = Q \qquad \underline{a} \cdot t = \underline{v}$$

Integral $\quad$ $\underline{\text{Sum}}$

* Slope ⇒ change ⇒ derivative
* Area ⇒ Integral

$$v(t) = \frac{dx}{dt} \Rightarrow \underline{\text{Slope of } x\text{-}t \Rightarrow v(t)}$$

$$\int v(t)\,dt = \int dx \Rightarrow \text{Integral [Area] of } v\text{-}t \Rightarrow \Delta x$$

$$a(t) = \frac{dv}{dt} \Rightarrow \underline{\text{Slope of } v\text{-}t \Rightarrow a(t)}$$

$$\int a(t)\,dt = \int dv \Rightarrow \Delta v \Rightarrow \text{Integ. [Area] of } a\text{-}t \Rightarrow \Delta v$$

Const. acc. 1-D motion :

Ex: Free fall motion ⇒ $g \cong 9.8\ m/s^2$

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = v_0 + a t$$

$x_0 = 0$

$\downarrow$ (+)

$g = 9.8\ m/s^2 \sim 10$

125m

$x(t)$

$$\boxed{\begin{matrix} x_0 = 0 \\ v_0 = 0 \end{matrix}}$$

$$x(t) = \cancel{0} + \cancel{0 \cdot t} + \frac{1}{2} g t^2$$

$\frac{1}{2} g = 5$

$$125 = 5 t^2$$

$$t = 5\ s.$$

$5t^2$

$5 \cdot 1^2 = 5$

$5 \cdot 2^2 = 20$

$5 \cdot 3^2 = 45$

80

125

$v=0$

$t = 1s$ $v=10$ 5

15

$t=2$ $v=20$

25

$t=3$ $v=30$

35

$t=4$ $v=40$

45

$t=5$ $v=50$

$$v(t) = \cancel{v_0} + a_0 t$$

$v_0 = 0$

$$v(t) = 10 \cdot t$$

$$[v(t)]^2 = (v_0 + a_0 t)^2$$

$$x(t) = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + \underline{2 v_0 \cdot a_0 t + a_0^2 t^2}$$

$$\boxed{v^2 = v_0^2 + 2a\Delta x}$$

$$2a_0 \underbrace{\left[v_0 t + \frac{1}{2} a t^2\right]}_{\Delta x}$$

$$(40)^2 = (30)^2 + 2\cancel{10}\,\Delta x$$

$$\frac{1600 - 900}{20} = \frac{70\cancel{0}}{2\cancel{0}} = 35$$

Time-dependent acceleration

a_1 $\qquad$ a_2

$$a(t) = \frac{dv}{dt} \Rightarrow \quad \int a(t)\,dt = \underbrace{\int dv}_{\Delta v}$$

$$dt \; v(t) = \frac{dx}{\cancel{dt}} \cdot \cancel{dt} \Rightarrow \qquad \Delta v = v(t) - v_0 = \int a(t)\,dt$$

$$a(t) = t - 3 \Rightarrow \qquad \Delta v = v(t) - v_0 = \int_0^t (t-3)\,dt$$

$$v(t) = v_0 + \frac{t^2}{2} - 3t$$

$$\int_{x_0}^{x(t)} dx = \int_0^t v(t)\,dt \Rightarrow \quad x(t) = x_0 + \int_0^t v(t)\,dt$$

$$\boxed{x(t) = x_0 + v_0 t + \frac{t^3}{6} - \frac{3t^2}{2}}$$

$$\int \cancel{e^{t^2}} \sin t \, dt$$

Chp-2: Motn. in 2D-3D

$$\vec{a}(t) = \frac{d\vec{v}}{dt} \qquad \vec{v}(t) = \frac{d\vec{x}}{dt} \rightarrow \boxed{\frac{d\vec{r}}{dt}}$$

$$\Delta\vec{v} = \int d\vec{v} = \int \vec{a}\, dt \qquad \Delta\vec{x} = \int_{x_0}^{\vec{x}_0} d\vec{x} = \int \vec{v}(t)\, dt$$

$$\frac{d\vec{r}}{dt} = \vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$\frac{d}{dt}\left[x\hat{i} + y\hat{j} + z\hat{k}\right] = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$v_i = \frac{di}{dt} \qquad i \Rightarrow x, y, z \qquad v_z = \frac{dz}{dt}$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

$$\frac{d\vec{v}}{dt} = \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\frac{d}{dt}\left[\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}\right] = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

Speed vs Velocity

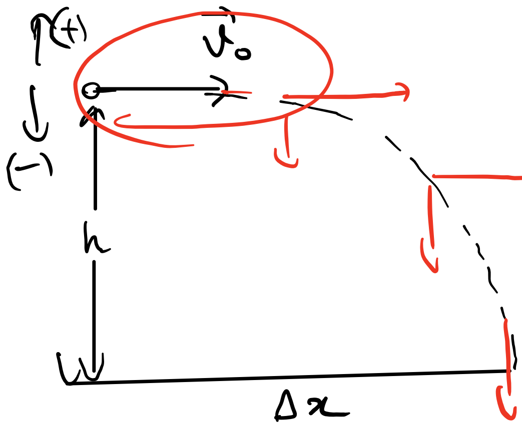
$$|\vec{v}| = \text{Speed} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

3 5 7

$$1\text{-D} \Rightarrow |\vec{v}| = |v| = v$$

Projectile Motion :

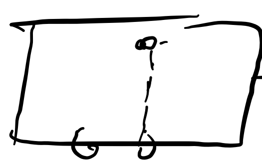
$\vec{a} = 0\hat{i} - g\hat{j}$

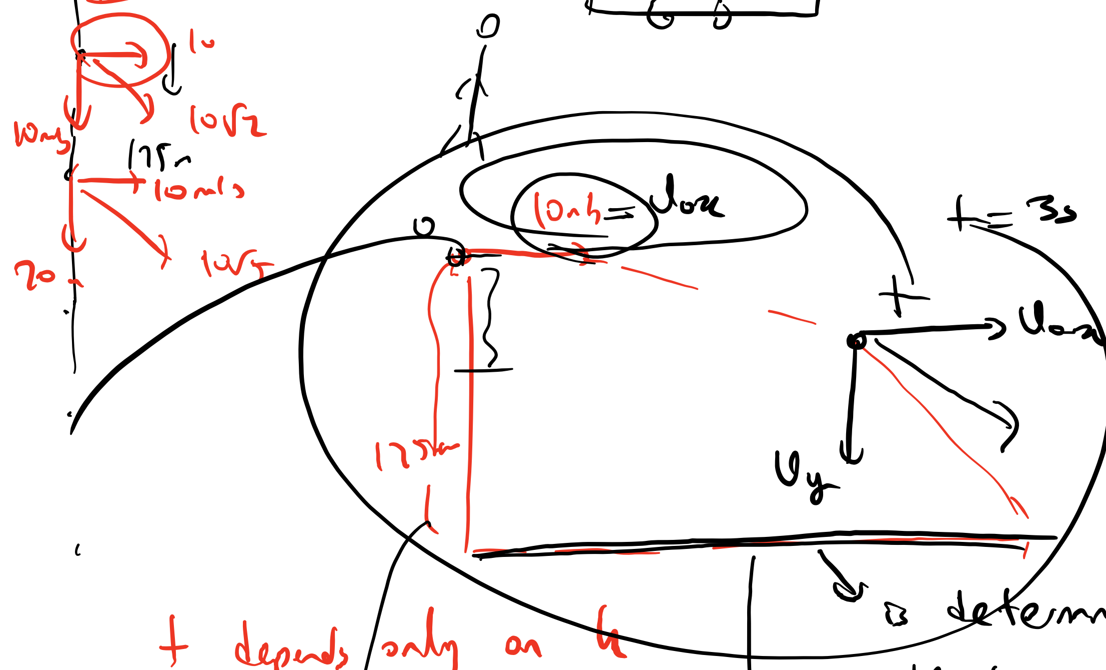


$\vec{v} = v_{0x}\hat{i} + \left[\cancel{v_{0y}} + \int_0^t a_y\,dt\right]\hat{j}$

$$\boxed{\therefore\ \vec{v} = v_{0x}\hat{i} - (gt)\hat{j}}$$

free-fall





t depends only on h

→ determined by both h and v_{0x}

$h = \frac{1}{2} g t^2$

$t^2 = \frac{2h}{g}$

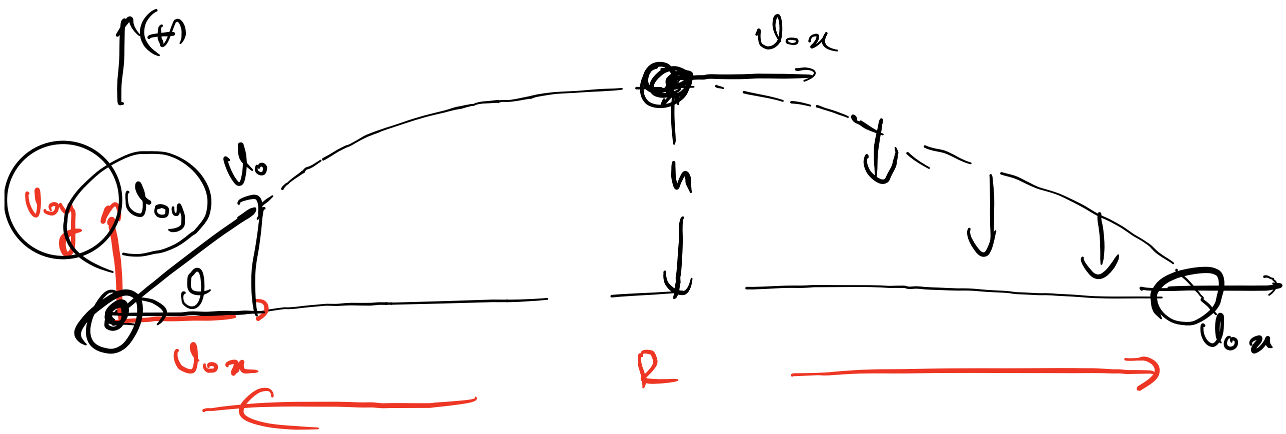
$\Delta x = v_{0x} \cdot t = v_{0x} \cdot \sqrt{\frac{2h}{g}}$

$|v_y| = gt \qquad \vec{v} = (v_{0x}\hat{i}) - gt\,\hat{j}$

$$|\vec{v}| = \sqrt{v_{0x}^2 + g^2t^2}$$

$$\sqrt{(10)^2 + 10^2 \cdot 3^2} = \sqrt{100 + 900}$$

$$\boxed{v(t=3) = 10\sqrt{10}\ m/s}$$



$$\boxed{\begin{aligned} v_{0x} &= v_0 \cdot \cos\theta \\ v_{0y} &= v_0 \sin\theta \end{aligned}}$$

$v_{fy} = v_{0y} - gt \quad -(1)$

$v_{fx} = v_{0x}$

$$\boxed{\vec{v} = v_{0x}\,\hat{i} + (v_{0y} - gt)\,\hat{j}}$$

$$\vec{x} = (v_{0x} \cdot t)\,\hat{i} + [v_{0y} \cdot t - \tfrac{1}{2}gt^2]\,\hat{j}$$

$$\boxed{\vec{a} = -g\,\hat{j}}$$

Top $\quad v_{fy} = 0$

$h \overset{?}{=}$

$$(1) \Rightarrow \quad 0 = v_{0y} - gt \Rightarrow t = \frac{v_{0y}}{g} = \frac{v_0 \sin\theta}{g}$$

$$t_{flight} = 2t = \boxed{\frac{2 v_0 \sin\theta}{g}}$$

$$h = \frac{1}{2} g t^2$$

$$v_{0y}\, t - \frac{1}{2} g t^2 = (gt)\cdot t - \frac{1}{2} g t^2 =$$

$$g t^2 - \frac{1}{2} g t^2 = \frac{1}{2} g t^2$$

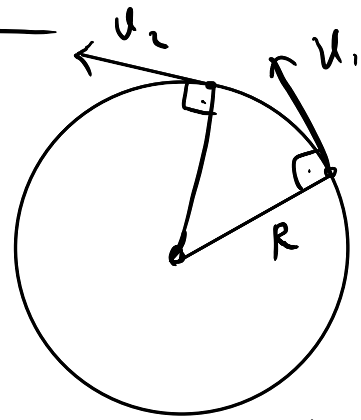
t^2

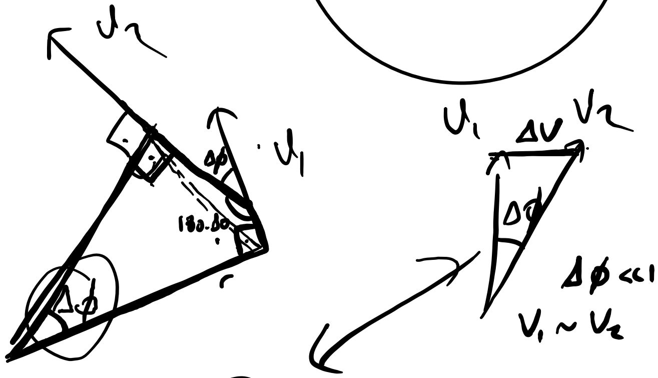
$$h = \frac{1}{2} g \cdot \frac{v_{0y}^2}{g^2} = \frac{v_0^2}{2g}$$

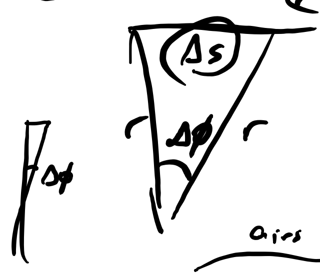
$$Range = v_{0x} \cdot t_{flight} = v_0 \cos\theta \cdot \frac{2 v_0 \sin\theta}{g}$$

$$\frac{v_0^2\, 2\sin\theta\cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g} = \text{Range}$$

Circuler Motn :







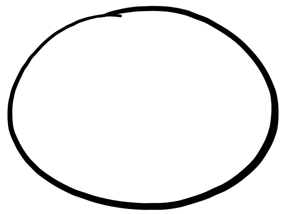
$$\frac{\Delta v}{v_1} = \frac{\Delta s}{r}$$

$$\Delta v = v_1 \frac{\Delta s}{r}$$

$$a_v = \frac{\Delta v}{\Delta t} \longrightarrow \overline{\lim_{\Delta t \to 0}}^{\,a_{rad}} \frac{\Delta v}{\Delta t} = \frac{v_1 \Delta s}{\Delta t\, r} \quad v_1$$

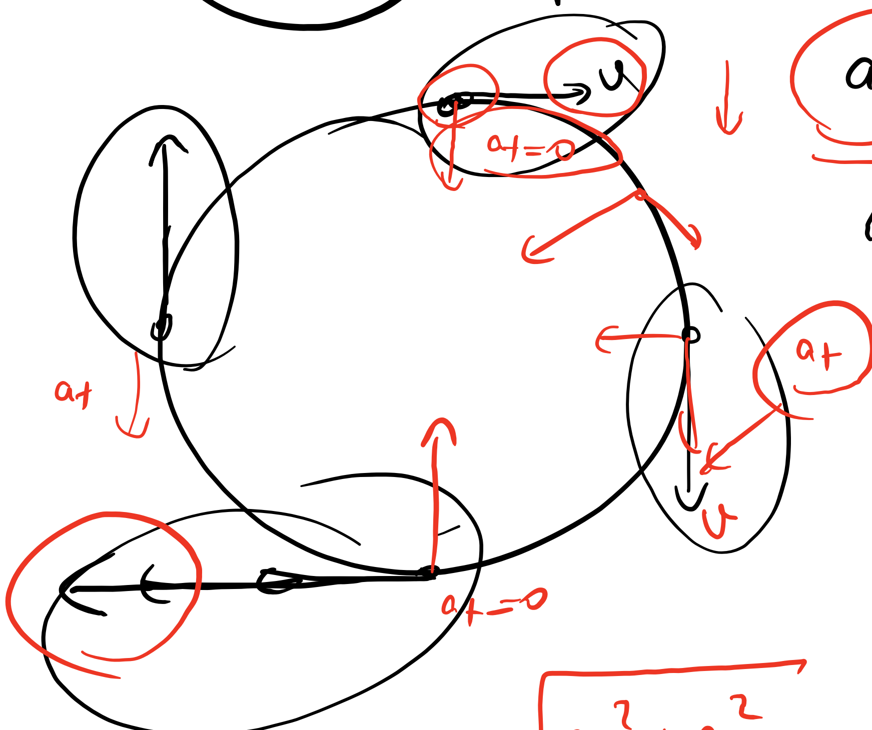
$$a_{rad} = \frac{v_1 \cdot v_1}{r} = \frac{v_1^2}{r} = \frac{v^2}{r}$$

$$a_r = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r^2}{r \cdot T^2} = \frac{4\pi^2 r}{T^2}$$



$$v \cdot T = 2\pi r$$

period



$$a_t = \frac{dv}{dt}$$

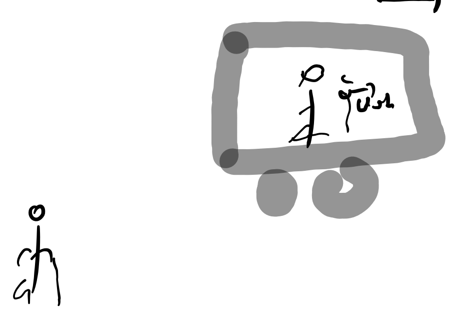
$\downarrow g$

$$a_r = v^2/r$$

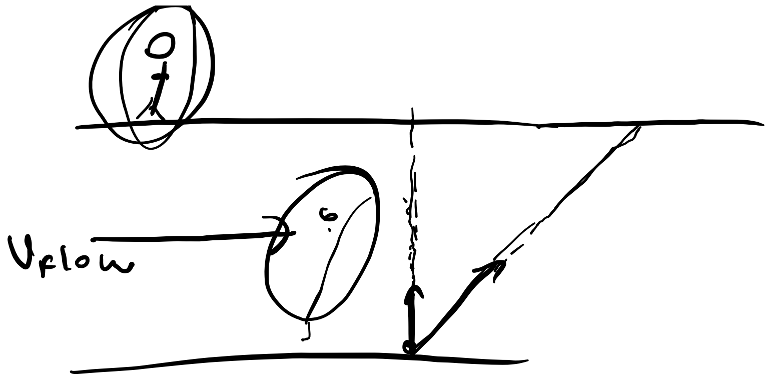
$$a_{total} = \sqrt{a_r^2 + a_t^2}$$

Relative Motion:

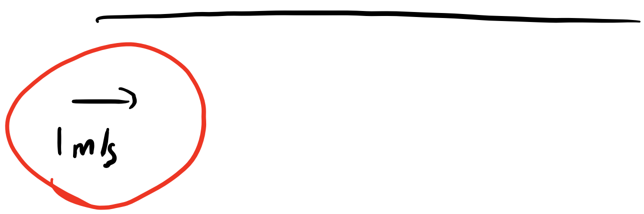
$\rightarrow v_{car} = const$



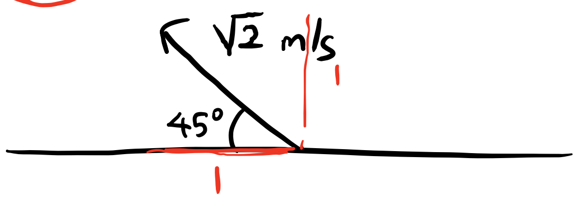
$$\vec{v}_{obs} = \vec{v}_{car} + \vec{v}_{obj}$$



$$\vec{v}_{obsr} = \vec{v}_{flow} + \vec{v}_{obj}$$



$\vec{v}_{obs} = ?$



$v_{obs} = 1\hat{j}$

$v_{obj} = \cancel{-1\hat{i}} + 1\hat{j}$

$+\, v_{flow} = \cancel{1\hat{i}}$