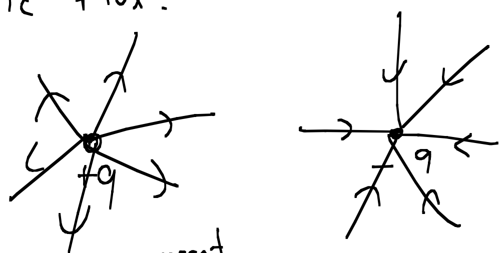
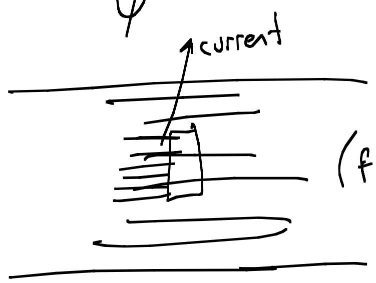
GAUSS' LAW

Note Title 10/4/2012

* Electric flux :

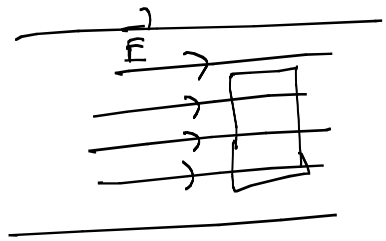


flux : flow
(latin) ↗



(flux) of water flow ↗

river



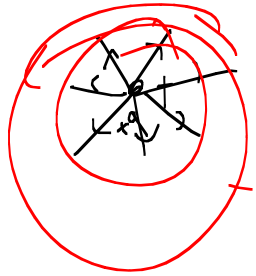
flux : Φ (phi)

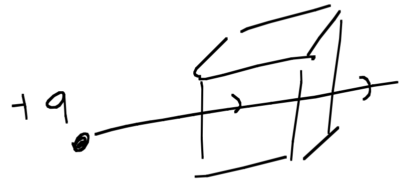
- E-field vectors passing out of the surface
  outward flux
- $\vec{E}$-field " " into the "
  inward flux

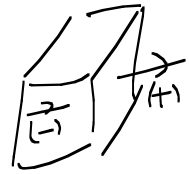
flow - flux analogy is misleading.

Gauss' Law

- Inward flux on a rectangular box = − charge
- Outward flux on a " " = + charge
- flux is indep. of the size of the box
- Charges outside the box have no effect



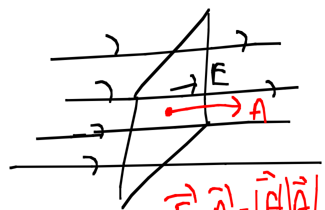




flux

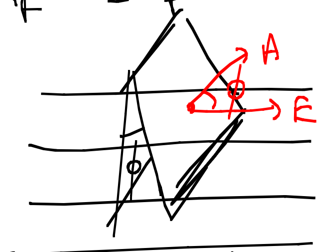
elc. field

$\Phi_E = \vec{E} \cdot \vec{A}$ area if $\vec{E}$ field is const.

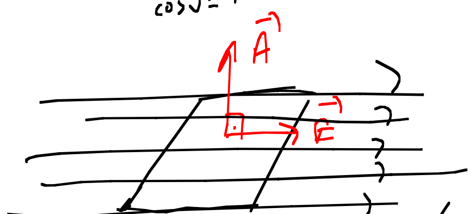


$\vec{E} \cdot \vec{A} = |\vec{E}||\vec{A}|$

$\cos\theta = 1$

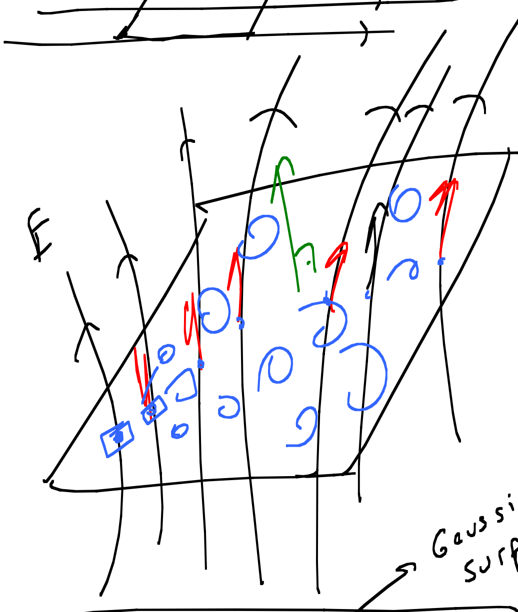


$\vec{E} \cdot \vec{A} = |\vec{E}||\vec{A}| \cos\phi$



$\Rightarrow \vec{E} \cdot \vec{A} = \Phi = 0$

$\cos 90°$ ensure



$\Phi = \vec{E} \cdot \vec{A}$

$\vec{A} = A\hat{n}$ → normal unit vector

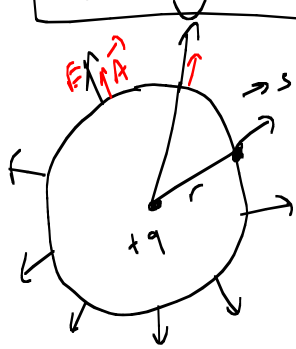
$\vec{A} \cdot \int d\vec{E}$ X

$$\boxed{\Phi = \int \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}}$$

Gaussian surface

Gauss' Law

$$\phi = \int \vec{E} \cdot d\vec{A} = \int |\vec{E}| dA$$



→ sphere

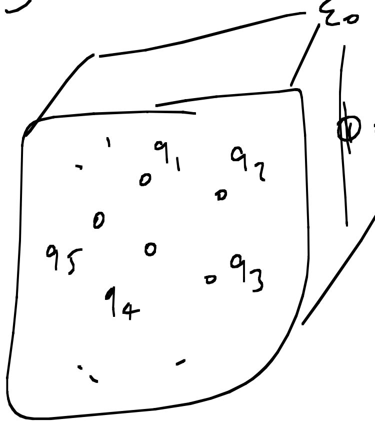
Coulomb's Law

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \Rightarrow |\vec{E}| \underbrace{\int dA}_{A} = \Phi$$

$$\Phi = \frac{1}{\cancel{4\pi}\varepsilon_0} \frac{q}{\cancel{r^2}} * \cancel{4\pi r^2} = \frac{q}{\varepsilon_0}$$

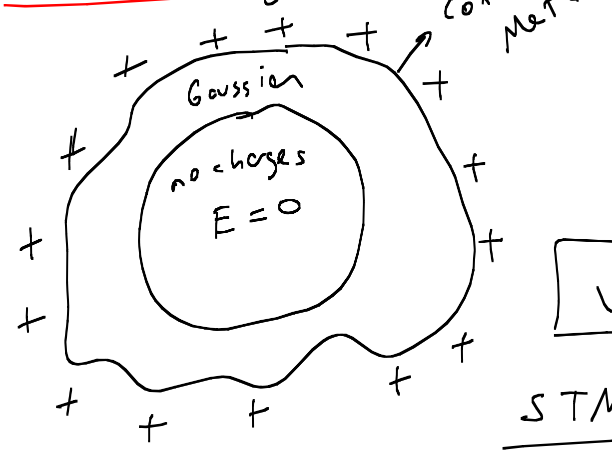
General Eqn

$$\Phi = \int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$ → enclosed



$$\Phi = \int \vec{E}\cdot d\vec{A} = \frac{Q_{En}}{\varepsilon_0} = \frac{q_1+q_2+q_3+\dots}{\varepsilon_0}$$

Conductivity



$$\Phi = \int \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

2×10^6 \$

vacuum room

clean room

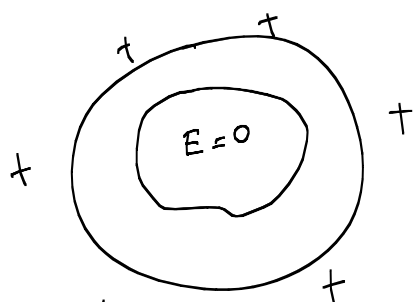
STM ⇒ transistors

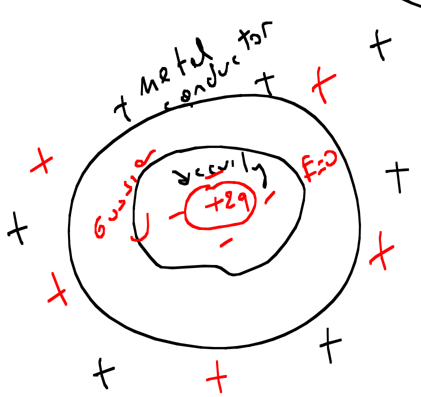
10^{-9} m ≈ nm

* Electrically neutral

conductivity idea

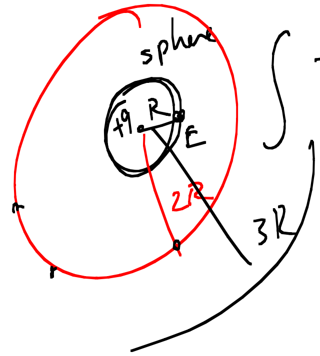
Cavity:





$\phi = \int E \cdot dA$ should be $= 0 = \frac{q}{\varepsilon_0}$

$2q - 2q = 0$ $\quad = \frac{q}{\varepsilon_0}$



$$\int \vec{E}\cdot d\vec{A} = E\int dA = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi R^2 = \frac{q}{\varepsilon_0}$$

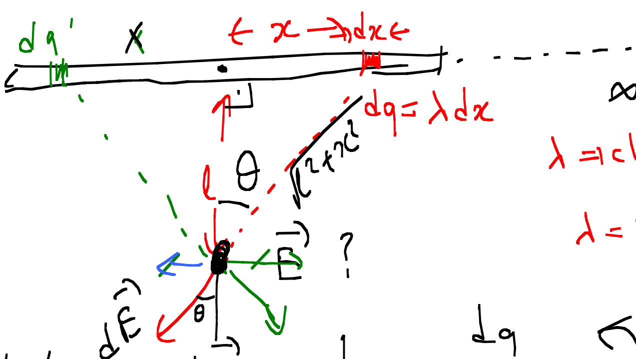
$$E_R = \frac{q}{4\pi\epsilon_0 R^2}$$

$$\Phi = \frac{q}{\varepsilon_0} = E\underbrace{\int dA}_{4\pi(2R)^2} \Rightarrow \frac{q}{\varepsilon_0} = 4\pi \cdot 4R^2 E$$

$$E_{2R} = \frac{1}{4\pi\epsilon_0}\frac{q}{4R^2}$$

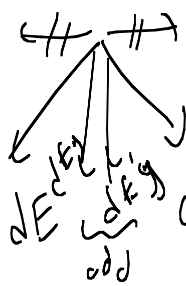
$$E_{3R} = \frac{1}{4\pi\epsilon_0}\frac{q}{9R^2}$$

Infin. charged line with (+)



λ = charge density

$$\lambda = \frac{Q}{L}$$



$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{l^2+x^2}$$ university

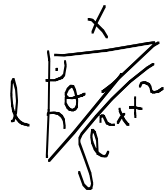
$$E = k\frac{q}{r^2}$$ high school

dEx components will cancel each other because of the symmetry of the system.

∴ We will just take y-comp. of each $d\vec{E}$ and add them up.

$$\int dE_y = \int dE\cos\theta$$

$$|d\vec{E}| = \frac{dq}{4\pi\epsilon_0}\frac{1}{l^2+x^2} \quad ; \quad dE_y = \frac{dq}{4\pi\epsilon_0(l^2+x^2)} * \cos\theta$$



$$\Rightarrow \cos\theta = \frac{l}{\sqrt{x^2+l^2}} \nearrow \text{const}$$

$$E_y = \int_{-\infty}^{\infty} dE_y = \int_{-\infty}^{\infty} \frac{\overbrace{dq}^{\lambda\, dx}}{4\pi\epsilon_0}\frac{1}{(l^2+x^2)}\frac{l}{(l^2+x^2)^{1/2}}$$

$-\infty$ same

$$E_y = \frac{\lambda l}{4\pi\epsilon_0} \underbrace{2 \cdot \int_0^{\infty}}_{\int_{-\infty}^{\infty}} \frac{dx}{(l^2+x^2)^{3/2}} = \frac{\lambda l}{2\pi\epsilon_0}\int_0^{\infty}\frac{dx}{l^3\left(1+\frac{x^2}{l^2}\right)^{3/2}}$$

$$E_y = \frac{\lambda l}{2\pi\epsilon_0 l^{\cancel{3}2}}\int_0^{\infty}\frac{dx}{\left(1+\frac{x^2}{l^2}\right)^{3/2}} \quad ; \quad \frac{x}{l} = \tan\theta$$

$$dx = l\, d(\tan\theta)$$

$$dx = \frac{l\, d\theta}{\cos^2\theta}$$

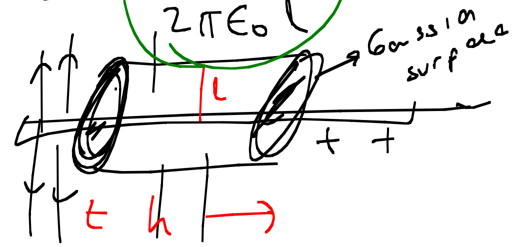
$$1 + \frac{x^2}{l^2} = 1 + \tan^2\theta = \frac{1}{\cos^2\theta}$$

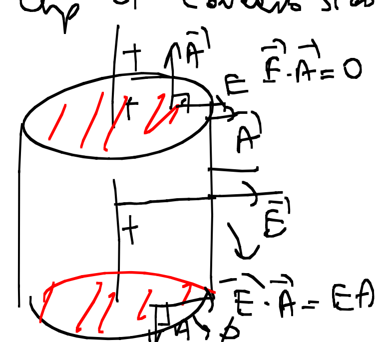
$$\left(1+\frac{x^2}{l^2}\right)^{3/2} = \frac{1}{\cos^3\theta}$$

$$\int \frac{l\, d\theta}{\cancel{\cos^2\theta}} * \cos^{\cancel{3}}\theta$$

$$l\int d\theta \cos\theta = l \sin\theta \Big|_0^{\pi/2} = l\left(\sin\frac{\pi}{2} - \cancel{\sin 0}\right) = l$$

$$\therefore E = \frac{\lambda}{2\pi\epsilon_0 l} \Rightarrow \text{Using Chp 21 coulomb's law}$$





$E \cdot A = \frac{q = \lambda \cancel{h}}{\varepsilon_0}$ h

$2\pi l \cdot \cancel{h}$

$2\pi l$

$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 l}}$$

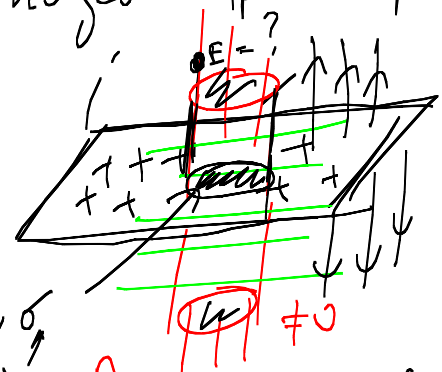
$\oint E \cdot dA = \frac{q}{\varepsilon_0}$

$E \int dA$

A

$$\boxed{\begin{aligned} Q &= \lambda \cdot l \Rightarrow 1d \\ Q &= \sigma * A \Rightarrow 2d \\ Q &= \rho \cdot V \Rightarrow 3d \end{aligned}}$$

Posit. Charged Inf. Surface

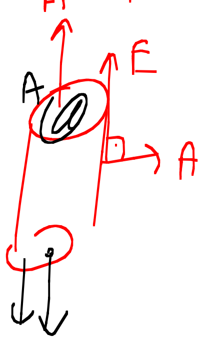


$\Phi = \int \vec{E} \cdot d\vec{A}$

$\int E \cdot dA_{lower}$

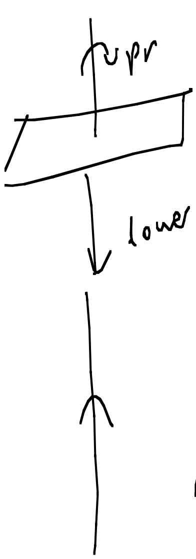
$+ \int E \cdot d\vec{A}_{upper}$

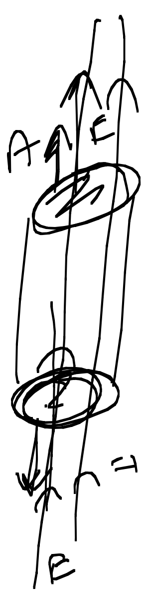
$+ \cancel{\int E \cdot d\vec{A}_{side}}$ 0



$\Phi = \frac{Q_{enc}}{\varepsilon_0} = E \int dA_{lower}$

$+ E \int dA_{upper}$





$\vec{E} \cdot \vec{A} = EA$

$\cos 0$

$A \quad n \Rightarrow \vec{E} \cdot \vec{A} = (-) EA$

$\cos 180^\circ$

$E \cdot A + E \cdot A$

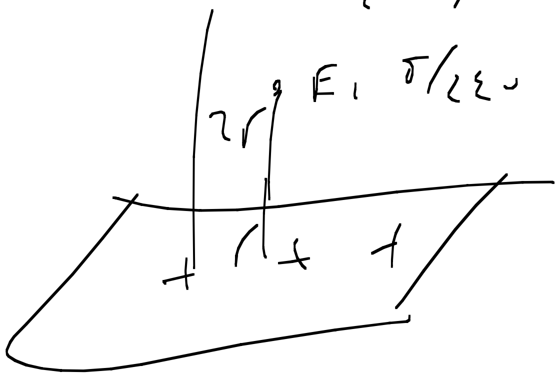
$2 EA = \Phi = \frac{Q_{en}}{\varepsilon_0}$

$\sigma \Rightarrow$ surface charge density

$\frac{Q}{A}$

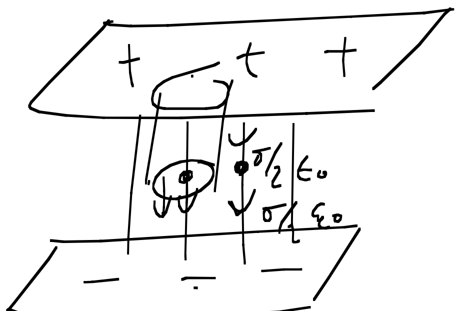
$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$

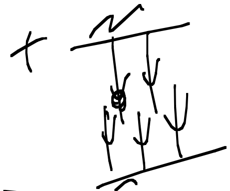
$$\boxed{E = \frac{\sigma}{2 \varepsilon_0}}$$



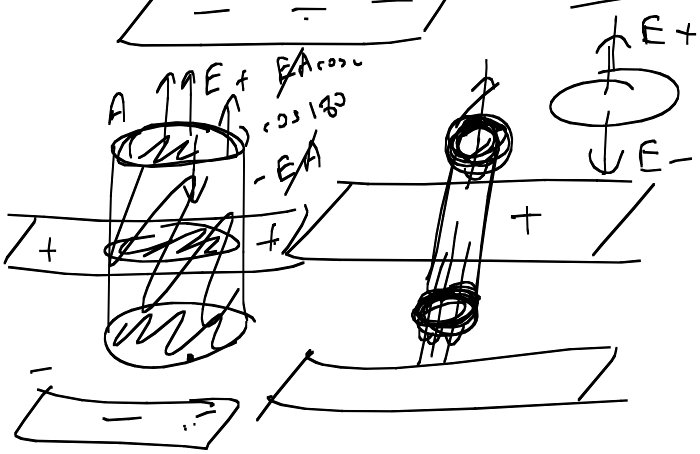
$F \sim \frac{1}{r^2} \quad \frac{1}{r^2}$

$E \quad \frac{1}{(2r)^2} \quad \frac{1}{4r^2}$





$$\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

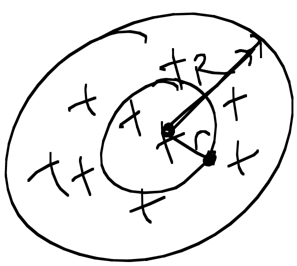


$$E \cdot \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

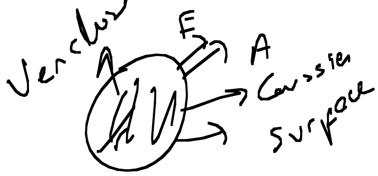
Charged solid sphere

volume charge density

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{Q}{V}$$



$$E_r = ? \qquad \int E\,dA = \int E \cdot dA = \frac{q_{enclosed}}{\varepsilon_0}$$



$$E \underbrace{\int dA}_{4\pi r^2} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\rho \underbrace{V_{enclosed}} = Q_{enclosed}$$

$$Q_{enc} = \rho \frac{4}{3}\pi r^3 = \frac{Q}{\cancel{\frac{4}{3}\pi} R^3} \times \cancel{\frac{4}{3}\pi} r^3 = Q\frac{r^3}{R^3}$$

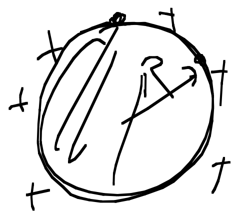
$$E \cdot \underbrace{\int dA} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E\, 4\pi \cancel{r^2} = \frac{Q}{\varepsilon_0} \frac{r^{\cancel{3}}}{R^3}$$

$r < R$

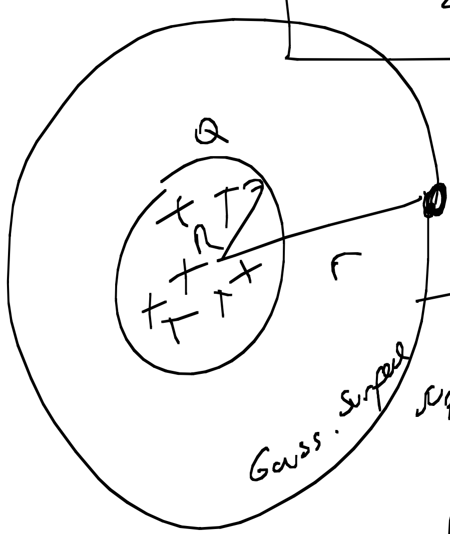
$$\boxed{E_r = \frac{Q\, r}{4\pi\varepsilon_0 R^3}} \Rightarrow r \to R$$

$E_{r=R} =$



$$E \overbrace{\int dA}^{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} = E \cdot 4\pi R^2$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 R^2}}$$



$r > R$

$$E \cdot \int dA = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

surface area $\rightarrow 4\pi r^2$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}} \Rightarrow$$