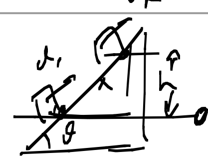
Chp 23 : Electric Potential

$\vec{F} = m\vec{a} \Rightarrow$ Work – En. $E_i = E_f$



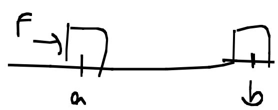
1) $v_f^2 = v_0^2 + 2a\Delta x$ $\quad mg\sin\theta = f = ma$

$f = 1 \quad a = g\sin\theta$

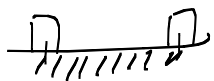
2) $\frac{1}{2}mv_i^2 + \cancel{PE_i}^{\,0} = \frac{1}{2}mv_f^2 + \underbrace{PE_f}_{mgh \to mg\,x\sin\theta}$

Electric Potential Energy:

* $W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$



* Conservative forces ⇒



friction is a non-cons

Gravity is a cons. force.

If F is conservative

⇒ F can be expressed interms of the potential energy.

$W_{ab} = -\Delta U = U_a - U_b$

$a - \downarrow dl \downarrow f \quad \downarrow mg$

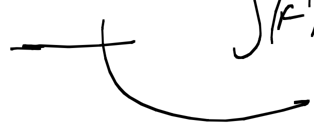
$b -$

$\int \vec{F}\cdot d\vec{l} = W$

$\int |\vec{F}|\,|d\vec{l}|\cos\theta$

$W_{ab} > 0 \quad U_a > U_b$

$W_{a\to b} = U_a - U_b$

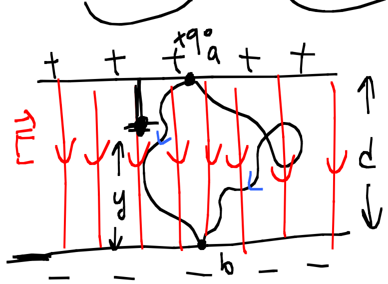


$\cos\theta = 1$

work-energy theorem:

$W = \Delta K \Rightarrow \Delta K = K_b - K_a = U_a - U_b$

$\therefore \underbrace{U_a + K_a}_{} = \underbrace{U_b + K_b}_{} \Rightarrow$ energy cons. eqn.



W is same and indep. of the path.

$W_{a\to b} = U_a - U_b$

If we define $U(b) = 0$

$U(y) = qEy$

$U(b) = 0$

If we define $U(a) = 0$

$U(y) = ?$

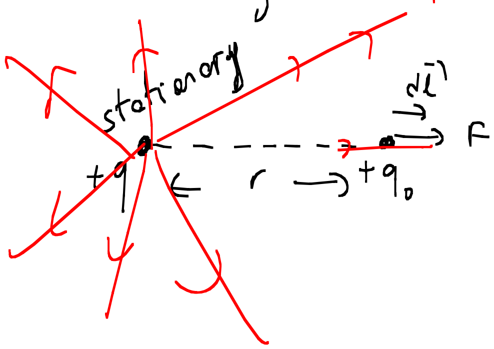
$-qE(d-y)$

$W = q_0 E d$

$= U_a - \cancel{U_b}^{\,0} =$

For two charges:

radial



$$F = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$$

$$W = \int \vec{F}\cdot d\vec{l}$$

$\cos 0 = 1$

$$W_{a\to b} = \int_a^b F\,dr = \frac{q q_0}{4\pi\epsilon_0}\underbrace{\int_a^b \frac{dr}{r^2}}$$

$$W = -\frac{q q_0}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = U_a - U_b \qquad -\frac{1}{r}\Big|_a^b$$

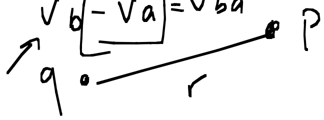
$$= \frac{q q_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = U_a - U_b$$

$$\boxed{U = \frac{q q_0}{4\pi\epsilon_0}\frac{1}{r}} \Rightarrow \text{Electric pot. energy} \Rightarrow \text{Joules}$$

$\vec{F} \to \vec{E} \Rightarrow$ Electric field $\Rightarrow$ force per unit charge

$\vec{E} = \frac{\vec{F}}{q_0}$ $\qquad \vec{F} = q\vec{E}$

$\boxed{V_b - V_a} = V_{ba}$



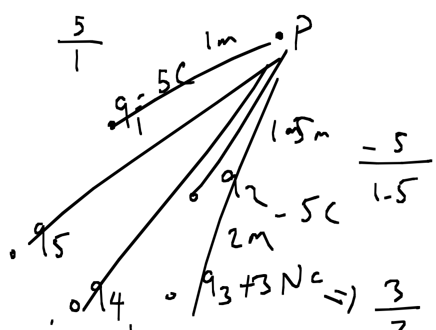
$V_p - \cancel{V_a}^{\,0}$

$U \Rightarrow V$: El. potential energy per unit charge

$$V = \frac{U}{q_0} \Rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \text{ : Electric potential}$$

$$\frac{\text{Joules}}{\text{Coulomb}} = \underline{\text{volt}}$$

Multiple charges



$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}}$$

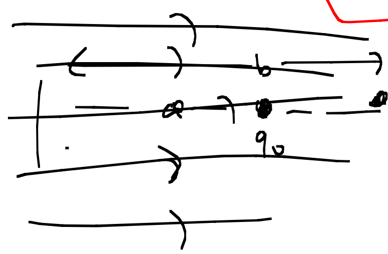
$U = q_0 V$

$$q_0(V_a - V_b) = U_a - U_b = W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$$

$$q_0\int_a^b \vec{E}\cdot d\vec{l}$$

$$\Rightarrow \quad q_0 (V_a - V_b) = q_0 \int_a^b \vec{E}\cdot d\vec{l}$$

$$V_a - V_b = \boxed{V_{ab} = \int_a^b \vec{E}\cdot d\vec{l}} = -\int_b^a \vec{E}\cdot d\vec{l}$$



$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}$$

Electron-volts : eV (unit energy of charge of electron at a pot. of 1V)

$U = qV$

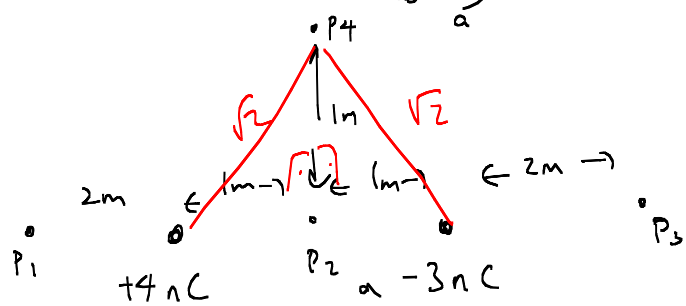
$e = 1.6 \times 10^{-19}$ C $\Rightarrow$ charge of a single electron

$V = 1$ V $\quad \Rightarrow \quad \boxed{1.6 \times 10^{-19} \text{ Joules } (V\,C) = 1\text{eV}}$

$1^{st} \Rightarrow$ Multiple charges $\quad V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$

$2^{nd} \Rightarrow$ Use $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \Rightarrow V = \int dV$

$3^{rd} \Rightarrow$ Use $V_{ab} = \int_a^b \vec{E}\cdot d\vec{l} \Rightarrow$ if we know $\vec{E}$



$V(p_1) = ?$

$V(p_2) = ?$

$V(p_3) = ?$

$V(p_4) = ?$

$$V(p_1) = \underbrace{\frac{1}{4\pi\epsilon_0}}_{9\times10^{9}} \sum \frac{q_i}{r_i} \Rightarrow \left[\frac{4\times10^{-9}}{2} - \frac{3\times10^{-9}}{4}\right]$$

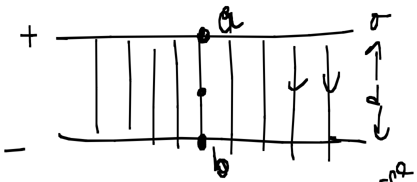
$$V(p_1) = \frac{1}{4\pi\epsilon_0} * 1.25\times10^{-9} \cong 11.25 \text{ V}$$

$$V(p_2) = \frac{1}{4\pi\epsilon_0} * \left(\frac{4\times10^{-9}}{1} - \frac{3\times10^{-9}}{1}\right) \cong 9\text{V}$$

$$V(p_3) = \frac{1}{4\pi\epsilon_0}\left(\frac{4\times10^{-9}}{4} - \frac{3\times10^{-9}}{2}\right) = -4.5\text{ V}$$

$$V(p_4) = \frac{1}{4\pi\epsilon_0}\left(\frac{4\times10^{-9}}{\sqrt{2}} - \frac{3\times10^{-9}}{\sqrt{2}}\right) = 6.36\text{ V}$$

Two conducting surfaces



$E = \frac{\sigma}{\varepsilon_0}$

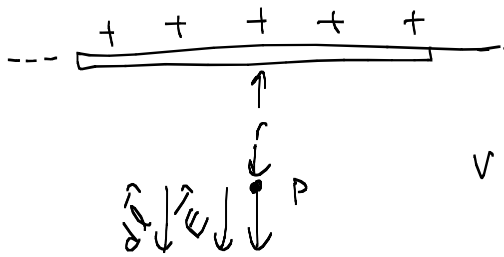
$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = E\int_a^b dl = Ed = V_{ab}$

$\int_a^b dl = d$

$E = \frac{\sigma}{\varepsilon_0} = \frac{V_{ab}}{d}$

surface charge density $\leftarrow$ $\boxed{\sigma = \frac{V_{ab}\,\varepsilon_0}{d}}$

Inf. long pos. charged line



$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$

$V = ?$

$V_a - V_b = V_{ab} = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\,dr$

$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\frac{r_b}{r_a}\right) = V_a - \cancel{V_b}$

$\ln r$

$V(b=\infty) = 0$

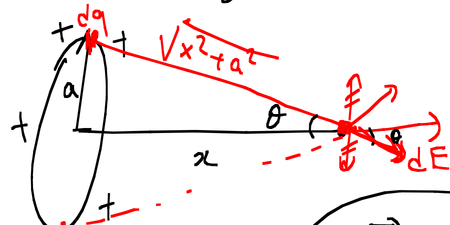
$\log b - \log a \qquad \log\frac{b}{a} \qquad r_b = \infty$

Don't (Can't) choose $r_b \to \infty \Rightarrow V_a \to \infty$

$V(r_0) = 0 \qquad V_a - V_{r_0} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r_a}\right)$

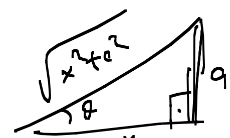
$V(r_0) = 0 \Rightarrow V_{ar_0} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r_a}\right)$

Ring of charge



$\vec{E}(x) = ?$

$V(x) = ?$



$\cos\theta = \frac{x}{\sqrt{x^2+a^2}}$

$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2+a^2}$

$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$

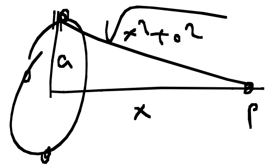
$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2+a^2} * \frac{x}{\sqrt{x^2+a^2}}$

$$E = \int dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}}\left(\int dq\right)Q = \frac{Q}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}}$$

$a \gg x$

$E \to 0$

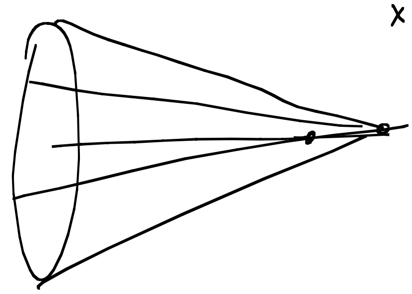
$V(x) =$



$$\Rightarrow dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+a^2}}$$

$$V = \int dV = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+a^2}}\underbrace{\int dq}_{Q} = \frac{Q}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+a^2}}$$

$$x \gg a \Rightarrow \frac{Q}{4\pi\epsilon_0}\frac{1}{x}$$



$$\frac{Q}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}}$$

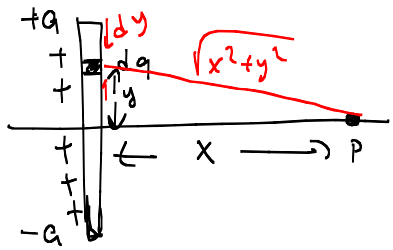
$$V = \int E\cdot dl$$

Harder to do!

<u>A rod of a finite size:</u>

$$V = \int \vec{E}\cdot d\vec{l}$$

Both ways are difficult



$$V = \int dV =$$

$\Downarrow$

λ

$\downarrow$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} \qquad dq = \frac{Q}{2a}*dy$$

$$\int dV = \cancel{\frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+y^2}}}\ \frac{\int dq}{Q}$$

$$\int dV = \frac{1}{4\pi\epsilon_0}\int_{-a}^{a}\frac{Q/2a\ dy}{\sqrt{x^2+y^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2+y^2}} = \ln\left(\sqrt{x^2+y^2}+y\right)$$

$$\frac{\partial}{\partial y}(x^2+y^2)^{1/2} = \frac{1}{\cancel{2}} * \cancel{2}y\,(x^2+y^2)^{-1/2}$$

$$\frac{\partial}{\partial y}\ln f(y) = \frac{f'(y)}{f(y)}$$

$$\frac{\partial}{\partial y}\ln\left(\sqrt{x^2+y^2}+y\right) = \frac{\frac{\partial}{\partial y}\left[\sqrt{x^2+y^2}+y\right]}{\sqrt{x^2+y^2}+y}$$

$$\frac{\left(\sqrt{x^2+y^2}\right)\cdot\frac{y}{\sqrt{\cancel{x^2}+y^2}}+1}{\left(\sqrt{x^2+y^2}\right)\left(\sqrt{x^2+y^2}+y\right)} = \frac{\left(y+\cancel{\sqrt{x^2+y^2}}\right)}{\sqrt{x^2+y^2}\left(\cancel{\sqrt{x^2+y^2}}+y\right)}$$

$$\boxed{V=\frac{1}{4\pi\epsilon_0}\frac{Q}{2a}*\ln\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)}$$

$\sqrt{x^2+y^2} =$

$\sqrt{x^2+y^2}$

$\sqrt{x^2+x^2\tan^2\theta}=x\sqrt{1+\tan^2\theta}$

$y = x\tan\theta$

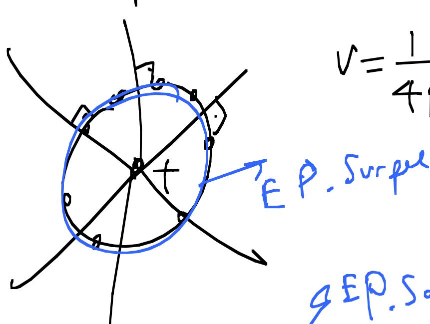
$dy = x\frac{d\theta}{\cos^2\theta}$

$$\int\frac{\cancel{x}\,d\theta}{\cos^2\theta\,\cancel{x}\underbrace{\sqrt{1+\tan^2\theta}}_{\cancel{1/\cos^2\theta}}} = \int\frac{d\theta}{\frac{\cos^2\theta}{\cos\cancel{\theta}}}$$

$$\underline{\int\frac{d\theta}{\cos\theta}} = \ln\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$$

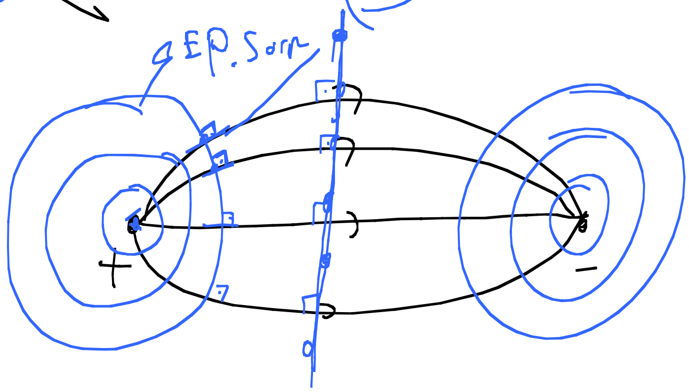
Equipotential Surfaces

Surfaces which have the same potential



$V=\frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$\vec{E}$ $\perp$ E.P. surface lines



$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\int_b^a dV = \int_a^b \vec{E}\cdot d\vec{l} \Rightarrow -\int_a^b dV = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\Rightarrow dV = -\vec{E}\cdot d\vec{l} = -\left(E_x\,dx + E_y\,dy + E_z\,dz\right)$$

$$\vec{E} = E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k}$$

$$d\vec{l} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\frac{\partial x}{\partial x} = 1 \qquad \frac{\partial y}{\partial x} = 0$$

$$\frac{dV}{\partial x} = -\left(E_x\,dx + E_y\,dy + E_z\,dz\right) \qquad \frac{dy}{dx} \neq 0$$

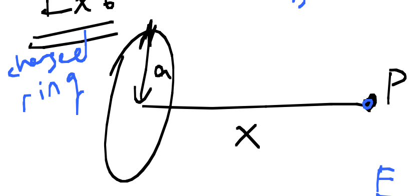
$$\frac{\partial V}{\partial x} = -E_x \ ; \quad \frac{\partial V}{\partial y} = -E_y \ ; \quad \frac{\partial V}{\partial z} = -E_z$$

$$\vec{E} = E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k}$$

$$\boxed{-\vec{\nabla} V = \vec{E}} \qquad \vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$-\frac{\partial}{\partial x}V\,\hat{i} - \frac{\partial}{\partial y}V\,\hat{j} - \frac{\partial}{\partial z}V\,\hat{k}$$

Ex:



$$V = \frac{Q}{4\pi\epsilon_0}\,\frac{1}{\sqrt{x^2+a^2}} \qquad \frac{\partial}{\partial x}(x^2+a^2)^{-1/2}$$

$E = ?$

$$E(x) = -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\epsilon_0} * -\frac{1}{\cancel{2}} * \cancel{2}x\,\frac{1}{(x^2+a^2)^{3/2}}$$

$$E(x) = \frac{Q}{4\pi\epsilon_0}\,\frac{x}{(x^2+a^2)^{3/2}}$$