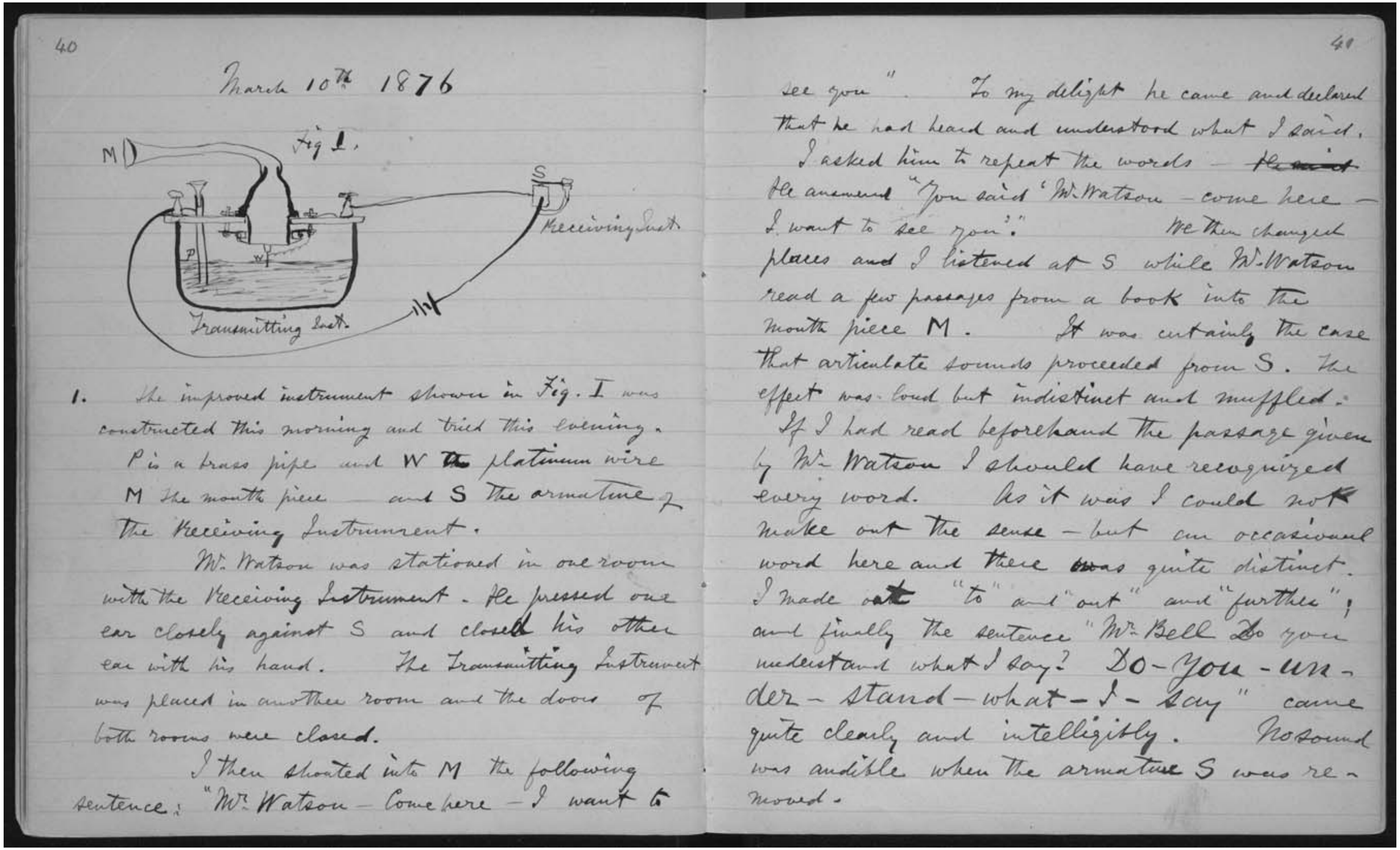
FIZ102E – Lecture 10
Inductance and Inductors

Alexandr Jonas

Department of Physics Engineering

İTÜ

What did we cover last week?



1\. The improved instrument shown in Fig. I was constructed this morning and tried this evening. P is a brass pipe and W the platinum wire M the mouth piece and S the armature of the Receiving Instrument.

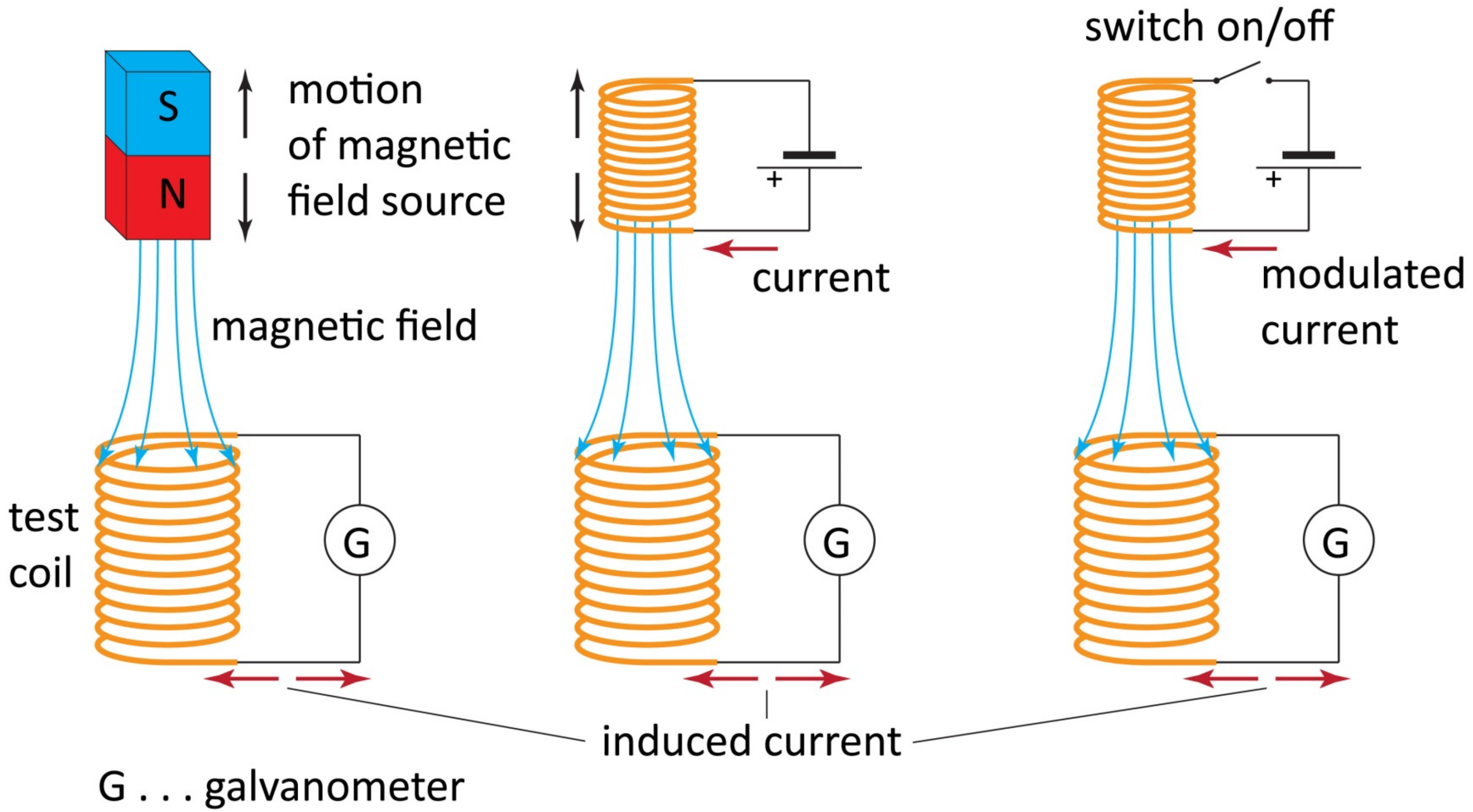
Mr. Watson was stationed in one room with the Receiving Instrument. He pressed one ear closely against S and closed his other ear with his hand. The Transmitting Instrument was placed in another room and the doors of both rooms were closed.

I then shouted into M the following sentence: "Mr. Watson — Come here — I want to see you". To my delight he came and declared that he had heard and understood what I said.

I asked him to repeat the words — ~~He said~~ He answered "You said 'Mr. Watson — come here — I want to see you'." We then changed places and I listened at S while Mr. Watson read a few passages from a book into the mouth piece M. It was certainly the case that articulate sounds proceeded from S. The effect was loud but indistinct and muffled.

If I had read beforehand the passage given by Mr. Watson I should have recognized every word. As it was I could not make out the sense — but an occasional word here and there was quite distinct. I made out "to" and "out" and "further"; and finally the sentence "Mr. Bell Do you understand what I say? Do — you — un-der — stand — what — I — say" came quite clearly and intelligibly. No sound was audible when the armature S was removed.

Induction experiments



Changing magnetic flux through a test coil leads to generation of induced current through the coil → <u>electromagnetic induction</u>

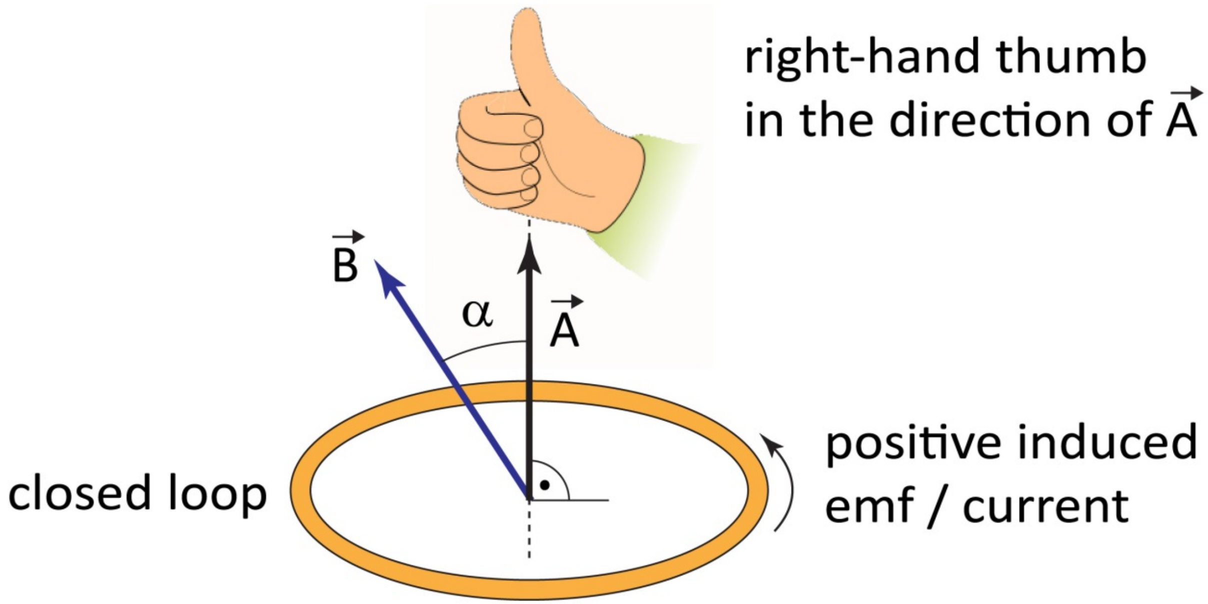
Faraday's law

"The induced electromotive force (emf) in a closed loop equals the negative of the time rate of change of magnetic flux through the loop"

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

For a coil formed by N identical loops

$$\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt}}$$



Magnetic flux $\Phi_B = \vec{B} \cdot \vec{A}$

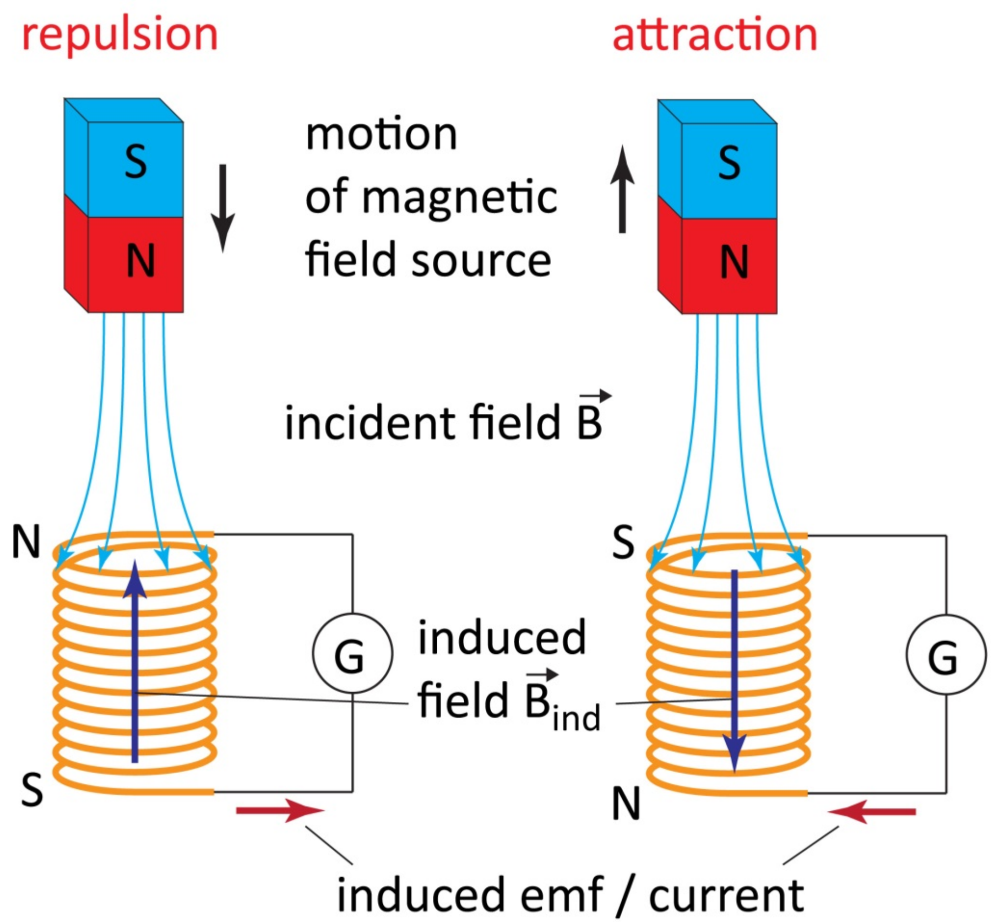
Changes due to changing magnitude and/or orientation of $\vec{B}$ and/or $\vec{A}$

Lenz's law

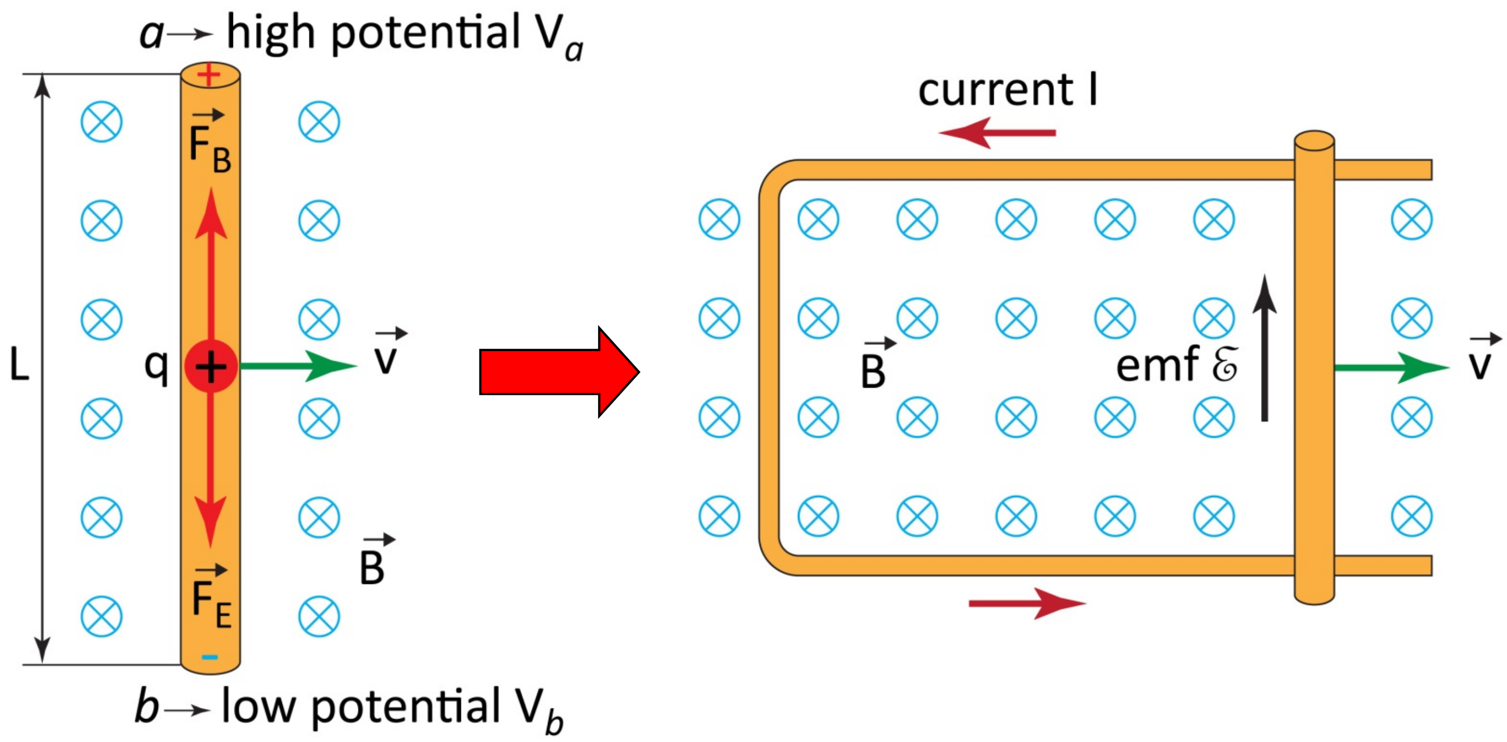
"The direction of any magnetic induction effect is such as to oppose the cause of the effect"

Induced current sets up its own magnetic field $\vec{B}_{ind}$

Induced field tends to preserve the current state by opposing any changes that caused the induction in the first place



Motional electromotive force



Magnetic force $\vec{F}_B$ causes charge separation → generation of electric field E = $(V_a - V_b)/L$ and electric force $\vec{F}_E$ opposing the magnetic force

In equilibrium: $\vec{F}_B = -\vec{F}_E \Rightarrow q(\vec{v}\times\vec{B}) = -q\vec{E} \Rightarrow E = vB \quad \wedge \quad E = (V_a - V_b)/L$

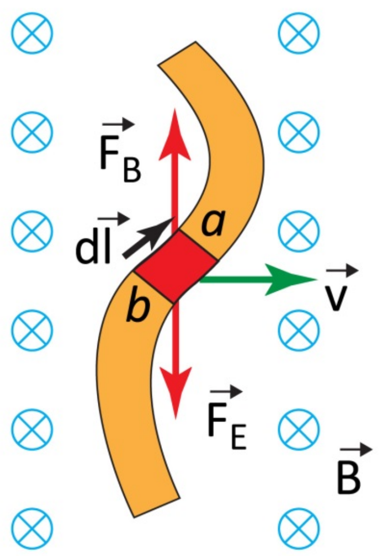
Motional electromotive force: $\boxed{\mathcal{E} \equiv (V_a - V_b) = v\,B\,L}$

Motional electromotive force

Motional electromotive force lifts charges within the moving conductor from low to high potential due to action of a non-electrostatic force

→ analogy to a battery

General expression for motional emf in a moving conductor



In equilibrium in a stationary magnetic field:

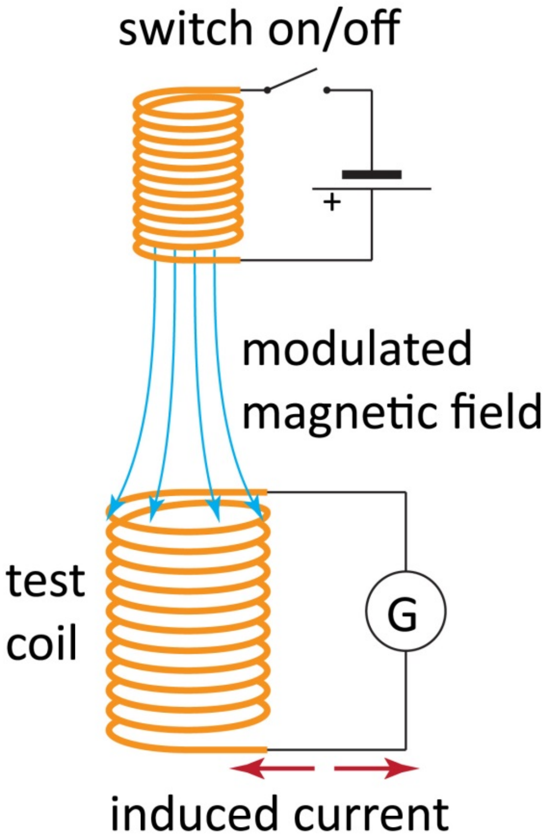
$$\vec{F}_B = -\vec{F}_E \Rightarrow q\left(\vec{v}\times\vec{B}\right) = -q\vec{E} \Rightarrow \vec{E} = -\left(\vec{v}\times\vec{B}\right)$$

Potential difference across conductor segment $d\vec{l}$:

$$dV_{ab} = -\vec{E}.d\vec{l} = \left(\vec{v}\times\vec{B}\right).d\vec{l}$$

Potential difference across the whole conductor: $\boxed{\mathcal{E} \equiv \left(V_a - V_b\right) = \int\left(\vec{v}\times\vec{B}\right).d\vec{l}}$

Induced electric fields



In a modulated magnetic field $\frac{d\Phi_B}{dt} \neq 0$

Electromotive force $\mathcal{E}$ is induced in the test coil

Origin of this induced electromotive force cannot be the motion of a conductor in a magnetic field

There has to be an <u>induced electric field</u> in the test coil due to the changing magnetic flux through the coil

Induced electric fields

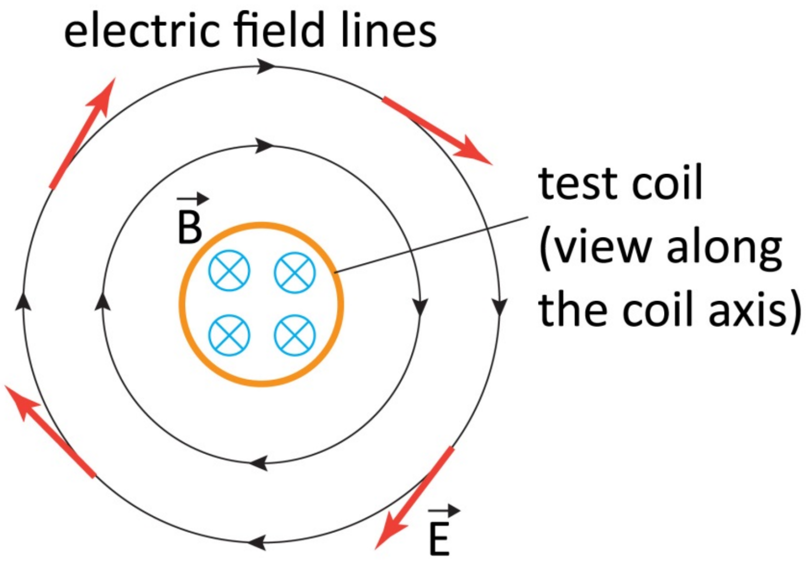
Induced electric field is non-conservative:

$$\oint_{\substack{closed\\loop}} \vec{E}\,.d\vec{l} = \mathcal{E}$$

From Faraday's law

for a stationary integration path:

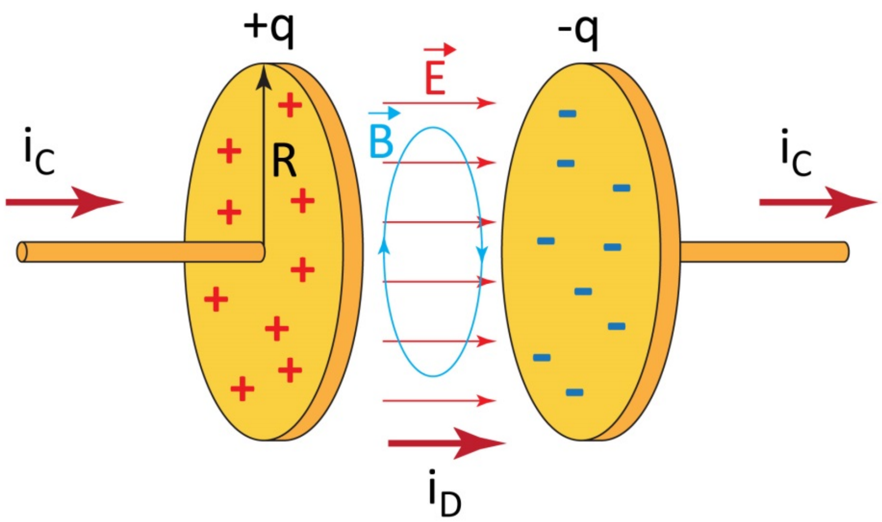
$$\oint_{\substack{closed\\loop}} \vec{E}\,.d\vec{l} = -\frac{d\Phi_B}{dt}$$



<u>Orientation of the induced electric field</u>

Electric field $\vec{E}$ induced in the test coil is tangential → it moves the charge around individual windings of the coil

Displacement current



Time-varying conduction current i_C charging a capacitor is related to the electric flux between the capacitor plates:

$$\boxed{i_C = \varepsilon_0 \frac{d\Phi_E}{dt}}$$

We can define <u>displacement current</u> i_D that mediates continuity of current flow between the capacitor plates:

$$\boxed{i_D \equiv i_C = \varepsilon_0 \frac{d\Phi_E}{dt}}$$

Modified Ampere's law including total current:

$$\boxed{\oint_{\substack{closed\\loop}} \vec{B}\cdot d\vec{l} = \mu_0 \left(i_C + i_D\right)_{encl}}$$

Maxwell's equations

Maxwell's equations summarize all of the relationships between electric and magnetic fields and their sources. In vacuum, they read as:

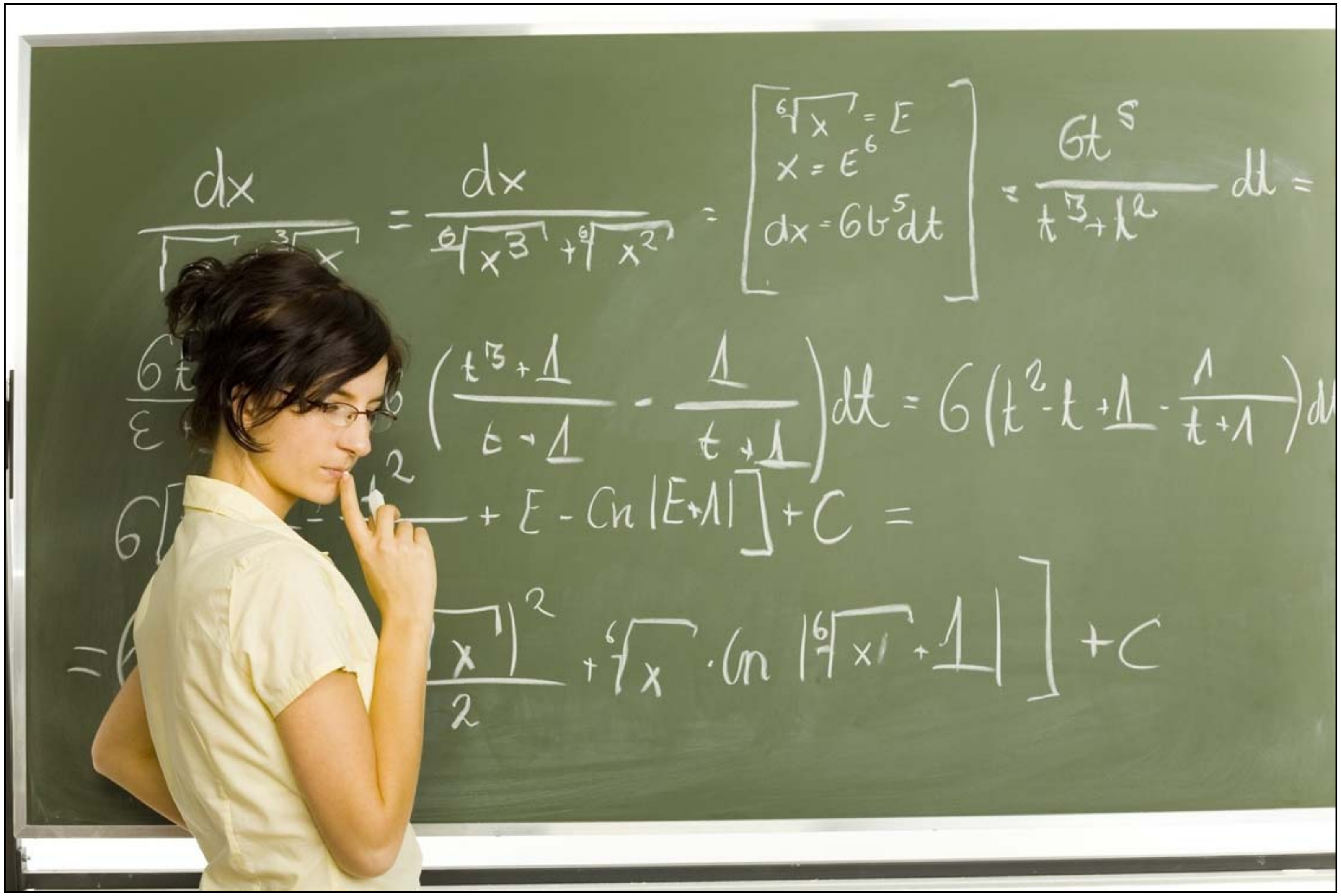
(1) $\oint \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ **Gauss's law for $\vec{E}$**

(2) $\oint \vec{B}.d\vec{A} = 0$ **Gauss's law for $\vec{B}$**

(3) $\oint \vec{B}.d\vec{l} = \mu_0 \left(i_C + \varepsilon_0 \frac{d\Phi_E}{dt} \right)_{encl}$ **Ampere's law**

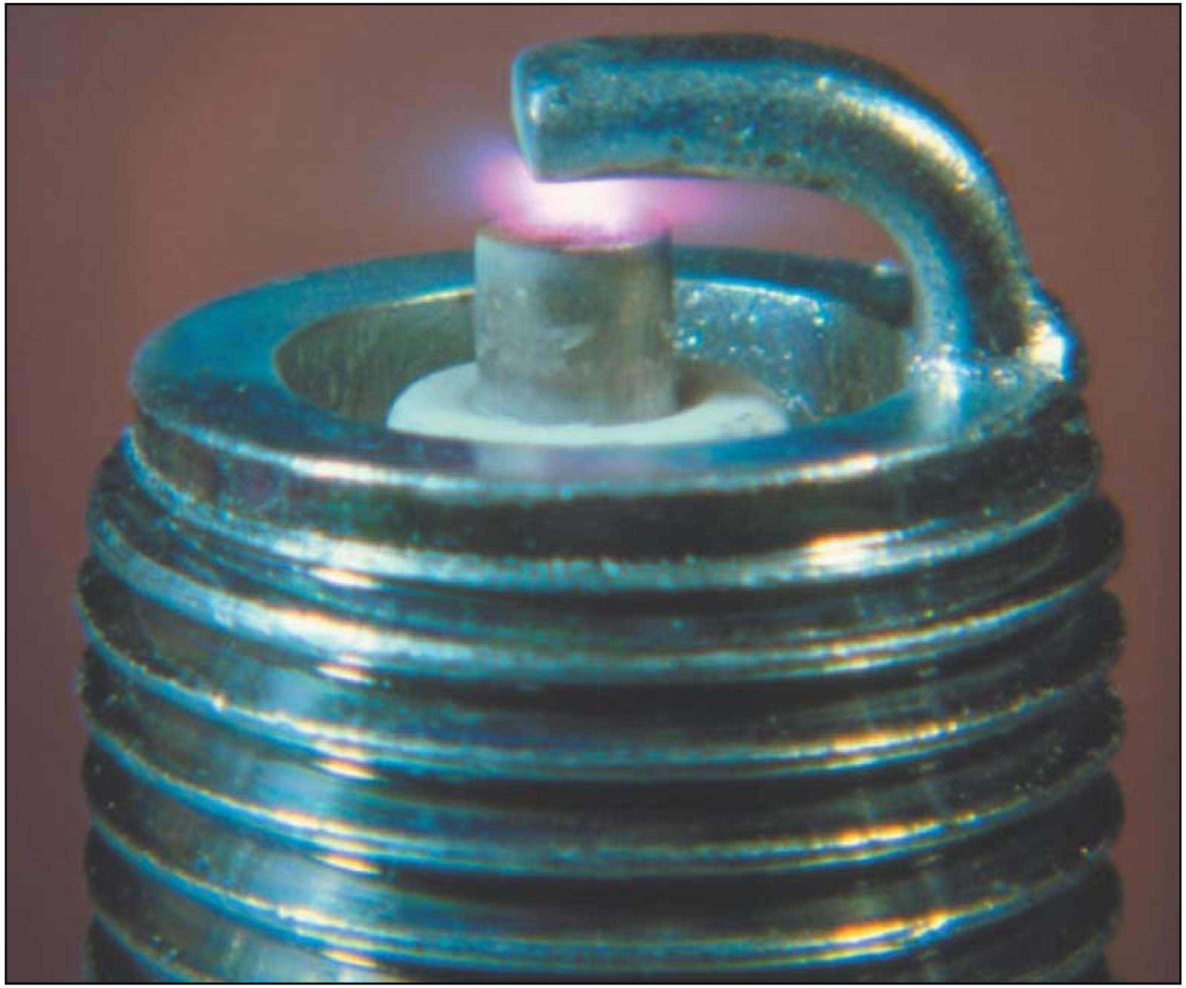
(4) $\oint \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt}$ **Faraday's law**

What will we cover today?



Mutual inductance, self-inductance, and inductors

Inductors and magnetic-field energy



The L-R-C circuits