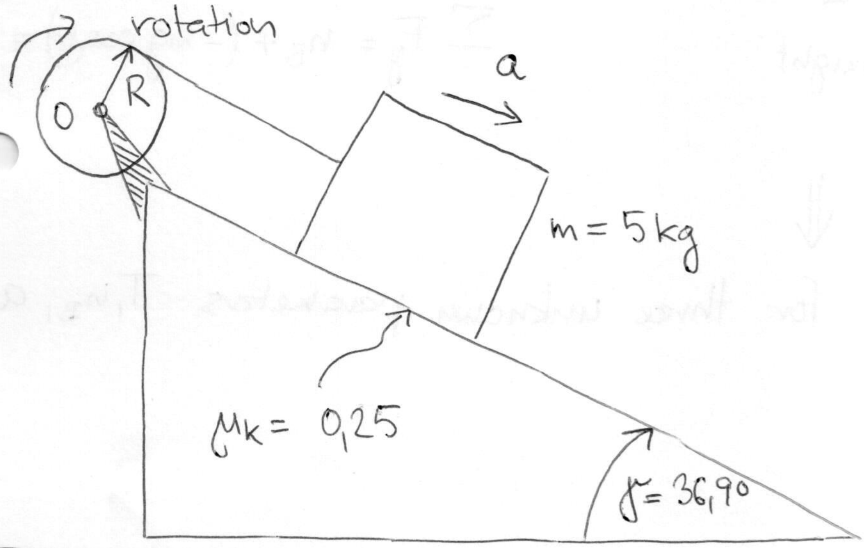
10.64 : A block with mass $m = 5{,}00\ kg$ slides down a surface inclined $36{,}9°$ to the horizontal (see picture). The coefficient of kinetic friction is $0{,}25$. A string attached to the block is wrapped around a flywheel (a solid disk) rotating about a fixed axis at point O. The flywheel has mass $M = 25\ kg$ and moment of inertia $I = 0{,}500\ kg m^2$ with respect to the axis of rotation. The string pulls without slipping at a perpendicular distance $R = 0{,}200\ m$ from that axis.

a) What is the acceleration of the sliding block?

b) What is the tension in the string?



Flywheel:
$M = 25 kg$
$I = 0{,}5\ kgm^2$
$R = 0{,}2 m$

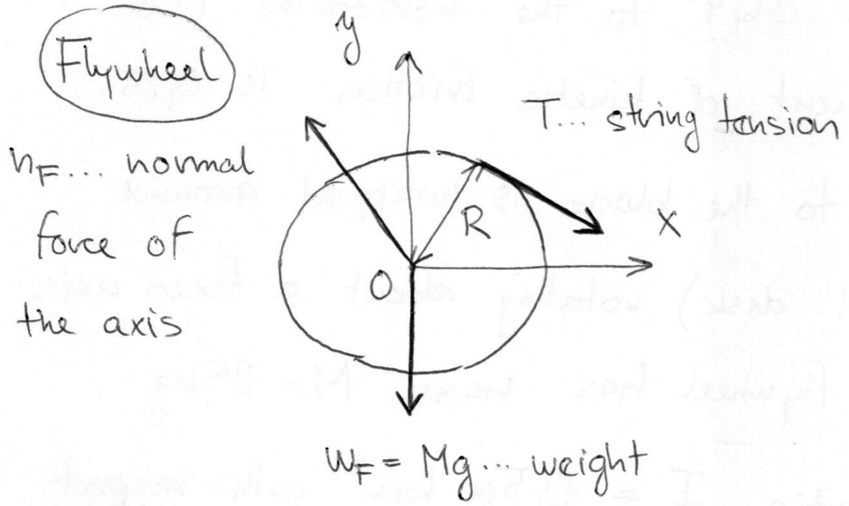
ad a)

String pulls without slipping over the flywheel surface

→ angular acceleration of the flywheel $\alpha_z = \frac{a}{R}$

where a is the block acceleration along the incline

Free-body diagrams



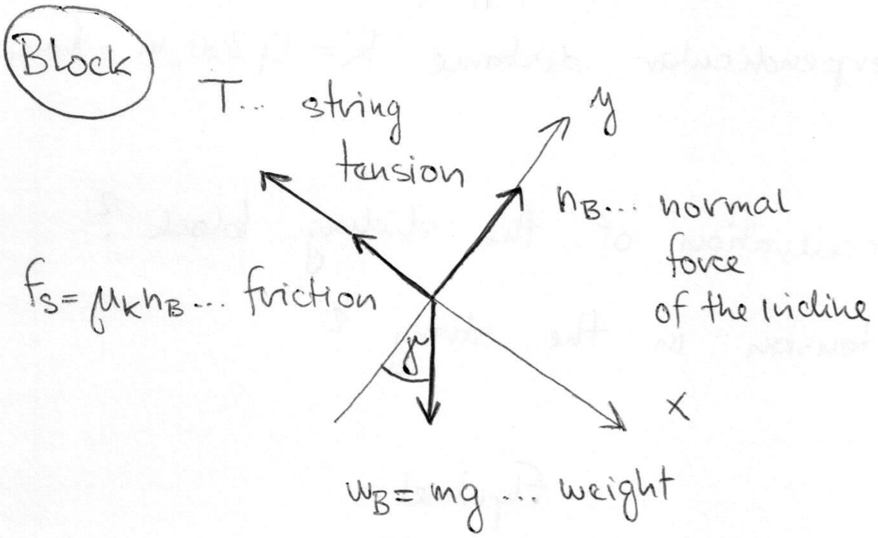
Rotational dynamics

Torque about the axis

$$\sum \tau_z = R \cdot T = I \alpha_z = I \frac{a}{R}$$

$$\Downarrow$$

$$T = \frac{Ia}{R^2}$$



2nd Newton's law

$$\sum F_x = mg \sin\gamma + (-f_s) + (-T) = ma$$

$$\sum F_y = n_B + (-mg\cos\gamma) = 0$$

$$\Downarrow$$

Set of three equations for three unknown parameters T, n_B, a:

I $\quad T = \dfrac{Ia}{R^2}$

II $\quad mg\sin\gamma - \mu_k n_B - T = ma$

III $\quad n_B - mg\cos\gamma = 0 \longrightarrow n_B = mg\cos\gamma$

- Inserting I and III into II gives

$$mg\sin\gamma - \mu_k mg\cos\gamma - \frac{Ia}{R^2} = ma$$

$$a\left(m + \frac{I}{R^2}\right) = mg\left(\sin\gamma - \mu_k\cos\gamma\right)$$

$$a = \frac{mg\left(\sin\gamma - \mu_k\cos\gamma\right)}{\left(m + \frac{I}{R^2}\right)} = \frac{5 \cdot 9{,}8 \cdot \left(\sin 36{,}9^\circ - 0{,}25\cos 36{,}9^\circ\right)}{5 + \frac{0{,}5}{(0{,}2)^2}} \frac{m}{s^2}$$

$$= \underline{\underline{1{,}12 \frac{m}{s^2}}}$$

ad b

- From I: $T = \frac{Ia}{R^2} = \frac{0{,}5 \cdot 1{,}12}{(0{,}2)^2} N = \underline{\underline{14\,N}}$

The block is moving down the incline with acceleration of $1{,}12 \frac{m}{s^2}$. The tension in the string pulling on the block is 14 N.

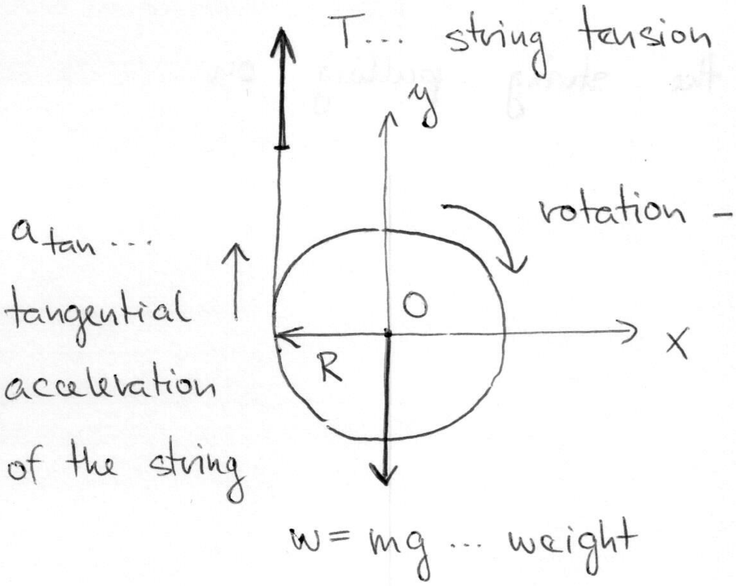
10.72: A massless string is wrapped several times around a small ring with radius $R = 0{,}08\,m$ and mass $m = 0{,}18\,kg$. The free end of the string is pulled upward so that the ring does not move vertically as the string unwinds (see picture)

a) Find the string tension as the string unwinds

b) Find the angular acceleration of the ring as the string unwinds

c) Find the upward acceleration of the hand that pulls on the free end of the string

d) How do the answers a) – c) change if we replace the ring with a solid disk of the same radius and mass?



Ring:

mass $m = 0{,}18\,kg$

radius $R = 0{,}08\,m$

moment of inertia $I = mR^2$

While pulling, the ring does not move vertically → acceleration a_{cm} of its center of mass located at point O is zero

The string unwinds from the ring without slipping:

tangential acceleration of the string $a_{tan} = R \cdot \alpha_z$

ad a) 2nd Newton's law

$$\sum F_y = T + (-mg) = m a_{cm} = 0 \implies T = mg = 0{,}18 \cdot 9{,}8\,N = \underline{\underline{1{,}76\,N}}$$

ad b) Rotational dynamics

$$\sum \tau_z = R \cdot T = I \alpha_z \implies \alpha_z = \frac{R \cdot T}{I} = \frac{\cancel{R} \cdot \cancel{m} \cdot g}{\cancel{m} R^{\cancel{2}}} = \frac{g}{R} = \frac{9{,}80}{0{,}08} \frac{rad}{s^2}$$

$$= \underline{\underline{122{,}5 \frac{rad}{s^2}}}$$

ad c) Acceleration of the hand is equal to the tangential acceleration of the string $\underline{\underline{a_{tan}}}$ at the ring surface:

$$a_{tan} = R \cdot \alpha_z = R \frac{g}{R} = g = \underline{\underline{9{,}80 \frac{m}{s^2}}}$$

ad d) For a solid disk replacing the ring, the only changing parameter is the moment of inertia $I = \frac{1}{2} m R^2$

$$\Downarrow$$

$$T = mg = \underline{\underline{1{,}76\,N}}$$

$$\alpha_z = \frac{RT}{I} = \frac{\cancel{R}\cancel{m}g}{\cancel{m}R^{\cancel{2}}/2} = \frac{2g}{R} = \underline{\underline{245 \frac{rad}{s^2}}}$$

$$a_{tan} = R \cdot \alpha_z = R \frac{2g}{R} = 2g = \underline{\underline{19{,}6 \frac{m}{s^2}}}$$

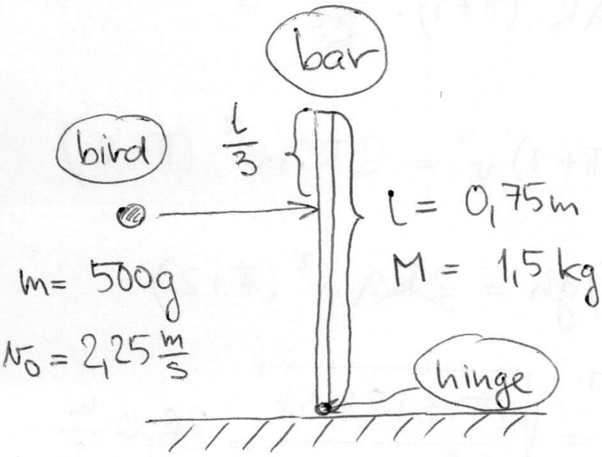
10.91 : A bird with mass $m = 500g$ is flying horizontally at $2{,}25 \frac{m}{s}$ when it hits a stationary vertical bar of length $l = 0{,}75m$ at a distance $l/3$ from the top of the bar. After the collision, the bird falls vertically down to the ground.

If the mass of the bar is $M = 1{,}5kg$, what is its angular velocity

a) just after it is hit by the bird?

b) just as it reaches the ground?

The bar has a frictionless hinge at the bottom



moment of inertia about the hinge

$$I = \frac{1}{3} M l^2$$

ada) During the collision of the bird with the bar, angular momentum of the system about the hinge is conserved:

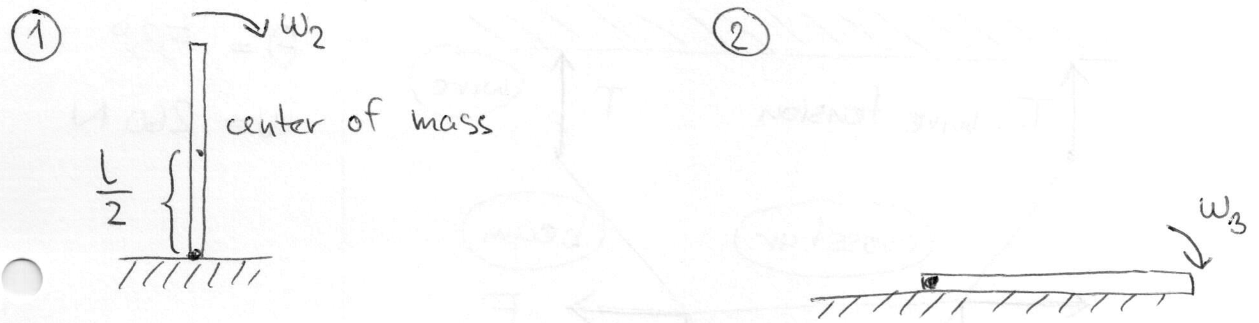
Before: $L_1 = m \cdot v_0 \cdot \frac{2l}{3}$ → only bird contributes, the bar is stationary

After : $L_2 = I\omega_2 = \frac{1}{3} ML^2\omega_2$ $\rightarrow$ after the collision, the bird is moving vertically toward the hinge and its angular momentum is zero

$$L_1 = L_2 \Rightarrow m v_0 \frac{2L}{3} = \frac{1}{3} ML^2 \omega_2$$

$$\omega_2 = \frac{2 m v_0}{ML} = \frac{2 \cdot 0{,}5 \cdot 2{,}25}{1{,}5 \cdot 0{,}75} \frac{rad}{s} = \underline{\underline{2 \frac{rad}{s}}}$$

ad b) When the bar starts falling down, only gravity does work $\rightarrow$ the total mechanical energy is conserved



$U_1 = M \cdot g \cdot \frac{L}{2}$

$K_1 = \frac{1}{2} I\omega_2^2$

$U_2 = 0$

$K_2 = \frac{1}{2} I\omega_3^2$

$$U_1 + K_1 = U_2 + K_2 \Rightarrow Mg\frac{L}{2} + \frac{1}{2}I\omega_2^2 = \frac{1}{2}I\omega_3^2$$

$$\Downarrow$$

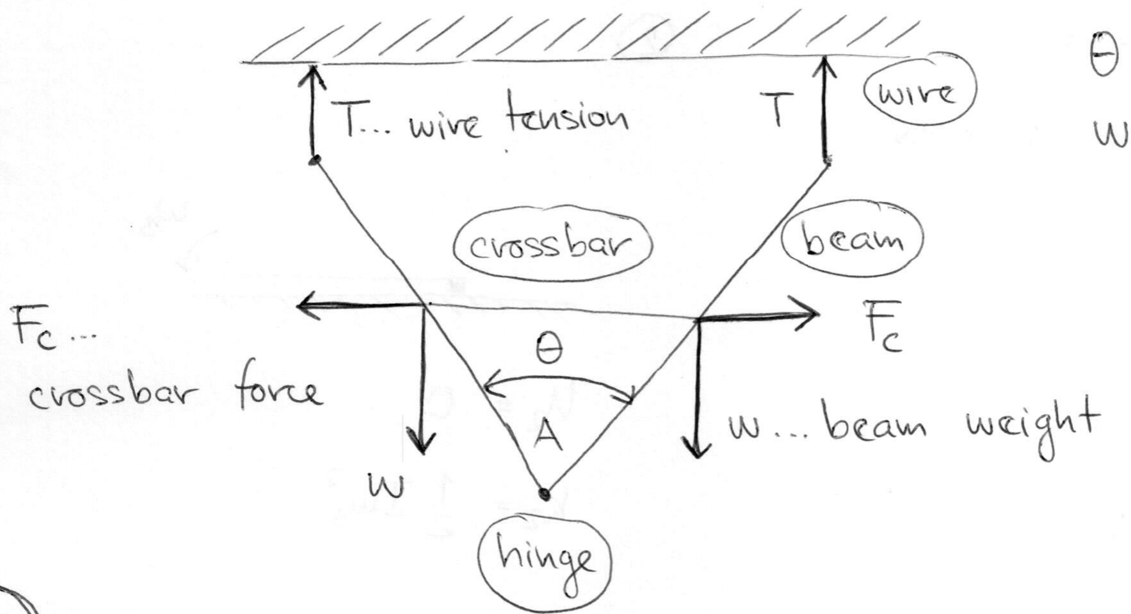
$$\omega_3 = \sqrt{\omega_2^2 + \frac{MgL}{I}} = \sqrt{\omega_2^2 + \frac{\cancel{M}g\cancel{L}}{\cancel{M}L^{\cancel{2}}/3}} = \sqrt{\omega_2^2 + \frac{3g}{L}} = \sqrt{2^2 + \frac{3 \cdot 9{,}8}{0{,}75}}$$

$$= \underline{\underline{6{,}57 \frac{rad}{s}}}$$

11.76 : Two identical uniform beams of weight 260 N each are connected at one end by a frictionless hinge and a light horizontal crossbar attached at the midpoints of the beams maintains an angle of 53° between the beams (see picture). The beams are suspended from the ceiling by vertical wires such that they form a "V".

a) What force does the crossbar exert on each beam?

b) What force (magnitude and direction) does the hinge at point A exert on each beam?



$\theta = 53°$

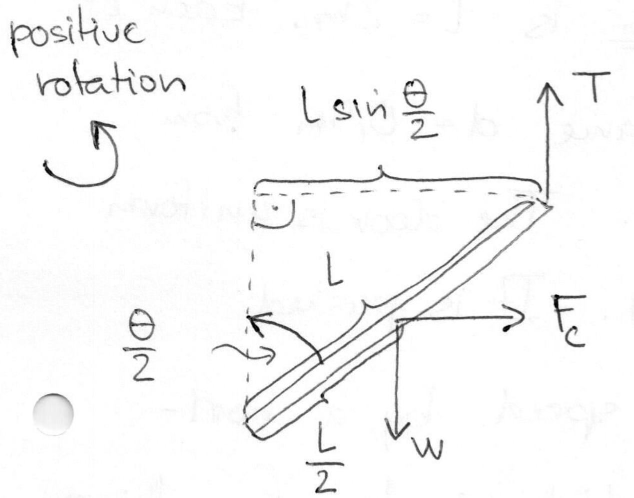
$W = 260$ N

ad a

- The whole hanging structure is in equilibrium

$\Rightarrow 2T - 2W = 0 \Rightarrow \underline{\underline{T = W}}$

- In equilibrium, net torque with respect to any point must be zero → express torque balance with respect to the hinge at point $\underline{\underline{A}}$ for the right beam:



$$\sum \tau_z = TL\sin\frac{\theta}{2} + \left(-F_c\frac{L}{2}\cos\frac{\theta}{2}\right) + \left(-W\frac{L}{2}\sin\frac{\theta}{2}\right) = 0$$

$$\Downarrow$$

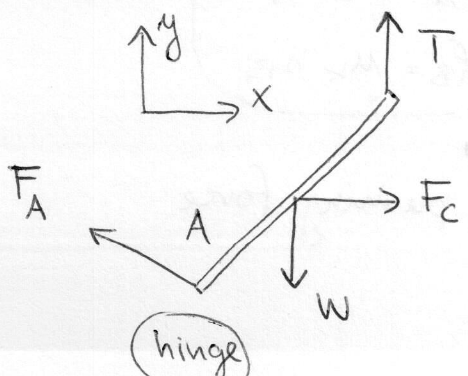
$$\frac{1}{2}F_c\cos\frac{\theta}{2} = \left(T - \frac{W}{2}\right)\sin\frac{\theta}{2}$$

Since $T = W$, torque balance gives

$$F_c = (2T - W)\cdot\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = W\tan\frac{\theta}{2} = 260\cdot\tan 26{,}5^\circ\ \text{N} = \underline{\underline{130\ \text{N}}}$$

Force $\underline{\underline{F_c}}$ exerted by the crossbar on the right beam pushes the beam to the right → the beam exerts a force of equal magnitude and opposite direction on the cross bar → the crossbar is under <u>compression</u>

ad b) Each beam of the structure must be in equilibrium



F_A ... force at the hinge

↳ since $T = W$, F_A cannot have any vertical component

$$\Downarrow$$

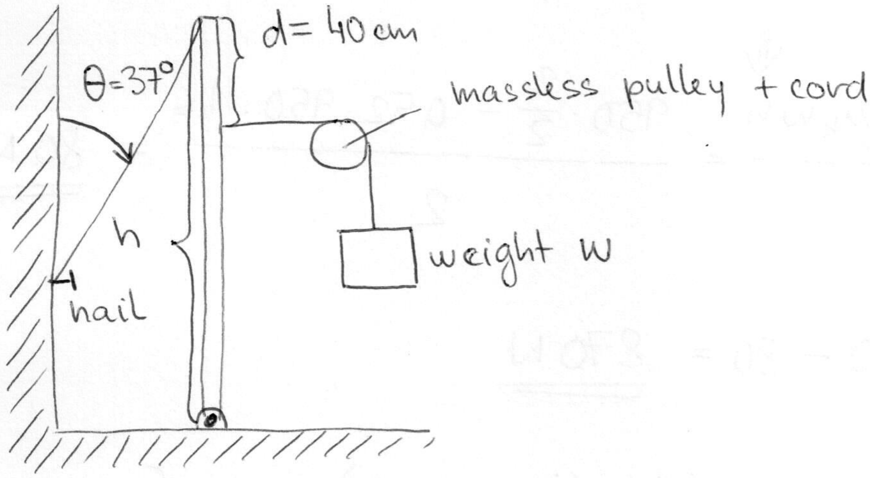
$$\sum F_x = F_c + (-F_A) = 0 \Rightarrow F_A = F_c = \underline{\underline{130\ \text{N}}}$$

opposite directions

11.82: A weight $\underline{\underline{W}}$ is supported as shown in the picture (the pulley has zero friction and mass and the cord is massless). The pole is pivoted about a hinge at its base, is 1,75 m tall, and weighs 55 N. A thin wire connects the top of the pole to a nail in a vertical wall; the nail will pull out if an outward force greater than 22 N acts on it.

a) What is the maximal weight that can be supported without pulling out the nail?

b) What is the magnitude of the force exerted on the pole by the hinge?



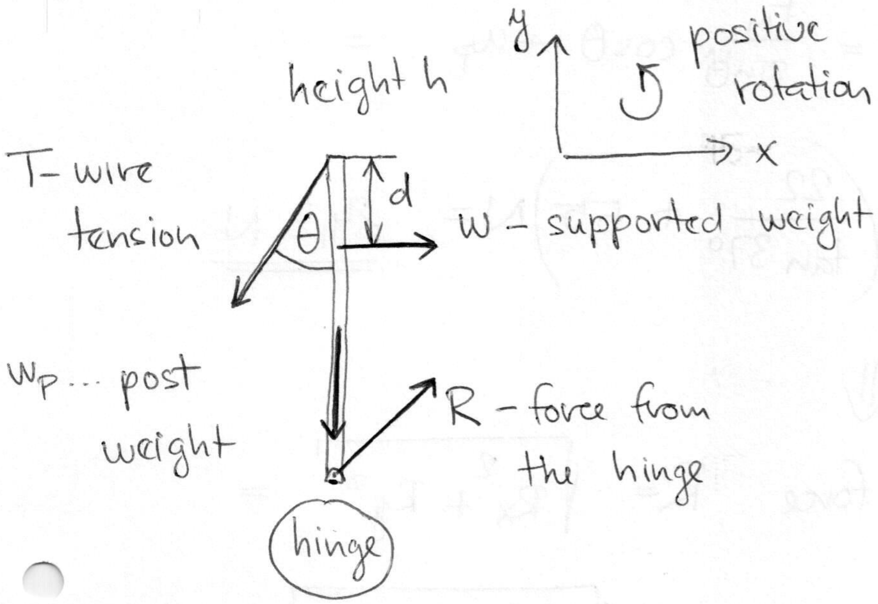
pole: height $h = 1{,}75\,m$

weight $W_p = 55\,N$

Maximal force on the nail (in horizontal direction)

$F_m = 22\,N$

Free-body diagram for the post



Maximal force on the nail

$$F_m = T \cdot \sin\theta$$

$$\Downarrow$$

$$T = \frac{F_m}{\sin\theta}$$

Equilibrium: torque balance about the hinge

$$\sum \tau_z = T \cdot \sin\theta \, h + (-w(h-d)) = 0$$

$$\Downarrow$$

$$w = \frac{T\sin\theta \cdot h}{(h-d)} = \frac{F_m \cdot h}{(h-d)} = \frac{22 \cdot 1{,}75}{(1{,}75-0{,}4)} N = \underline{\underline{28{,}5 N}}$$

ad b) Force balance for the post:

$$\sum F_x = R_x + w - T\cdot\sin\theta = 0 \Rightarrow R_x = T\cdot\sin\theta - w = 22N - 28{,}5N$$

$$= -\underline{\underline{6{,}5N}}$$

- actual direction of $\underline{\underline{R_x}}$ is opposite to that shown in the free-body diagram

$$\sum F_y = R_y - T\cdot\cos\theta - w_P = 0$$

$$R_y = T\cos\theta + w_p = \frac{F_m}{\sin\theta}\cos\theta + w_p =$$

$$= \frac{F_m}{\tan\theta} + w_p = \left(\frac{22}{\tan 37^\circ} + 55\right) N = \underline{\underline{84{,}2\ N}}$$

$$\Downarrow$$

Magnitude of the hinge force $R = \sqrt{R_x^2 + R_y^2} =$

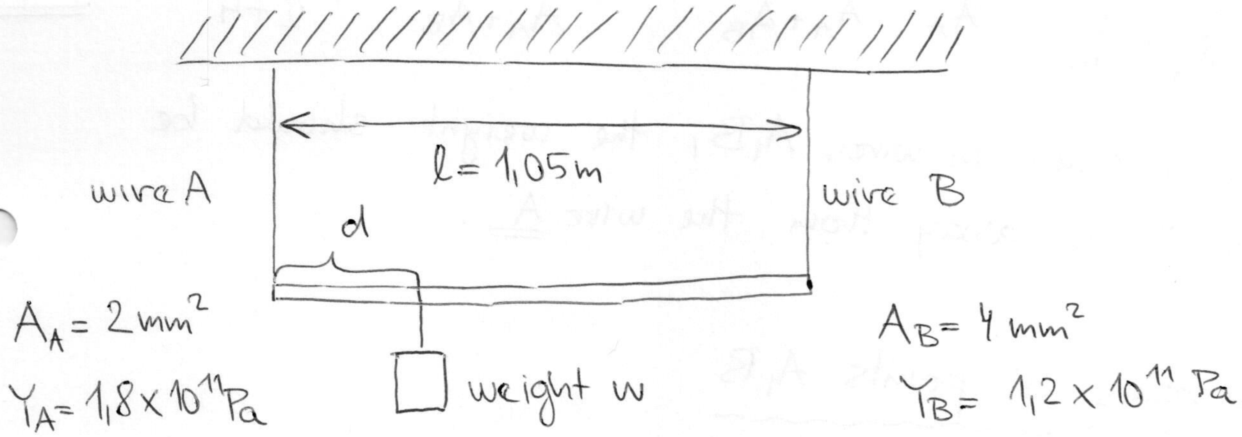
$$= \sqrt{6{,}5^2 + 84{,}2^2}\ N =$$

$$= \underline{\underline{84{,}5\ N}}$$

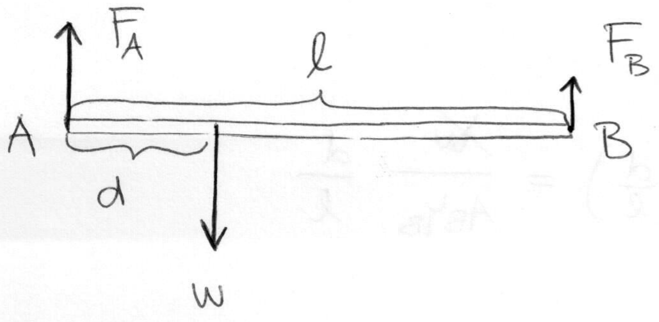
11.87: A 1,05 m long rod of negligible weight is supported at its ends by wires A and B of equal length (see picture). The cross-sectional area of A is 2,00 mm^2 and that of B is 4,00 mm^2. Young's modulus for wire A is $Y_A = 1{,}80 \times 10^{11}$ Pa and that for B is $Y_B = 1{,}20 \times 10^{11}$ Pa. At what point along the rod should a weight w be suspended to produce:

a) equal stresses in A and B

b) equal strains in A and B



Free-body diagram



Force balance:

$F_A + F_B - w = 0 \Rightarrow F_A + F_B = w$

Torque balance about point A:

$F_B \cdot l - w \cdot d = 0 \Rightarrow F_B l = wd$

$$\left.\begin{array}{ll} \mathrm{I} & F_A + F_B = w \\ \mathrm{II} & w d = F_B \ell \end{array}\right\} \quad F_B = w \cdot \frac{d}{\ell}$$

$$F_A = w - F_B = w\left(1-\frac{d}{\ell}\right)$$

ad a) Stresses at points A, B

$$(\text{stress})_A = \frac{F_A}{A_A} = \frac{w}{A_A}\left(1-\frac{d}{\ell}\right)$$

$$(\text{stress})_B = \frac{F_B}{A_B} = \frac{w}{A_B}\,\frac{d}{\ell}$$

$$(\text{stress})_A = (\text{stress})_B \Rightarrow \frac{\cancel{w}}{A_A}\left(1-\frac{d}{\ell}\right) = \frac{\cancel{w}}{A_B}\,\frac{d}{\ell}$$

$$\frac{1}{A_A} = \left(\frac{1}{A_A}+\frac{1}{A_B}\right)\frac{d}{\ell}$$

$$d = \frac{\ell}{A_A}\,\frac{1}{\frac{1}{A_A}+\frac{1}{A_B}} = \frac{\ell}{\cancel{A_A}}\,\frac{\cancel{A_A}A_B}{A_A+A_B} = \frac{\ell A_B}{A_A+A_B} = \frac{1{,}05\cdot 4}{2+4} = 0{,}7\text{ m}$$

For equal stresses in wires A, B, the weight should be suspended 0,7 m away from the wire A.

ad b) Strains at points A, B

$$(\text{strain})_A = \frac{F_A}{A_A Y_A} = \frac{w}{A_A Y_A}\left(1-\frac{d}{\ell}\right)$$

$$(\text{strain})_B = \frac{F_B}{A_B Y_B} = \frac{w}{A_B Y_B}\,\frac{d}{\ell}$$

$$(\text{strain})_A = (\text{strain})_B \Rightarrow \frac{\cancel{w}}{A_A Y_A}\left(1-\frac{d}{\ell}\right) = \frac{\cancel{w}}{A_B Y_B}\,\frac{d}{\ell}$$

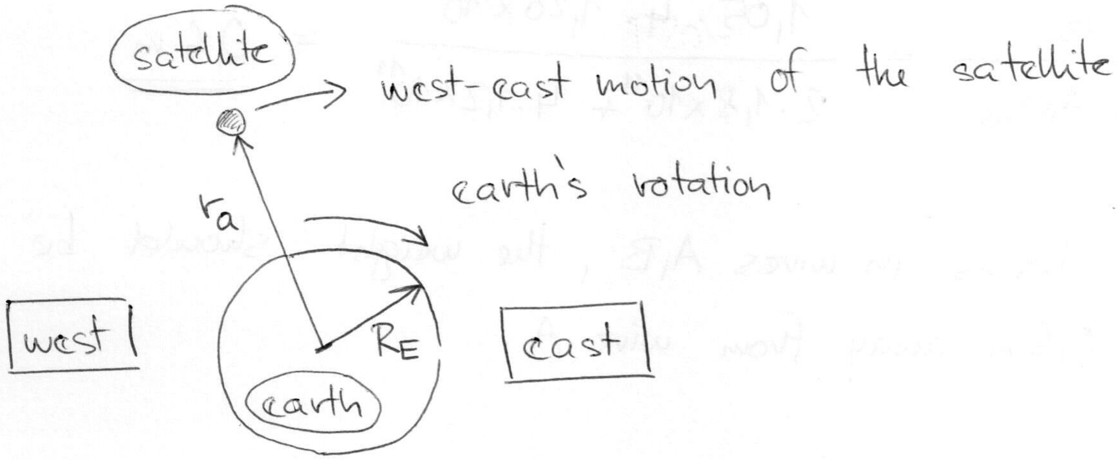
$$\frac{1}{A_A Y_A} = \left(\frac{1}{A_A Y_A} + \frac{1}{A_B Y_B}\right)\frac{d}{\ell}$$

$$d = \frac{\ell}{A_A Y_A}\,\frac{1}{\frac{1}{A_A Y_A} + \frac{1}{A_B Y_B}} = \frac{\ell}{\cancel{A_A Y_A}}\,\frac{\cancel{A_A Y_A}\,A_B Y_B}{(A_A Y_A + A_B Y_B)}$$

$$= \frac{\ell\,A_B Y_B}{A_A Y_A + A_B Y_B} = \frac{1{,}05 \cdot 4 \cdot 1{,}20\times10^{11}}{2\cdot 1{,}8\times10^{11} + 4\cdot 1{,}2\times10^{11}} = \underline{\underline{0{,}6\text{ m}}}$$

For equal strains in wires A, B, the weight should be suspended 0,6m away from wire A.

12.59: (a) Suppose you are on the earth's equator and observe a satellite passing directly overhead and moving from west to east in the sky. Exactly 12 hours later, you again observe this satellite to be directly overhead. How far above the earth's surface is the satellite orbit?



- Satellite moves in the direction of the earth's rotation

↳ in 12 hours, the earth rotates by half-revolution (180°) and the satellite has to complete one full orbit (360°) + additional half-orbit (180°) to catch up with the observer standing at the equator

⇓

Satellite's orbital period $\underline{\underline{T_a}}$ fulfills $\frac{3}{2} T_a = 12h$

$$T_a = \frac{2}{3} 12h = \underline{\underline{8h}}$$

- For a satellite in a circular orbit with a radius $\underline{\underline{r}}$, the orbital period $T = \dfrac{2\pi r^{3/2}}{\sqrt{G m_E}}$

$$\Rightarrow r = \left(\frac{T\sqrt{Gm_E}}{2\pi}\right)^{2/3} = \left(\frac{T}{2\pi}\right)^{2/3}(Gm_E)^{1/3}$$

$$r_a = \left(\frac{T_a}{2\pi}\right)^{2/3}(Gm_E)^{1/3} = \left(\frac{8\cdot 3600}{2\pi}\right)^{2/3}\left(6{,}67\times10^{-11}\cdot 5{,}97\times10^{24}\right)^{1/3}\ m$$

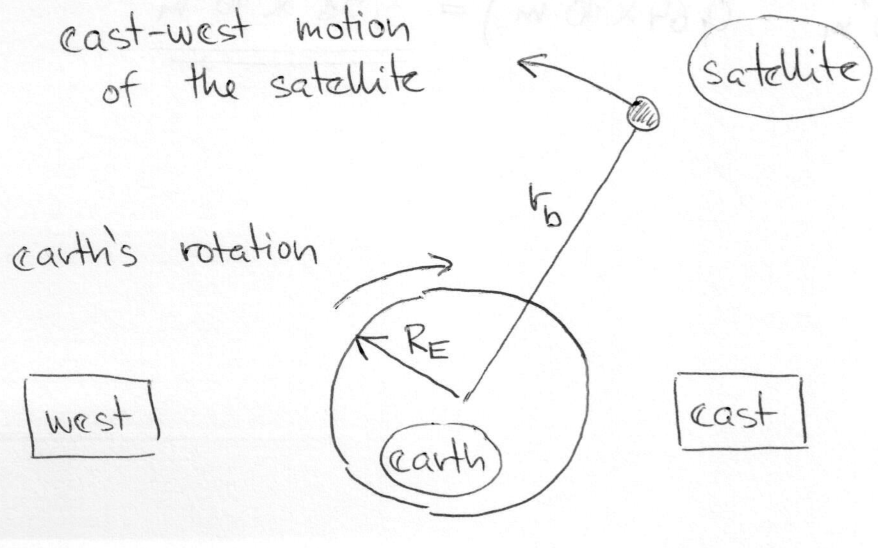
$$= \underline{\underline{2{,}03\times10^{7}\ m}}$$

$\Rightarrow$ distance $\underline{\underline{h_a}}$ from the earth's surface is

$$h_a = r_a - R_E = \left(2{,}03\times10^{7}m - 0{,}64\times10^{7}m\right) =$$

$$= \underline{\underline{1{,}39\times10^{7}\ m}}$$

(b) You observe another satellite directly overhead and traveling east to west. This satellite is again overhead in 12 hours. How far is this satellite's orbit above the earth's surface?



- Satellite moves opposite to the direction of the earth's rotation

↳ in 12 hours, the earth rotates by half-revolution (180°) and the satellite completes a half-orbit (180°) to catch up with the observer standing at the equator

$$\Downarrow$$

Satellite's orbital period $\underline{\underline{T_b}}$ fulfills $\frac{1}{2} T_b = 12\,h$

$$T_b = 2 \cdot 12\,h = \underline{\underline{24\,h}}$$

$$\Downarrow$$

$$r_b = \left(\frac{T_b}{2\pi}\right)^{2/3} \left(G \cdot m_E\right)^{1/3} = \left(\frac{24 \times 3600}{2\pi}\right)^{2/3} \cdot \left(6{,}67 \times 10^{-11} \cdot 5{,}97 \times 10^{24}\right)^{1/3} m$$

$$= \underline{\underline{4{,}22 \times 10^{7}\,m}}$$

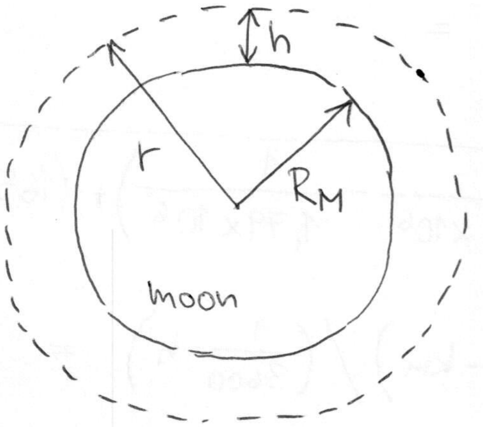
$$\Downarrow$$

distance $\underline{\underline{h_b}}$ from the earth's surface is

$$h_b = r_b - R_E = \left(4{,}22 \times 10^{7} m - 0{,}64 \times 10^{7} m\right) = \underline{\underline{3{,}58 \times 10^{7}\,m}}$$

12.65: A spacecraft is in a circular orbit around the moon at an altitude of 50 km above the moon's surface. Suddenly, the speed of the spacecraft decreases accidentally by $20\,\frac{m}{s}$. If nothing is done to correct the spacecraft's orbit, with what speed will it crash into the lunar surface?

orbit



moon: mass $m_M = 7{,}35 \times 10^{22}\,kg$

radius $R_M = 1{,}74 \times 10^6\,m$

height of the orbit: $h = 50\,km$

$\Downarrow$

$r = R_M + h = 1{,}79 \times 10^6\,m$

- Initially, the spacecraft is in a stable orbit with radius $r = R_M + h$

  ↳ spacecraft's speed $v_0 = \sqrt{\frac{G m_M}{r}} = \sqrt{\frac{6{,}67 \times 10^{-11} \cdot 7{,}35 \times 10^{22}}{1{,}79 \times 10^6}}\,\frac{m}{s}$

$$= 1655\,\frac{m}{s}$$

- If the speed is suddenly decreased to $v_1 = (v_0 - \Delta v)$ where $\Delta v = 20\frac{m}{s}$, the satellite is no longer in a stable orbit and starts falling toward the moon

↳ during the fall, the total mechanical energy is conserved

Position 1 (orbit) : $U_1 = -G\dfrac{m_M m}{r}$; $K_1 = \dfrac{1}{2} m v_1^2 = \dfrac{1}{2} m (v_0 - \Delta v)^2$

Position 2 (moon's surface) : $U_2 = -G\dfrac{m_M m}{R_M}$; $K_2 = \dfrac{1}{2} m v_2^2$

$$\Downarrow$$

$$-G\frac{m_M \cancel{m}}{r} + \frac{1}{2}\cancel{m}(v_0 - \Delta v)^2 = -G\frac{m_M \cancel{m}}{R_M} + \frac{1}{2}\cancel{m} v_2^2$$

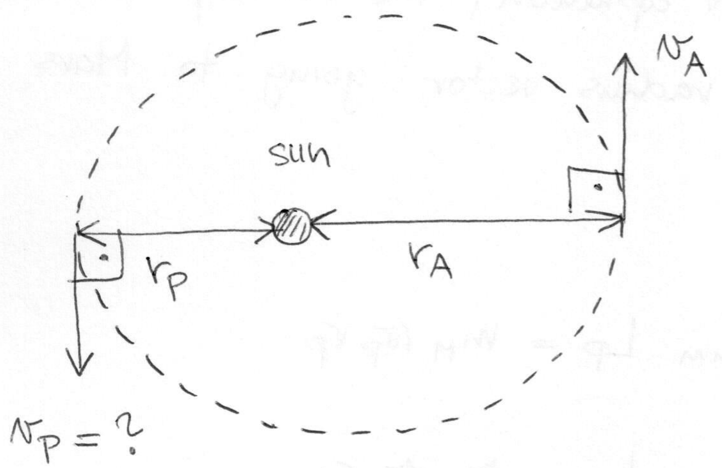
$$v_2 = \sqrt{2Gm_M\left(\frac{1}{R_M} - \frac{1}{r}\right) + (v_0 - \Delta v)^2} =$$

$$= \sqrt{2 \cdot 6{,}67\times10^{-11} \cdot 7{,}35\times10^{22} \cdot \left(\frac{1}{1{,}74\times10^6} - \frac{1}{1{,}79\times10^6}\right) + (1655-20)^2}\ \frac{m}{s}$$

$$= 1682\ \frac{m}{s} = 1682 \cdot \left(\frac{1}{1000}\,km\right) / \left(\frac{1}{3600}\,h\right) =$$

$$= 6055\ \frac{km}{h}$$

12.76 : As Mars orbits the sun in its elliptical orbit, its distance of the closest approach to the center of the sun (at perihelion) is $r_P = 2{,}067 \times 10^{11}$ m and its maximum distance from the center of the sun (at aphelion) is $r_A = 2{,}492 \times 10^{11}$ m. If the orbital speed of Mars at aphelion is $v_A = 2{,}198 \times 10^{4}\ \frac{m}{s}$, what is its orbital speed at perihelion? Ignore influence of other planets.



Sun:

mass $m_S = 1{,}99 \times 10^{30}$ kg

Mars:

mass $m_M = 6{,}42 \times 10^{23}$ kg

Two possible strategies of solution:

(a) Using conservation of the total mechanical energy

Perihelion: $U_P = -G\dfrac{m_S m_M}{r_P}$; $K_P = \dfrac{1}{2} m_M v_P^2$

Aphelion: $U_A = -G\dfrac{m_S m_M}{r_A}$; $K_A = \dfrac{1}{2} m_M v_A^2$

$$-G\frac{m_S \cancel{m_M}}{r_P}+\frac{1}{2}\cancel{m_M}v_P^2=-G\frac{m_S\cancel{m_M}}{r_A}+\frac{1}{2}\cancel{m_M}v_A^2$$

$$v_P=\sqrt{v_A^2+2Gm_S\left(\frac{1}{r_P}-\frac{1}{r_A}\right)}=\sqrt{(2{,}198\times10^4)^2+2\cdot6{,}67\times10^{-11}\cdot1{,}99\times10^{30}\cdot\left(\frac{1}{2{,}067\times10^{11}}-\frac{1}{2{,}492\times10^{11}}\right)}$$

$$=\underline{\underline{26\,500\ \frac{m}{s}}}$$

(b) Using conservation of angular momentum

- At both perihelion and aphelion, the velocity of Mars is perpendicular to the radius vector going to Mars from the sun

$\Downarrow$

Perihelion: angular momentum $L_P = m_M v_P r_P$

Aphelion: angular momentum $L_A = m_M v_A r_A$

- Gravitational force has zero torque about the sun

$\rightarrow$ angular momentum is conserved

$$L_P = L_A \Rightarrow m_M v_P r_P = m_M v_A r_A \Rightarrow$$

$$v_P = v_A\frac{r_A}{r_P}=\left(2{,}198\times10^4\cdot\frac{2{,}492\times10^{11}}{2{,}067\times10^{11}}\right)\frac{m}{s}=\underline{\underline{26\,500\ \frac{m}{s}}}$$