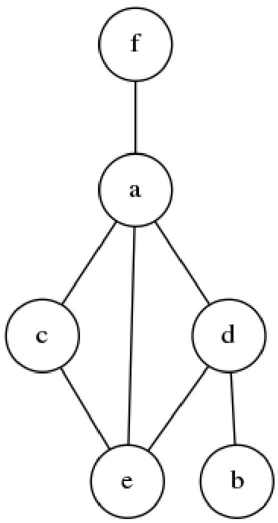
Introduction to Topology Quiz 3, April 19th, 2016

Name:____________________

Number:__________________

1. Consider the following graph for this question:



	a	b	c	d	e	f
a	0	2	1	1	1	1
b	2	0	3	1	2	3
c	1	3	0	2	1	2
d	1	1	2	0	1	2
e	1	2	1	1	0	2
f	1	3	2	2	2	0

	a	b	c	d	e	f
Eccentricity	2	3	3	2	2	3

   (a) Fill in the distance matrix above.

   (b) Calculate the eccentricities of the vertices:

   (c) Calculate the radius and the diameter of the graph.

   Solution: Radius is 2, and the diameter is 3.

2. If we have two different metrics $d(x,y)$ and $f(x,y)$ on a set X, we said d and f are *metrically equivalent* $d \underset{ME}{\sim} f$ if there are positive real numbers $m, M \in (0,\infty)$ such that

$$m \cdot f(x,y) \leq d(x,y) \leq M \cdot f(x,y)$$

for all $x, y \in X$. Show that this relation is symmetric.

Solution: Since $m \cdot f(x,y) \leq d(x,y)$ we must have $f(x,y) \leq \frac{1}{m} \cdot d(x,y)$. But we also have $d(x,y) \leq M \cdot f(x,y)$ which also implies $\frac{1}{M} \cdot d(x,y) \leq f(x,y)$. Combining these two we get

$$\frac{1}{M} \cdot d(x,y) \leq f(x,y) \leq \frac{1}{m} \cdot d(x,y)$$

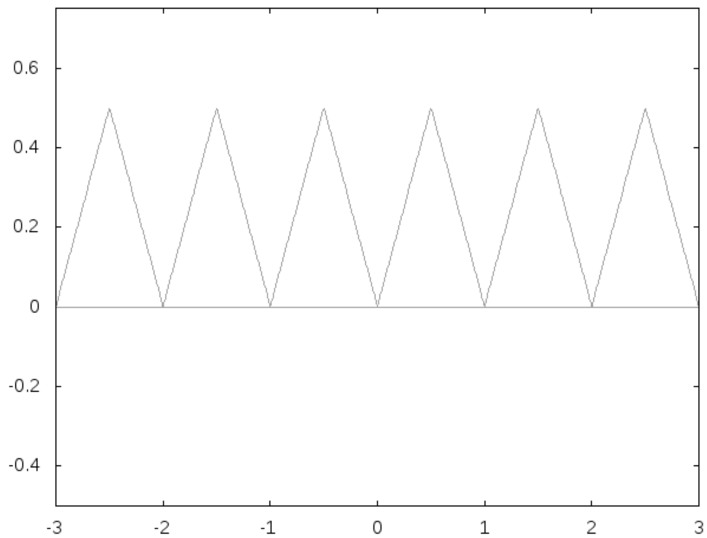
In other words, $f \underset{ME}{\sim} d$.

3. Let $\mathbb{Z}$ be the set of integers $\{0, \pm 1, \pm 2, \ldots\}$.

   (a) Sketch a graph of the function

$$f(x) = d(x, \mathbb{Z}) = \inf_{z \in \mathbb{Z}} |x - z|$$

Solution:



   (b) Consider the region Ω between the graph of the function $f(x) = d(x, \mathbb{Z})$ and the x-axis . Write a description of the geodesic distance function $g((a,b),(c,d))$ for every $(a,b),(c,d) \in \Omega$.

Solution: Assume $(a,b),(c,d) \in \Omega$ and WLOG $a < c$. If $\lfloor a \rfloor = \lfloor c \rfloor$ then

$$d((a,b),(c,d)) = \sqrt{(a-c)^2 + (b-d)^2}$$

If $\lfloor a \rfloor \neq \lfloor c \rfloor$, then

$$d((a,b),(c,d)) = \sqrt{(a - \lceil a \rceil)^2 + b^2} + \lfloor c \rfloor - \lceil a \rceil + \sqrt{(c - \lfloor c \rfloor)^2 + d^2}$$