Integral
FIZ 102E

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Contents

The problem

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How?

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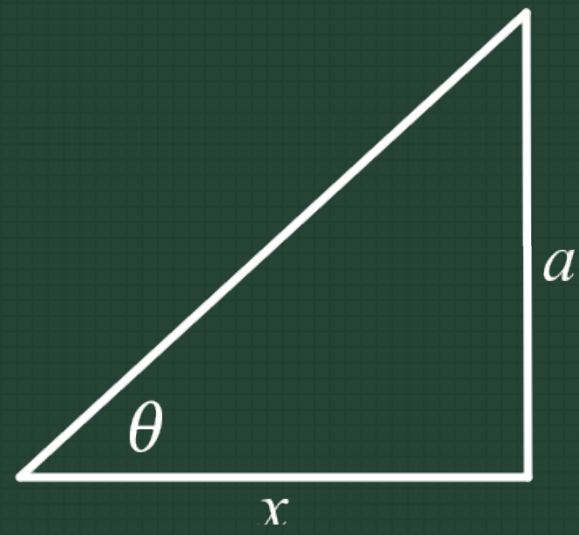

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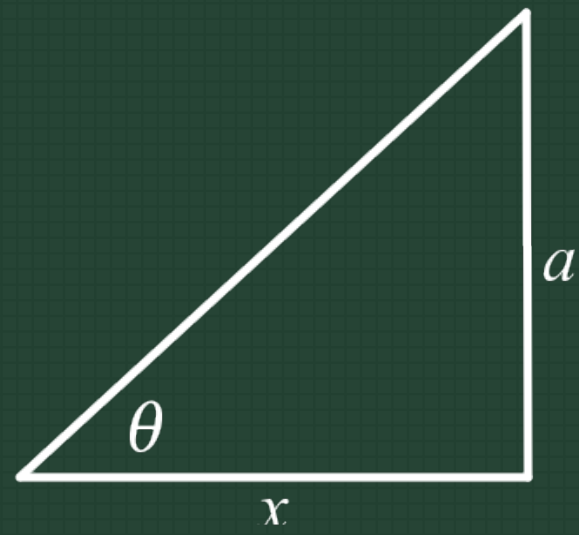

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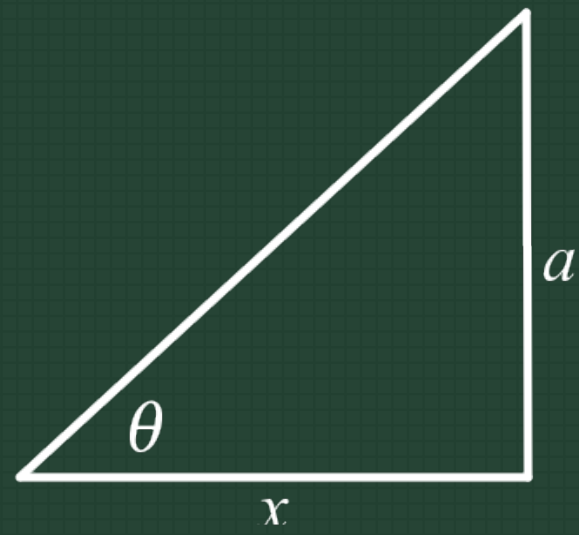

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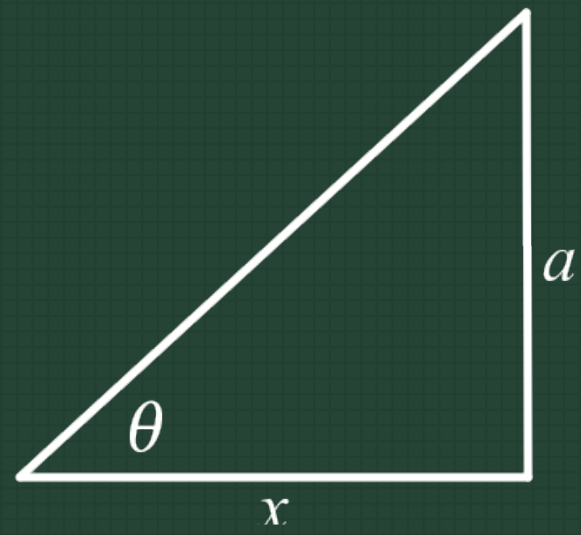
$$I = \int \frac{-a\,\mathrm{cosec}^2\theta\,d\theta}{(a^2 + a^2\cot^2\theta)^{3/2}}$$



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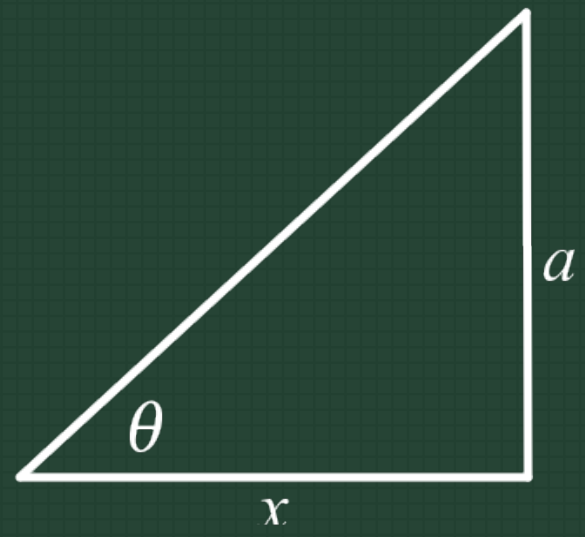

- Using $1 + \cot^2\theta = \mathrm{cosec}^2\theta$ we obtain

$$I = \int \frac{-a\,\mathrm{cosec}^2\theta\,d\theta}{a^3\mathrm{cosec}^3\theta}$$

$$= -\frac{1}{a^2}\int \sin\theta\,d\theta$$

$$= \frac{1}{a^2}\cos\theta = \frac{1}{a^2}\frac{x}{\sqrt{a^2 + x^2}}$$