Capacitance and Dielectrics

FIZ102E: Electricity & Magnetism

Yavuz Ekşi

İTÜ, Fizik Müh. Böl.

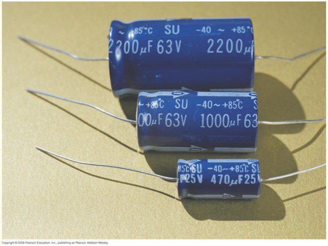
Outline

Learning Goals

- The nature of capacitors, and how to calculate a quantity that measures their ability to store charge.
- How to analyze capacitors connected in a network.
- How to calculate the amount of energy stored in a capacitor.
- What dielectrics are, and how they make capacitors more effective.
- How a dielectric inside a charged capacitor becomes polarized.
- How to use Gauss's laws when dielectrics are present.

Introduction

- A capacitor is a device that stores electric potential energy and electric charge.
- Any two conductors insulated from each other form a capacitor.
- To store energy in this device, transfer charge from one conductor to the other so that one has a negative charge and the other has an equal amount of positive charge.



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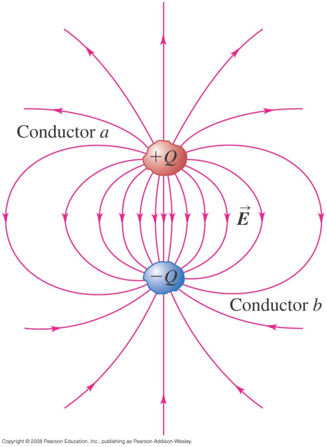

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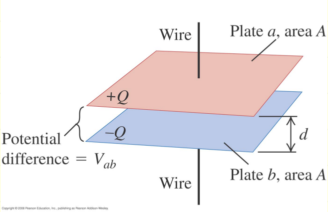

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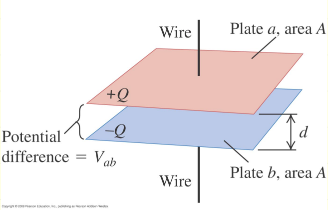

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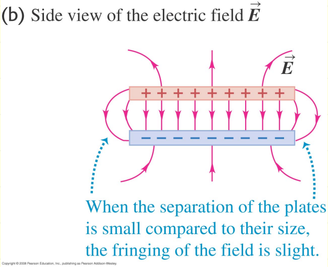
- Work must be done to move the charges through the resulting potential difference between the conductors.
- The work done is stored as electric potential energy.



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- Capacitors have many practical applications: electronic flash units for photography, mobile phones, airbag sensors for cars, and radio and television receivers.
- For a particular capacitor, the ratio of the charge on each conductor (Q) to the potential difference between the conductors (V) is a constant, called the capacitance (C).
- The capacitance depends on
  - the sizes of the conductors
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  - on the insulating material (if any)

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$$C = \frac{Q}{V}$$

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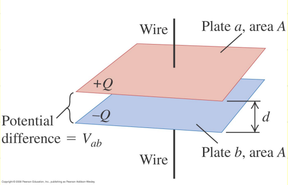

$$C = \epsilon_0 \frac{A}{d}$$

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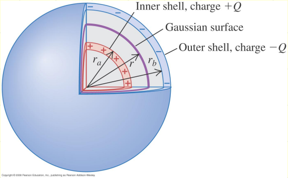

$$C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

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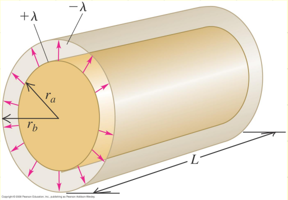

$$C = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

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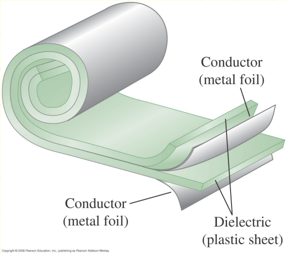
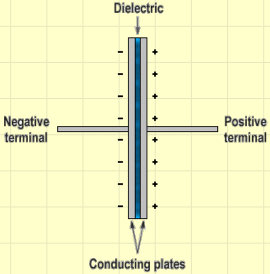


$$C = KC_0, \quad K > 1$$

- Compared to the case in which there is only vacuum between the conductors, the capacitance increases when an insulating material (a dielectric) is present.
- This happens because a redistribution of charge, called *polarization*, takes place within the insulating material.



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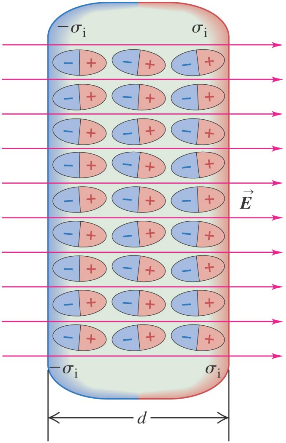

Capacitors and Capacitance

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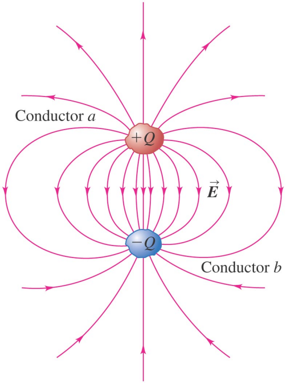
- Any two conductors separated by an insulator (or a vacuum) form a capacitor
- In most practical applications, each conductor initially has zero net charge and electrons are transferred from one conductor to the other; this is called *charging* the capacitor.
- One common way to charge a capacitor is to connect these two wires to opposite terminals of a battery.



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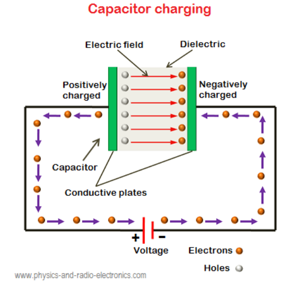

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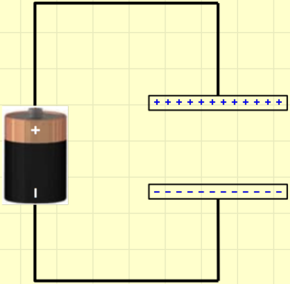
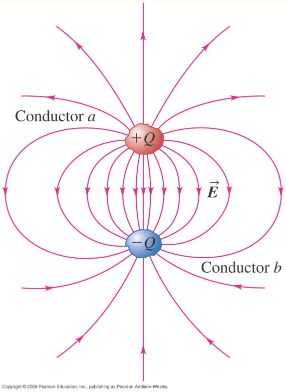
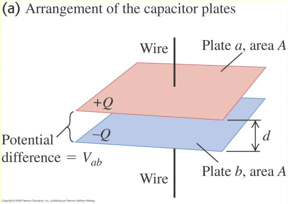


Capacitors and Capacitance

- The two conductors of a charged capacitor have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole remains zero.
- When we say that a capacitor has charge Q, or that a charge Q is stored on the capacitor, we mean that the conductor at higher potential has charge $+Q$ and the conductor at lower potential has charge $-Q$



- The prototype of a capacitor is the conducting parallel plates.
- In circuit diagrams a capacitor is represented by either of these symbols:
- In practice cylindirical capacitors are easier to manufacture.



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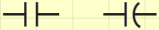

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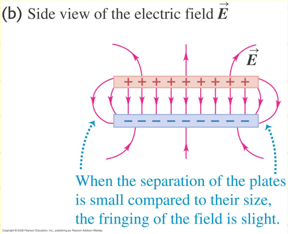

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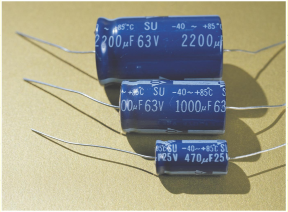
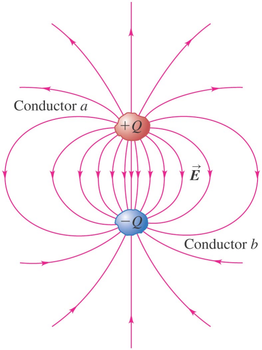


Capacitors and Capacitance

- E at any point in the region between the conductors is proportional to the magnitude Q of charge on each conductor.
- It follows that V_{ab} between the conductors is also proportional to Q.
- If we double the magnitude of charge on each conductor, E at each point doubles, and V_{ab} doubles.
- However, Q/V_{ab} does not change and is called the capacitance of the capacitor:

$$\boxed{C = \frac{Q}{V_{ab}}}$$



- Capacitance is a measure of the ability of a capacitor to store energy.

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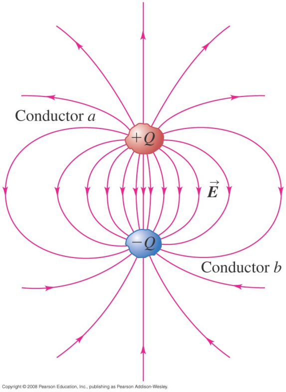


- The greater the capacitance C of a capacitor, the greater the magnitude Q of charge on either conductor for a given potential difference V_{ab} and hence the greater the amount of stored energy.

Unit of Capacitance

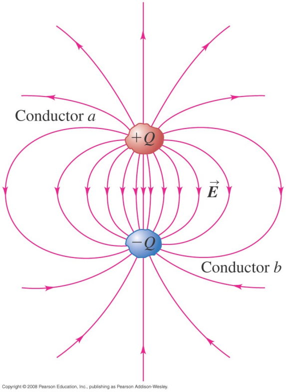
- The SI unit of capacitance is called one farad (1 F),
- The definition of capacitance

$$C = \frac{Q}{V_{ab}}$$

implies that

$$1\,\mathrm{F} = 1\,\mathrm{coulomb/volt}$$

- 1 F is a very large capacitance. In many applications $1\,\mu\mathrm{F}$ or 1 pF is more convenient.



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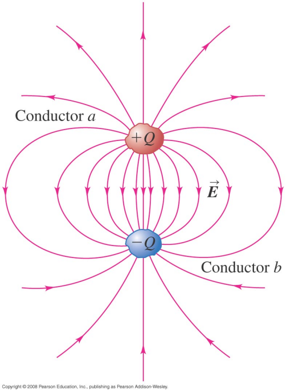

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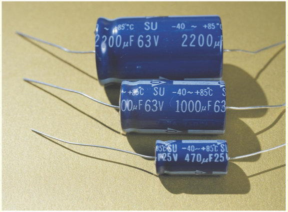

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Calculating Capacitance: Capacitors in Vacuum

- We can calculate the capacitance C of a given capacitor by finding the potential difference V_{ab} between the conductors for a given magnitude of charge Q and then using

$$C = \frac{Q}{V_{ab}}$$

- For now we'll consider only capacitors in vacuum; that is, empty space separates the conductors that make up the capacitor.
- We will calculate the capacitance of parallel, spherical and cylindrical capacitors.

Parallel-plate capacitor

- $E = \sigma/\epsilon_0$
- $\sigma = Q/A$
- $E = Q/A\epsilon_0$
- $V_{ab} = Ed$

$$V_{ab} = \frac{Q}{A\epsilon_0}d$$

- Using $C = Q/V_{ab}$

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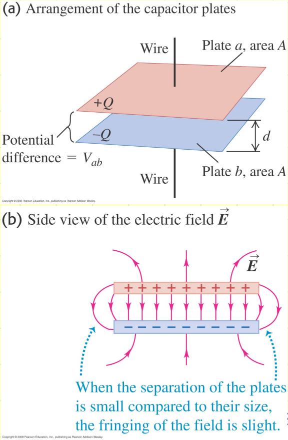

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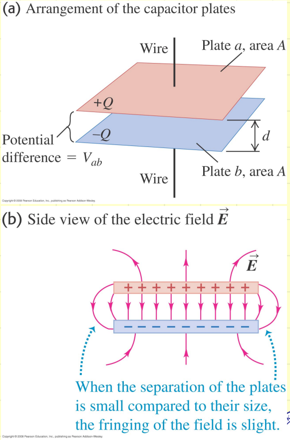

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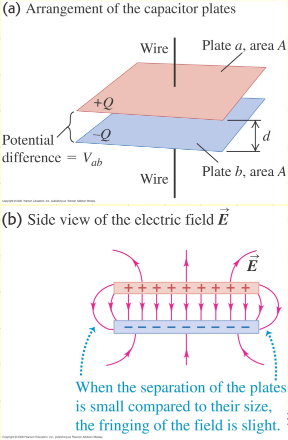

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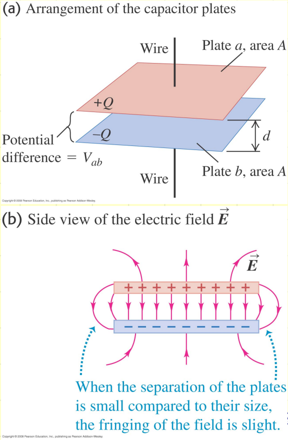

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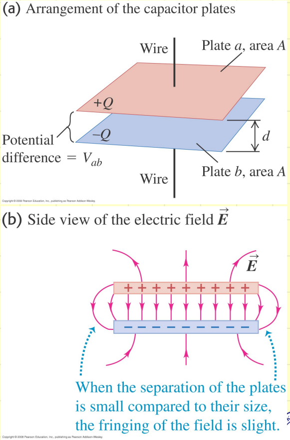


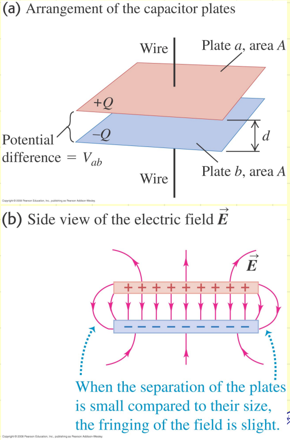
Parallel-plate capacitor

- The capacitance

$$C = \epsilon_0 \frac{A}{d}, \quad \text{parallel plate capacitor}$$

depends on only the geometry of the capacitor;

- it is directly proportional to the area A of each plate and
- inversely proportional to their separation d.
- Recall that $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$. Check that $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$.



Exercise: $1.0\,\mathrm{F}$ capacitor

Question:

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Solution:

Applying

$$C = \epsilon_0 \frac{A}{d}$$

one finds that $A = 1.1 \times 10^8\,\mathrm{m}^2$ which correspond to a square of $\sim 10\,\mathrm{km}$. Thus $1.0\,\mathrm{F}$ is indeed a large capacitance!

Exercise: Properties of a parallel-plate capacitor

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The plates of a parallel-plate capacitor in vacuum are $5.00\,\mathrm{mm}$ apart and $2.00\,\mathrm{m}^2$ in area. A $10.0\,\mathrm{kV}$ potential difference is applied across the capacitor. Compute (a) the capacitance; (b) the charge on each plate; and (c) the magnitude of the electric field between the plates.

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Solution:

- (a) $C = \epsilon_0 \frac{A}{d}$ gives $C = 3.54\,\mathrm{pF}$.
- (b) $Q = CV_{ab}$ gives $Q = 35.4\,\mu\mathrm{C}$.
- (c) $E = \sigma/\epsilon_0$ gives $E = 2.00 \times 10^6\,\mathrm{N/C}$
- We can also find E by $E = V_{ab}/d$.

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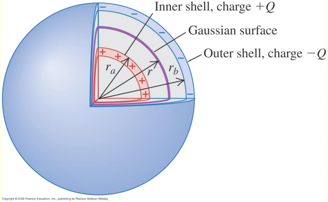
Condenser microphone

Inside a condenser microphone is a capacitor with one rigid plate and one flexible plate. The two plates are kept at a constant potential difference V_{ab}. Sound waves cause the flexible plate to move back and forth, varying the capacitance C and causing charge to flow to and from the capacitor in accordance with the relationship $C = Q/V_{ab}$. Thus a sound wave is converted to a charge flow that can be amplified and recorded digitally.

Exercise: A spherical capacitor

Question:

Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge $+Q$ and outer radius r_a, and the outer shell has charge $-Q$ and inner radius r_b . Find the capacitance of this spherical capacitor.

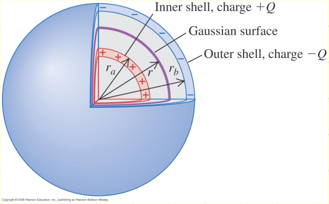


Exercise: A spherical capacitor

Solution:

- The potential at any point between the spheres is $V = Q/4\pi\epsilon_0 r$.
- Hence the potential of the inner $(+)$ conductor at $r = r_a$ wrt that of the outer $(-)$ conductor at $r = r_b$ is

$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$
$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



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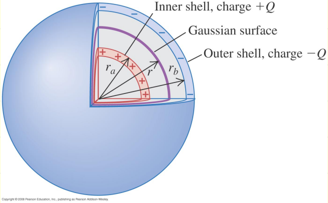
- $$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$
  $$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



- Using $C = Q/V_{ab}$ gives

$$C = \frac{4\pi\epsilon_0}{\frac{1}{r_a} - \frac{1}{r_b}} = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

Exercise

How is this result

$$C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

related to

$$C = \epsilon_0 \frac{A}{d}$$

for the parallel plate capacitor?

Consider a spherical capacitor with inner sphere close to the outer sphere: $r_a \simeq r_b$ while $r_b - r_a = d$.

The area of the sphere is $A \simeq 4\pi r_b^2$ and we get $C \simeq \epsilon_0 \frac{A}{d}$!

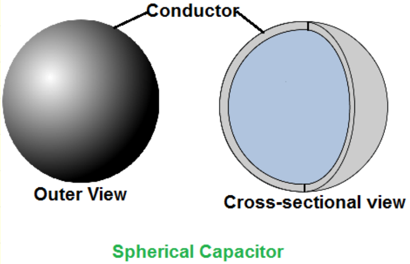
Capacitance of an isolated sphere

- Capacitance of an isolated sphere can be found by considering $r_b \to \infty$ in

$$C = \frac{4\pi\epsilon_0}{\frac{1}{r_a} - \cancelto{0}{\frac{1}{r_b}}}$$

which gives

$$C = 4\pi\epsilon_0 r_a$$

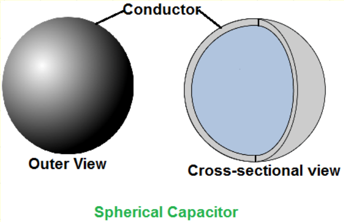


- The result is independent of whether this is a spherical shell or a solid sphere.

Capacitance of the Earth

- The radius of the Earth is $6300\,\mathrm{km}$. This gives

$$\begin{aligned} C &= 4\pi\epsilon_0 r_a \\ &= r_a/k = \frac{6.3\times 10^6\,\mathrm{m}}{9\times 10^9\,\mathrm{m/F}} \\ &\sim 10^{-3}\,\mathrm{F} \end{aligned}$$



- $1\,\mathrm{F}$ is indeed a large capacitance!

Exercise: charged balls

Question:

Two charged balls of charges $q_1 = 4Q$ and $q_2 = -Q$ with radii $r_1 = a$ and $r_2 = 2a$ touch each other and seperated afterwards. What are the final charges?

Exercise: charged balls

Question:

Two charged balls of charges $q_1 = 4Q$ and $q_2 = -Q$ with radii $r_1 = a$ and $r_2 = 2a$ touch each other and seperated afterwards. What are the final charges?

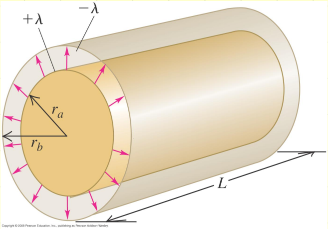
Question:

The total charge is $3Q$. The balls will share the charges in proportion to their capacitance which is proportional to their radii. Thus $q_1' = Q$ and $q_2' = 2Q$.

Exercise: A cylindrical capacitor

Question:

Two long, coaxial cylindrical conductors are separated by vacuum. The inner cylinder has outer radius r_a and linear charge density $+\lambda$. The outer cylinder has inner radius r_b and linear charge density $-\lambda$. Find the capacitance per unit length for this capacitor.



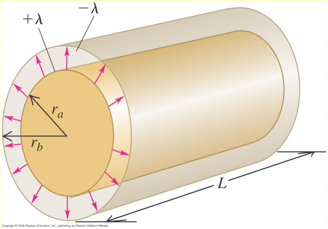
Solution:

- The potential in the space between the cylinders is:

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}$$

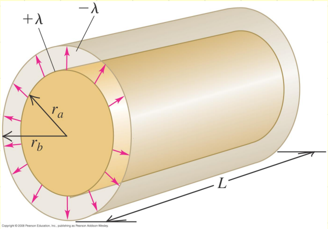
- Then the potential difference between the cylinders is

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$



Solution:

- $$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$



- The total charge Q in a length L is $Q = \lambda L$ and so

$$C = \frac{Q}{V_{ab}} = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

Exercise

How is this result

$$C = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

related to

$$C = \epsilon_0 \frac{A}{d}$$

for the parallel plate capacitor?

Consider a cylindirical capacitor with inner scylinder close to the outer cylinder: $r_a \simeq r_b$ while $r_b - r_a = d \ll r_b$.

$$C = \frac{2\pi\epsilon_0 L}{\ln \frac{r_a + d}{r_a}} = \frac{2\pi\epsilon_0 L}{\ln\left(1 + \frac{d}{r_a}\right)} \simeq \frac{2\pi\epsilon_0 L}{\frac{d}{r_a}}$$

where we used Taylor expansion $\ln(1+x) = x$ if $x \ll 1$.

The area of the cylinder is $A \simeq 2\pi r_a L$ and we get $C \simeq \epsilon_0 \frac{A}{d}$!

Check your understanding

A capacitor has vacuum in the space between the conductors. If you double the amount of charge on each conductor, what happens to the capacitance?

(i) It increases;
(ii) it decreases;
(iii) it remains the same;
(iv) the answer depends on the size or shape of the conductors.

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Capacitors in series and parallel

Capacitors in series and parallel

- Capacitors are manufactured with certain standard capacitances and working voltages
- However, these standard values may not be the ones you actually need in a particular application.
- You can obtain the values you need by combining capacitors; many combinations are possible, but the simplest combinations are a series connection and a parallel connection.

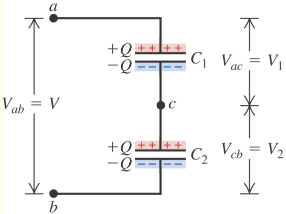
Capacitors in series

- In a series connection the magnitude of charge on all plates is the same.
- $V_{ab} = V_{ac} + V_{cb}$ or $V = V_1 + V_2$
- $V_{ac} = V_1 = Q/C_1$ &
  $V_{cb} = V_2 = Q/C_2$
- $V_{ab} = V = Q(\frac{1}{C_1} + \frac{1}{C_2})$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

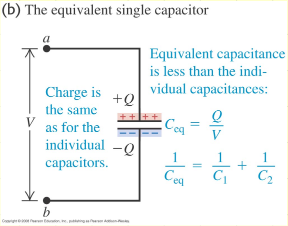
(a) Two capacitors in series

Capacitors in series:
- The capacitors have the same charge Q.
- Their potential differences add: $V_{ac} + V_{cb} = V_{ab}$.



Capacitors in series

- The equivalent capacitance C_{eq} of the series combination is defined as the capacitance of a single capacitor for which the charge Q is the same as for the combination, when the potential difference V is the same.
- The combination can be replaced by an equivalent capacitor of capacitance $C_{\text{eq}} = Q/V$.



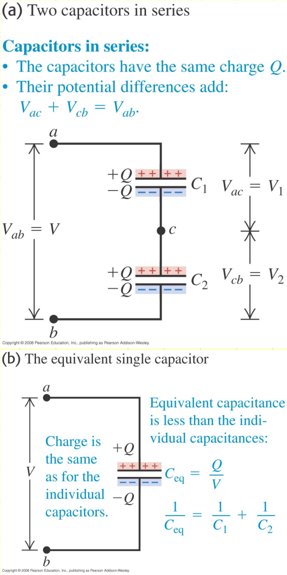
Capacitors in series

- $$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}, \qquad C_{\mathrm{eq}} = \frac{Q}{V}$$

  implies

  $$\boxed{\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

- The reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.



Capacitors in series

- If there are N capacitors in series

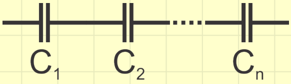
$$\boxed{\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}}$$

- If capacitors all have equal capacitances $(C = C_1 = C_2 = \cdots = C_N)$

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C} + \frac{1}{C} + \cdots \frac{1}{C} = \frac{N}{C}$$

and so $C_{\mathrm{eq}} = C/N$.

- In a series connection the equivalent capacitance is always *less* than any individual capacitance.



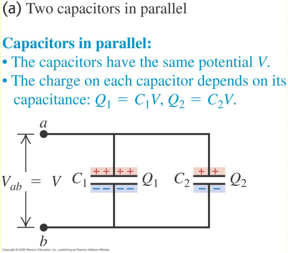
Example:

4 capacitors of $2\,\mu\mathrm{F}$ connected in series has the equivalent capacitance of $0.5\,\mu\mathrm{F}$.

Capacitors in Parallel

- In a parallel connection the potential difference for all individual capacitors is the same and is equal to $V_{ab} = V$.
- $Q_1 = C_1 V$ & $Q_2 = C_2 V$
- The total charge Q of the combination, and thus the total charge on the equivalent capacitor, is $Q = Q_1 + Q_2 = (C_1 + C_2)V$

$$\frac{Q}{V} = C_1 + C_2$$



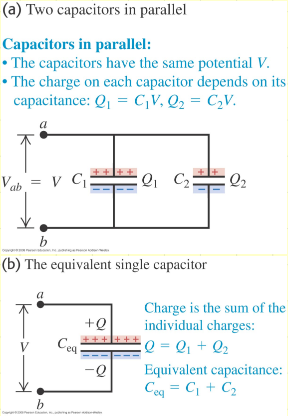
Capacitors in Parallel

- The parallel combination is equivalent to a single capacitor with the same total charge $Q = Q_1 + Q_2$ and potential difference V as the combination

$$C_{\text{eq}} = \frac{Q}{V}$$

- Thus

$$\boxed{C_{\text{eq}} = C_1 + C_2}$$



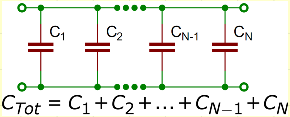
Capacitors in Parallel

- In the same way we can show that for N capacitors in parallel

$$\boxed{C_{\text{eq}} = C_1 + C_2 + \cdots + C_N}$$

The equivalent capacitance of a parallel combination equals the sum of the individual capacitances.

- If capacitors all have equal capacitances ($C = C_1 = C_2 = \cdots = C_N$) $C_{\text{eq}} = NC$
- In a parallel connection the equivalent capacitance is always *greater* than any individual capacitance.



Exercise:

Let $C_1 = 6.0\,\mu\text{F}$, $C_2 = 3.0\,\mu\text{F}$, and $V_{ab} = 18\,\text{V}$. Find the equivalent capacitance and the charge and potential difference for each capacitor when the capacitors are connected

- (a) in series
- (b) in parallel

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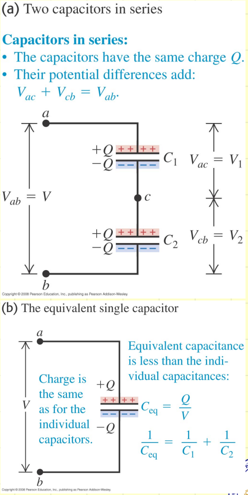

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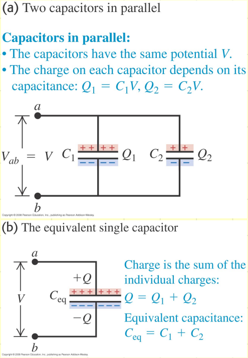


Solution:

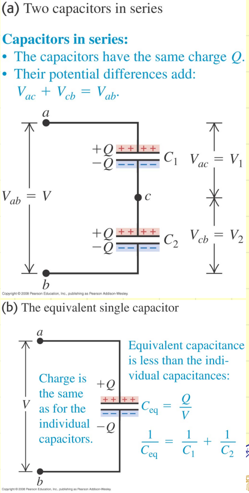
- For a series combination

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6.0\,\mu\text{F}} + \frac{1}{3.0\,\mu\text{F}}$$

$$\Rightarrow C_{\rm eq} = 2.0\,\mu\text{F}$$

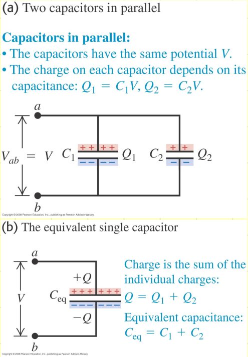
- The charge Q on each capacitor in series is the same as that on the equivalent capacitor:
$Q = C_{\rm eq}V = (2.0\,\mu\text{F})(18\,\text{V}) = 36\,\mu\text{C}$

- The potential difference across each capacitor is inversely proportional to its capacitance:
$V_{ac} = V_1 = \dfrac{Q}{C_1} = \dfrac{36\,\mu\text{C}}{6.0\,\mu\text{F}} = 6\,\text{V}$ and similarly $V_{cb} = \dfrac{Q}{C_2} = \dfrac{36\,\mu\text{C}}{3.0\,\mu\text{F}} = 12\,\text{V}$



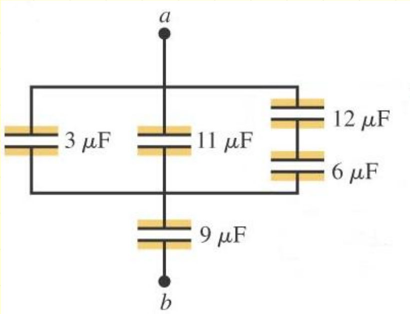
Solution:

- For a parallel combination
  $C_{\text{eq}} = C_1 + C_2 = 6.0\,\mu\text{F} + 3.0\,\mu\text{F} = 9.0\,\mu\text{F}$
- The potential difference across each of the capacitors is the same as that across the equivalent capacitor, $18\,\text{V}$.
- The charge on each capacitor is directly proportional to its capacitance:
  $Q_1 = C_1 V = (6.0\,\mu\text{F})(18\,\text{V}) = 108\,\mu\text{C}$
  $Q_2 = C_2 V = (2.0\,\mu\text{F})(18\,\text{V}) = 54\,\mu\text{C}$



Exercise: A capacitor network

- Find the equivalent capacitance of the five-capacitor network shown.
- $12\,\mu$F and $6\,\mu$F series combination yields $C' = 4\,\mu$F.
- $3\,\mu$F, $11\,\mu$F & $4\,\mu$F paralllel combination yields $C'' = 18\,\mu$F.
- Finally, the $18\,\mu$F and $9\,\mu$F capacitors in series gives $6\,\mu$F



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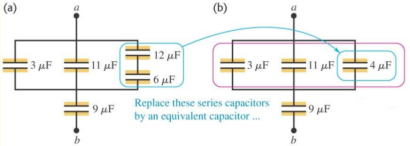

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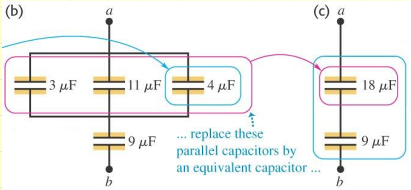

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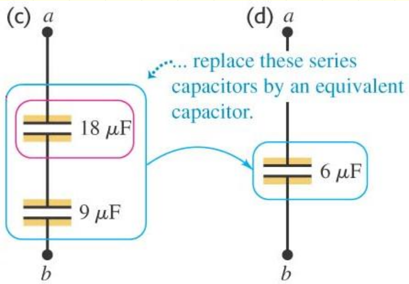


Energy storage in capacitors and electric-field energy

Energy storage in capacitors and electric-field energy

energy stored in capacitor = work required to charge it.

discharge: stored energy is recovered as electrical work.

During charging, at the moment when the charge is q and the potential difference is $v = q/C$, the work $\mathrm{d}W$ required to transfer an additional charge $\mathrm{d}q$ is

$$\mathrm{d}W = v\mathrm{d}q = \frac{q\mathrm{d}q}{C}$$

The total work W needed to increase q from zero to Q is

$$W = \int_0^W \mathrm{d}W = \frac{1}{C}\int_0^Q q\mathrm{d}q = \frac{Q^2}{2C}$$

Potential energy stored in a capacitor

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Energy in capacitor - analogy with strechted spring

Potential energy stored in a capacitor

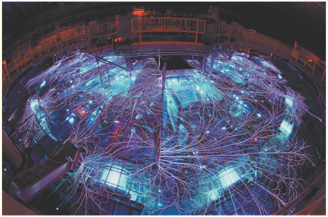
$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- Capacitance measures the ability of a capacitor to store both energy and charge.
- At fixed V, increasing C gives a greater charge Q and greater stored energy $U = \frac{1}{2}CV^2$.
- To transfer fixed Q, W is inversely proportional to C, i.e. the greater C, the easier it is to charge Q

Recall: elastic potential energy $U = \frac{1}{2}kx^2$

Identify: $Q \Leftrightarrow x$ and $1/C \Leftrightarrow k$

Applications of Capacitors: Energy Storage



The Z machine has a large number of capacitors in parallel. Arcs are produced during discharge into a spool of thread. Heats the target to $T > 2 \times 10^9$K. For a brief space of time: 80 times the power output of all the electric power plants on earth combined!

In other applications, the energy is released more slowly: Similar to springs in the suspension of an automobile a capacitor in an electronic circuit can smooth out unwanted variations in voltage due to power surges.

Think of the energy as being stored *in the field* in the region between the capacitor plates.

The energy density in a vacuum

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(\epsilon_0 A/d)(Ed)^2}{Ad} = \frac{1}{2}\epsilon_0 E^2$$

Valid for any electric-field configuration in vacuum.

Remember: Electric-field energy is electric potential energy.

energy as being a shared property of all the charges

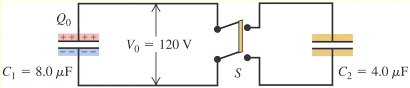
OR

energy as being a property of the E-field that the charges create

Example: Transferring charge/energy between capacitors

Question:

We connect a capacitor $C_1 = 8.0\,\mu\text{F}$ to a power supply, charge it to a potential difference $V_0 = 120\,\text{V}$, and disconnect the power supply. Switch S is open.



(a) What is the charge Q_0 on C_1?

(b) What is the energy stored in C_1?

(c) Capacitor $C_2 = 4.0\,\mu\text{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor?

(d) What is the final energy of the system?

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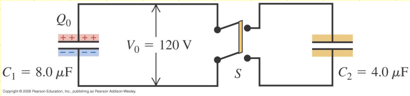

(a) What is the charge Q_0 on C_1?

Solution a:

The initial charge Q_0 on C_1 is

$$Q_0 = C_1 V_0 = (8.0\,\mu\text{F})(120\,\text{V}) = 960\,\mu\text{C}$$

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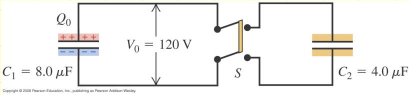
We connect a capacitor $C_1 = 8.0\,\mu\text{F}$ to a power supply, charge it to a potential difference $V_0 = 120\,\text{V}$, and disconnect the power supply. Switch S is open.

(b) What is the energy stored in C_1?



Solution b:

The energy initially stored in C_1 is

$$U_{\text{initial}} = \frac{1}{2} Q_0 V_0 = \frac{1}{2}(960 \times 10^{-6}\,\text{C})(120\,\text{V}) = 0.058\,\text{J}$$

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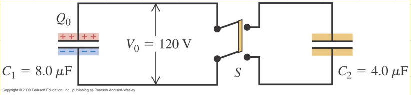

(c) Capacitor $C_2 = 4.0\,\mu\text{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor?

Solution c: 1 st step

When we close the switch, the positive charge Q_0 is distributed over the upper plates of both capacitors and the negative charge $-Q_0$ is distributed over the lower plates.

Example: Transferring charge/energy between capacitors

Question:

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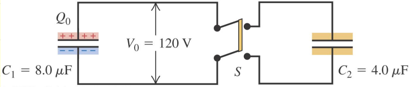


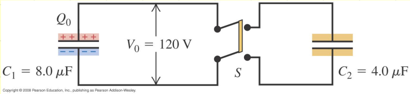
(c) Capacitor $C_2 = 4.0\,\mu\text{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor?

Solution c: 2nd step

Let Q_1 and Q_2 be the magnitudes of the final charges on the capacitors. Conservation of charge requires that $Q_1 + Q_2 = Q_0$. The potential difference V between the plates is the same for both capacitors because they are connected in parallel, so the charges are $Q_1 = C_1 V$ and $Q_2 = C_2 V$.

Question:

We connect a capacitor $C_1 = 8.0\,\mu\text{F}$ to a power supply, charge it to a potential difference $V_0 = 120\,\text{V}$, and disconnect the power supply.



(c) Capacitor $C_2 = 4.0\,\mu\text{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor?

Solution c: 3rd step

We now have three independent equations relating the three unknowns Q_1, Q_2, and V. Solving these, we find

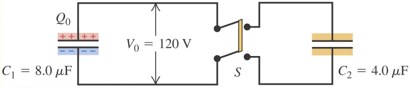
$$V = \frac{Q_0}{C_1 + C_2} = 80\,\text{V}$$

$Q_1 = 640\,\mu\text{C}$ and $Q_2 = 320\,\mu\text{C}$

Example: Transferring charge/energy between capacitors

Question:

We connect a capacitor $C_1 = 8.0\,\mu\mathrm{F}$ to a power supply, charge it to a potential difference $V_0 = 120\,\mathrm{V}$, and disconnect the power supply.



(d) What is the final energy of the system?

Solution d:

The final energy of the system is

$$U_{\text{final}} = \frac{1}{2}Q_1V + \frac{1}{2}Q_2V = \frac{1}{2}Q_0V = 0.038\,\mathrm{J}$$

Note: $U_{\text{final}} < U_{\text{initial}}$; the difference was converted to energy of some other form. The conductors become a little warmer because of their resistance, and some energy is radiated as EM waves.

Example: Electric-field energy

Question: What is the magnitude of the electric field required to store $1.00\,\mathrm{J}$ of electric potential energy in a volume of $1.00\,\mathrm{m}^3$ in vacuum?

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The E-field can be calculated from the energy density

$$E = \sqrt{\frac{2u}{\epsilon_0}} = \sqrt{\frac{2\left(1.00\mathrm{J/m}^3\right)}{8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2}} = 4.75 \times 10^5\,\mathrm{V/m}$$

Question: If the field magnitude is 10 times larger than that, how much energy is stored per cubic meter?

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Question: If the field magnitude is 10 times larger than that, how much energy is stored per cubic meter?

u is proportional to E^2,

if E increases by a factor of 10, then u increases by $10^2 = 100$.

Example: Electric field energy stored in a uniform sphere of charge Q and radius a

Hint:

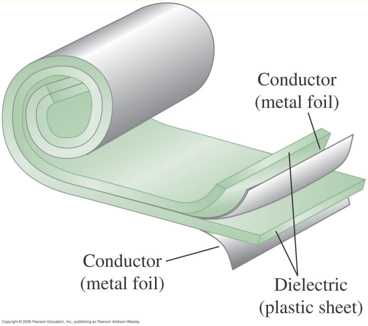
- Outside the sphere $E = Q/4\pi\epsilon_0 r^2$
- and so $u = \frac{1}{2}\epsilon_0 E^2 \propto 1/r^4$.
- The total energy is $U = \int_V u \, \mathrm{d}V$ where $\mathrm{d}V = 4\pi r^2 \, \mathrm{d}r$.
- $U_{\text{out}} = \int_a^\infty u 4\pi r^2 \, \mathrm{d}r = ?$
- Inside the sphere $Qr/4\pi\epsilon_0 a^3$
- and so $u = \frac{1}{2}\epsilon_0 E^2 \propto r^2$.
- $U_{\text{in}} = \int_0^a u 4\pi r^2 \, \mathrm{d}r = ?$
- $U = U_{\text{in}} + U_{\text{out}} = \frac{3}{5}\frac{Q^2}{4\pi\epsilon_0 a}$

Dielectrics

Dielectrics

Most capacitors have a nonconducting material (dielectric) between their conducting plates. Why?

- maintains two large metal sheets at a very small separation without actual contact
- prevents dielectric breakdown
- capacitance is greater with dielectric compared to vacuum



A common type of capacitor uses dielectric sheets to separate the conductors.

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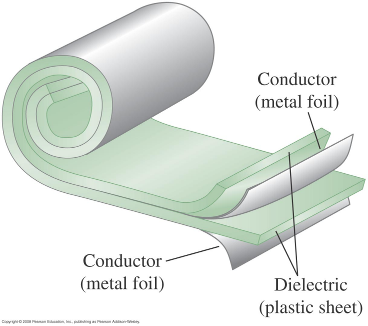

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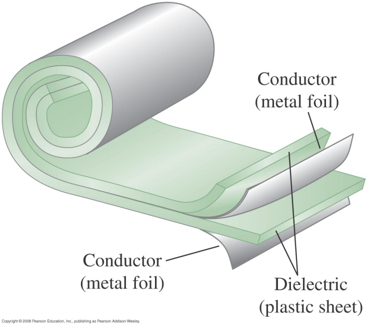

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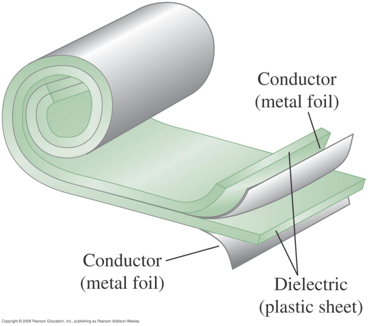


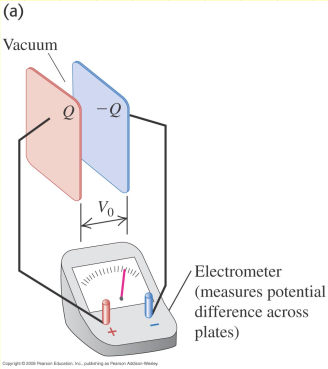
A common type of capacitor uses dielectric sheets to separate the conductors.

Dielectric constant of a material K

A sensitive *electrometer* is a device that measures the potential difference between two conductors without letting any appreciable charge flow from one to the other.

- without dielectric: charge Q and potential V_0
- with dielectric: charge Q and potential $V < V_0$
- $C = Q/V$ increases $\Rightarrow C > C_0$
- dielectric constant of the material

$$K = C/C_0 \quad \text{and} \quad V = V_0/K$$



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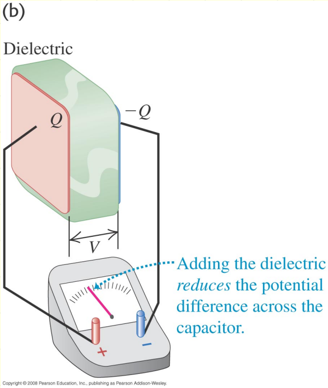

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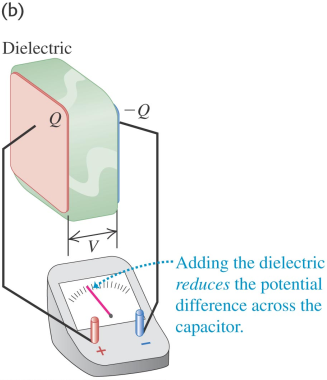

$$K > 1$$

Values of Dielectric Constant K at $20°C$

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Water as a dielectric?

- While water has a very large value of $K = 80$!
- Would we use it in capacitors?
- No! Why?
- While pure water is a very poor conductor, it is also an excellent ionic solvent.
- Any ions that are dissolved in the water will cause charge to flow between the capacitor plates, so the capacitor discharges.

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Leakage Current

- No real dielectric is a perfect insulator.
- Hence there is always some leakage current between the charged plates of a capacitor with a dielectric.
- We tacitly ignored this effect when we derived expressions for the equivalent capacitances of capacitors in series ($C^{-1} = C_1^{-1} + C_2^{-1}$) and in parallel ($C = C_1 + C_2$).
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Induced Charge and Polarization

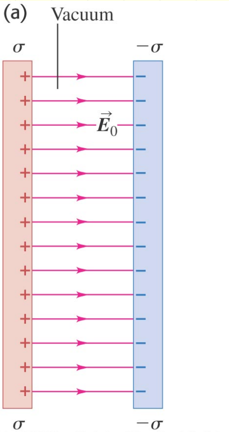
Induced Charge and Polarization

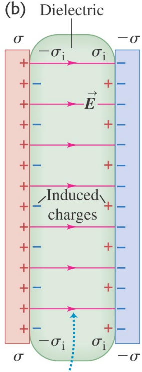
- Vacuum E_0, E with dielectric

  $$E = E_0/K$$

  $K > 1$

- induced charge of the opposite sign appears on each surface of the dielectric





For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

$$E_0 = \frac{\sigma}{\epsilon_0} \quad \text{and} \quad E = \frac{\sigma - \sigma_i}{\epsilon_0} \quad \text{so that} \quad \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

Capacitance and energy with dielectric

The permittivity of a dielectric is denoted by ϵ.

$$\epsilon = K\epsilon_0, \qquad K > 1$$

Capacitance of a parallel plate capacitor with dielectric

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

Electric energy density in a dielectric

$$u = \frac{1}{2} K\epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

A capacitor with and without a dielectric

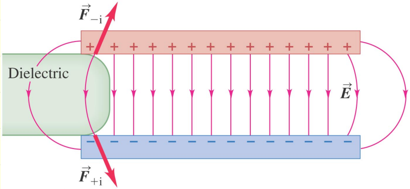
Adding a dielectric between the plates of a capacitor increases the capacitance by a factor of K.

For a given amount of charge, adding the dielectric also reduces V, E-field, the electric energy density, and the total stored energy, all by a factor of $1/K$.

Energy storage with and without a dielectric

$$U_0 = \frac{1}{2}\frac{Q^2}{C_0} > \frac{1}{2}\frac{Q^2}{C} = U \Rightarrow \quad \text{energy decreases at fixed} \quad Q$$

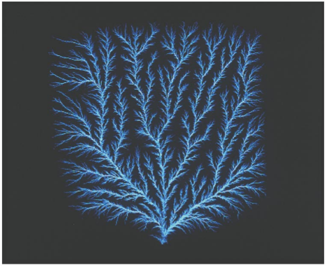
As a result there is force pulling the dielectric slab into the capacitor.



Dielectric Breakdown

Dielectric Breakdown

- When a dielectric is subjected to a sufficiently strong electric field, dielectric breakdown takes place and the dielectric becomes a conductor.
- Lightning is a dramatic example of dielectric breakdown in air.



Dielectric breakdown occurs when the E-field is so strong that e^- are ripped loose from their molecules and crash into other molecules, liberating even more e^-s. This avalanche of moving charge forms a spark or arc discharge.

The maximum E-field magnitude that a material can withstand without the occurrence of breakdown is called its **dielectric strength**.

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

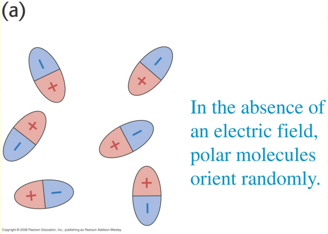
This quantity is affected significantly by temperature, trace impurities, small irregularities in the metal electrodes, and other factors that are difficult to control.

The dielectric strength of dry air is about 3×10^6 V/m.

Molecular model of induced charge

Polar molecules in E-field

(a) When no electric field is present in a gas or liquid with polar molecules, the molecules are oriented randomly



(b) In an electric field, however, they tend to orient themselves. Because of thermal agitation, the alignment of the molecules with $\vec{\mathbf{E}}$ is not perfect.

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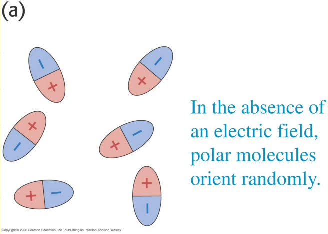

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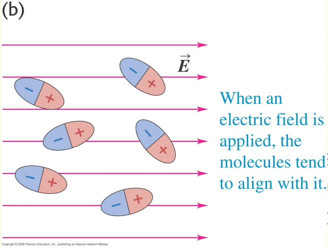
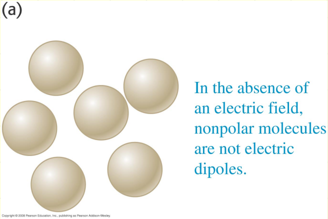


Nonpolar molecules in E-field

(a) A molecule that is not ordinarily polar will become a dipole when placed in an $\vec{\mathbf{E}}$ field because $\vec{\mathbf{E}}$ the positive and negative charges in the molecules in opposite directions.

(b) This causes a redistribution of charge within the molecule. Such dipoles are called *induced* dipoles.



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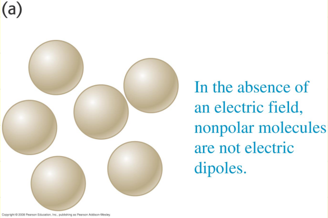
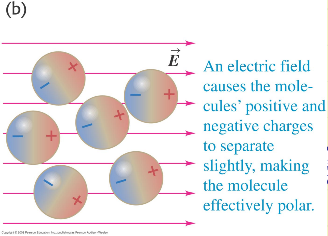
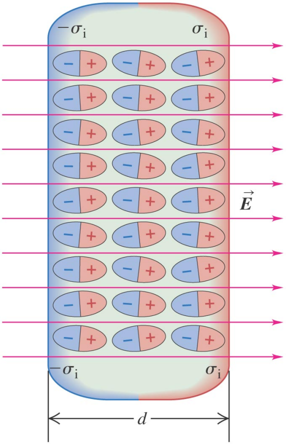




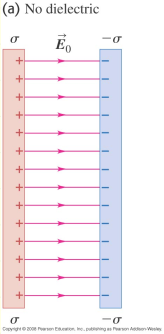
Molecular model of induced charge

With polar/nonpolar molecules, the redistribution of charge caused by the field leads to the formation of a layer of charge on each surface of the dielectric material.

- surface charge densities σ_i
- these charges are not free to move indefinitely
- called *bound charges*
- in the interior of the material the net charge per unit volume remains zero
- this redistribution of charge is called *polarization*
- the material is said to be polarized.

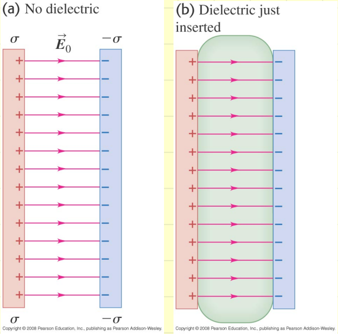


Capacitor with dielectric



(a) the original field inside the capacitor without dielectric slab

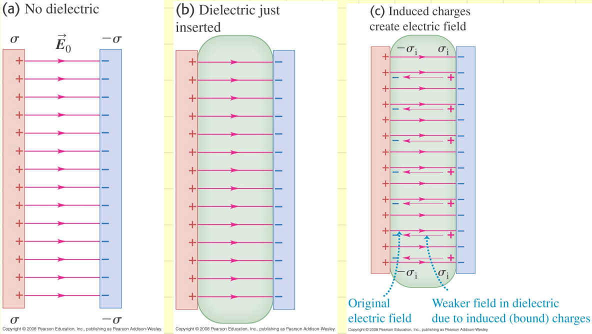
Capacitor with dielectric



(a) the original field inside the capacitor without dielectric slab

(b) dielectric has been inserted but no rearrangement of charges

Capacitor with dielectric



(a) the original field inside the capacitor without dielectric slab

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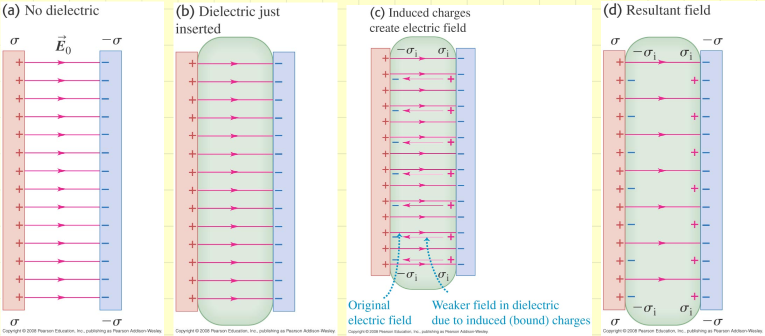
Capacitor with dielectric



(a) the original field inside the capacitor without dielectric slab

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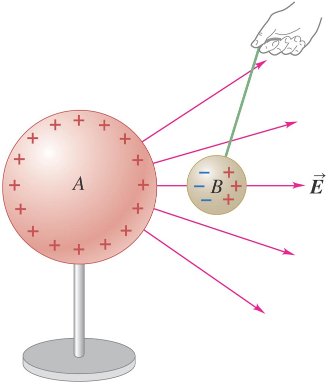
(c) additional field set up in the dielectric by its induced surface charges (opposite to the original field

(d) the resultant field in the dielectric, decreased in magnitude

Force on an uncharged object by a charged object

- induced positive charges on B experience a force toward the right
- force on the induced negative charges is toward the left
- the negative charges are closer to A, and thus are in a stronger field
- the force toward the left is stronger (attraction)
- works similarly for conductors



Gauss' law in dielectrics

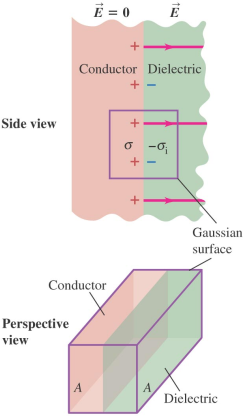
Gauss' law in dielectrics

Apply Gauss' law to the rectangular box shown by purple lines

- The left side is embedded in the conductor $\Rightarrow E = 0$
- The right side is embedded in the dielectric with field magnitude E
- $E_\perp = 0$ on other four sides
- $Q_{\text{encl}} = (\sigma - \sigma_i)A$
- $EA = Q_{\text{encl}}/\epsilon_0$ gives

$$EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0}$$

Not clear since σ_i and E in dielectric unknown.



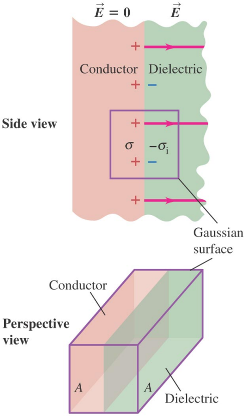
Gauss' law in dielectrics

Apply Gauss' law to the rectangular box shown by purple lines

- Recall $\sigma_i = \sigma(1 - 1/K)$ which is equivalent to $\sigma - \sigma_i = \sigma/K$
- Using this with $EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0}$ we get $KEA = \sigma A/\epsilon_0$
- We thus write Gauss' law in dielectrics as

$$\boxed{\oint K\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}-\text{free}}}{\epsilon_0}}$$

- Equation says that the flux of KE, not E, through the Gaussian surface is equal to the enclosed *free* charge σA divided by ϵ_0.



A spherical capacitor with dielectric

Problem: Use Gauss's law to find the capacitance of the spherical capacitor of if the volume between the shells is filled with an insulating oil with dielectric constant K.

Consider a spherical Gaussian surface of radius $r_a < r < r_b$.

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Using $E(r) = Q/\left(4\pi K\epsilon_0 r^2\right)$, the potential difference is

$$V = \frac{Q}{4\pi K\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

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and the capacitance follows as

$$C = \frac{Q}{V} = \frac{4\pi K\epsilon_0 r_a r_b}{r_b - r_a}$$

Exercise: Repeat when capacitor is partially filled.