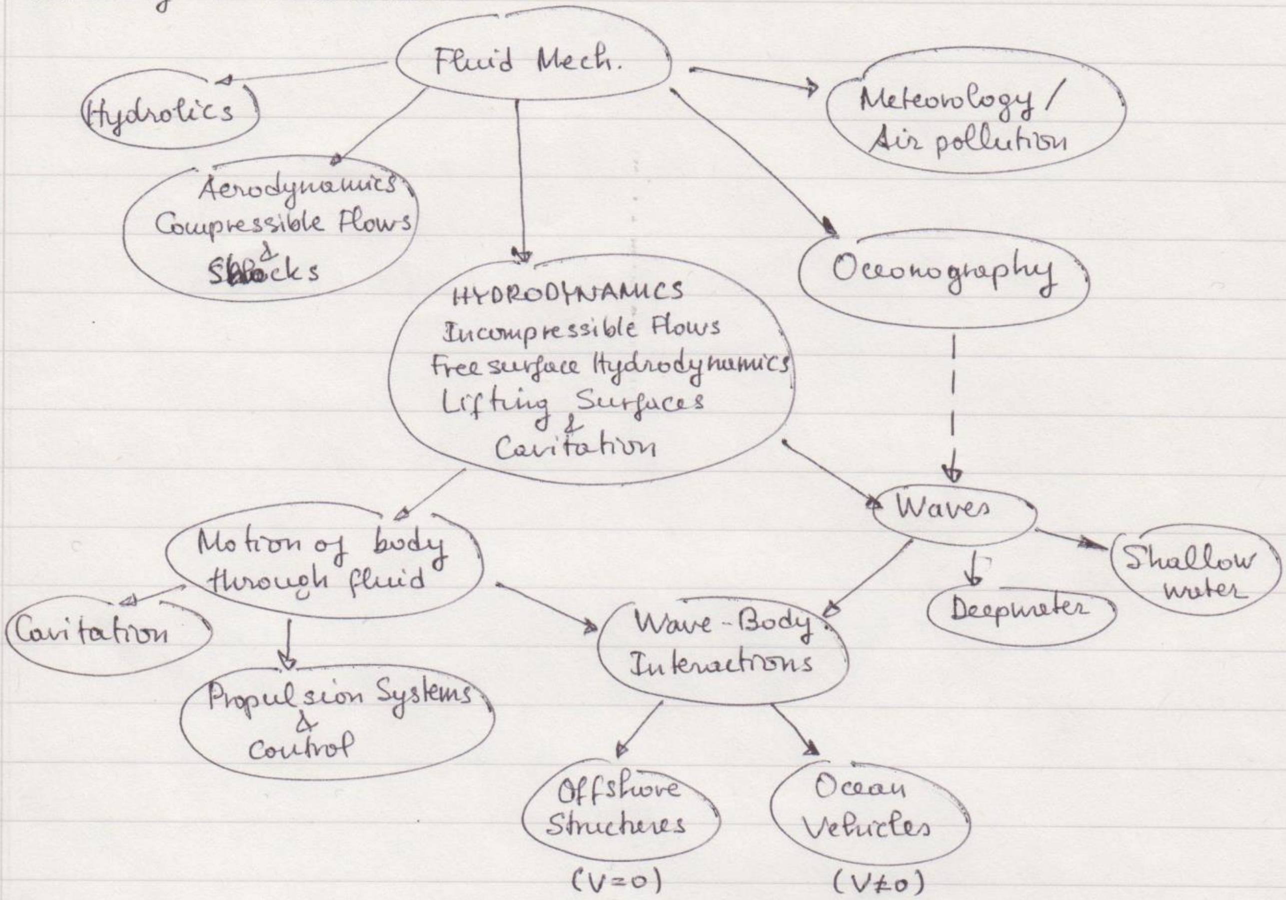
FLOATING BODY HYDRODYNAMICS

Introduction

Floating Body Hydrodynamics, more generally, marine hydrodynamics is the branch of Fluid Mechanics :



In the present course, we limit our attention to the following context :

- Physics, experiments and similarity
- Equation of motions for real and ideal fluids
- Modeling techniques in ideal fluid flow
- Free surface effects and wave-body interaction

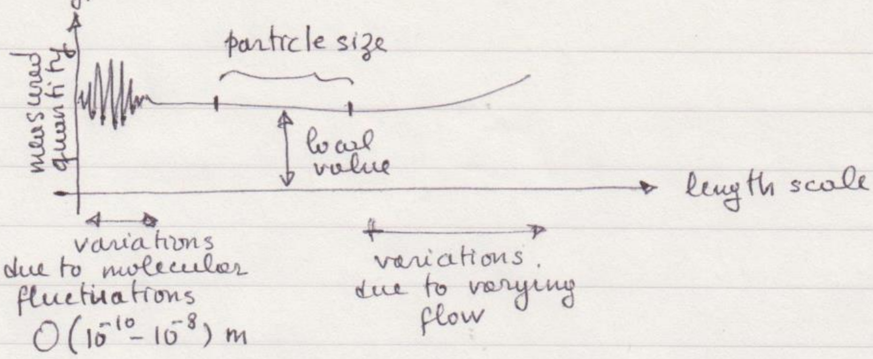
Unlike a solid, a fluid represents both liquids and gases whose particles move easily among themselves. Fundamental laws of mechanics and continuum hypothesis apply to fluids and solids whereas constitutive laws differs in solids and fluids. To be more expressive, fundamental laws are

i) Conservation of mass

ii) Conservation of momentum

iii) Conservation of energy are applied to solids as well as fluids.

Continuum hypothesis states that



To obtain local value; smallest length scale $\sim 10^{-5}$ m $\Rightarrow$ volume $\sim 10^{-15} m^3$ which corresponds 10^{13} molecules of water

On the other hand, solids and fluids obey different constitutive laws. Constitutive laws relate dynamic properties (force, stress, etc) to kinematic properties (position, velocity, etc.). For example in a mass-spring system, Hook's law relate force to displacement as $F = kx$. In a liquid, Archimedes principle relates hydrostatic pressure to the position:

$$p = \rho g z \quad .$$

Dimensional Analysis and similitude

In most cases a physical phenomenon is studied by dimensional analysis. This would be helpfull as well in formulating the relations which link 'model scale experiments to full scale phenomenon. Pi-theorem is derived for this purpose: Denote the physical quantity as Q and consider the 3 fundamental physical units; mass (M), length (L) and time (T). If Q depends on N-1 significant parameters, there will be a total of N interrelated dimensional quantities including Q. If we intend to non-dimensionalize the parameters including Q, then there will be N-3 non-dimensional quantities which can be interrelated. Take for example of free-fall of a body in a vacuum. Mathematically; Q in this case is the vertical displacement of the body which is interrelated with $y = f(t, g, m)$. Total 4 parameters give normally $4-3=1$ non-dimensional quantity. By using a mathematical approach: $y = \mathcal{E} g^a t^b m^c$. Thus

$$L = \mathcal{E}(LT^{-2})^a (T)^b (M)^c$$

$\therefore$

$$\left.\begin{aligned} L&: \quad 1 = a \\ T&: \quad 0 = -2a + b \Rightarrow b = 2 \\ M&: \quad 0 = c \end{aligned}\right\} \text{This gives } y = \mathcal{E} g t^2 \text{ and } \frac{y}{gt^2} = \mathcal{E}.$$

Now, let's start with 3 basic forces meaningfull in hydrodynamics.
The order of magnitude of inertial force may be given by $\rho U^2 L^2$
" " " gravitational " " " $\rho g L^3$
" " " " viscous " " " " $\mu U L$
where U is speed and kinematic viscousity $\nu = \mu/\rho$ $[M^2/s]$. We can compare the forces by forming the ratios:

$$\frac{\text{Inertial F.}}{\text{Gravitational F}} = \frac{U^2}{gL} = F_r$$

$$\frac{\text{Inertial F.}}{\text{Viscous F.}} = \frac{\rho U L}{\mu} = \frac{UL}{\nu} = R_e$$

$$\frac{\text{Gravitational F.}}{\text{Viscous F.}} = \frac{gL^2}{\nu U}$$

In a wave motion U may be replaced by ωL (where ω is the circular frequency), then $F_r = \frac{\omega^2 L^2}{gL} = \frac{\omega^2 L}{g}$

Now, for example, take vortex shedding from a circular cylinder. Investigating a circular cylinder moving with a constant velocity implies that there are two physical quantities in question, namely; lift force and frequency. Let's study them independently: maximum lift, L_{max}, by intuition, depend on ρ, U, d, μ. Thus

$$L_{max} = C(\rho, \mu, U, d)$$

Since there are 5 Interrelated dimensional quantities which yields according to Pi-theorem two non-dimensional quantities:

$$L_{max} / \rho U^2 d^2 = C\left(\frac{Ud}{\nu}\right) = C_L(R_e)$$

On the other hand, frequency $f = \omega/2\pi$ where ω denotes circular frequency. We may introduce dimensional parameters U, μ, d, ρ which are interrelated with frequency f. Thus;

$$f = S(U, \mu, d, \rho)$$

We expect that there are two non-dimensional quantities such that

$$\frac{f\,d}{U} = S(R_e)$$

where non-dimensional quantity S is called Strouhal number.

Now let's study wave forces on a fixed body in waves;

Force F (our quantity Q in this problem), may depend on 9 significant parameters such as ρ (water density), g (gravity), ν (kin. viscousity), A (wave amplitude), λ (wave length), β (angle of incidence of the waves), h (depth of water), L (characteristic length) and t (time). In this case we end up with 7 non-dimensional parameters according to Pi-theorem:

$$\frac{F}{\rho g L^3} = C_F\left(A/L,\ h/\lambda,\ L/\lambda,\ \beta,\ \frac{\omega L A}{\nu},\ \omega t\right)$$

Here, ω is not a new parameter, since it depends on g and λ; $\omega^2 = \frac{2\pi g}{\lambda}$ (in deep water).

Now we can investigate the meaning of the non-dimensional parameters and relate them to the basic forces in hydrodynamics. To do this, take the ratio of $\frac{\text{viscous force}}{\text{inertial force}} \propto \frac{U^2}{\dot{U}L} = \frac{(\omega A)^2}{(\omega^2 A)L} = \frac{A}{L}$. This is an interesting result which states that if this ratio is small, then viscous effects could be negligable. On the other hand, if A/λ and A/h is small, then linearized version of the waves could be used. Thus, under these circumstances force coefficient can be given as:

$$C_F = C_{F_a}\cos(\omega t + \tau) = C_M \dot{U} + C_d U$$ where C_m and C_d denotes hydrodynamic mass and damping coefficients.

When $L/\lambda \ll 1$, hydrostatic force's gains importance and then mass coefficient, C_m, is given as: $C_M = (m_a + \rho\nabla)/\rho g L^3$.

On the other hand, when A/L can no longer be accepted as small quantity, body force is governed by viscous drag as:

$$F = \frac{\rho}{2} L^2 U|U| C_D(R_e).$$

When $A/L = O(1)$, that is inertial and viscous forces are of the same order of magnitude, the force is expressed, namely by Morison's equation, as;

$$F = (m_a + \rho\nabla)\dot{U} + \frac{\rho}{2} L^2 U|U| C_D(R_e).$$

Note that the above Morison's equation ignores the inherent interaction between viscous and inertial effects.

HW/1.1 Natural frequency of the sloshing water in a swimming pool of a cruiser ship is studied by an experimental set-up with a scale of 1/10. If the natural frequency is found 3.5 Hz, then what is the natural frequency for full scale?

> Reading Assignment ① (for the Quiz I) Mar. Tech. Vol. 43, No. 4, Oct. 2006, pp 170-179

BASIC FLOW DESCRIPTIONS

Lagrangian Description : When a flow is modeled by fluid particles which carry flow properties, such as density, pressure, velocity etc., and if we follow the advance of the particles (its properties may change in time meanwhile), then we end up with Lagrangian description. The method of Smoothed Particle Hydrodynamics which represents the fluid flow by a large number of particles with properties $\rho_i(t)$, $\vec{v}_i(t)$, $p_i(t)$ strictly uses Lagrangian description and follow the trajectories of the fluid particles. In that sense it is simple to model the flow - as conservation of mass and momentum apply to each particle -, but its computational cost is too high!

Eulerian Description : Instead of tracking each fluid particle in a fluid flow, Eulerian description is focused on the evolution of the flow properties at every point in space as time advances, such as $\vec{v}(\vec{x},t)$, $\rho(\vec{x},t)$, $p(\vec{x},t)$.

For example ; a wave probe fixed in space is indeed an Eulerian measuring device, whereas a neutrally bouyant wave probe is a Lagrangian measuring device, i.e. fixed probe provides records in $\xi(\vec{x},t)$ - which defines wave elevation at position $\vec{x}$ and time t, and bouyant probe provides data in pairs ; $\xi(t)$ and $\vec{x}(t)$. A streamline on which the fluid velocity $\vec{v}$ at a given time is tangent is an Eulerian definition. A path line which is the trajectory of a given particle in time is a Lagrangian definition.

Continuous Flow : A continuous flow requires the following ; $\vec{v}(\vec{x},t)$ should be finite and a continuous function of $\vec{x}$ and t, that is $\nabla\vec{v}$ and $\frac{\partial\vec{v}}{\partial t}$ are finite. Thus (without proof), if the flow is continuous two particles that are neighbors will always remain neighbors.

In a finite fluid volume - if no segment of fluid can be joined or broken apart - ; it is called material volume and will remain material. The interface between two material volumes is a material surface and will remain material. A free surface - between air and water - is an example of a material surface.