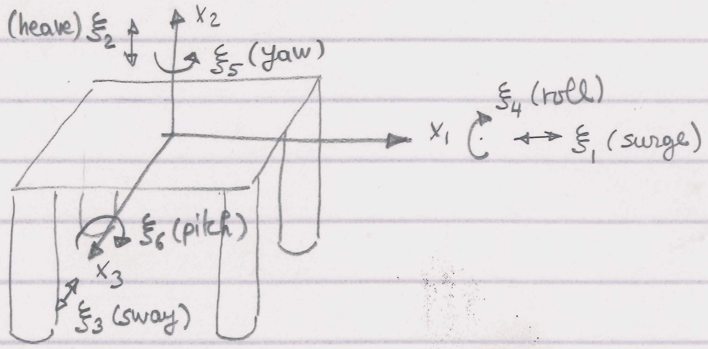
Body Response in Regular Waves

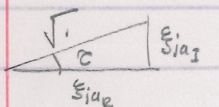
Let us first define 6 degrees of freedom of motion:



We assume plane progressive waves of amplitude ξ_a with frequency ω. If the excitation frequency is ω, then the resultant motion has a frequency ω but with a phase shift. We can now consider the six motions $\xi_j(t) = Re\{\xi_{ja}\, e^{i\omega t}\}$

ξ_{ja} denotes the complex amplitude of the j^{th} mode of motion. Note that:

$$\xi_{ja} = \xi_{jaR} + i\,\xi_{jaI} \quad ; \quad \xi_j = \xi_{jaR}\cos\omega t - \xi_{jaI}\sin\omega t$$



$$= \sqrt{\xi_{jaR}^2 + \xi_{jaI}^2}\left[\frac{\xi_{jaR}}{\sqrt{\xi_R^2+\xi_I^2}}\cos\omega t - \frac{\xi_{jaI}}{\sqrt{\xi_R^2+\xi_I^2}}\sin\omega t\right]$$

$$= |\xi_{ja}|\cos(\omega t + \tau)$$

where $\tau = \arctan\dfrac{\xi_{jaI}}{\xi_{jaR}}$ is the phase shift with respect to wave forcing.

Then the corresponding velocities are $U_j(t) = Re\{i\omega\,\xi_{ja}\, e^{i\omega t}\}$

If we assume small amplitude waves, we can decompose the total velocity potential into:

$$\Phi(x,y,z;t) = Re\Big\{\Big(\underbrace{\sum_{j=1}^{6}\Phi_j(x,y,z)}_{\text{Radiation pot.}} + \underbrace{\Phi_D(x,y,z)}_{\text{Diffraction pot.}}\Big)e^{i\omega t}\Big\}$$

Radiation potentials are obtained in the absence of incident waves and boundary conditions defined on the body surface are given as;

$$\frac{\partial\Phi_j}{\partial n} = i\omega\,\xi_{ja}\, n_j \quad ; \quad j=1,2,3 \qquad \text{and}$$

$$\frac{\partial\Phi_j}{\partial n} = i\omega\,\xi_{ja}(\vec{r}\times\vec{n})_{j-3} \quad ; \quad j=4,5,6$$

These forced motion problems are also called radiation problem, since the related potential solutions must satisfy also radiation condition at infinity which ensures the uniqueness of the solution. In 2-D, the radiation condition

$$\Phi_j \propto e^{\mp ikx} \quad ; \quad x \to \pm\infty$$

in 3-D

$$\Phi_j \propto R^{-1/2}\, e^{-ikR} \quad ; \quad R \to \infty$$

On the other hand diffraction ~~potenti~~ solution is characterized by the kinematic b.c. on the body $\frac{\partial \Phi_D}{\partial n} = 0$ where $\Phi_D = \Phi_0 + \Phi_7$

Here Φ_0 denotes the incident waves and Φ_7 represents the scattered waves due to the body fixed among the waves. In the ~~rao~~ diffraction problem, the incident waves potential is known, thus the diffraction condition turns out to be: $\frac{\partial \Phi_7}{\partial n} = -\frac{\partial \Phi_0}{\partial n}$ on the body.

Both radiation and diffraction potentials are governed by

$\nabla^2 \Phi_j = 0$ throughout the fluid domain. The other condition that must be satisfied is the linearized free surface b.c.

$$-\frac{\omega^2}{g}\Phi_j + \frac{\partial \Phi_j}{\partial z} = 0 \qquad \text{on } z=0 \quad ; j = 0, 1, \ldots 7.$$

From the diffraction problem we obtain excitation loads (or forces), and from the radiation problem we obtain addes mass (which is in phase with acceleration) and damping ~~coeffici~~ (which is phase with velocity) and restoring forces and moments (which is in phase with displacement). To calculate first-order force and moment;

$$p = -\rho\left(\frac{\partial \Phi}{\partial t} + gz\right) = -\rho \,\mathrm{Re}\left\{\left(\sum_{j=1}^{6}\Phi_j + (\Phi_0 + \Phi_7)\right) i\omega e^{i\omega t}\right\} - \rho g z$$

Then $\vec{F}$ and $\vec{M}$ are obtained;

$$\begin{pmatrix}\vec{F}\\ \vec{M}\end{pmatrix} = -\rho g \iint_{S_B}\begin{pmatrix}\vec{n}\\ \vec{r}\times\vec{n}\end{pmatrix} z\, dS - \rho\,\mathrm{Re}\sum_{j=1}^{6} i\omega e^{i\omega t}\iint_{S_B}\begin{pmatrix}\vec{n}\\ \vec{r}\times\vec{n}\end{pmatrix}\Phi_j\, dS$$

added mass and damping terms

$$-\rho\,\mathrm{Re}\; i\omega e^{i\omega t}\iint_{S_B}\begin{pmatrix}\vec{n}\\ \vec{r}\times\vec{n}\end{pmatrix}(\Phi_0 + \Phi_7)\, dS.$$

Hydrostatic force & moment

exciting forces & moments.

The added mass and damping is extracted from the force

$$\vec{F_i} = -\sum_{j=1}^{6}\left(a_{ij}\dot{U}_j + b_{ij}U_j\right) \qquad (a_{ij}\ \&\ b_{ij} \text{ are symmetric } 6\times 6 \text{ matrices}).$$

Thus for a full ~~motion~~ wave responce analysis we have

$$(m_{ij} + a_{ij})\ddot{\xi}_j(t) + b_{ij}\dot{\xi}_j(t) + c_{ij}\xi_j(t) = \mathrm{Re}\underbrace{\left\{A e^{i\omega t}\right\}}_{\text{Exciting force}}$$

body mass/moment matrix; added masses; damping coefficients; restoring force; Exciting force. (due to the incoming waves).

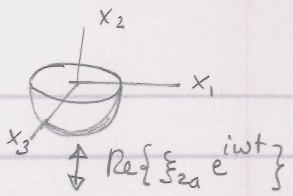
where the matrix m_{ij} is defined by

$$m_{ij} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & -mz_G \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & mz_G & 0 & 0 \\ 0 & 0 & mz_G & I_{11} & I_{12} & I_{13} \\ 0 & 0 & 0 & I_{21} & I_{22} & I_{23} \\ -mz_G & 0 & 0 & I_{31} & I_{32} & I_{33}\end{bmatrix}$$

mass moments of inertia (ρ_B : mass density)

$$I_{ij} = \iiint_V \rho_B\left[\vec{x}\cdot\vec{x}\,\delta_{ij} - x_i x_j\right] dV \qquad ; i,j = 1, 2, 3.$$

For example, take the heave motion of a sphere



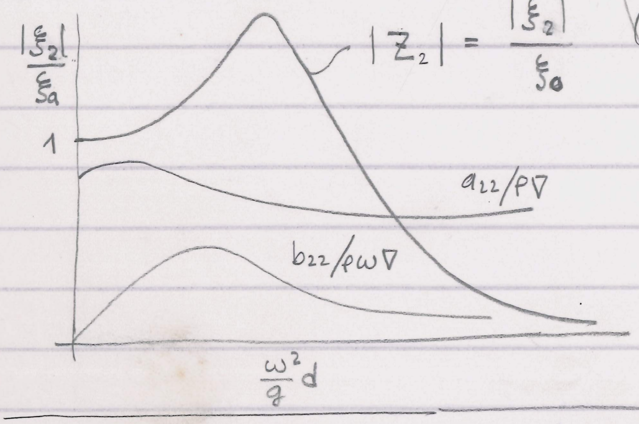
$$(m_{22}+a_{22})\,\ddot{\xi}_2(t) + b_{22}\,\dot{\xi}_2(t) + c_{22}\,\xi_2(t) = Re\{A\,e^{i\omega t}\}$$

$m_{22} = m$ = mass of the sphere , a_{22} = added mass in z direction due to the motion along z-direction. Here we assumed that other motions are decoupled.

b_{22} = damping coeff. ; $c_{22} = \rho g S$

where $S = \dfrac{\pi d^2}{4}$

> First solve radiation and diffraction problems in mode ②.
Then substitute the hydrodynamic coefficients in the above equation; obtain heave-response in waves;



(Transfer function) or (RAO)

$$\xi_2 = \frac{A}{-\omega^2(m_{22}+a_{22}) + i\omega b_{22} + c_{22}}$$

A resonance occurs when $\omega = \omega_n = \left[c_{22}/(a_{22}+m_{22})\right]^{1/2}$

Reading assignment : MH (Newman) 6.17 Damping and Added mass , pp. 295 – 300.

M.Sc. Thesis . ? (B.G.)

(10.1) For home : For a two-dimensional, floating body, show that a suitable linear combination of heave and sway oscillations at the same frequency will result in outgoing waves only at $x \to \infty$, with no waves at $x \to -\infty$. By reversing the sign of time t, deduce that the corresponding body motions act to absorb an incident wave system with 100 % efficiency.

Instead of : (10.1) A half submerged sphere of 1 m diameter in deep sea-water is subject to plane progressive monochromatic waves of length 4 m and height of 0.2 m. Determine the amplitude of the response of the sphere in heave. What is the amplitude of the vertical exciting force? Use the graphs in Fig. 6.24 of J. Newman's "Marine Hydrodynamics".