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Computer Engineering Department

DIGITAL CIRCUITS SOLUTIONS TO EXAMPLARY EXAM QUESTIONS

SOLUTION 1:

a. B is negative, result is negative, there is an overflow, and operation is subtraction
i) Overflow condition: pos – neg = neg , therefore A must be **positive**.

ii) A = 0xxx xxxx		0xxx xxxx	
B= 1001 1101	2's comp.	+ 0110 0011	smallest possible A= 0001 1101
R= 1xxx xxxx		1xxx xxxx	

The same solution by thinking in decimal:
B= $(-99)_{10}$, to generate an overflow result must be at least +128. (Note that result seems to be negative, but due to overflow the real sign of the result is positive.)
A-99=128, smallest possible **A=$(29)_{10}$ = 0001 1101**

b. The carry bit is 1. It means **no borrow**. Therefore A>B.

SOLUTION 2:

a)

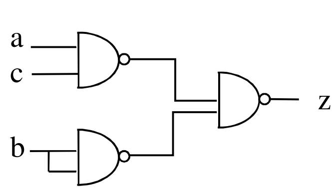
Expression:
(a+E)(a'+F) or (a'+E)(a+F)

b)

(a+E)(a'+F)(E+F)	= (a+E)(a'+F)(E+F+aa')	Inverse and identity
	= (a+E)(a'+F)(E+F+a)(E+F+a')	Distribution
	= (a+E)(1+F)(a'+F)(1+E)	Identity
	= (a+E)(a'+F)	

c)

z	= ab'c+acd+ab+a'b	
	= ab'c+acd+b(a'+a)	Distribution
	= acd+ab'c+b	Inverse
	= acd+ab'c+b+ac	Consensus
	= ac(d+b'+1)+b	Distribution and identity
	= ac+b	



SOLUTION 3:

a)

f(A,B,C,D)=A’B’CD+AB’CD+AC’D+AC’D’+A’B’CD’+ABCD+ACD’

=(A’+A)B’CD + AC’(D+D’) + A’B’CD’ + ABCD + ACD’ (Inverse)

=(B’+AB)CD + AC’ + A’B’CD’ + ACD’ (absorbtion)

= B’CD + A(CD + C’) + (A’B’+A) CD’ (absorbtion)

= B’CD + AD + AC’ + B’CD’ + ACD’

= B’CD + AD + B’CD’ + A(C’+CD’) (absorbtion)

= B’CD + AD + B’CD’ + AC’ + AD’

= B’C(D + D’) + A(D+C’+D’) (inverse)

= B’C + A

b)

f

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	1
11	Φ	0	Φ	1
10	1	1	1	1

By considering 0 and Φ points we can obtain complement of f.

$\bar{f} = B\bar{C} + BD$

De Morgan:

$f = \overline{B\bar{C} + BD} = \overline{B\bar{C}} \cdot \overline{BD}$

$= (\bar{B} + C) \cdot (\bar{B} + \bar{D})$

Or by considering true (1) points:

$f = \bar{B} + (C\bar{D})$

Distributive Law:

$f = (\bar{B} + C) \cdot (\bar{B} + \bar{D})$

SOLUTION 4 :

a) Maxterms (0 generating inputs): 0001, 0101, 1100, 1101, 1110, 1001

ab \ cd	00	01	11	10
00	1		1	1
01	1		1	1
11			1	
10	1		1	1

Prime Implicants:

b’d’ , a’d’, a’c, b’c, cd

b)

False (0) points of function f are true (1) points of the **complement** ($\bar{f}$(a,b,c,d)).

Num	abcd
1	0001√
5	0101√
9	1001√
12	1100√
13	1101√
14	1110√

Num	abcd
1,5	0-01√
1,9	-001√
5,13	-101√
9,13	1-01√
12,13	110- **X**
12,14	11-0 **X**

Num	abcd	
1,5,9,13	- - 01	**X**

Prime Implicants:
abc' , abd', c'd

SOLUTION 5 :
Step 1. Point 4 is a distinguished point, and F is an essential prime implicant. F is selected, points 4, 7, 13 and 14 are removed.
Step 2. C covers D with equal cost. D is removed.
Step 3. True point 2 is a distinguished point, and C is an essential prime implicant. C is selected, points 2, 5, 8 and 10 are removed.
Step 4. B covers E with less cost. E is removed.
Step 5. True point 11 is a distinguished point, and B is an essential prime implicant. B is selected.

Hence B+C+F is the minimal covering sum (sufficient base) with 26 unit cost.

SOLUTION 6:
a)

Z: AB \ CD	00	01	11	10
00	1	0	1	1
01	1	1	1	0
11	Φ	0	0	0
10	1	0	1	1

Set of all prime implicants:

$\bar{C}\bar{D},\ \bar{B}C,\ \bar{B}\bar{D},\ \bar{A}B\bar{C},\ \bar{A}BD,\ \bar{A}CD$
A B C D E F

b) **Prime Implicant Chart:**

		0	2	3	4	5	7	8	10	11	Cost
√	A	X			X			X			6
√	B		X	X					X	X	5
	C	X	X					X	X		6
	D				X	X					8
√	E					X	X				7
	F			X			X				7

c) Cheapest sufficient set of prime implicants:
A + B + E: Cost=18
Cheapest expression: $Z = \bar{C}\bar{D} + \bar{B}C + \bar{A}BD$

SOLUTION 7:

a.

Truth table:

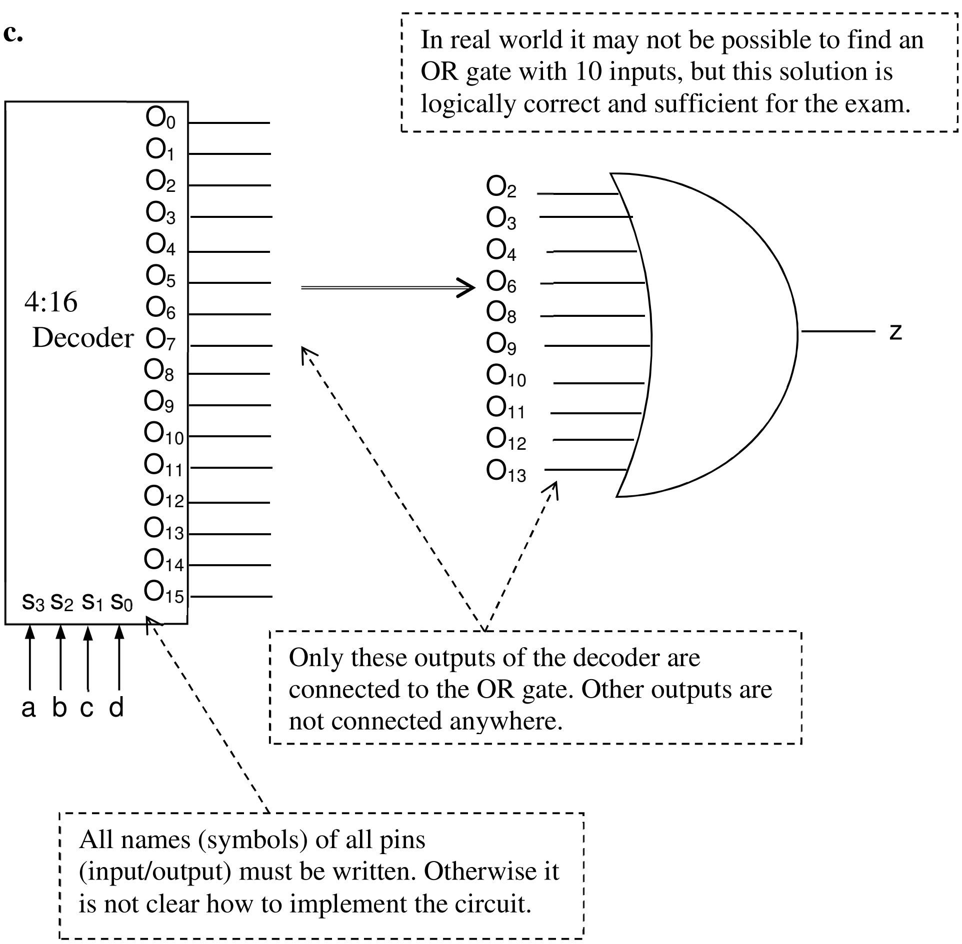
a	b	c	d	z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

From the truth table we can obtain minterms and write the expression of the function in the 1st canonical form.
Remember, in minterms all variables (literals) appear once.
1st canonical form:
z= a'b'cd'+ a'b'cd + a'bc'd'+ a'bcd'+ ab'c'd'+ ab'c'd + ab'cd' + ab'cd + abc'd'+ abc'd

b.

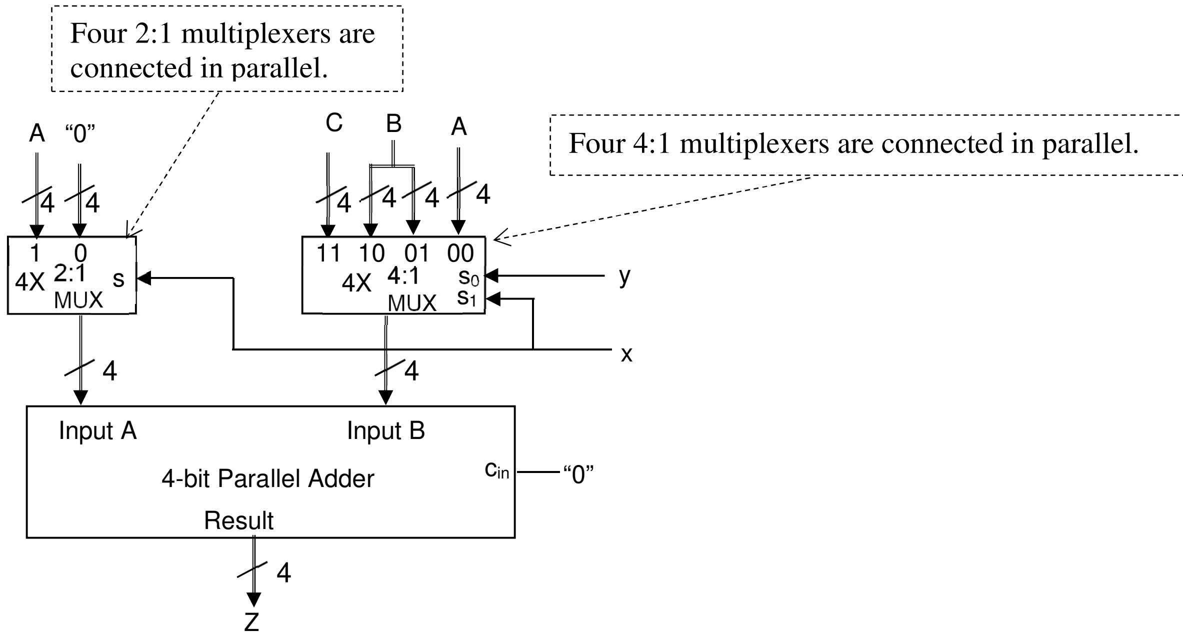
There are different ways to minimize the expression in the 1st canonical form.
Minimized expression:
z= a'bd' + ac' + b'c

c.

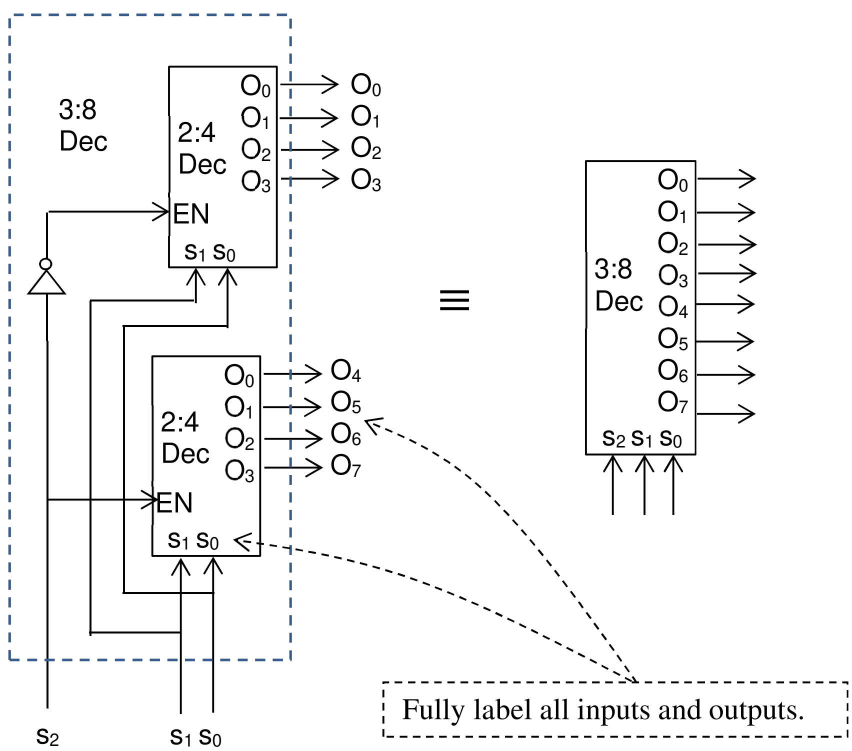


SOLUTION 8:

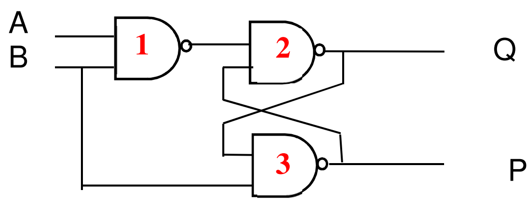
There are different possible proper solutions. One of them is given below.



SOLUTION 9:



SOLUTION 10:



A	B	Q	P	
0	0	0	1	If B=0 then P = 1. The output of NAND1 is 1 and Q is 0 (stable)
0	1	0	1	After (A=0, B=0), Q=0 from previous state and P = 1. The output of NAND1 is 1and Q = 0 (stable)
1	0	0	1	If B=0 then P = 1. The output of NAND1 is 1 and Q is 0 (stable)
1	1	1	0	The output of NAND1 is 0. Therefore the output of NAND2, Q= 1 and P=0 (stable)
0	1	1	0	After (A=1, B=1), The output of NAND1 is 1. P = 0 from previous state and Q = 1, P=0 (stable)

A	B	
0	0	Reset
1	0	
0	1	Don't change
1	1	Set

The circuit is stable and can be switched to another state and has set, reset and don't change conditions. Therefore it can be used as a memory unit.

A	**B**	**Q(t)**	**Q(t+1)**
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

AB\Q(t)	**0**	**1**
00	0	0
01	0	1
11	1	1
10	0	0

Q(t+1) = BQ(t) + AB

SOLUTION 11:

a) $Q_1^+ = A\bar{Q}_1 + \bar{B}Q_1$

$Q_0^+ = 1 \oplus Q_0 = \bar{Q}_0$

$Z = \bar{S}_1 S_0 + S_1 \bar{S}_0$

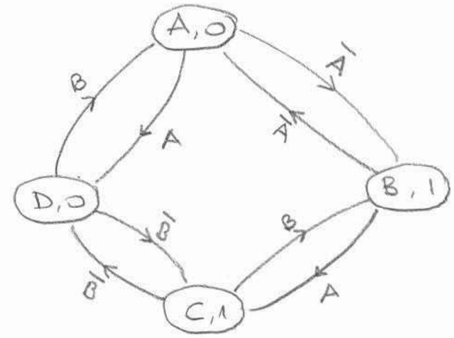
$= S_1 \oplus S_0 = Q_1 \oplus Q_0$

Output is a function of states $\longrightarrow$ Moore Model

$Q_1^+ Q_0^+$

Q_1Q_0 \ AB	00	01	10	11	Z
00	01	01	11	11	0
01	00	00	10	10	1
10	11	01	11	01	1
11	10	00	10	00	0

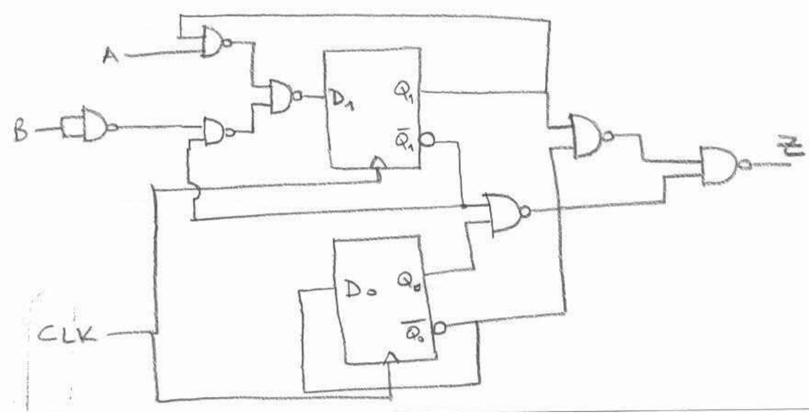
A : 00
B : 01
C : 10
D : 11



b) $Q_1^+ = D_1 = A\bar{Q}_1 + \bar{B}Q_1$

$Q_0^+ = D_0 = \bar{Q}_0$

$Z = Q_1 \oplus Q_0$



SOLUTION 12:

a)

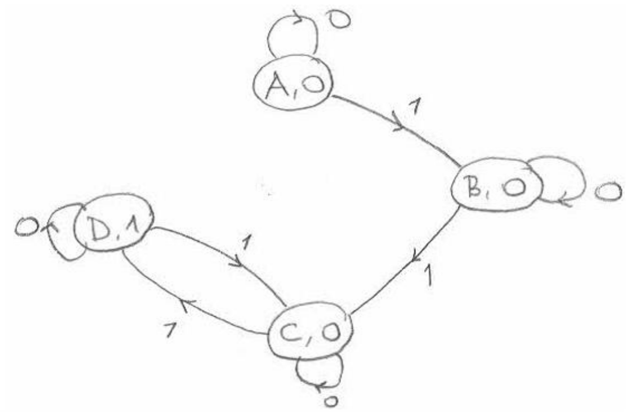
States

A: zero ones

B: one ones

C: two or even ones

D: (three or more) and odd ones



State/output table

$Q_1^+Q_0^+, Z$

Q_1Q_0 \ X	0	1	Z
A	A	B	0
B	B	C	0
C	C	D	0
D	D	C	1

Codes

A: 00

B: 01

C: 10

D: 11

$Q_1^+Q_0^+$

Q_1Q_0 \ X	0	1
00	00	01
01	01	10
10	10	11
11	11	10

b)

Symbol	Transition	J	K
0	$0 \to 0$	0	ϕ
α	$0 \to 1$	1	ϕ
β	$1 \to 0$	ϕ	1
1	$1 \to 1$	ϕ	0

Q_1^+

Q_1Q_0 \ X	0	1
00	0	0
01	0	α
10	1	1
11	1	1

Q_0^+

Q_1Q_0 \ X	0	1
00	0	α
01	1	β
10	0	α
11	1	β

J_1

Q_1Q_0 \ X	0	1
00	0	0
01	0	1
11	ϕ	ϕ
10	ϕ	ϕ

K_1

Q_1Q_0 \ X	0	1
00	ϕ	ϕ
01	ϕ	ϕ
11	0	0
10	0	0

J_0

Q_1Q_0 \ X	0	1
00	0	1
01	ϕ	ϕ
11	0	1
10	ϕ	ϕ

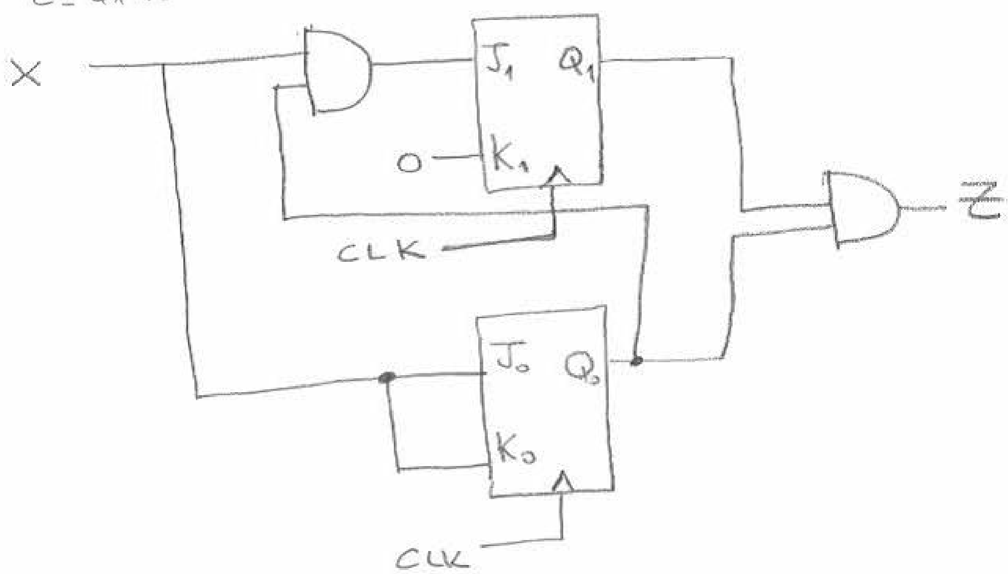
K_0

Q_1Q_0 \ X	0	1
00	ϕ	ϕ
01	0	1
11	ϕ	ϕ
10	0	1

$J_1 = Q_0 X$ $\quad$ $J_0 = X$

$K_1 = 0$ $\quad$ $K_0 = X$

$Z = Q_1 Q_0$



SOLUTION 13:

A	Q1	Q2	Q3	Q4	Z
L	Off	On	On	Off	L
H	On	Off	Off	On	H

The expression for the function Z = f(A) = A (buffer).