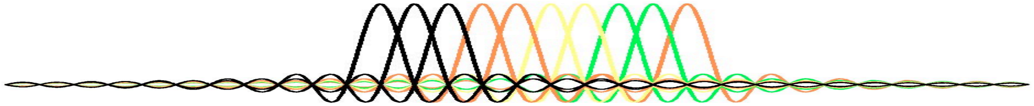


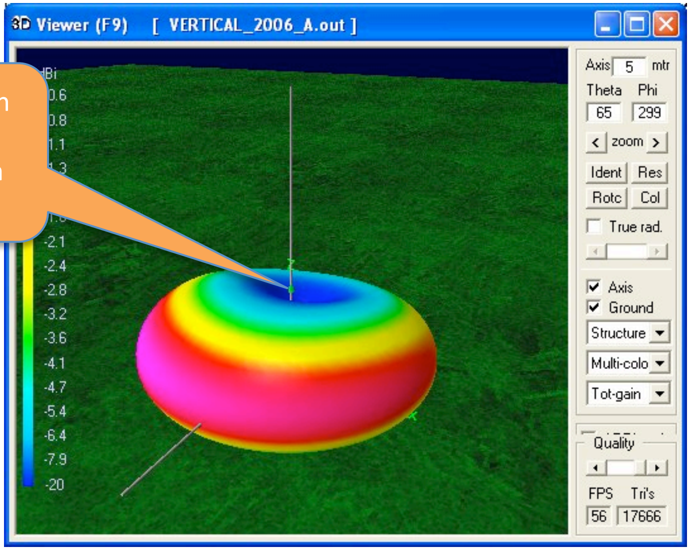
EHB 453, Introduction to Mobile Communications

Lecture 5: Wireless Channel

Prof. Mustafa Ergen

Outline

- After digital modulation, the constructed signal is sent to antenna to be transmitted over the air.
- A voltage is applied to an antenna and it creates electromagnetic field that propagates according to Maxwell’s equation



3D Viewer (F9) [VERTICAL_2006_A.out]

Axis 5 mtr

Theta 65 Phi 299

☐ True rad.

☑ Axis

☑ Ground

Structure

Multi-colo

Tot-gain

FPS 56 Tri's 17666

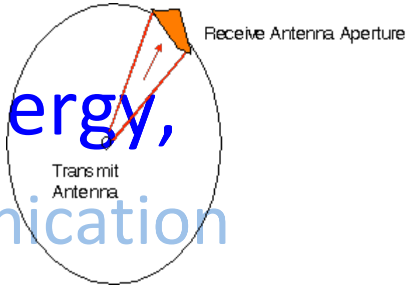
Vertical antenna: Propagation is flat on the ground with more than 10 dB reduction in the vertical direction

Antenna Size

- Electromagnetic waves in all direction.
- Waves induce electric currents in the receiver’s antenna.
- Energy it creates for a given voltage of a given frequency is directly coupled with antenna size.
- Antenna size is directly coupled with the field’s wavelength (**λ**)
- **λ** is inversely proportional with the carrier frequency **$\lambda=c/f_c$** where **c** is speed of light.

Dynamics

- Pathloss
  - – Due to conservation of energy,
    - Enabler of cellular communication



- Shadowing
  - – Due to blocking ob

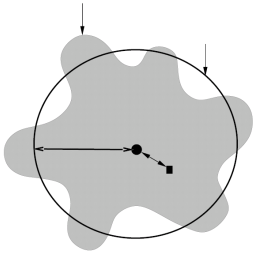


Figure 2.10: Contours of Constant Received Power.

- Fading
  - – Due to multipath comp

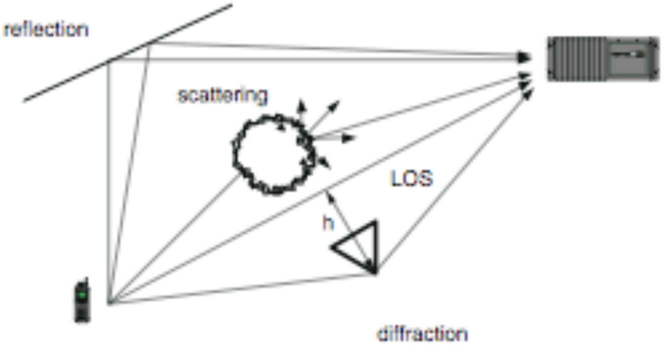
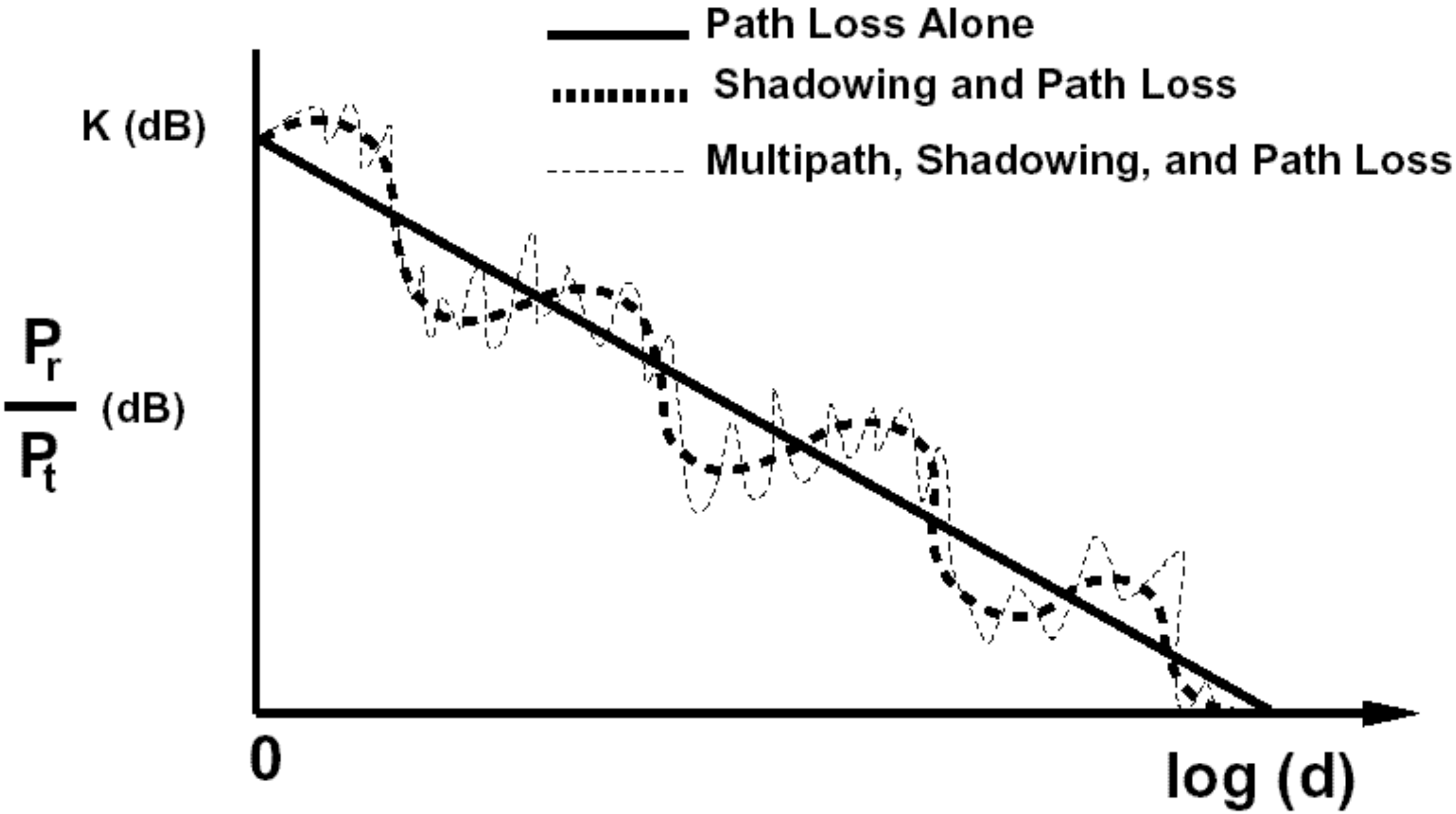
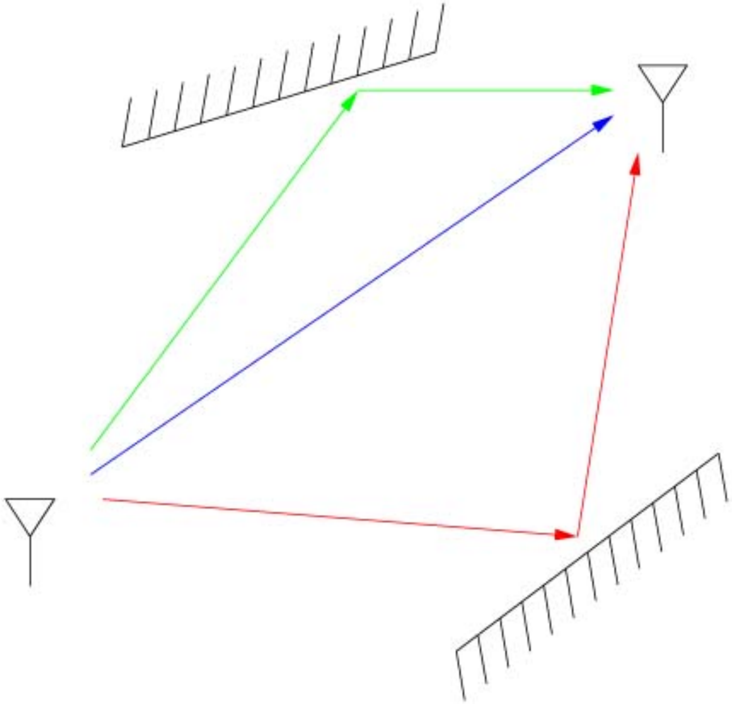


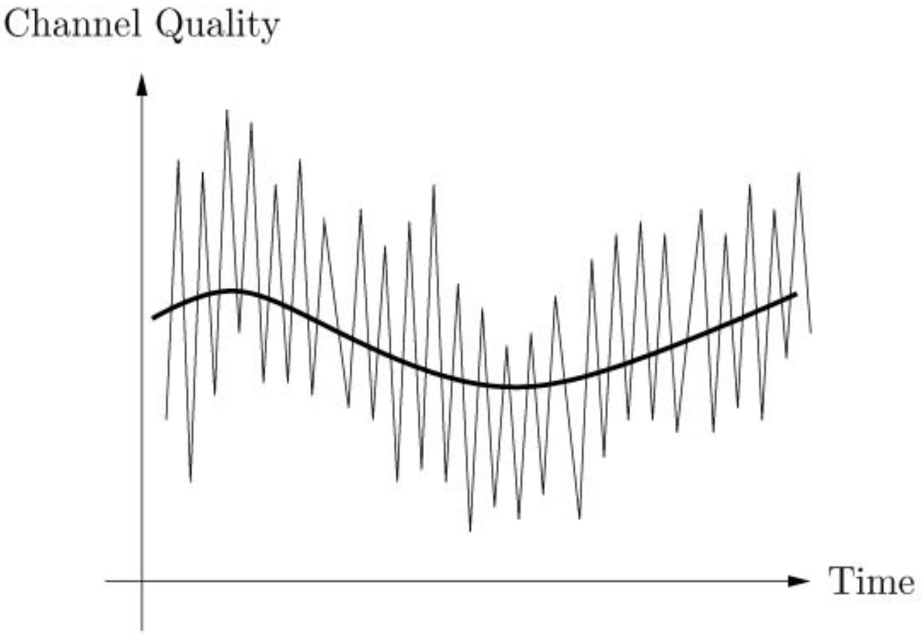
Fig. 2.15 Multipath components

Overall


Path Loss Alone
Shadowing and Path Loss
Multipath, Shadowing, and Path Loss
K (dB)
$\frac{P_r}{P_t}$ (dB)
0
log (d)

Wireless Multipath Channel

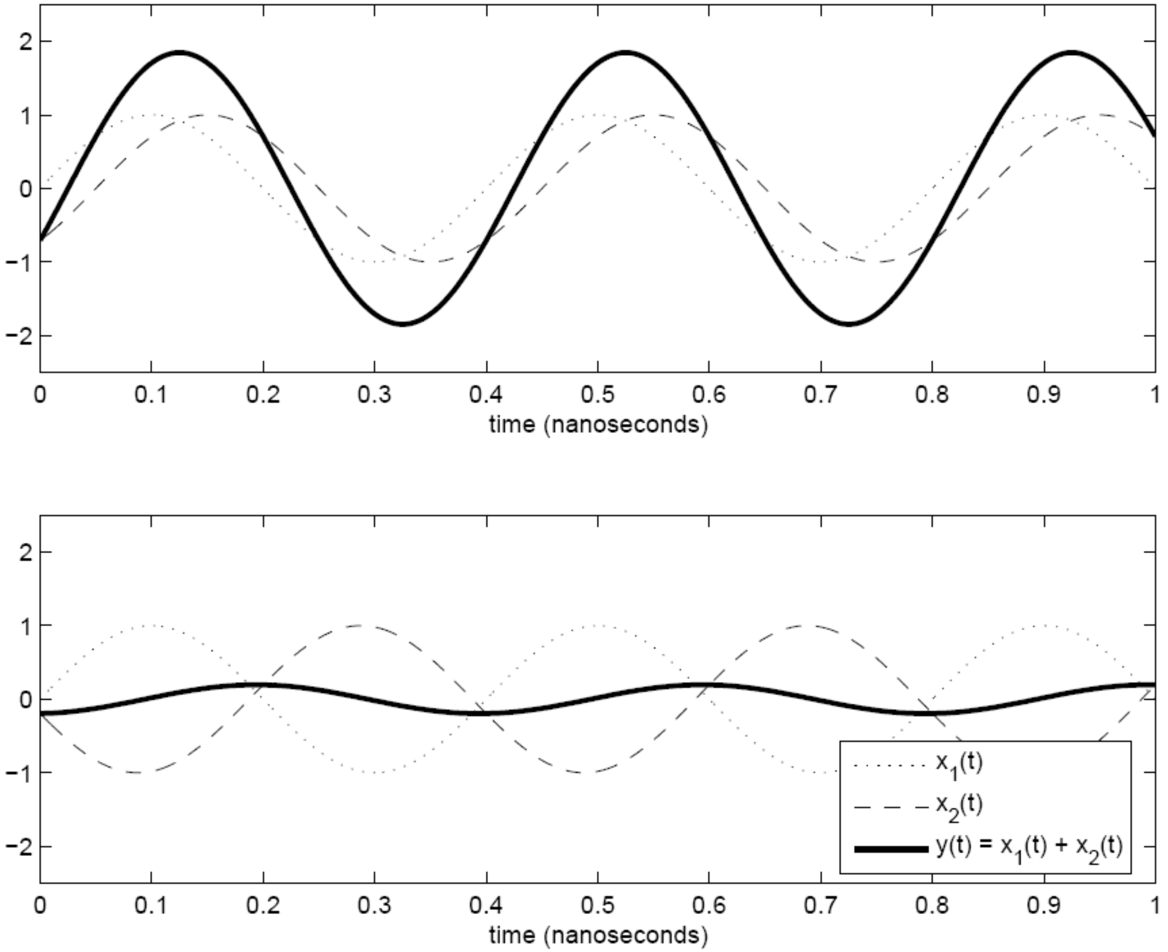




Channel varies at two spatial scales:

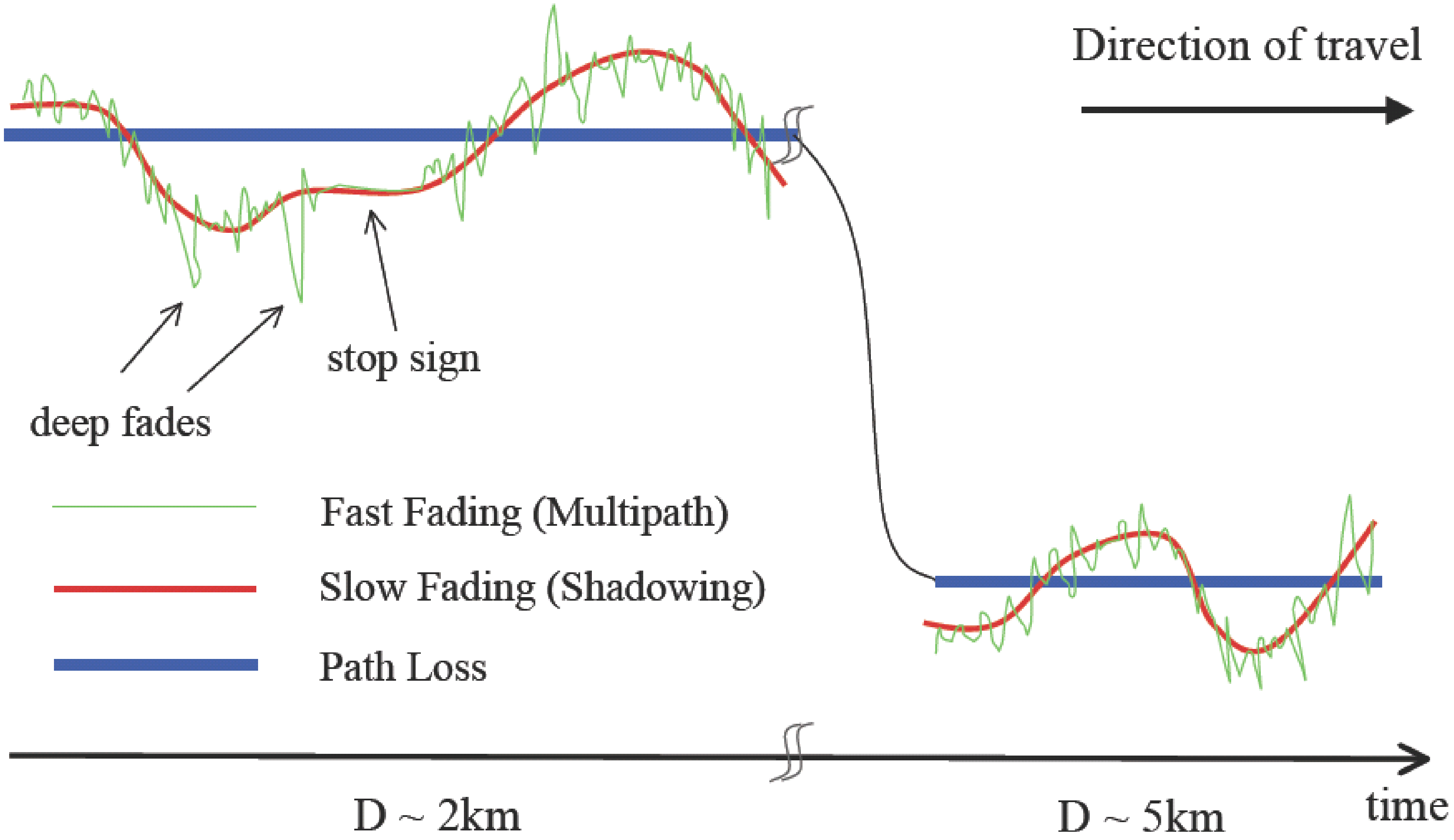
- * Large scale fading: path loss, shadowing
- * Small scale fading:
  - Multi-path fading (frequency selectivity, coherence b/w, ~500kHz),
  - Doppler (time-selectivity, coherence time, ~2.5ms)

MultiPath Interference: Constructive & Destructive



The difference between constructive interference (top) and destructive interference (bottom) at $f_c = 2.5$ GHz is less than 0.1 nanoseconds in phase, which corresponds to about 3 cm.

Mobile Wireless Channel w/ Multipath



Goal

- We wish to understand how physical parameters such as
  - carrier frequency
  - mobile speed
  - bandwidth
  - delay spread
  - angular spread

  impact how a wireless channel behaves from the cell planning and communication system point of view.
- We start with deterministic physical model and progress towards statistical models, which are more useful for design and performance evaluation.

Large-scale fading: Cell-Site Planning

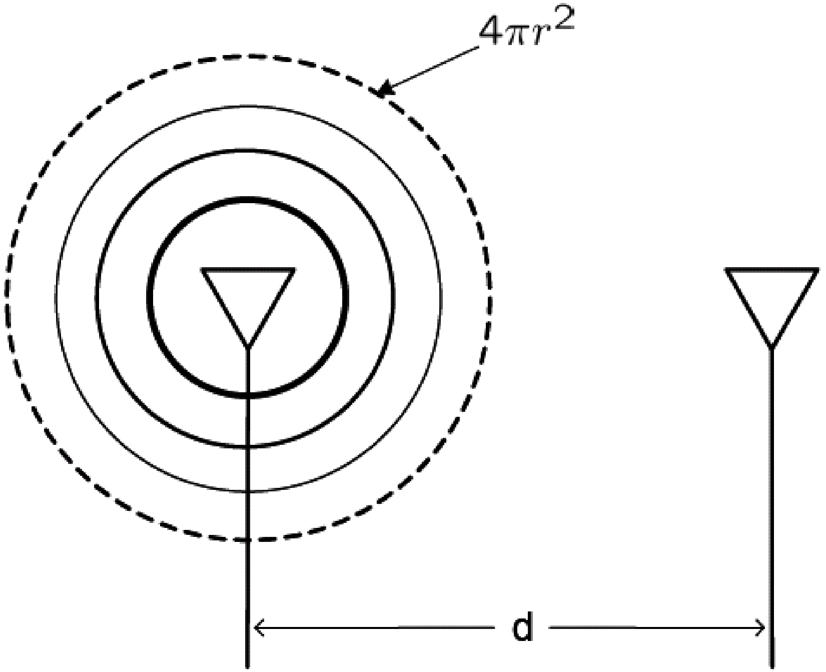
- In free space, received power attenuates like $1/r^2$.
- With reflections and obstructions, can attenuate even more rapidly with distance. Detailed modelling complicated.
- Time constants associated with variations are very long as the mobile moves, many seconds or minutes.
- More important for cell site planning, less for communication system design.

Path Loss Modeling

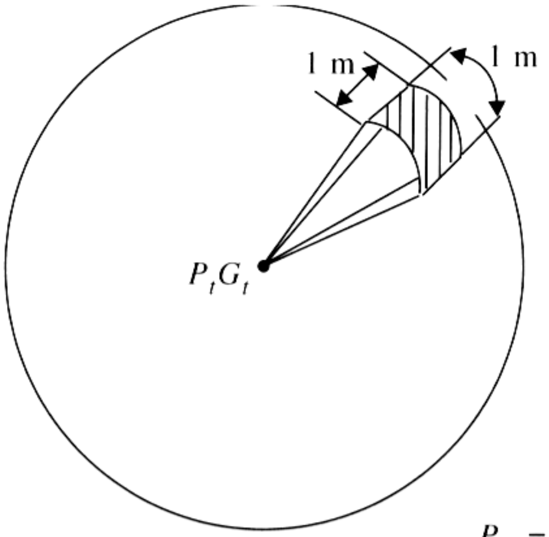
- Maxwell’s equations
  - Complex and impractical
- Free space path loss model
  - Too simple
- Ray tracing models
  - Requires site-specific information
- Empirical Models
  - Don’t always generalize to other environments
- Simplified power falloff models
  - Main characteristics: good for high-level analysis

Free-Space-Propagation

- If oscillating field at transmitter, it produces three components:
  - The electrostatic and inductive fields that decay as $1/d^2$ or $1/d^3$
  - The EM radiation field that decays as $1/d$ (power decays as $1/d^2$)



Free-space and received fields: Path Loss



(a)

$$P_d = \frac{P_t G_t}{4\pi d^2} = \frac{EIRP}{4\pi d^2} = \frac{|E|^2}{120\pi} \ \text{W/m}^2$$

(power flux density P_d)

<u>Note</u>: Electric Field (E) decays as *1/r*, but
Power (P_d) decays as *$1/r^2$*

Path Loss in dB:

$$P_L \ \text{dB} = 10 \log_{10} \frac{P_t}{P_r} \ \text{dB}. \qquad \frac{P_r}{P_t} = \left[\frac{\sqrt{G_l}\lambda}{4\pi d}\right]^2.$$

$\sqrt{G_l}$ is the product of the transmit and receive antenna field radiation patterns in the LOS direction.

Decibels: dB, dBm

- **dB** (Decibel) = $10 \log_{10} (Pr/Pt)$
  Log-ratio of two signal levels. Named after Alexander Graham Bell. For example, a cable has 6 dB loss or an amplifier has 15 dB of gain. System gains and losses can be added/subtracted, especially when changes are in several orders of magnitude.

- **dBm (dB milliWatt)**
  Relative to 1mW, i.e. 0 dBm is 1 mW (milliWatt). Small signals are -ve (e.g. -83dBm). Typical 802.11b WLAN cards have +15 dBm (32mW) of output power. They also spec a -83 dBm RX sensitivity (minimum RX signal level required for 11Mbps reception).

For example,

  - 33dBm is 2W – Macro Base Station
  - 20dBm is 100mW - WiFi
  - 7dBm is 5mW - 3G Femtocell

Path Loss: *Range* vs *Bandwidth* Tradeoff

- Frequencies < 1 GHz are often referred to as “beachfront” spectrum. Why?

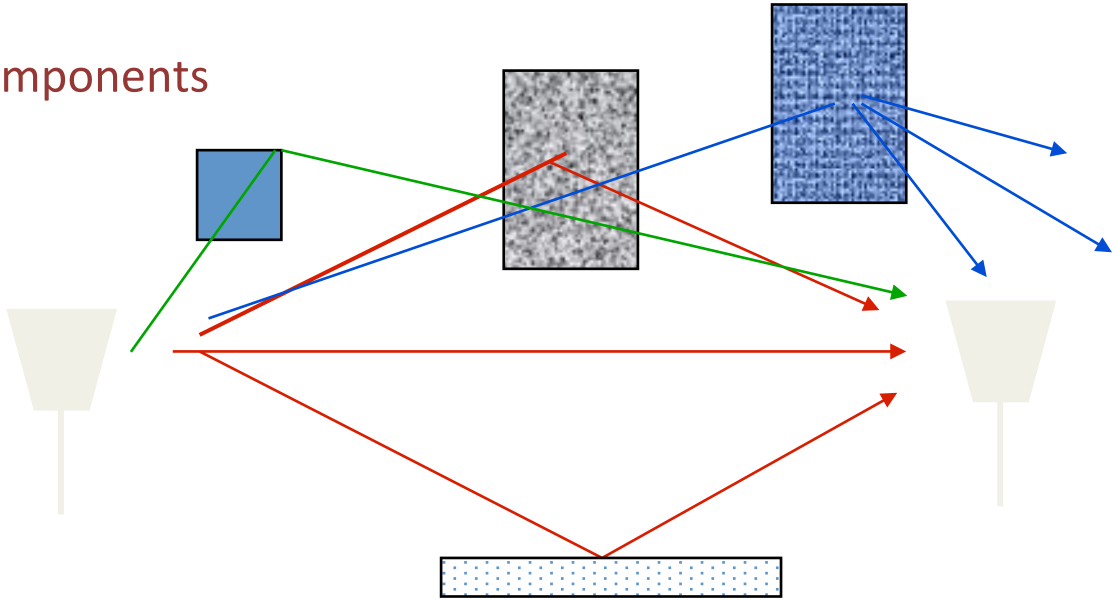
- 1. High frequency RF electronics have traditionally been harder to design and manufacture, and hence more expensive. [less so nowadays]
- 2. Pathloss increases ~ O(f_c^2)
  - A signal at 3.5 GHz (one of WiMAX’s candidate frequencies) will be received with about 20 times less power than at 800 MHz (a popular cellular frequency).
  - Effective path loss exponent also increases at higher frequencies, due to increased absorption and attenuation of high frequency signals

- Tradeoff:
  - Bandwidth at higher carrier frequencies is more plentiful and less expensive.
  - Does *not* support large transmission ranges.
  - (also increases problems for mobility/Doppler effects etc)

- WIMAX Choice:
  - Pick any two out of three: *high data rate, high range, low cost*.

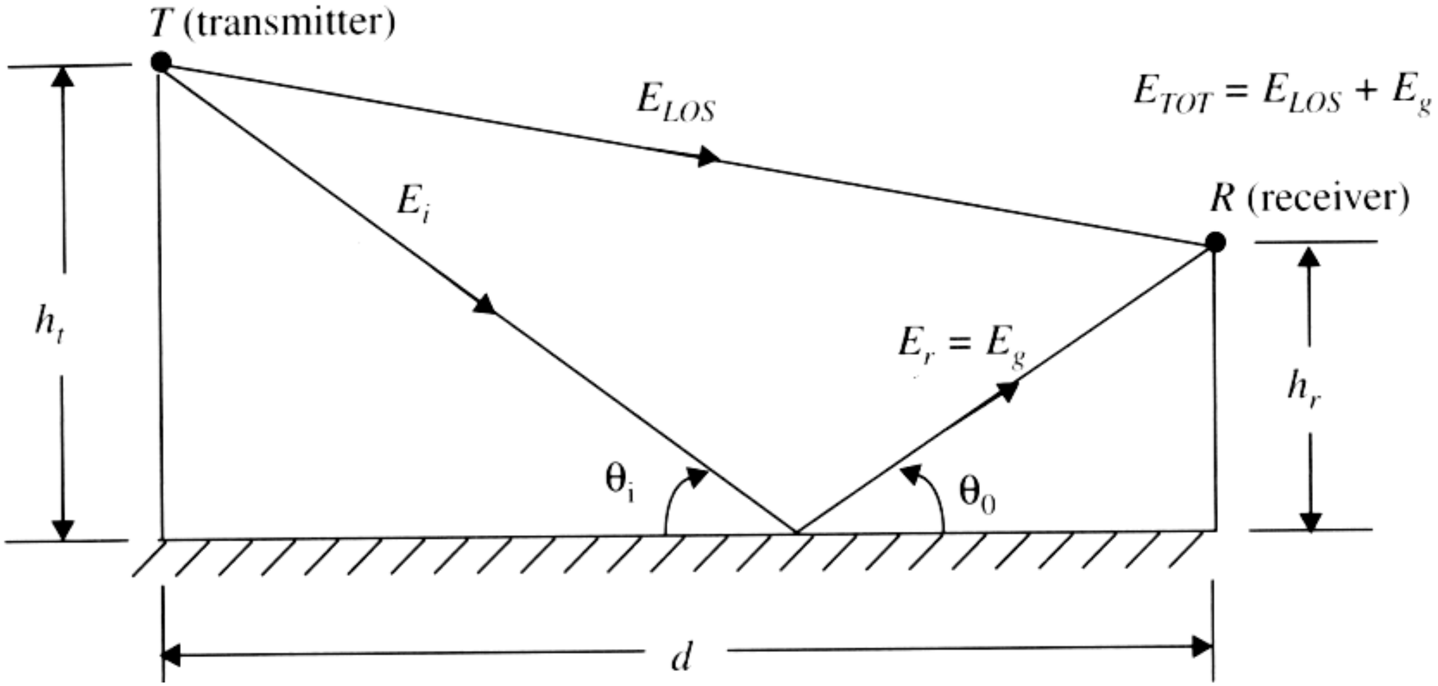
Ray Tracing

- Models all signal components
  - Reflections
  - Scattering
  - Diffraction



- Diffraction: signal “bends around” an object in its path to the receiver:
  - Diffraction Path loss exceeding 100 dB
- Error of the ray tracing approximation is smallest when the receiver is many wavelengths from the nearest scatterer, and all the scatterers are large relative to a wavelength and fairly smooth.
  - Good match w/ empirical data in rural areas, along city streets (Tx/Rx close to ground), LAN with adjusted diffraction coefficients

Classical 2-ray Ground Bounce model



$$P_r \approx \left[\frac{\lambda\sqrt{G_l}}{4\pi d}\right]^2 \left[\frac{4\pi h_t h_r}{\lambda d}\right]^2 P_t = \left[\frac{\sqrt{G_l} h_t h_r}{d^2}\right]^2 P_t,$$

Source: A. Goldsmith book (derivation in book)

$$P_r \text{ dBm} = P_t \text{ dBm} + 10\log_{10}(G_l) + 20\log_{10}(h_t h_r) - 40\log_{10}(d).$$

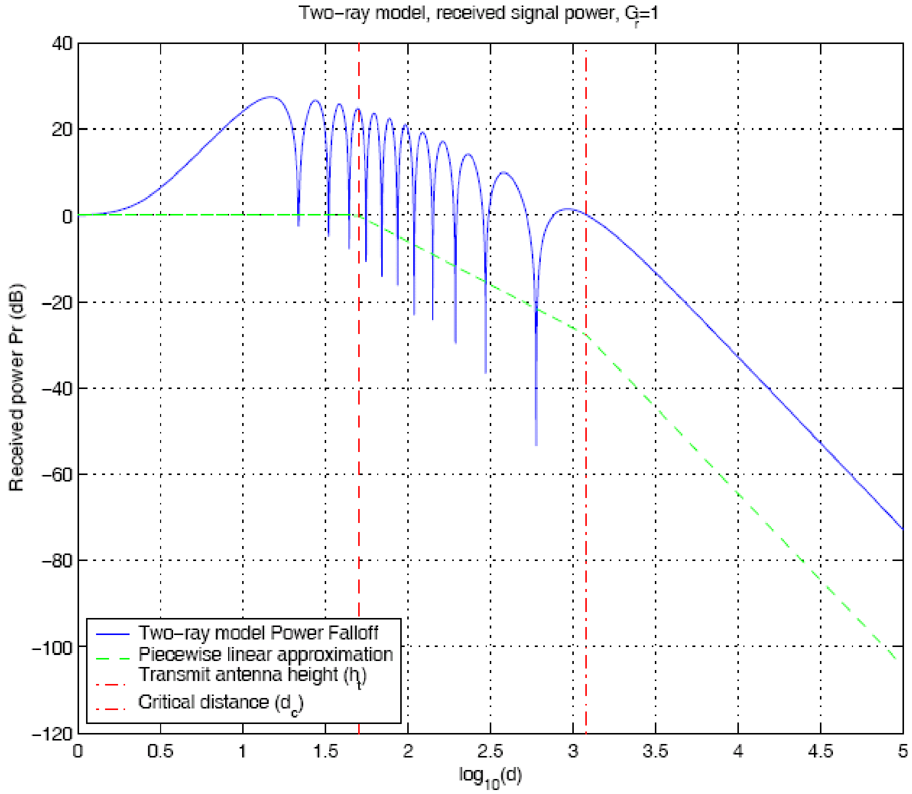
2-ray model: distance effect, critical distance

- $d < h_t$: constructive i/f
- $h_t < d < d_c$: constructive and destructive i/f (multipath fading upto critical distance)
- $d_c < d$: only destructive interference

$$d_c = 4h_t h_r/\lambda,$$

- Piecewise linear approximation w/ slopes 0, -20 dB/decade, -40 dB/ decade

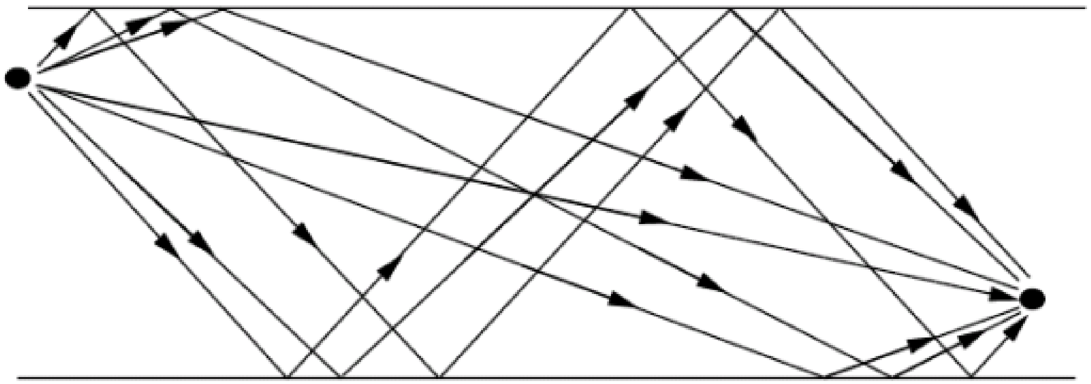
$f = 900\text{MHz}$, R=-1, $h_t = 50\text{m}$, $h_r = 2\text{m}$, $G_l = 1$, $G_r = 1$



Received Power versus Distance for Two-Ray Model.

Source: A. Goldsmith book

10-Ray Model: Urban Microcells



- Ground and 1-3 wall reflections
- Falloff with distance squared (d^{-2})!
  - Dominance of the multipath rays which decay as d^{-2}, …
  - … over the combination of the LOS and ground-reflected rays (the two-ray model), which decays as d^{-4}.
- *Empirical studies:* $d^{-\gamma}$, where γ lies anywhere between two and six

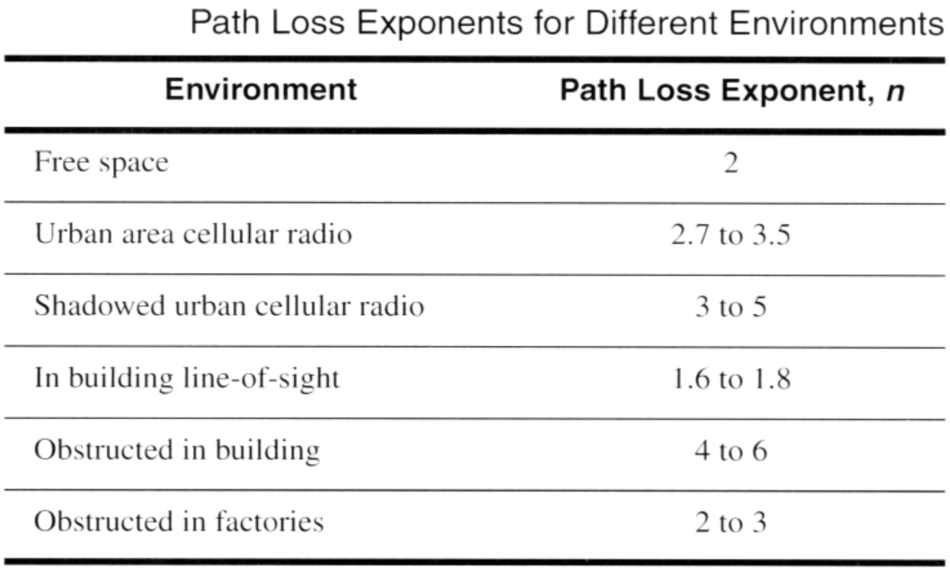
Simplified Path Loss Model

- Used when path loss dominated by reflections.
- Most important parameter is the path loss exponent γ, determined empirically.

$$P_r = P_t K \left[\frac{d_0}{d}\right]^{\gamma}, \qquad 2 \le \gamma \le 8$$

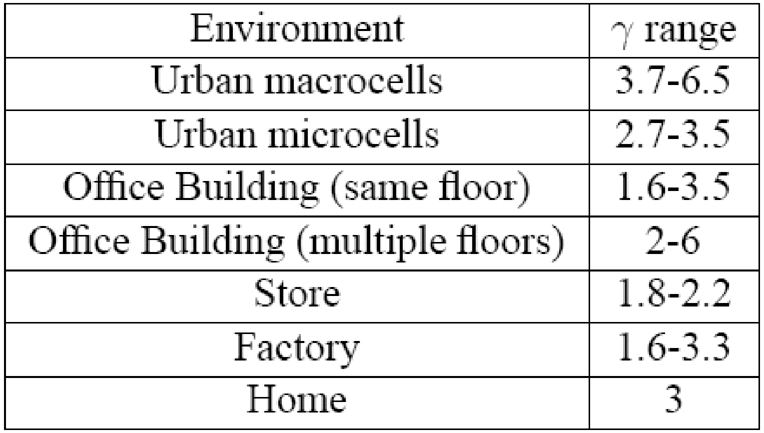
- <u>Cell design impact:</u> If the radius of a cell is reduced by half when the propagation path loss exponent is 4, the transmit power level of a base station is reduced by 12dB (=10 log 16 dB).
  - Costs: More base stations, frequent handoffs

Typical large-scale path loss



Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3



Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Source: Rappaport and A. Goldsmith books

Empirical Models

- Okumura model
  - Empirically based (site/freq specific)
  - Awkward (uses graphs)
- Hata model
  - Analytical approximation to Okumura model
- Cost 136 Model:
  - Extends Hata model to higher frequency (2 GHz)
- Walfish/Bertoni:
  - Cost 136 extension to include diffraction from rooftops
- Erceg:
  - Extends Hata-Okumura with empirical data

Commonly used in cellular system simulations

Empirical Path Loss: Okamura, Hata, COST231

- Empirical models include effects of path loss, shadowing and multipath.
  - Multipath effects are averaged over several wavelengths: local mean attenuation (LMA)
  - Empirical path loss for a given environment is the average of LMA at a distance d over all measurements
- **Okamura**: based upon Tokyo measurements. 1-100 lm, 150-1500MHz, base station heights (30-100m), median attenuation over free-space-loss, 10-14dB standard deviation.

$$P_L(d)\ \text{dB} = L(f_c,d) + A_{mu}(f_c,d) - G(h_t) - G(h_r) - G_{AREA}$$

- **Hata**: closed form version of Okamura

$$P_{L,urban}(d)\ \text{dB} = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d). \quad (2.31)$$

- **COST 231:** Extensions to 2 GHz

$$P_{L,urban}(d)\text{dB} = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d) + C_M, \quad (2.34)$$

Source: A. Goldsmith book

Erceg Model.

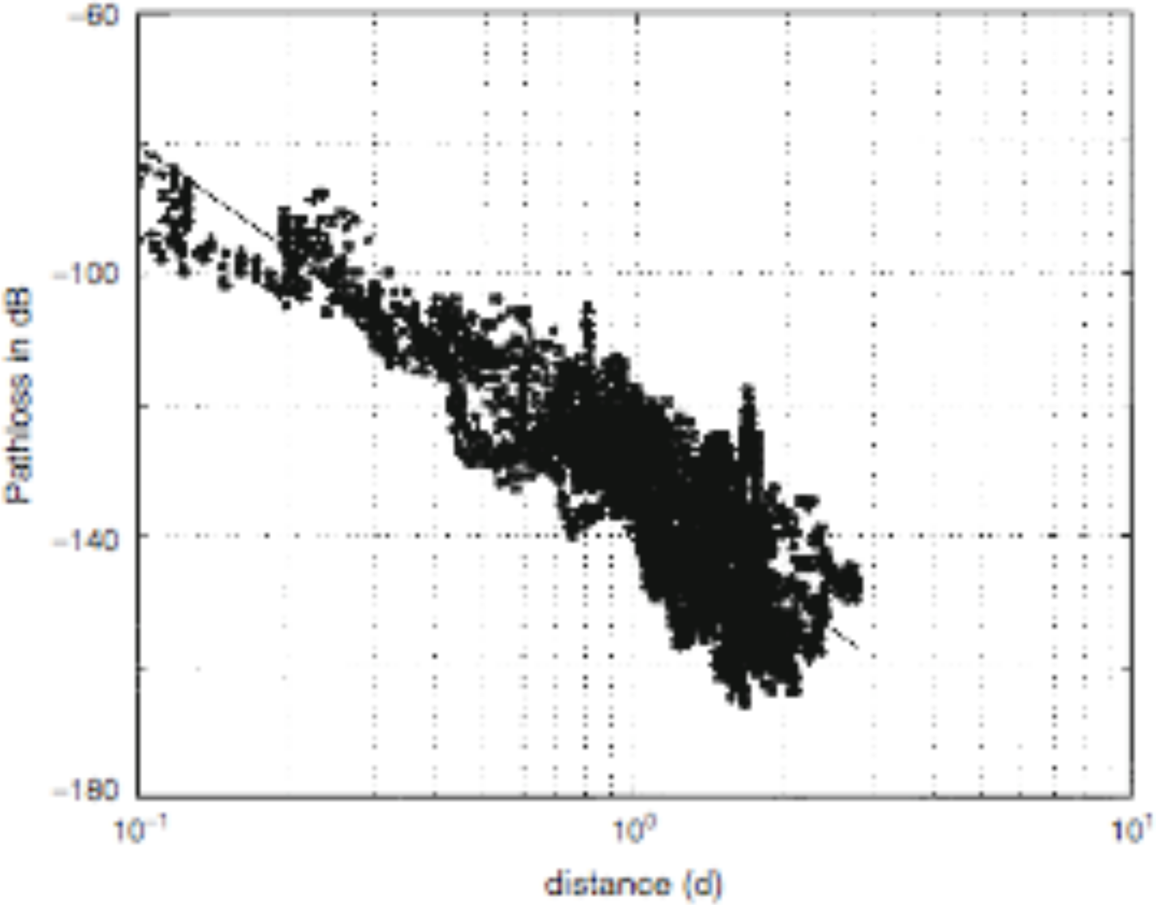
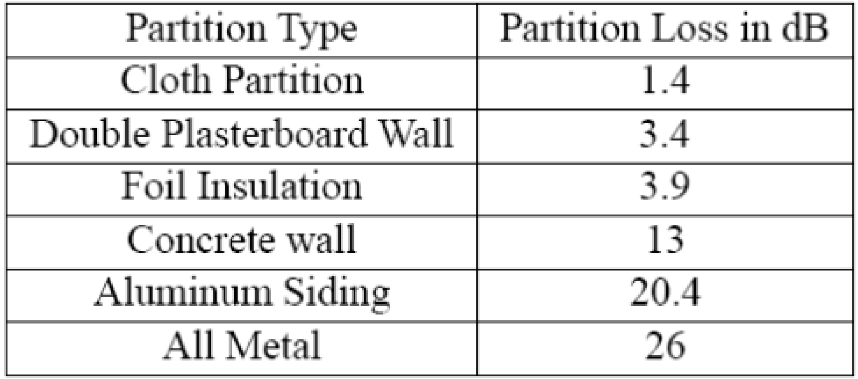


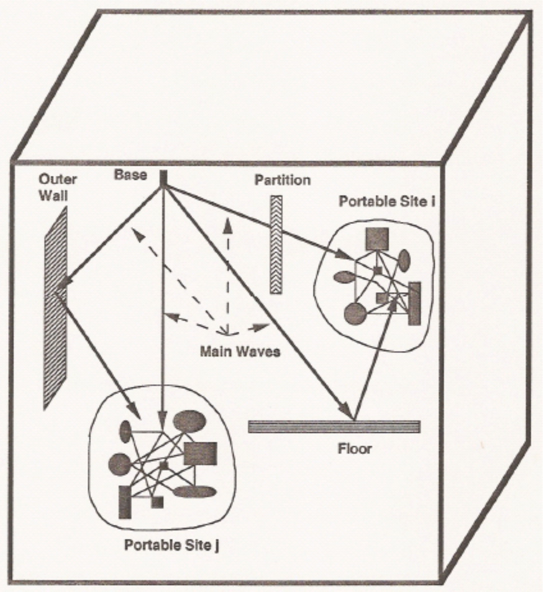
Fig. 2.17 Pathloss for a macrocell in the Seattle area: base station height is 25 m. Source: Erceg, IEEE JSAC, 1999

Indoor Models

- 900 MHz: 10-20dB attenuation for 1-floor, 6-10dB/floor for next few floors (and frequency dependent)
- Partition loss each time depending upon material (see table)
- Outdoor-to-indoor: building penetration loss (8-20 dB), decreases by 1.4dB/floor for higher floors. (reduced clutter)
- Windows: 6dB less loss than walls (if not lead lined)



Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26



Shadowing

$$P_r = P_t P_o \chi \left(\frac{d_o}{d}\right)^{\alpha},$$

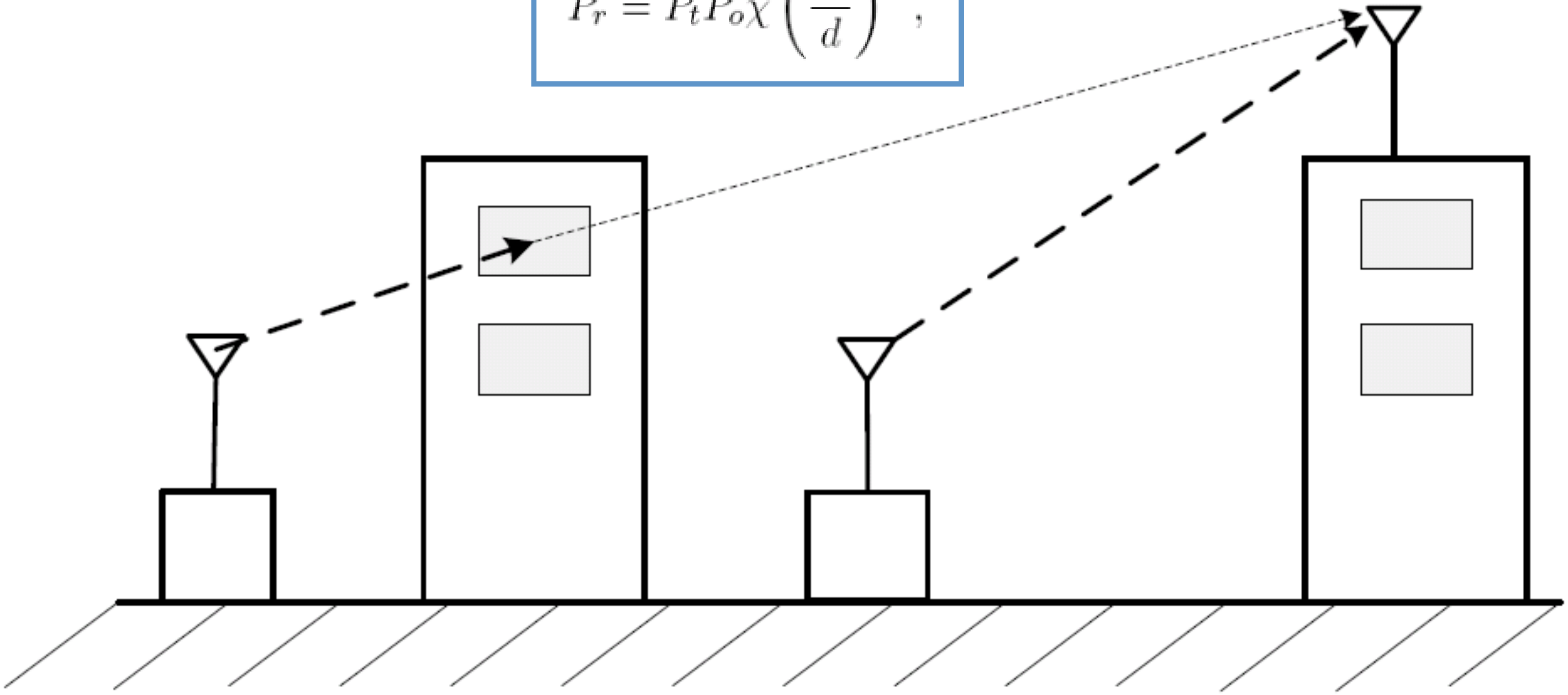


Figure 3.3: Shadowing can cause large deviations from path loss predictions.

- Log-normal model for shadowing r.v. (χ)

Log-Normal Shadowing

- Assumption: shadowing is dominated by the attenuation from blocking objects.
- Attenuation of for depth *d:*

  $s(d) = e^{-\alpha d}$,

  (α*:* attenuation constant).
- Many objects:

  $s(d_t) = e^{-\alpha \sum di} = e^{-\alpha dt}$,

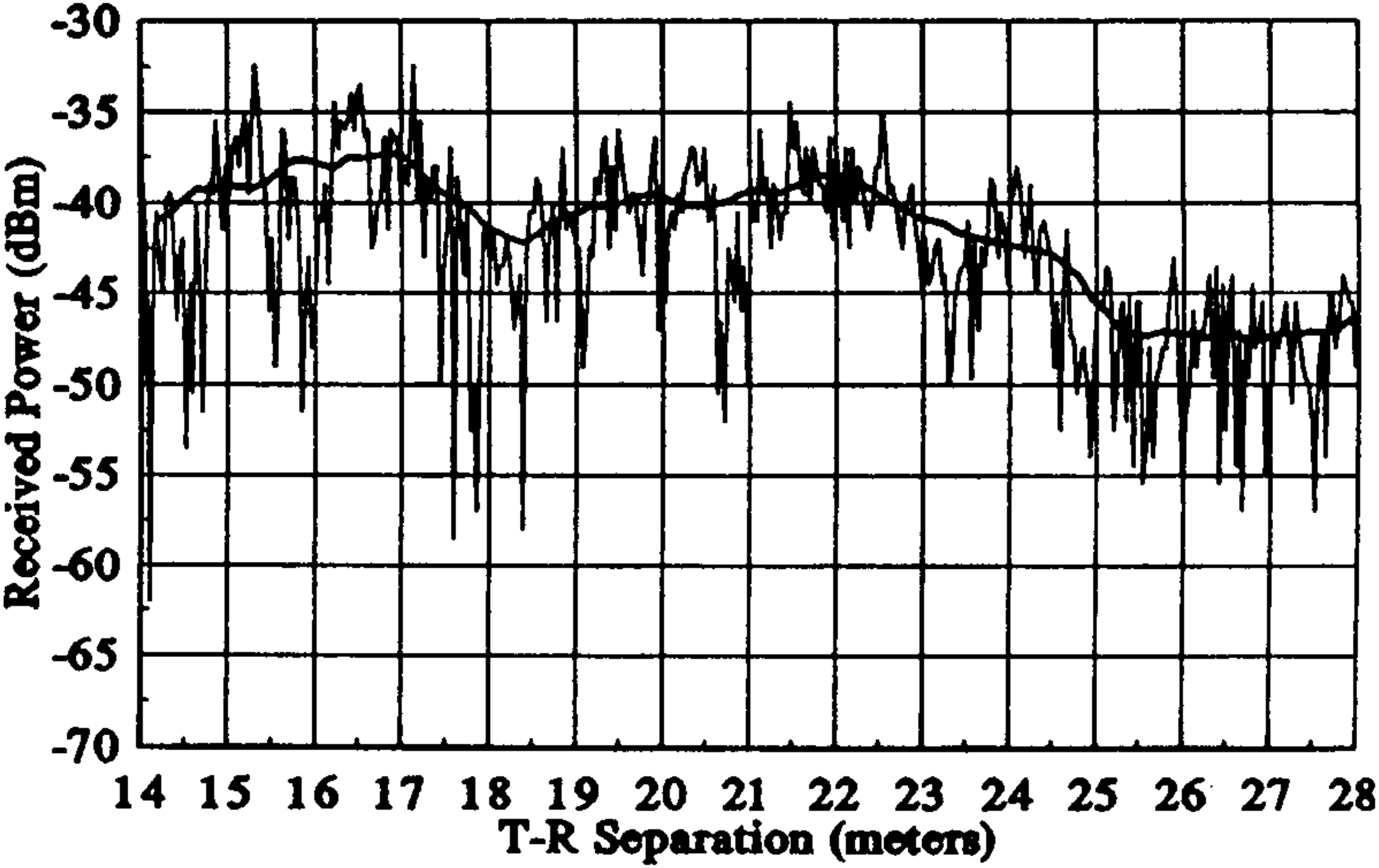
  $d_t = \sum d_i$ is the sum of the random object depths

- Cental Limit Theorem (CLT): $\alpha d_t = \log s(d_t) \sim \mathrm{N}(\mu, \sigma)$.
  - log $s(d_t)$ is therefore log-normal

Small-scale Multipath fading: System Design

- Wireless communication typically happens at very high carrier frequency. (eg. f_c = 900 MHz or 1.9 GHz for cellular)
- Multipath fading due to constructive and destructive interference of the transmitted waves.
- Channel varies when mobile moves a distance of the order of the carrier wavelength. This is about 0.3 m for 900 Mhz cellular.
- For vehicular speeds, this translates to channel variation of the order of 100 Hz.
- *Primary driver* behind wireless communication system design.

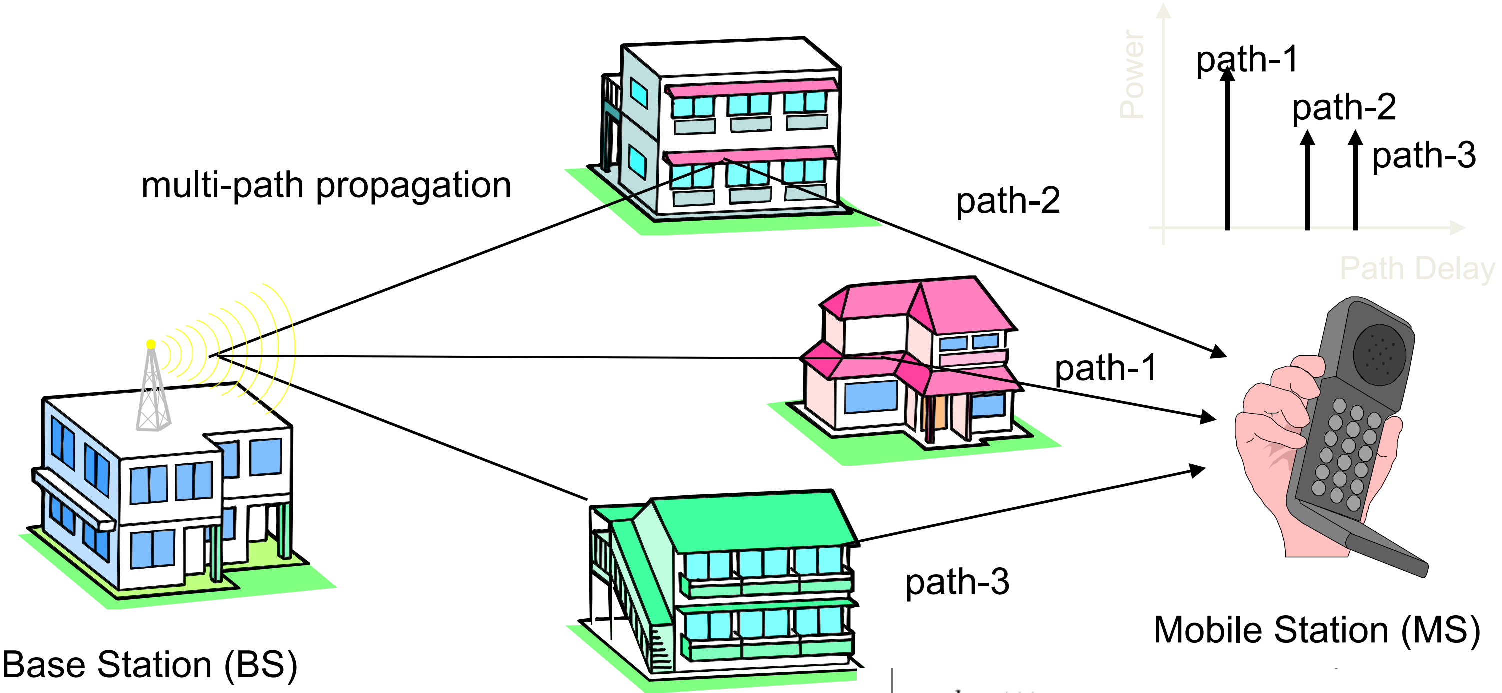
Fading: Small Scale vs Large Scale



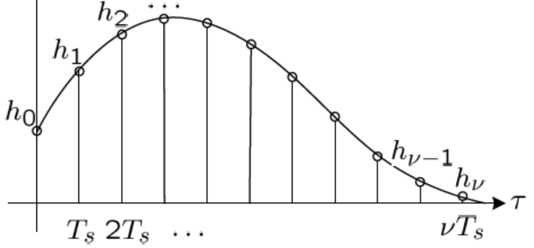
Source #1: Single-Tap Channel: Rayleigh Dist'n

- Path loss, shadowing => average signal power loss
  - Fading around this average.
  - Subtract out average => fading modeled as a zero-mean random process
- Narrowband Fading channel: Each symbol is long in time
  - The channel h(t) is assumed to be uncorrelated across symbols => single "tap" in time domain.
- Fading w/ many scatterers: Central Limit Theorem
  - In-phase (cosine) and quadrature (sine) components of the snapshot r(0), denoted as r_I (0) and r_Q(0) are independent Gaussian random variables.
  - Envelope Amplitude: $|r| = \sqrt{r_I^2 + r_Q^2}$ is Rayleigh.
  - Received Power: $|r|^2 = r_I^2 + r_Q^2$ is exponentially distributed.

Source #2: Multipaths: Power-Delay Profile



Channel Impulse Response:
Channel amplitude |h| <u>correlated</u> at delays τ.
Each "tap" value @ kTs Rayleigh distributed
(actually the sum of several sub-paths)



Eg: Power Delay Profile (WLAN/ indoor)

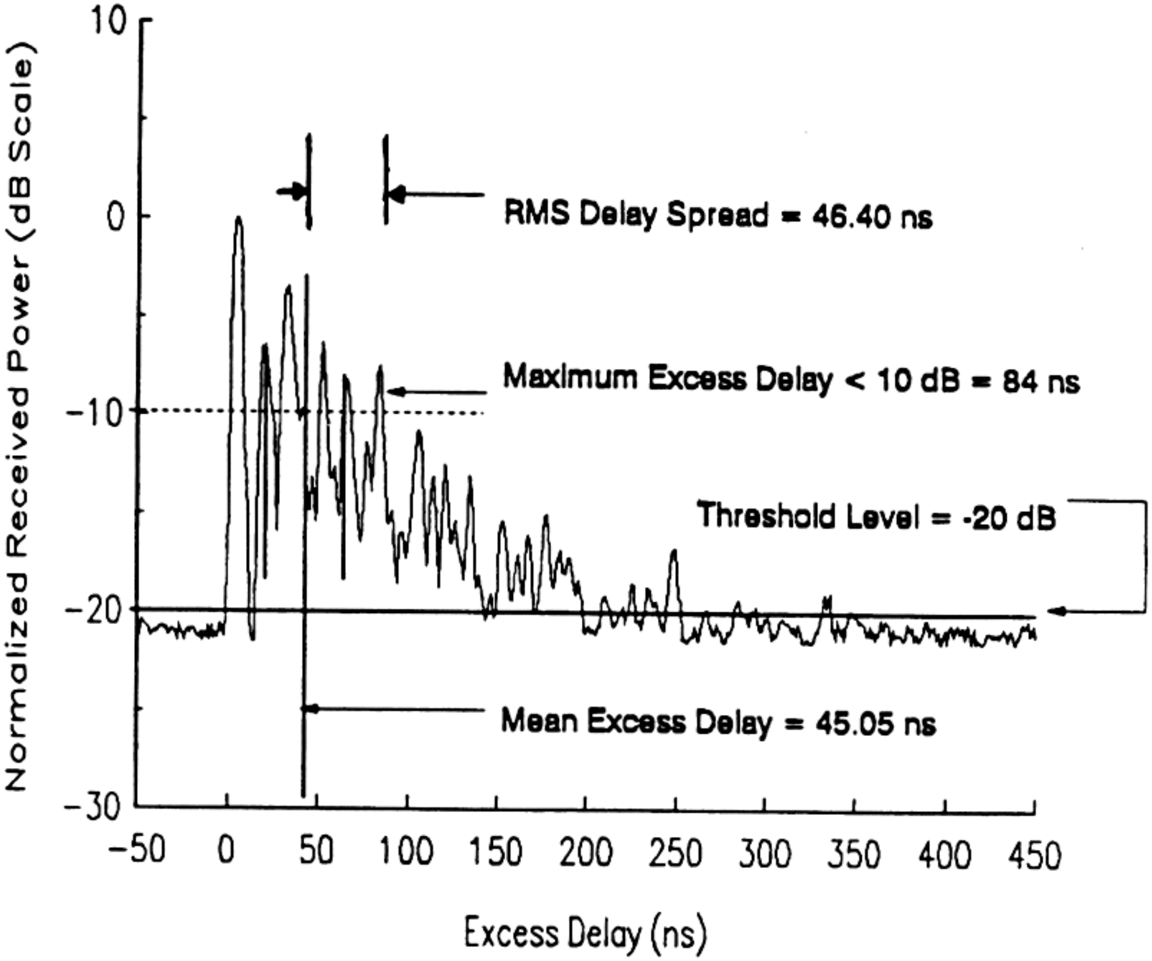
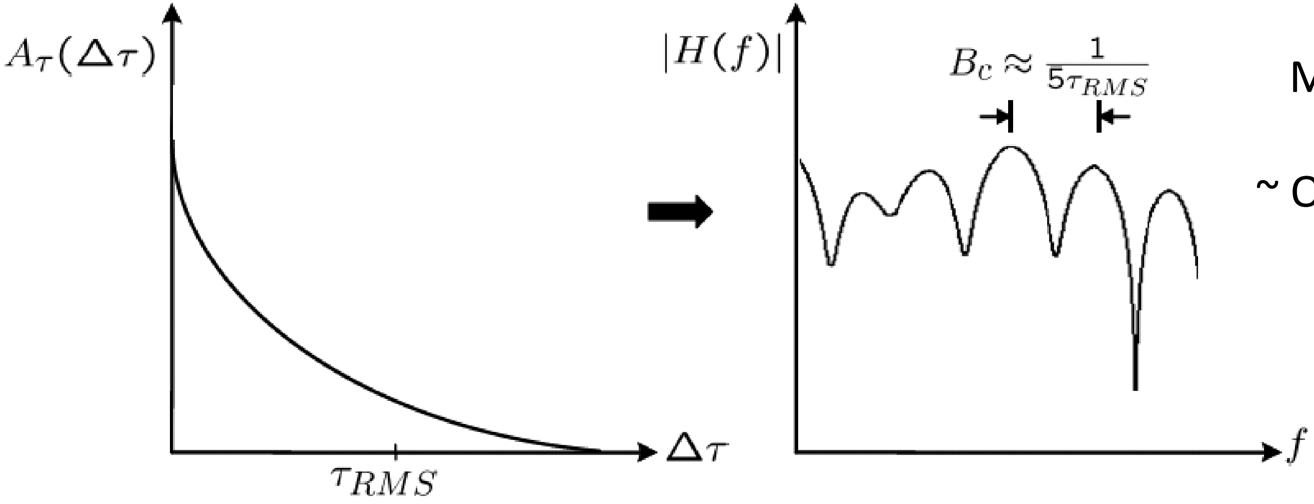


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

Multipath: Time-Dispersion => Frequency Selectivity

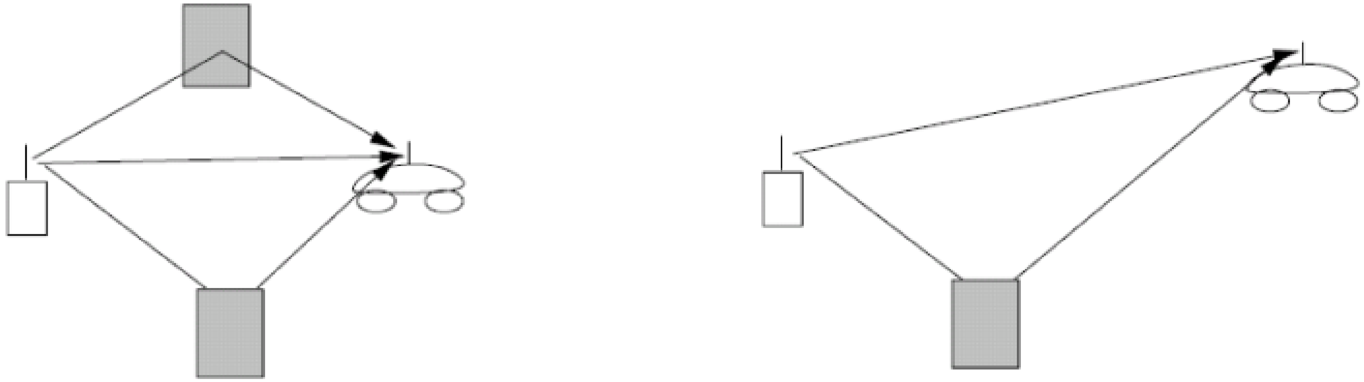
- The impulse response of the channel is correlated in the time-domain (sum of “echoes”)
  - Manifests as a power-delay profile, dispersion in channel autocorrelation function A(Δτ)
- Equivalent to “selectivity” or “deep fades” in the frequency domain
- **Delay spread:** τ ~ *50ns (indoor) – 1μs (outdoor/cellular).*
- **Coherence Bandwidth**: Bc = *500kHz (outdoor/cellular) – 20MHz (indoor)*
- Implications: High data rate: symbol smears onto the adjacent ones (ISI).

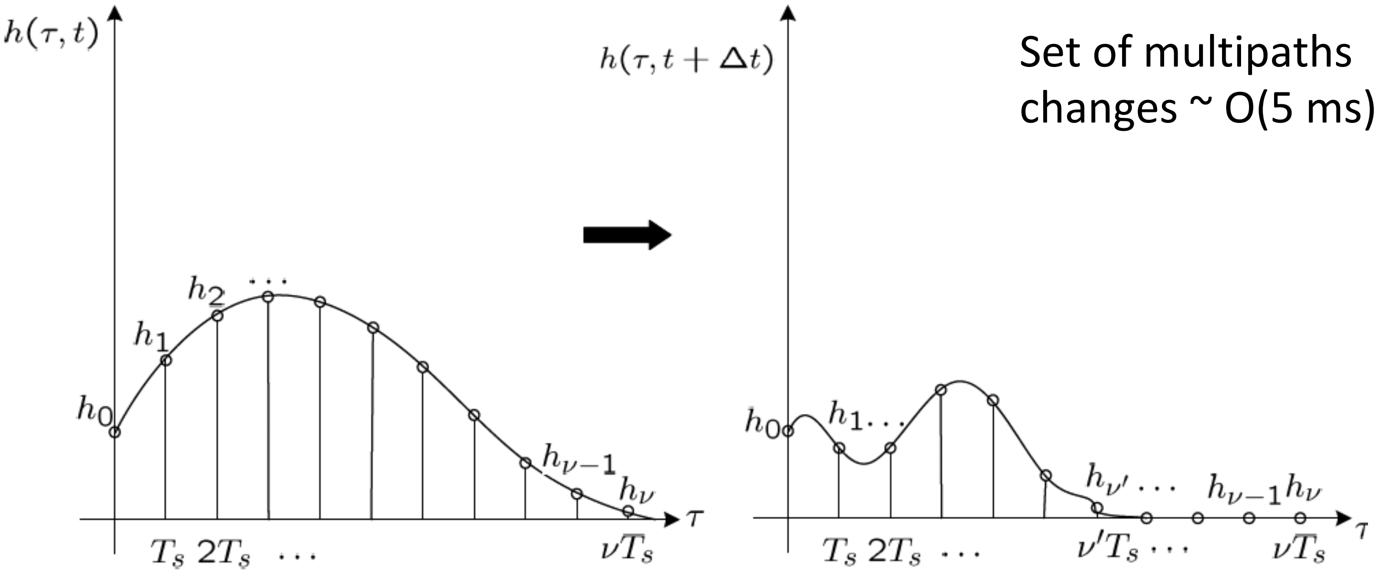


Multipath effects ~ O(1μs)

the shape of the multipath intensity profile $A_\tau(\Delta\tau)$ determines the correlation pattern of the channel frequency response (bottom)

Source #3: Doppler: Non-Stationary Impulse Response.





Set of multipaths changes ~ O(5 ms)

Doppler: Dispersion (Frequency) => Time-Selectivity

- The doppler power spectrum shows dispersion/flatness ~ doppler spread (100-200 Hz for vehicular speeds)
  - Equivalent to “selectivity” or “deep fades” in the time domain correlation envelope.
  - Each envelope point in time-domain is drawn from Rayleigh distribution. But because of Doppler, it is not IID, but correlated for a time period ~ Tc (correlation time).
- **Doppler Spread:** Ds ~ 100 Hz (vehicular speeds @ 1GHz)
- **Coherence Time**: Tc = 2.5-5ms.
- Implications: A deep fade on a tone can persist for 2.5-5 ms! Closed-loop estimation is valid only for 2.5-5 ms.

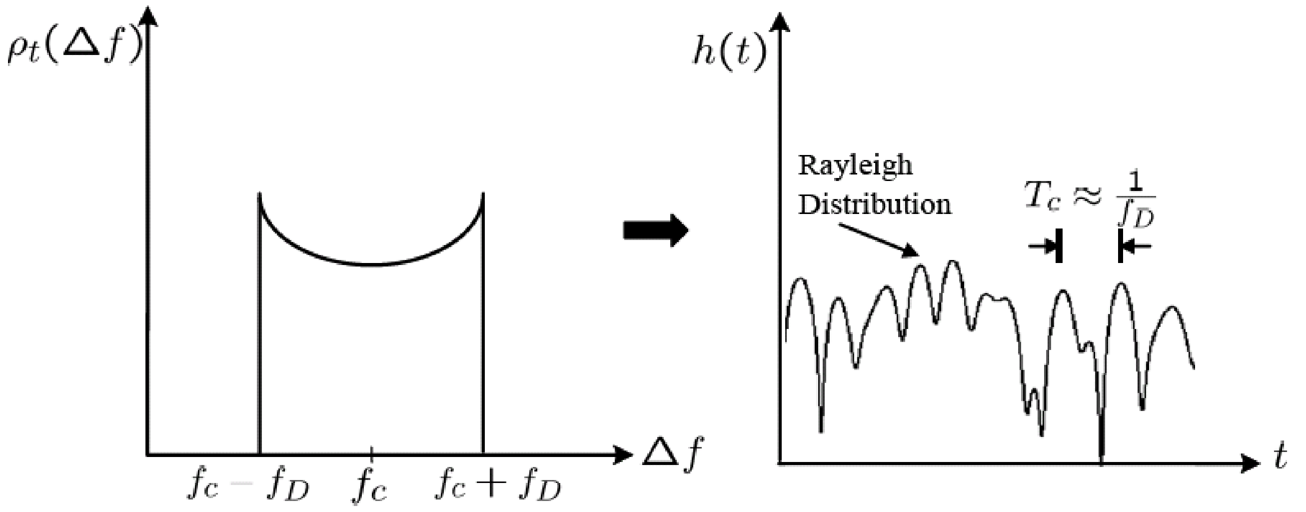


Figure 3.18: The shape of the Doppler power spectrum $\rho_t(\Delta f)$ determines the correlation envelope of the channel in time (top).

Fading Summary: Time-Varying Channel Impulse Response

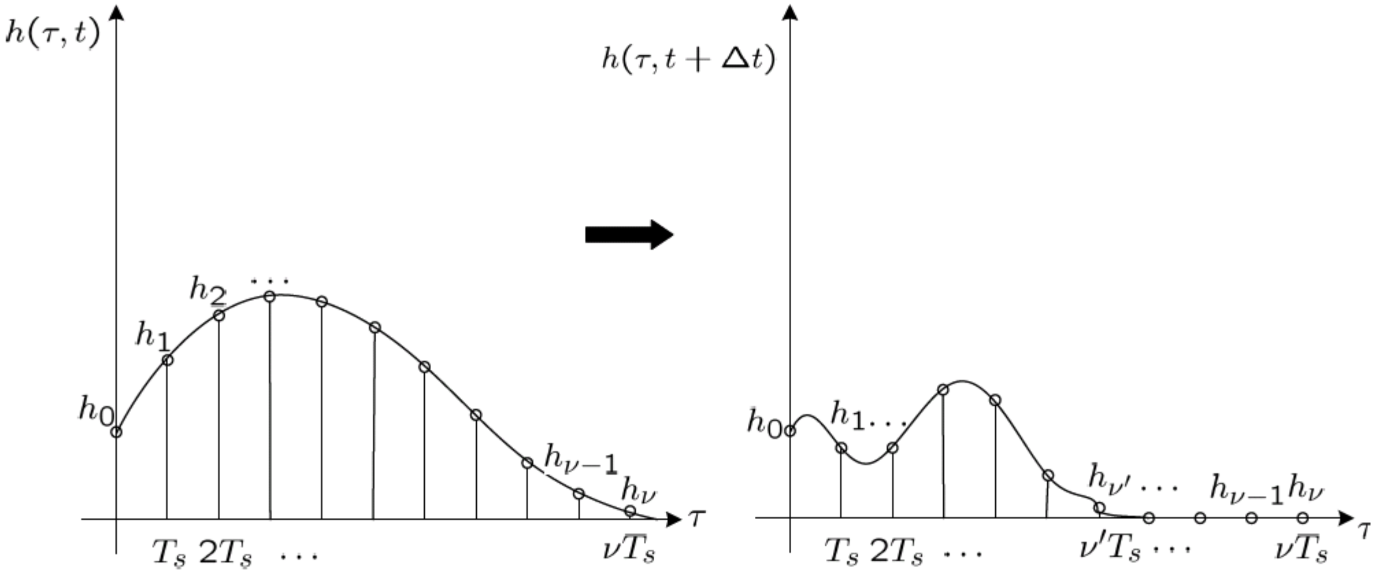


Figure 3.12: The delay τ corresponds to how *long* the channel impulse response lasts. The channel is time varying, so the channel impulse response is also a function of time, i.e. $h(\tau, t)$, and can be quite different at time $t + \Delta t$ than it was at time t.

- **#1:** At each tap, channel gain |h| is a Rayleigh distributed *r.v.*. The random *process* is not IID.
- **#2:** Response spreads out in the time-domain (τ), leading to inter-symbol interference and deep fades in the frequency domain: "***frequency-selectivity***" caused by multi-path fading
- **#3:** Response completely vanish (deep fade) for certain values of t: "***Time-selectivity***" caused by doppler effects (frequency-domain dispersion/spreading)

Dispersion-Selectivity Duality

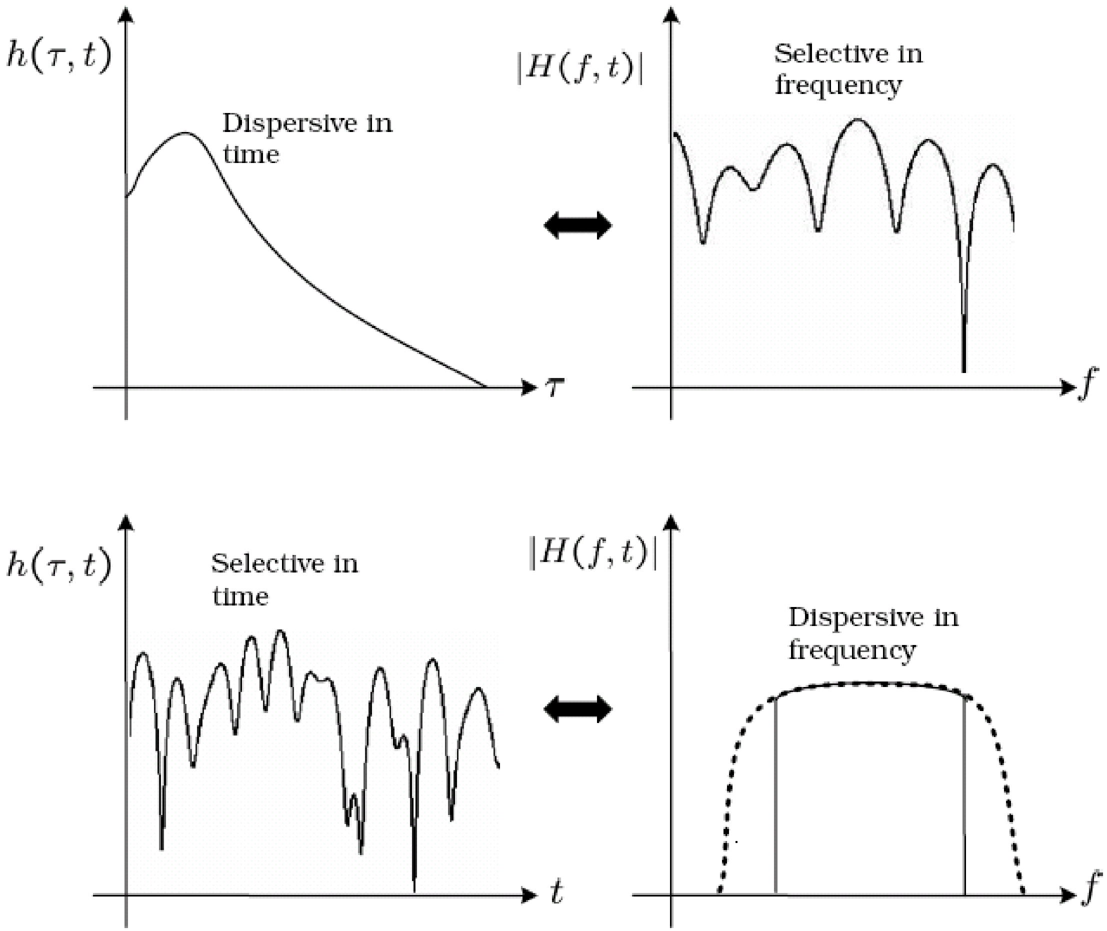


Figure 3.19: The dispersion–selectivity duality: Dispersion in time causes frequency selectivity, while dispersion in frequency causes time selectivity.

Dispersion-Selectivity Duality (Contd)

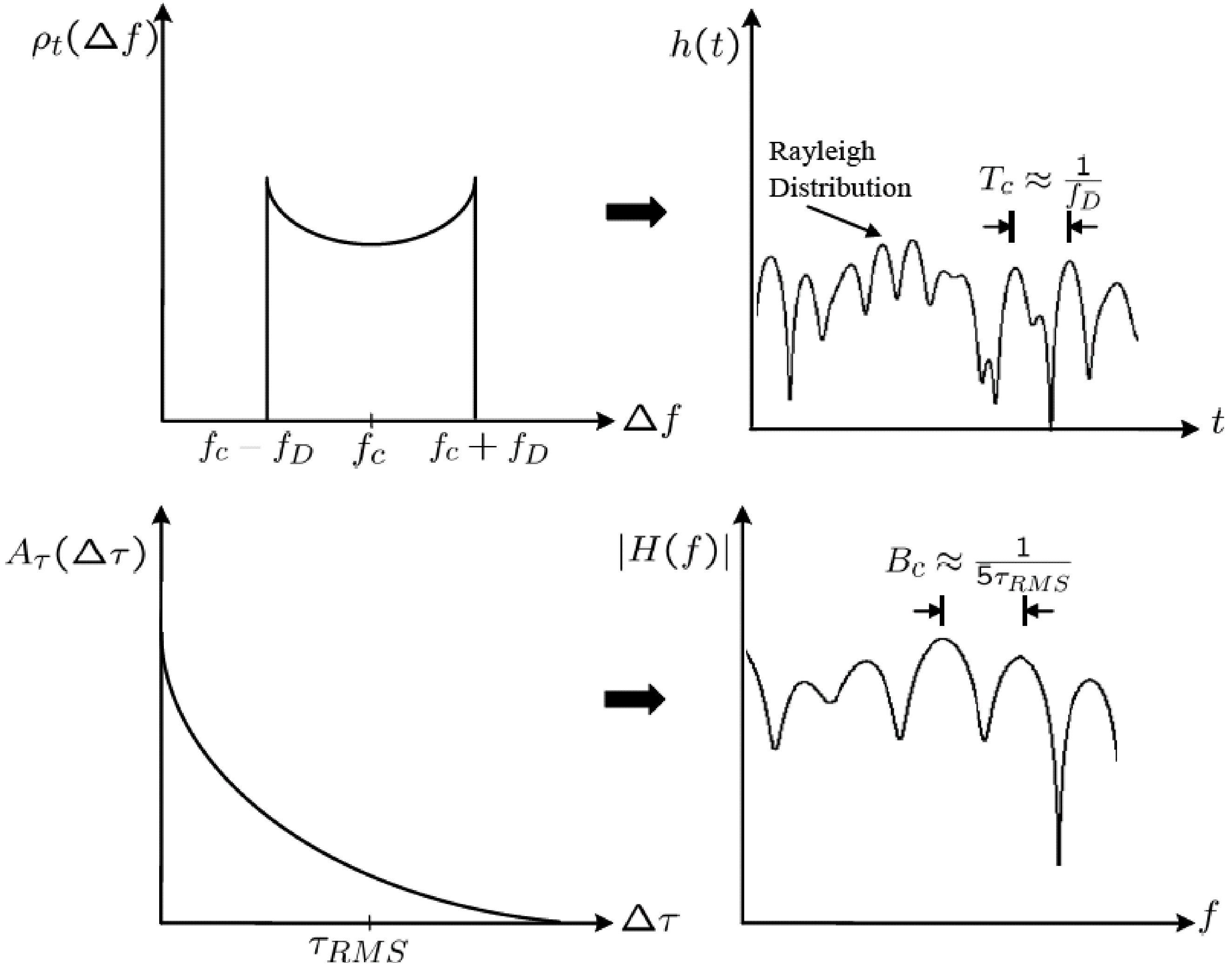


Figure 3.18: The shape of the Doppler power spectrum $\rho_t(\Delta f)$ determines the correlation envelope of the channel in time (top). Similarly, the shape of the multipath intensity profile $A_\tau(\Delta\tau)$ determines the correlation pattern of the channel frequency response (bottom)

Fading: Jargon

- **Flat fading**: no multipath ISI effects.
  - Eg: narrowband, indoors
- **Frequency-selective fading:** multipath ISI effects.
  - Eg: broadband, outdoor.

- **Slow fading:** no doppler effects.
  - Eg: indoor Wifi home networking
- **Fast Fading:** doppler effects, time-selective channel
  - Eg: cellular, vehicular

- Broadband cellular + vehicular => Fast + frequency-selective

Rayleigh, Ricean, Nakagami-m fading

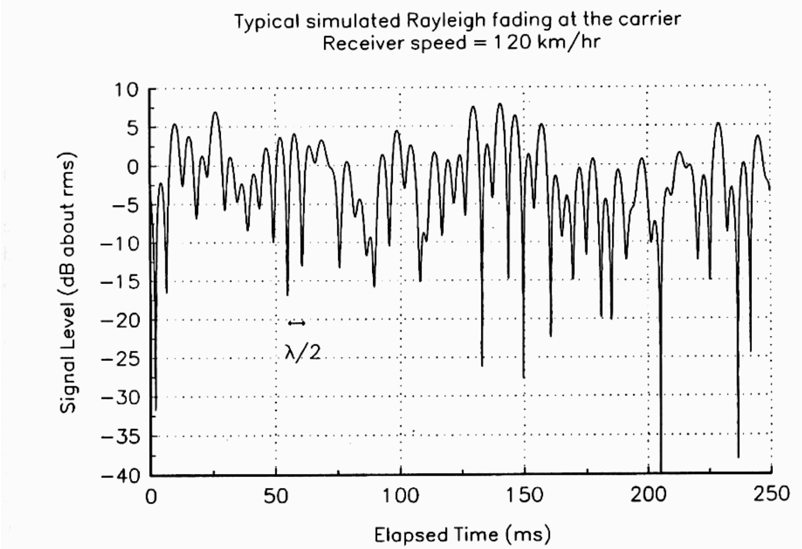


Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

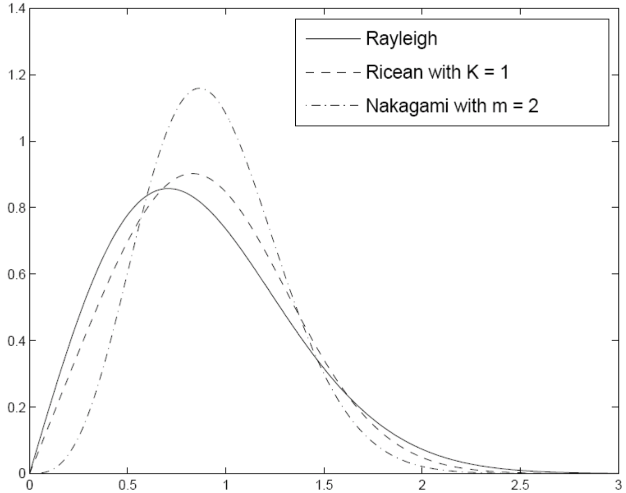


Figure 3.16: Probability distributions $f_{|r|}(x)$ for Rayleigh, Ricean w/ $K = 1$, and Nakagami with $m = 2$. All have average received power $P_r = 1$.

<u>Ricean</u> used when there is a dominant LOS path.
K parameter: strength of LOS to non-LOS. K = 0 => Rayleigh

<u>Nakagami-m</u> distribution can in many cases be used in tractable analysis of fading channel performance. More general than Rayleigh and Ricean.

Effect of Rayleigh Fading

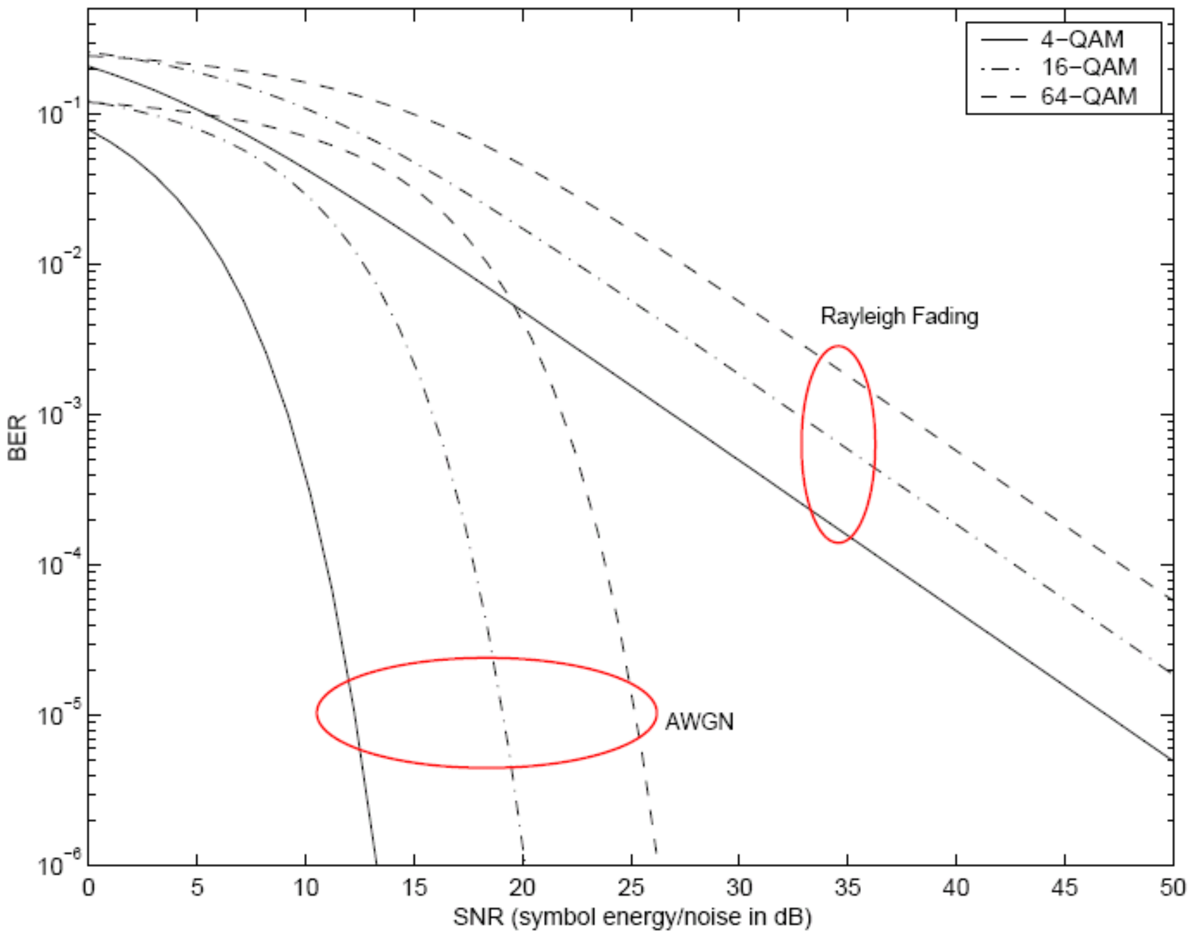
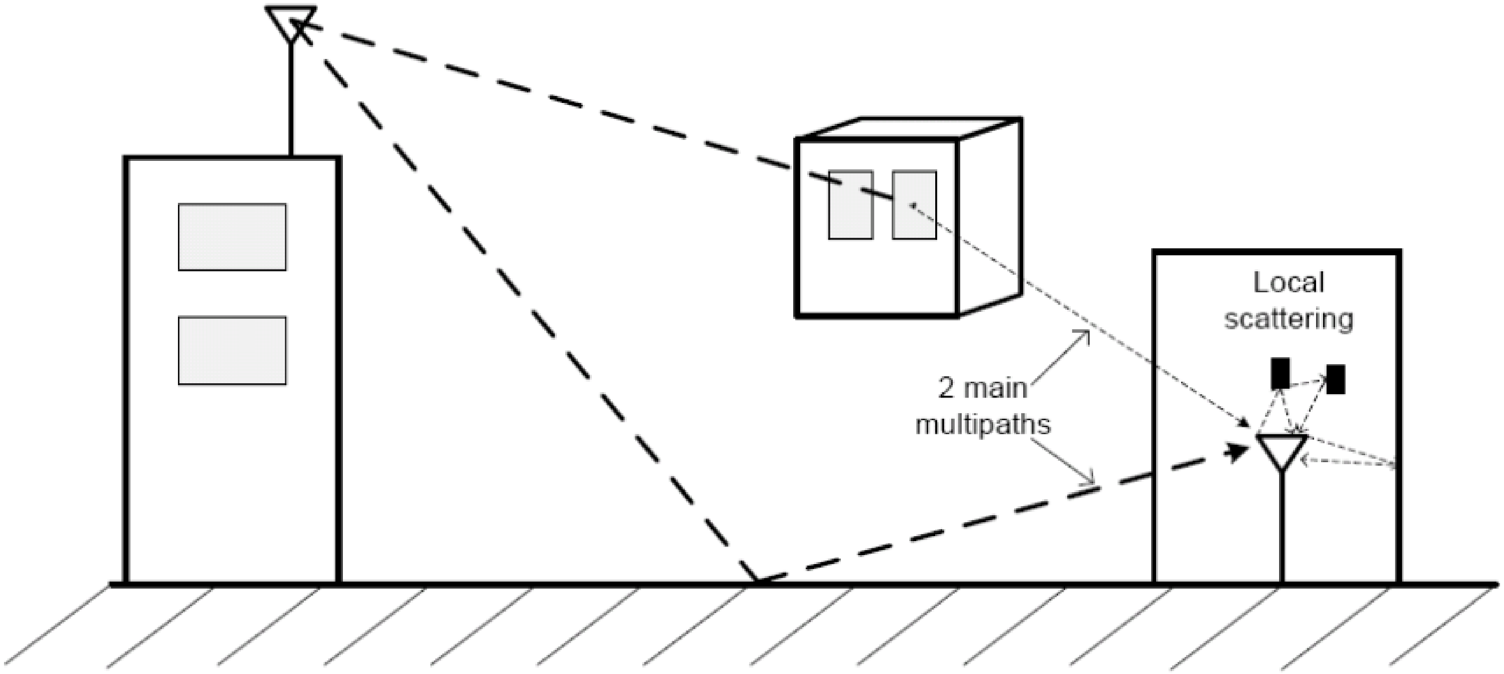


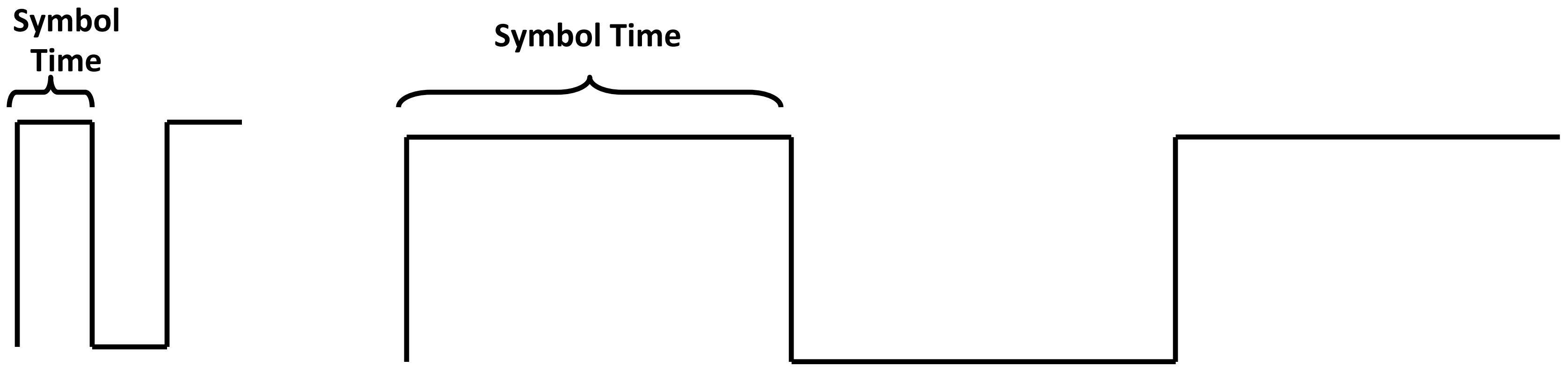
Figure 3.22: Flat fading causes a loss of at least 20-30 dB at reasonable BER values.

Broadband Fading: Multipath Frequency Selectivity



- A few major multipaths, and lots of local scatterers => each channel sample “tap” can be modeled as Rayleigh
  - A “tap” period generally shorter than a symbol time.
- Correlation between tapped values.

Power Delay Profile => Inter-Symbol interference



Symbol Time

Symbol Time

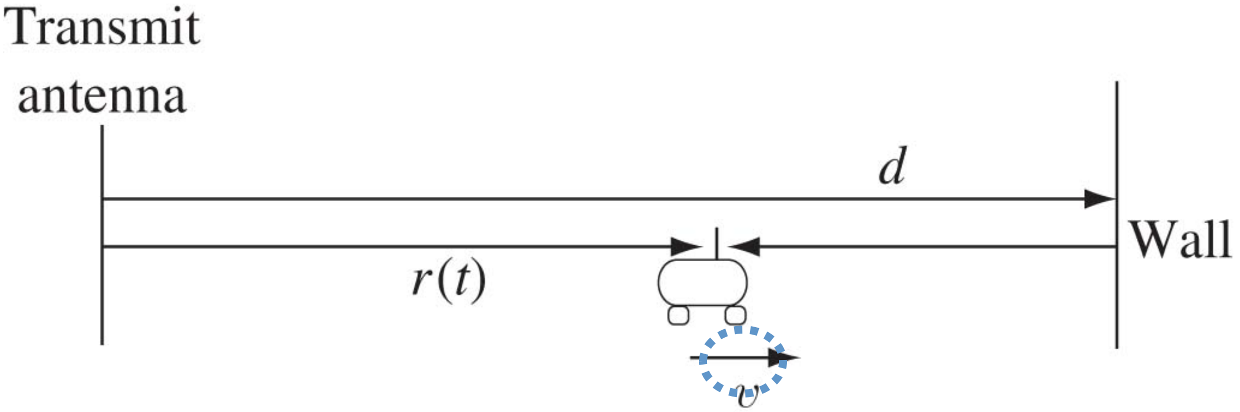
- Higher bandwidth => higher symbol rate, and smaller time per-symbol
- Lower symbol rate, more time, energy per-symbol
- If the delay spread is longer than the symbol-duration, symbols will “smear” onto adjacent symbols and cause symbol errors

Delay spread
~ 1 µs

Symbol Error!
If symbol rate
~ Mbps

No Symbol Error! (~kbps)
(energy is collected
over the full symbol period
for detection)

Doppler: Reflecting Wall, Moving Antenna



$$E_r(f,t) = \frac{\alpha \cos 2\pi f[(1-\frac{v}{c})t - \frac{r_0}{c}]}{r_0 + vt} - \frac{\alpha \cos 2\pi f\left[(1+\frac{v}{c})t + \frac{r_0 - 2d}{c}\right]}{2d - r_0 - vt}.$$

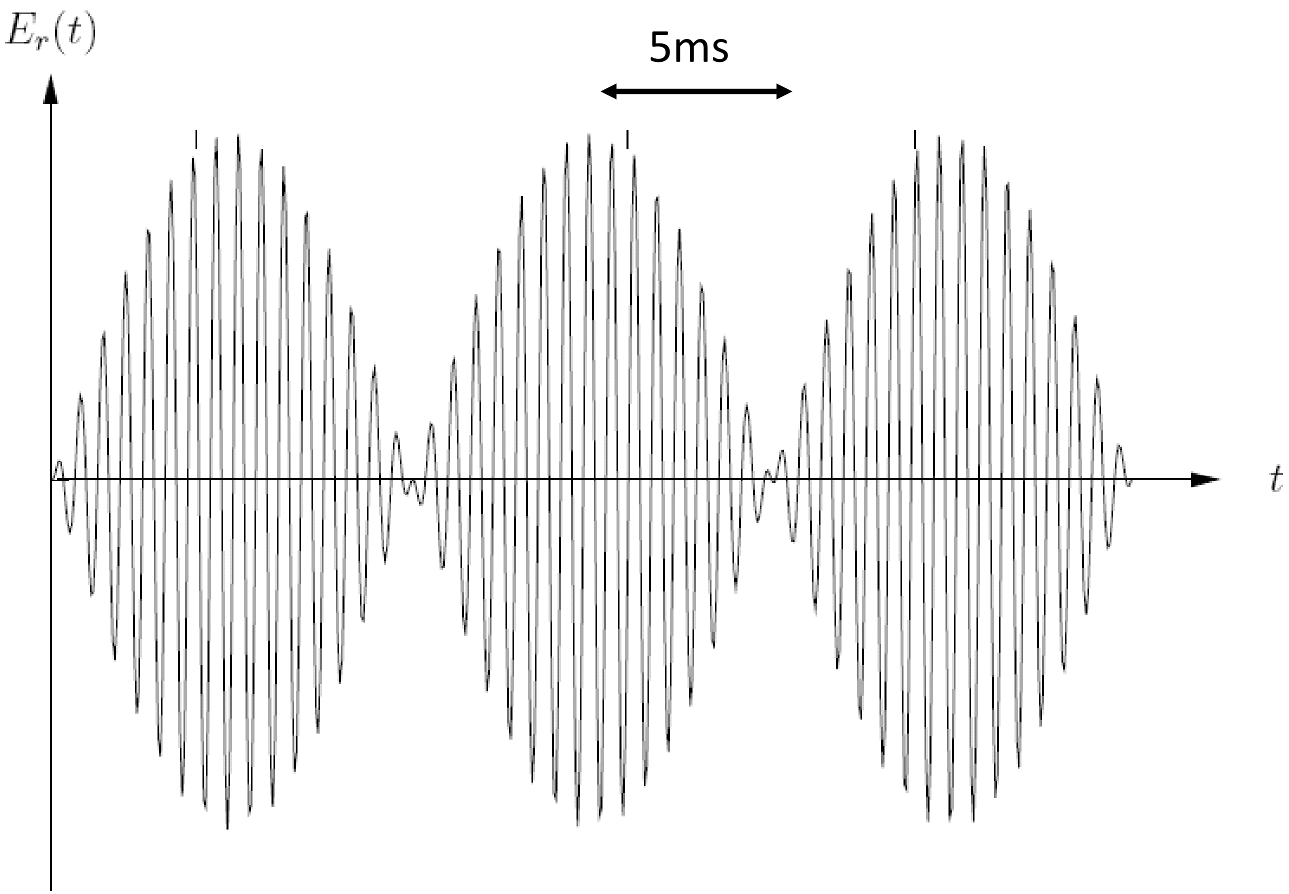
- *Doppler spread:* $\boxed{D_s := D_2 - D_1}$

$$D_1 := -fv/c.$$

$$D_2 := +fv/c$$

- Note: opposite sign for doppler shift for the two waves
- Effect is roughly like the *product of two sinusoids*

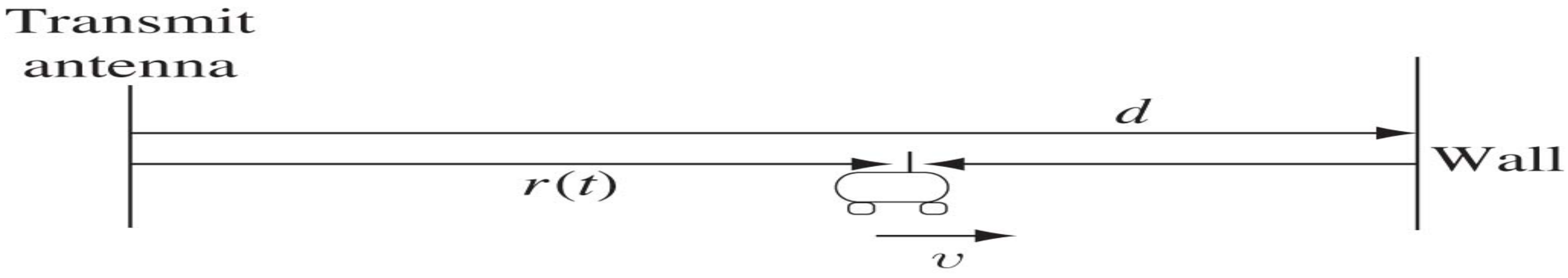
Doppler Spread: Effect



- Fast oscillations of the order of GHz
- Slow envelope oscillations order of 50 Hz => peak-to-zero every 5 ms
- A.k.a. **Channel coherence time (Tc**) = *c/4fv*

Two-path (mobile) Example

- v= 60 km/hr, fc = 900 MHz:
- Direct path has Doppler shift of roughly -50 Hz = -fv/c
- Reflected path has shift of +50 Hz
- Doppler spread = 100 Hz



Doppler Spread: Effect

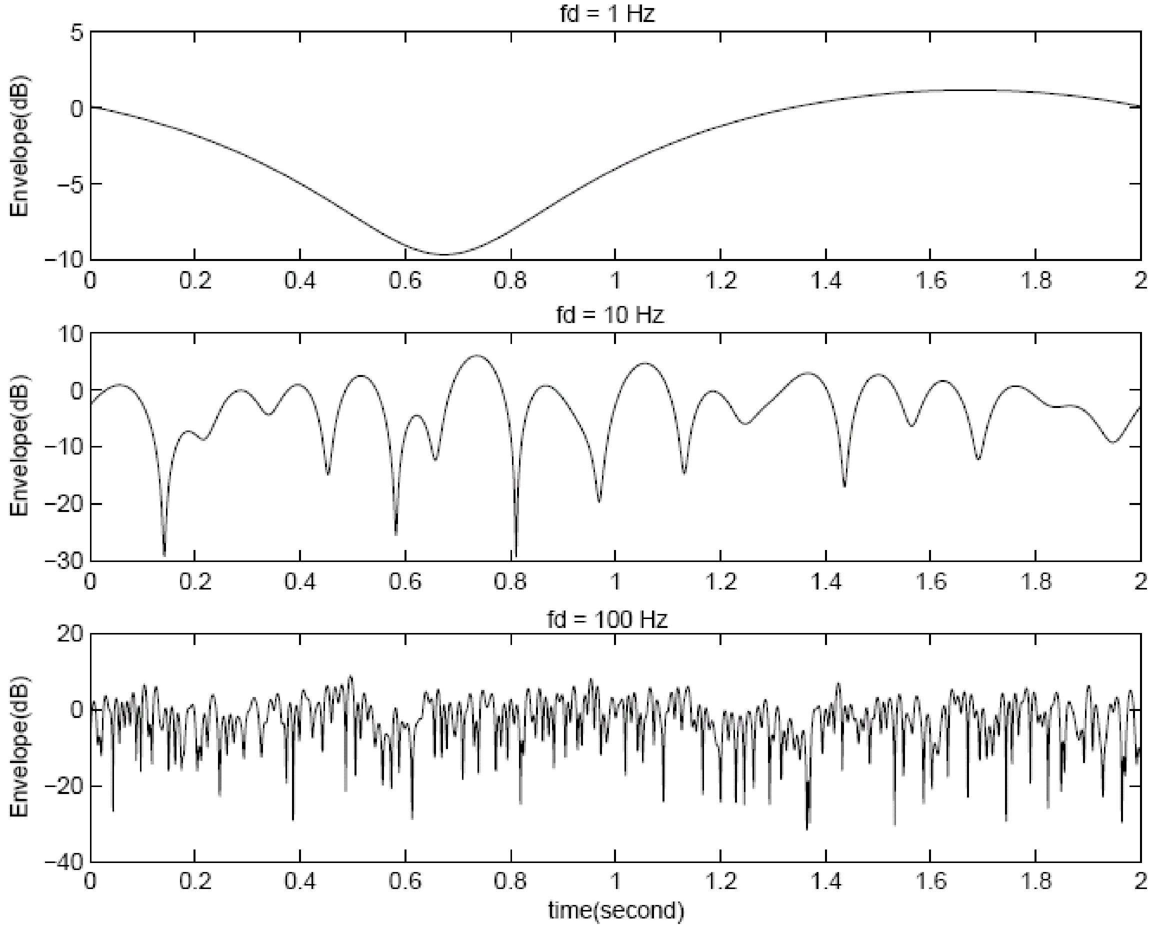


Figure 3.20: A sample output of the provide Rayleigh fading Matlab function for Doppler frequencies of $f_D = 1, 10$, and 100 Hz.

Angular Spread: Impact on Spatial Diversity

- Space-time channel models:
  - Mean/RMS angular spreads (similar to multipath delay spread)
  - The time-varying impulse response model can be extended to incorporate AOA (angle-of-arrival) for the array.
  - A(θ): average received signal power as a function of AoA θ.
  - Needs appropriate linear transformation to achieve full MIMO gains.

$$\overline{a}(\theta_n(t)) = [e^{-j\psi_{n,1}}, \ldots, e^{-j\psi_{n,M}}]^T$$

$$\mu_\theta = \frac{\int_{-\pi}^{\pi} \theta A(\theta) d\theta}{\int_{-\pi}^{\pi} A(\theta) d\theta},$$

$$\sigma_\theta = \sqrt{\frac{\int_{-\pi}^{\pi} (\theta - \mu_\theta)^2 A(\theta) d\theta}{\int_{-\pi}^{\pi} A(\theta) d\theta}},$$

Angular Spread and Coherence Distance

- θRMS: RMS angular spread of a channel
  - Refers to the statistical distribution of the angle of the arriving energy.
- Large θRMS => channel energy is coming in from many directions,
  - Lot of local scattering, and this results in more statistical diversity in the channel based upon AoA
- Small θRMS => received channel energy is more focused.
  - More focused energy arrival results in less statistical diversity.

- The dual of angular spread is coherence distance, Dc.
  - As the angular spread↑, the coherence distance ↓, and vice versa.
  - A coherence distance of d means that any physical positions separated by d have an essentially uncorrelated received signal amplitude and phase.

↑freq =>
better angular diversity!

Key Wireless Channel Parameters

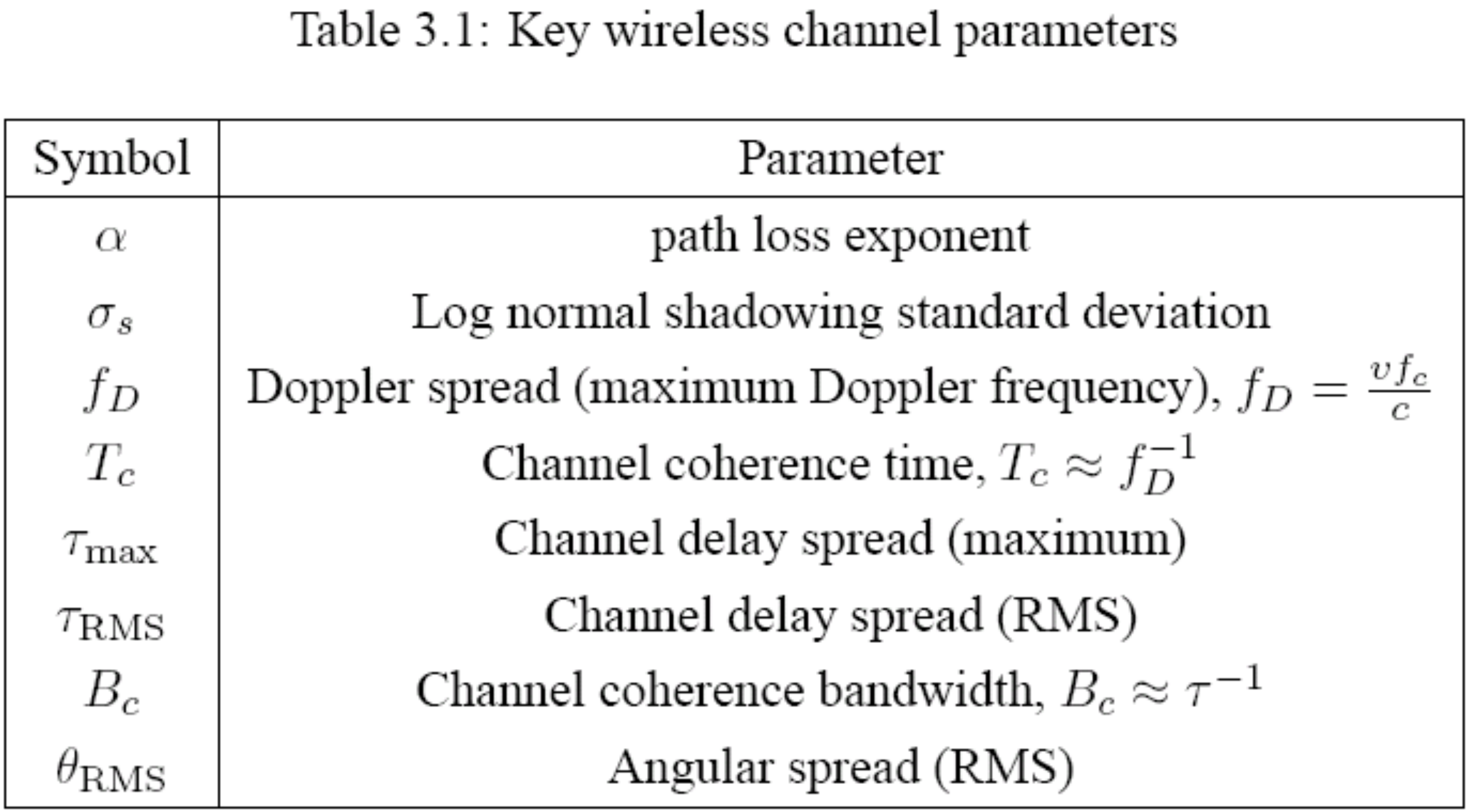


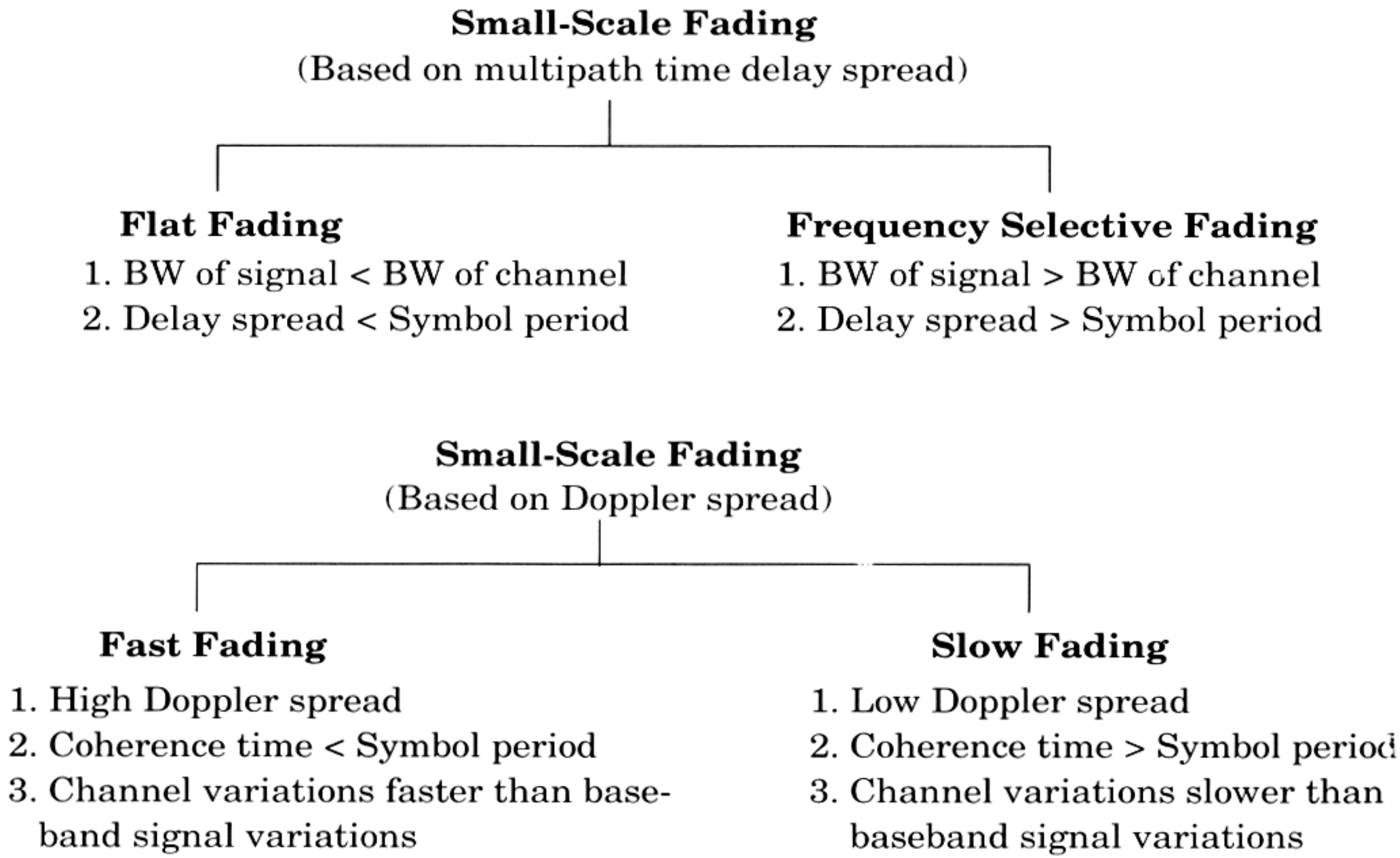
Table 3.1: Key wireless channel parameters

Symbol	Parameter
α	path loss exponent
σ_s	Log normal shadowing standard deviation
f_D	Doppler spread (maximum Doppler frequency), $f_D = \frac{v f_c}{c}$
T_c	Channel coherence time, $T_c \approx f_D^{-1}$
$\tau_{\max}$	Channel delay spread (maximum)
τ_{RMS}	Channel delay spread (RMS)
B_c	Channel coherence bandwidth, $B_c \approx \tau^{-1}$
θ_{RMS}	Angular spread (RMS)

Fading Parameter Values

Key channel parameters and time-scales	Symbol	Representative values
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between transmitter and receiver	d	1 km
Velocity of mobile	v	64 km/h
Doppler shift for a path	$D = f_c v/c$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
Time-scale for change of path amplitude	d/v	1 minute
Time-scale for change of path phase	$1/(4D)$	5 ms
Time-scale for a path to move over a tap	$c/(vW)$	20 s
Coherence time	$T_c = 1/(4D_s)$	2.5 ms
Delay spread	T_d	1 μs
Coherence bandwidth	$W_c = 1/(2T_d)$	500 kHz

Small-Scale Fading Summary



Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

Fading: Design Impacts (Eg: Wimax)

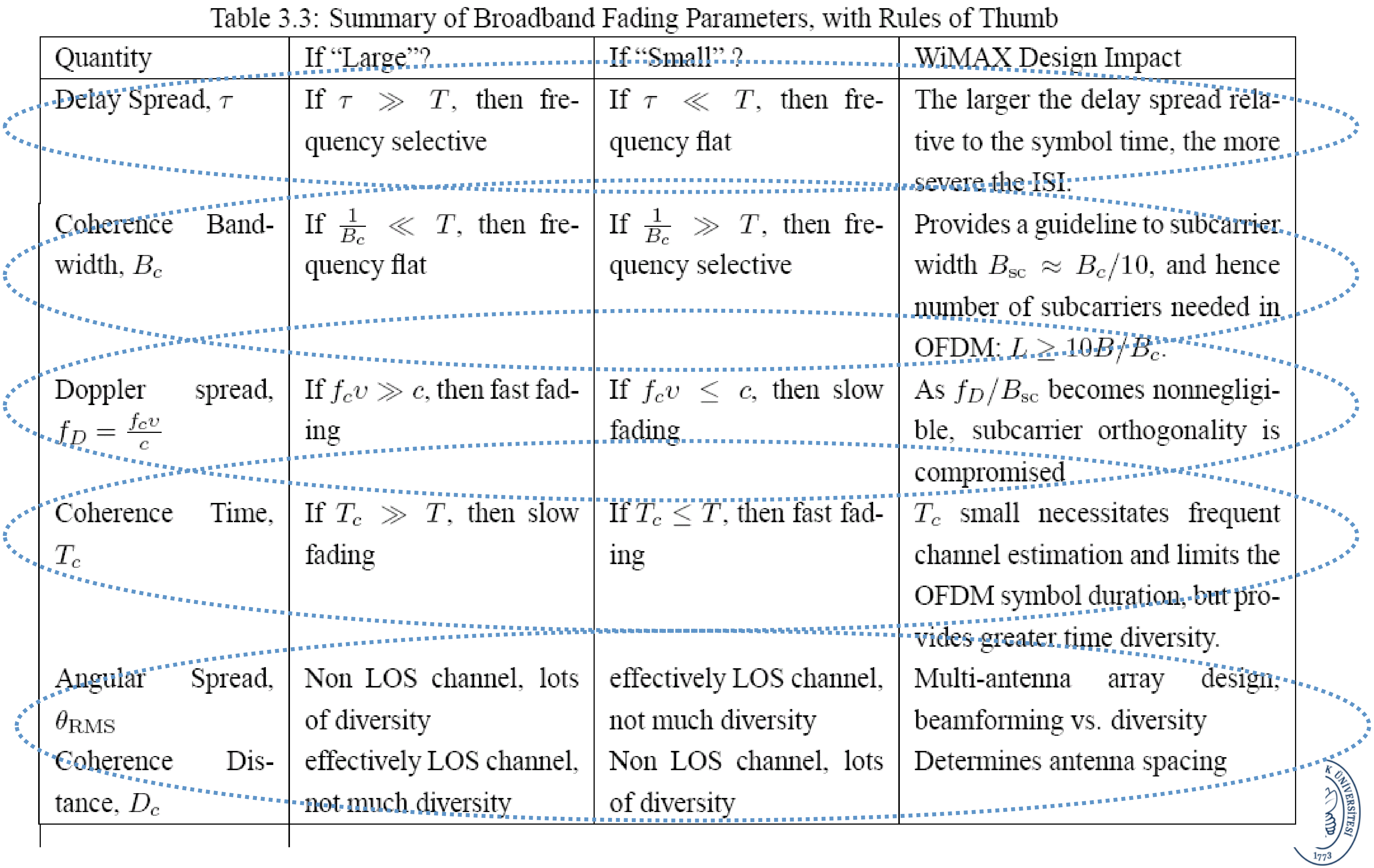


Table 3.3: Summary of Broadband Fading Parameters, with Rules of Thumb

Quantity	If "Large"?	If "Small" ?	WiMAX Design Impact
Delay Spread, τ	If $\tau \gg T$, then frequency selective	If $\tau \ll T$, then frequency flat	The larger the delay spread relative to the symbol time, the more severe the ISI.
Coherence Bandwidth, B_c	If $\frac{1}{B_c} \ll T$, then frequency flat	If $\frac{1}{B_c} \gg T$, then frequency selective	Provides a guideline to subcarrier width $B_{\mathrm{sc}} \approx B_c/10$, and hence number of subcarriers needed in OFDM: $L \geq 10B/B_c$.
Doppler spread, $f_D = \frac{f_c v}{c}$	If $f_c v \gg c$, then fast fading	If $f_c v \leq c$, then slow fading	As f_D/B_{sc} becomes nonnegligible, subcarrier orthogonality is compromised
Coherence Time, T_c	If $T_c \gg T$, then slow fading	If $T_c \leq T$, then fast fading	T_c small necessitates frequent channel estimation and limits the OFDM symbol duration, but provides greater time diversity.
Angular Spread, θ_{RMS}	Non LOS channel, lots of diversity	effectively LOS channel, not much diversity	Multi-antenna array design, beamforming vs. diversity
Coherence Distance, D_c	effectively LOS channel, not much diversity	Non LOS channel, lots of diversity	Determines antenna spacing

Time-Invariance Assumption: Typical Channels are Underspread

- Coherence time Tc depends on carrier frequency and vehicular speed, of the order of milliseconds or more.
- Delay spread Td depends on distance to scatterers, of the order of nanoseconds (indoor) to microseconds (outdoor).
- Channel can be considered as time-invariant over a long time scale (“underspread”).
  - Transfer function & frequency domain methods can still be applied to this approximately LTI model

Doppler Spread

$$D_s := \max_{i,j} |f_c \tau_i'(t) - f_c \tau_j'(t)|$$

- Doppler spread is proportional to:
- the carrier frequency fc;
- the angular spread of arriving paths.

$$\tau_i'(t) = \frac{v}{c} \cos \theta_i$$

- where θi is the angle the direction of motion makes with the i th path.

Degrees of Freedom (Complex Dimensions)

- Discrete symbol $x[m]$ is the mth sample of the transmitted signal; there are W samples per second.
- Continuous time signal $x(t)$, 1 s ≡ W discrete symbols
- Each discrete symbol is a complex number;
  - It represents one (complex) dimension or degree of freedom.
  - Bandlimited $x(t)$ has W degrees of freedom per second.
  - Signal space of complex continuous time signals of duration T which have most of their energy within the frequency band $[-W/2, W/2]$ has dimension approximately WT.
- Continuous time signal with bandwidth W can be represented by W complex dimensions per second.
- Degrees of freedom of the channel to be the dimension of the received signal space of $y[m]$

Summary

- We have understood both qualitatively and quantitatively the concepts of path loss, shadowing, fading (multi-path, doppler), and some of their design impacts.

- We have understood how time and frequency selectivity of wireless channels depend on key physical parameters.

- We have come up with linear, LTI and statistical channel models useful for analysis and design.

References

- Mobile Broadband by Ergen
- ECSE6961 Slides of Kalyanaraman
- Wireless Communication by Goldsmith
- Fundamentals of Wireless by Tse