

3 Find the eigenvectors of the matrix **A** given below:

3.) $\det(\lambda I - A) = 0$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 3 & 2 \end{bmatrix}$$

$$\begin{vmatrix} \lambda - 2 & 0 & 0 \\ 0 & \lambda - 2 & -3 \\ 0 & -3 & \lambda - 2 \end{vmatrix} = 0$$

$$(\lambda-2)[(\lambda-2)^2 - 9] = 0 \quad (\lambda-2)(\lambda^2 - 4\lambda + 4 - 9) = 0$$
$$(\lambda-2)(\lambda^2 - 4\lambda - 5) = 0$$
$$\qquad\qquad\qquad \underbrace{\hspace{1cm}}_{-5 \quad 1}$$

$$(x-2)(x^2-4x-5)=0$$

$$\lambda_1 = 2$$

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$$\lambda_2 = -1 \quad \lambda_3 = 5$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$x_2 = x_3 = 0$$

$$x_1 = c \neq 0$$

$$[C]$$

$$\lambda_2 = -1 \quad \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 0$$

$$x_2 + x_3 = 0$$

$$x_2 = -x_3$$

$$x_2 = c \neq 0$$

$$x_3 = -2$$

$$\lambda_3 = 5 \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 0$$

$$x_2 - x_3 = 0$$

$$x_2 = x_3 = cf_0$$

$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -c & c \end{bmatrix}$$

[cc]