Note Title 4/18/2012

Dynamics of Rotation

$\vec{F} = m\vec{a}$ lin

kg $\frac{m}{s^2}$ kg m/s²

$\vec{\tau} = I\vec{\alpha}$ ½² rotational motion

md^2 kg m²

kg $\frac{m^2}{s^2}$

$\vec{p} = m\vec{v}$ kg m/s Lin. mom. $\frac{1}{2}mv^2$

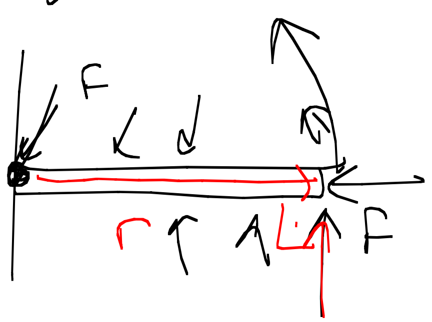
$\vec{L} = I\vec{\omega}$ 1/s ang. mom $\frac{1}{2}I\omega^2$

kg m² kg m/s kg m²/s

$\vec{r} \times \vec{p} \to \vec{L}$

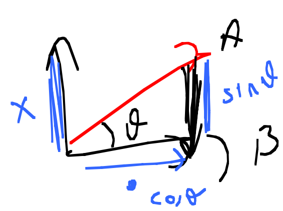
$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

↳ Ability of a force to rotate an object



$\tau \propto r$

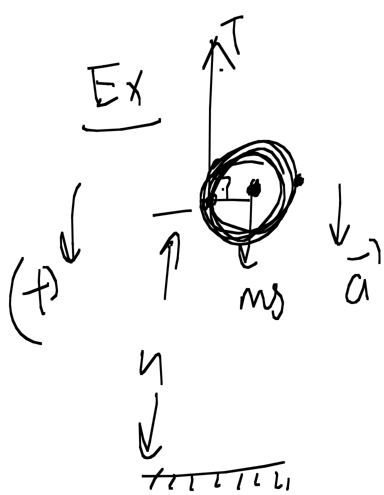
$\tau \propto F$



$A \times B = AB \sin\theta$

$A \cdot B = AB \cos\theta$

Ex



Yo-yo

$\vec{F} = m\vec{a}$

$\vec{\tau} = I\vec{\alpha}$

$\sum \vec{\tau} = I\vec{\alpha}$

$\sum \vec{F} = m\vec{a}$

$\tau = \vec{r} \times \vec{F}$

$$T * R = \tau_{tension}$$
$$\tau_{mg} = mg * 0 = 0$$

Pully

Cylindrical

$$-T + mg = ma \quad (1)$$
$$T * R = I\alpha \quad (2)$$

$$\boxed{I = \frac{1}{2} M R^2 \quad (3)}$$

$$\alpha = \frac{a}{R}$$

(1) $mg - T = ma$ (Force)

(2) $T * \cancel{R} = \frac{m}{2} \cancel{R^2} \frac{a}{\cancel{R}}$ (Torque)

$$mg - \cancel{T} = ma \quad (1)$$
$$\cancel{T} = \frac{m}{2} a \quad (2)$$

$$T = ? \quad \frac{m}{\cancel{2}} * \frac{\cancel{2} g}{3}$$

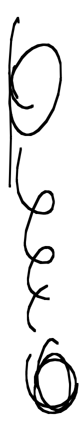
$$\boxed{\frac{mg}{3} = T}$$

$+$

$$\cancel{m} g = \frac{3}{2} \cancel{m} a \Rightarrow a = \frac{2g}{3}$$

$$v^2 = 2ah \Rightarrow$$

$$v^2 = 2 * \frac{2g}{3} h \Rightarrow v = \sqrt{\frac{4gh}{3}}$$

2) 

$$w = \frac{v}{R}$$

$$\frac{1}{2} m v^2 + \frac{1}{2} I w^2 = mgh$$

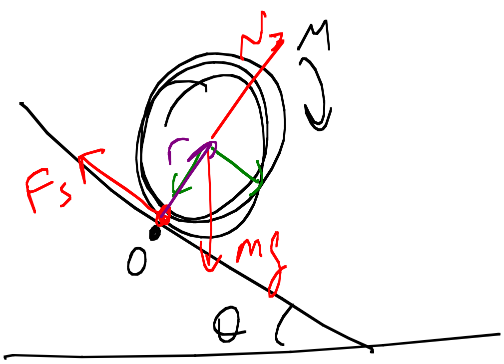
$$\frac{1}{2} \cancel{m} v^2 + \frac{1}{2} * \frac{1}{2} \cancel{m} \cancel{R^2} * \frac{v^2}{\cancel{R^2}} = \cancel{m} g h$$

$$\frac{v^2}{2}\Big[\underbrace{1+\frac{1}{2}}_{3/2}\Big] = gh$$

$$\frac{3v^2}{4} = gh \rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

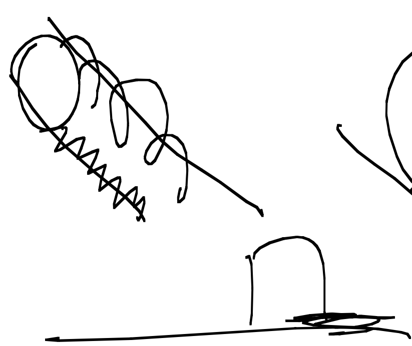
$F_s \le \underbrace{\mu_s N}_{\mu_s mg\cos\theta}$

Ex:



$a = ?$

$F_s = ?$



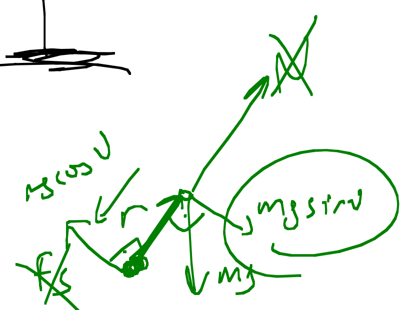
$\frac{2}{5}MR^2 = I_s$

$I_{obj} + Md^2$

$\frac{2}{5}MR^2 + MR^2$

(1) $\sum \tau_O = \underline{mg\sin\theta} \times \underline{R}$

$= I\alpha = \frac{7}{5}MR^2\frac{a}{R}$



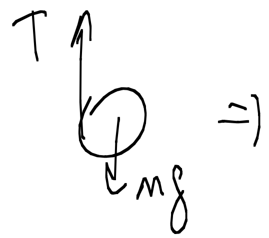
$I = I_{obj} + Md^2$

$$I_{sp} = \frac{2}{5}MR^2 + MR^2$$

$$\frac{7}{5}MR^2$$

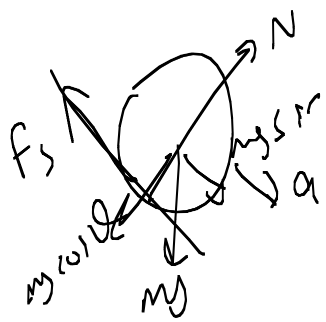
(1) $mg\sin\theta\,\cancel{R} = \frac{7}{5}M\cancel{R^2}\times\frac{a}{\cancel{R}}$

$\cancel{m}g\sin\theta = \frac{7}{5}\cancel{m}a$ (1)



$\Rightarrow \quad mg - T = ma$

$mg\sin\theta - f_s = ma \rightarrow \frac{2}{5}Ma$



$f_s = mg\sin\theta - ma \quad (2)$

$(1)\; a = \frac{5}{7} g\sin\theta$

$(2) \Rightarrow f_s = mg\sin\theta - \frac{5}{7}mg\sin\theta$

$f_s = mg\sin\theta\left[1 - \frac{5}{7}\right] = \frac{2}{7}mg\sin\theta$

2nd way

$\sum \vec{\tau}_p = \cancel{\tau}_{mg}^{\,0} + \cancel{\tau}_N^{\,0}$

τ_{fs}

$f_s * R = I\alpha = \frac{2}{5}MR^2\frac{a}{R}$

$f_s \cancel{R} = \frac{2}{5}M\cancel{R^2}\frac{a}{\cancel{R}}$

$\boxed{f_s = \frac{2}{5}Ma}$ $\quad a = \frac{5}{7}g\sin\theta$

$f_s \quad \frac{2}{\cancel{5}} * \frac{\cancel{5}}{7} * Mg\sin\theta$

$$\boxed{f_s = \frac{2}{7} M g \sin\theta}$$

Ex

$u = m, g, h, M$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} \frac{1}{2} M \cancel{r^2} \frac{v^2}{\cancel{r^2}}$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{4} M v^2$$

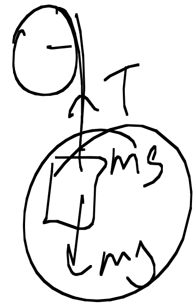
$$mgh = \frac{v^2}{2} \left[m + \frac{M}{2} \right]$$

$$v^2 = \frac{2mgh \,/m}{\frac{m}{m} + \frac{M}{2} \,/m} \Rightarrow v^2 = \frac{2gh}{1 + \frac{M}{2m}}$$

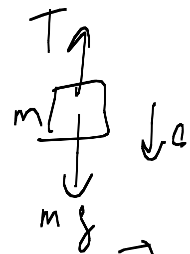
$$v = \sqrt{\frac{2gh}{1 + \frac{2M}{2m}}}$$

$M \to 0$

$\sqrt{2gh}$



$$\Rightarrow \quad mg - T = ma \quad (1)$$



$$\tau = \vec{r} \times \vec{F}$$

$$T * R = I \alpha \quad (2)$$

$$T * \cancel{R} = \frac{1}{2} M R^{\cancel{2}} \frac{a}{\cancel{R}}$$

$$\cancel{T} = \frac{M}{2} a \quad (2)$$

$$mg - \cancel{T} = ma \quad (1)$$

$$+$$

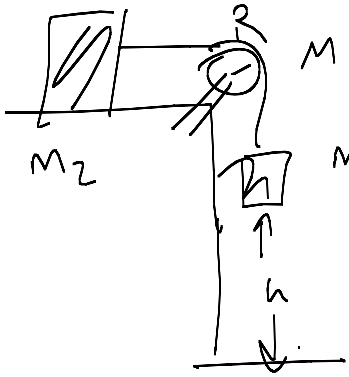
$$mg = ma + \frac{M}{2} a$$

$$a\left(m + \frac{M}{2}\right) = mg$$

$$\boxed{a = \frac{m}{m + \frac{M}{2}} g}$$

$$v^2 = 2ah = 2 \frac{mg}{m + \frac{M}{2}} * h \quad \begin{matrix} 1/m \\ 1/m \end{matrix}$$

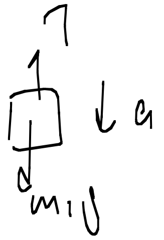
$$v^2 = \frac{2gh}{1 + \frac{M}{2m}}$$



$a = ?$

$v = ?$

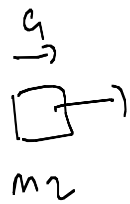
$$\boxed{a = \frac{m_1}{m_1+m_2} g}$$



$$m_1 g - \cancel{T} = m_1 a$$

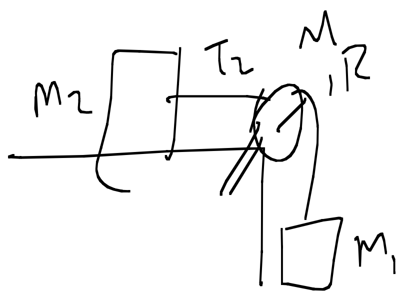
$v^2 = 2h * a$

$v^2 = \frac{2hm_1}{m_1+m_2}$



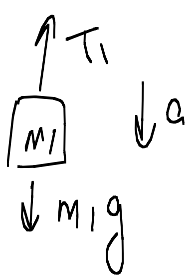
$T \Rightarrow + \quad \cancel{T} = M_2 a$

$$m_1 g = (m_1 + m_2) a$$



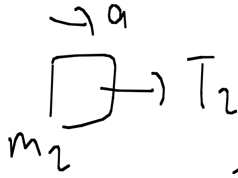
$v^2 = 2ah$

$$v = \sqrt{2 \frac{m_1}{\left(m_1+m_2+\frac{M}{2}\right)} g h}$$



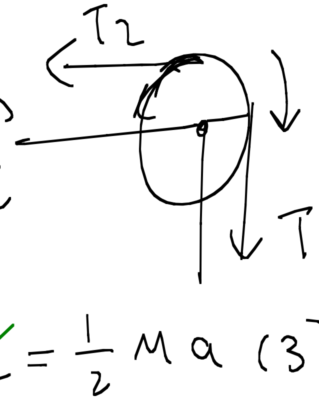
$\Rightarrow \quad m_1 g - \cancel{T_1} = m_1 a \quad (1)$

$\cancel{T_2} = m_2 a \quad (2)$



$\Rightarrow$

$$\sum \tau = T_1 R - T_2 R$$



$a = \left(\frac{m_1}{m_1+M_2+\frac{M}{2}}\right) g$

clock wise (+) $= I\alpha$

$$= \frac{1}{2} M \cancel{R^2} * \frac{a}{\cancel{R}} = (T_1 - T_2)\cancel{R}$$

$$\cancel{T_1} - \cancel{T_2} = \frac{1}{2} M a \quad (3)$$

$(1)+(2)+(3) \quad m_1 g = \left(m_1 + m_2 + \frac{M}{2}\right) a$

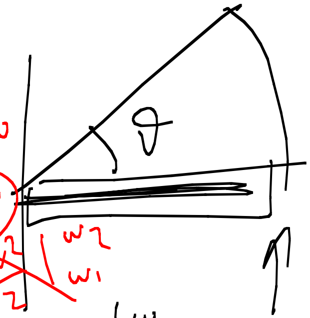
$$W = \int \vec{F} \cdot d\vec{x}$$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

$I \frac{d\omega}{dt} \, d\theta$ ω

$I \omega \, d\omega$

$I \int \omega \, d\omega$ $\frac{\omega^2}{2} \Big|_{\omega_1}^{\omega_2}$



$F = ma$

$$\tau = I\alpha = I \frac{d\omega}{dt}$$

$$W = \underline{\frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2} = \Delta KE$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_1^2 = \Delta KE$$

$$dW = F \cdot dx \qquad W = \cancel{\int} \tau \, d\theta$$

$$dW = \tau \, d\theta \Rightarrow \quad dW \quad Fv = Pow$$

$$\text{Power } \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau \omega = Pow$$

$\frac{kg\,m}{s}$ $\vec{P} = mu$

$\frac{kg\,m^2}{s}$ $\vec{L} = I\omega \rightarrow 1/s$ $\qquad \underline{\vec{L} = \vec{r} \times \vec{p}}$

$\downarrow \qquad \llcorner kg\,m^2$

Angular momentum $\qquad \vec{\tau} = \vec{r} \times \vec{F}$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left[\vec{r} \times \vec{p}\right] = \frac{d}{dt}\left[\vec{r} \times (m\vec{v})\right]$$

$$\underbrace{\frac{d\vec{r}}{dt}} \times m\vec{v} + \underbrace{\vec{r} \times m \frac{d\vec{v}}{dt}}_{} \quad \vec{a}$$

$$\underline{\vec{v} \times m\vec{v}} \qquad \vec{r} \times m\vec{a}$$

0 $\qquad \vec{L}$

$\hat{i} \times \hat{i} =$

$\hat{i} \cdot \hat{i} = 1$

$\hat{k}$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{p} = \vec{\tau}$$

$$\sum \vec{\tau} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} \text{ is conserved}$$

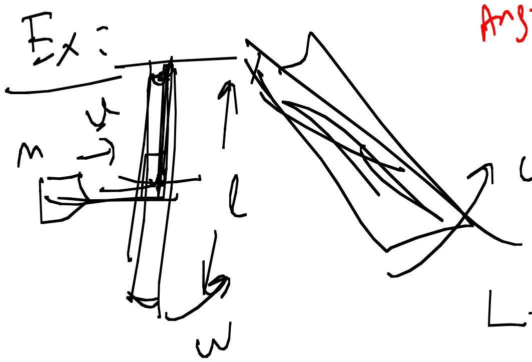
$$\boxed{\vec{p}_i = \vec{p}_f} \Leftarrow \frac{d\vec{p}}{dt} = \vec{F}$$

$$\sum \vec{F} = 0 \Rightarrow \vec{p} = \text{const}$$

Ang-Momentum conservation $\Leftarrow \frac{d\vec{L}}{dt} = \vec{\tau} = 0$

$$\boxed{\vec{L}_i = \vec{L}_f}$$

Ex:



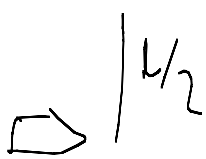
$$\vec{L} = \vec{r} \times \vec{p}$$

$$P = mu$$

$$L = I\omega$$

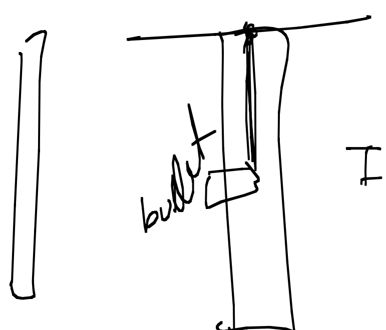
$L_i = L_f$ cons. of ang. mom

$I\omega$



$$mu * \frac{l}{2} = I_{door} * \omega$$

$$\Rightarrow \omega = \frac{mul}{2I_{door}}$$



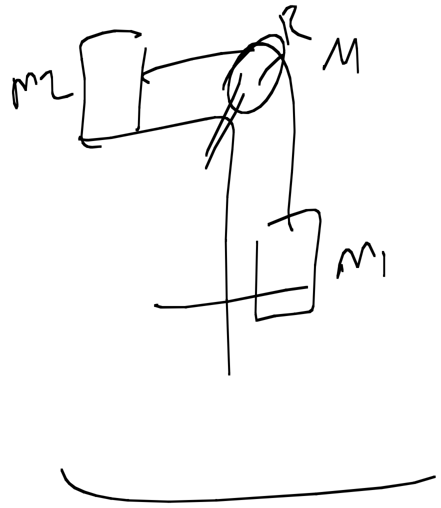
$$I_{door} + I_{bullet} = I_{total}$$

$$\frac{1}{3}Ml^2 + \cancel{m\left(\frac{l}{2}\right)^2}$$

$$\frac{1}{3}Ml^2$$

$u = 400\,m/s$ $\quad l = 1$ $\quad m = 10\,g$ $\quad M = 15\,kg$

$$\omega = \frac{10 \times 10^{-3} * 4 \times 10^{2} * l}{2 * \frac{1}{3} * 15 * l^2} = \boxed{0.4\ \frac{rad}{s}}$$



$$m_1 g h = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I \omega^2$$

$\frac{1}{2} M \cancel{R^2}$

$\omega \to \frac{v}{\cancel{R}}$

$$m_1 g h = \frac{1}{2}(m_1+m_2) v^2 + \frac{1}{4} M v^2$$

$$\frac{v^2}{2}\left[m_1+m_2+\frac{M}{2}\right] = m_1 g h$$

$$v = \sqrt{\frac{2 m_1 g h}{m_1+m_2+\frac{M}{2}}}$$