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 ( not pen).

## Questions 1-10

1. What is the unit of angular momentum?
(a) $\mathrm{kgm}^{2} / \mathrm{s}$
(b) $\mathrm{kgm} / \mathrm{s}$
(c) $\mathrm{kgm} / \mathrm{s}^{2}$
(d) $J s^{-} 1$
(e) $J s^{2}$
2. Three identical masses $m$ are at the edges of an equilateral triangle (side length $a$ ). This triangle rotates with constant angular velocity $\omega$ about the center of the triangle. Axis of rotation is perpendicular to the plane of the triangle. $G$ is gravitational constant. Which following relation is true?
(a) $\omega^{2}=\frac{3 G m}{a^{3}}$
(b) $\omega^{2}=\frac{3 G m}{a^{3}} \cos (\pi / 6)$
(c) $\omega^{2}=\frac{3 G m}{a^{3}} \sin (\pi / 6)$
(d) $\omega^{2}=\frac{3 G m}{a^{3}} / \cos (\pi / 6)$
(e) $\omega^{2}=\frac{3 G m}{a^{3}} / \sin (\pi / 6)$
3. Potential energy of a particle in $x y$ plane $(z=0)$ is $\frac{k}{2}\left(x^{2}+y^{2}\right)$. What is time derivative of its angular momentum with respect to origin? (Hint: Note that the potential enegy is a function of distance to the origin. What is then the direction of the force derived from this potential?)
(a) 0
(b) $k x^{2}$
(c) $-k x^{2}$
(d) $-k x y$
(e) $-k y^{2}$
4. An ice skater spins with angular velocity $\omega$. Her angular momentum is $I \omega$. She extends her arms and her moment of inertia becomes $\frac{4}{3} I$. What is the change in her mechanical energy(neglect fiction with ice)
(a) $-\frac{1}{8} I \omega^{2}$.
(b) $\frac{1}{8} I \omega^{2}$.
(c) $-\frac{1}{3} I \omega^{2}$.
(d) 0 .
(e) $-\frac{1}{3} I \omega^{2}$.
5. Potential energy of a point particle is $V=V_{0} \cos (\pi x / L)$ where $V_{0}$ is a positive constant. Which one of the following statement is true?
(a) $x=0$ is an unstable equilibrium position and $x=L$ is a stable equilibrium position. (b) $x=L$ is an unstable equilibrium position and $x=0$ is a stable equilibrium position. (c) $x=L$ and $x=0$ are both stable equilibrium position. (d) neither of $x=L$ and $x=0$ is an equilibrium position. (e) $x=L$ and $x=0$ are both unstable equilibrium position.
6. Radius of the orbit of a satellite orbiting around earth is $R$. The orbit is circular and angular velocity is $\omega$ with $\omega^{2}=g(R) / R$. What will be the apparent weight of an astronaut of mass $m$ in the satellite( as measured by a dynamometer in the satellite) ?
(a) 0
(b) $m\left(g(r)+\omega^{2} R\right)$
(c) $m g(R)$
(d) $m \omega^{2} R$
(e) $m\left(g(r)+\omega^{2} R\right) / \sqrt{2}$
7. A cylindrical shell of radius $R$, mass $m$ is rolling down an inclined plane without slipping. The inclined plane makes an angle $\theta$ with horizontal plane. The static friction coefficient between cylindrical shell and the inclined plane is $\mu$. The cylindrical shell has a negligible thickness(inner and outer radius of the shell are almost equal). What is the friction force between the shell and inclined plane?
(a) $m g \sin \theta / 2$
(b) $\mu m g \cos \theta$.
(c) $m g \sin \theta$
(d) $m g \sin \theta / 3$
8. The moment of inertia of a homogeneous sphere of radius $R$ is $\frac{2}{5} M R^{2}=\frac{2}{5} \rho \frac{4 \pi}{3} R^{5}$ where $M$ is the mass of the sphere and $\rho$ its density. What is moment of inertia of a spherical shell of mass $M$ and radius $R$. The shell has a negligible thickness. Hint: moment of inertia of a shell of thickness $d R$ is equal to the difference of intertias between sphere of radius $R+d R$ and $R$. Spheres and shell are made of the same material.
(a) $M R^{2} 2 / 3$
(b) $M R^{2}$
(c) $M R^{2} / 2$
(d) $M R^{2} / 3$
(e) $M R^{2} 2 / 5$
9. A car is accelerating with constant linear acceleration $a$. The radius of its wheels is $R$. A piece of mud is stuck on the tyre. When the car reaches the instantaneous velocity $v_{0}$ the piece of mud is at its heighest point on its trajectorys( $2 R$ from the ground). What is magnitude of linear acceleration of the piece of mud at this instant?(the car does not slide)
(a) $\sqrt{v_{0}^{4} / R^{2}+4 a^{2}}$
(b) $\sqrt{v_{0}^{4} / R^{2}+a^{2}}$
(c) $v_{0}^{2} / R$
(d) 0
(e) $2 a$
10. Height of a high rising building is $L$. Axis $x$ is in upward directed. What can be said on difference $x_{c m}-x_{c g}$ where $x_{c m}$ and $x_{c g}$ are respectively center of mass and gravity of the building (gravity decreases in the upward direction)? (a) $x_{c m}-x_{c g}>0$.
(b) $x_{c m}-x_{c g}<0$
(c) $x_{c m}-x_{c g}=0$.
(d) sign of $x_{c m}-x_{c g}$ depends on the height of the building
(e) $x_{c m}-x_{c g}>L$.

## Questions 11-15

A heterogeneous horizontal rod of length $L$ is hinged to the vertical wall. Mass per unit of length of the rod is variable and it is given by $8 M x^{3} / L^{4}$ where $x$ is the distance to the hinge $(x \leq L)$ and $M$ is constant. The distance between point mass $m$ and the hinge is $L / 2$. The rope makes an angle $30^{\circ}$ with the horizontal.
11. What is mass $M_{\text {rode }}$ of the rode?
(a) $2 M$
(b) $M$
(c) $8 M / 3$
(d) $M / 2$
(e) $2 M / 3$
12. Find distance $L_{G}$ between the hinge and the centre of gravity of the rode (do not take into account point mass $m$ ).
(a) $4 L / 5$
(b) $L / 3$
(c) $2 L / 3$
(d) $L / 5$
(e) $3 L / 4$
13. Find the tension in the rope(as mass $m$ is much smaller than the mass of the rode neglect mass $m$ )
(a) $\frac{g M_{\text {rode }} L_{G}}{L \sin \left(30^{0}\right)}$
(b) $\frac{g M_{\text {rode }} L_{G}}{L \cos \left(30^{0}\right)}$
(c) $\frac{g M_{\text {rode }} L_{G}}{\operatorname{Ltan}\left(30^{0}\right)}$
(d) $\frac{g M_{\text {rode }} L_{G} \sin \left(30^{0}\right)}{L}$
(e) $\frac{g M_{\text {rode }} L}{L_{G} \sin \left(30^{\circ}\right)}$
14. What is moment of inertia of the rode $\left(I_{0}\right)$ with respect to the hinge (neglect $m$ )?
(a) $\frac{4}{3} M L^{2}$.
(b) $M L^{2}$.
(c) $\frac{4}{5} M L^{2}$.
(d) $\frac{7}{3} M L^{2}$
(e) $M L_{G}^{2}$.
15. The rope breaks off at $t=0$. What is the magnitude of the normal force that the rode applies to mass $m$ for $t \rightarrow 0^{+}$?
(a) $m g\left(1-\frac{L L_{G} M_{\text {rode }}}{2 I_{0}}\right)$
(b) $m g$
(c) $m g L_{G} / L$
(d) 0
(e) $m g\left(1+\frac{L L_{G} M_{\text {rode }}}{2 I_{0}}\right)$

## Questions 16-20

A cubic block, with side length of $L$ and mass $m$ starts sliding with initial speed $v_{0}$ on a frictionless surface. Just before it reaches the end of the table it collides with a small curb and the block starts rotating as it is shown in the figure.
16. What is the initial angular momentum of the box with respect to the small curb?
(a) $m v_{0} L / 2$
(b) $m v_{0} L$
(c) $2 m v_{0} L$
(d) $3 m v_{0} L / 2$
(e) $m v_{0} L / 3$
17. If this is a cubic box, calculate the moment of inertia $I_{0}$ of it when it rotates around one of its edges.
(a) $\frac{2}{3} m L^{2}$
(b) $\frac{1}{3} m L^{2}$
(c) $\frac{1}{6} m L^{2}$
(d) $\frac{1}{2} m L^{2}$
(e) $\frac{2}{5} m L^{2}$
18. What is the angular velocity of the box after the colliding with the curb?
(a) $\frac{m v_{0} L}{2 I_{0}}$
(b) $\frac{m v_{0} L}{I_{0}}$
(c) $\frac{2 m v_{0} L}{I_{0}}$
(d) $\frac{3 m v_{0} L}{2 I_{0}}$
(e) $\frac{m v_{0} L}{3 I_{0}}$
19. What is the minimum initial velocity of the box in order for it to pass the falling point See explanation in the figure)?
(a) $\sqrt{\frac{4 I_{0} g(\sqrt{2}-1)}{m L}}$
(b) $\sqrt{\frac{2 I_{0} g(\sqrt{2}-1)}{m L}}$
(c) $\sqrt{\frac{I_{0} g(\sqrt{2}-1)}{4 m L}}$
(d) $\sqrt{\frac{2 I_{0} g(\sqrt{2}-1)}{3 m L}}$
(e) $\sqrt{\frac{I_{0} g(\sqrt{2}-1)}{m L}}$
20. Which one of the following is a true statement?(in the following choices angular momentum means angular momentum with respect to the curb)
(a) Angular momentum is conserved during the collision but not conserved after the collision. (b) Since only conservative forces act on the box angular momentum is conserved after the collision. (c) Angular momentum is not conserved since the collision is inelastic. (d) Since this is not a closed system, angular momentum is not conserved throughout the whole motion.
(e) Both energy and angular momentum of the system is conserved throughout the whole motion.

## Questions 21-25

Three masses in space $m_{1}, m_{2}, m_{3}$ are aligned along $x$ axis. They are respectively at $x 1=0, x_{2}=L$ and $x_{3}=2 L$. Masses $m_{2}$ and $m_{3}$ are attached to each other by a massless rope. Gravitational constant is $G$. Take $m_{1}=4 m$ and $m_{2}=m_{3}=m$.
21. What is the net force applied to the system $S$ which consists of $m_{2}$ and $m_{3} . \vec{i}$ is unit vector along $x$ axis and $L>0$.
(a) $-5 G m^{2} / L^{2} \vec{i}$
(b) $-(8 / 2.25) G m^{2} / L^{2} \vec{i}$
(c) $-6 G m^{2} / L \vec{i}$
(d) 0
(e) $5 G m^{2} / L^{2} \vec{i}$
22. What is the acceleration of centre of mass of the system $S\left(S\right.$ consists of $\left.m_{2}, m_{3}\right)$ ?
(a) $-(5 / 2) G m / L^{2} \vec{i}$
(b) $(5 / 2) G m / L^{2} \vec{i}$
(c) $-5 G m / L^{2} \vec{i}$
(d) $-(8 / 5) G m^{2} / L^{2} \vec{i}$
(e) 0
23. What is tension on the rope?(The purpose of this problem is to show that under lunar gravity, high tides occurs at opposite faces of the earth ).
(a) $(3 / 2) G m^{2} / L^{2}$
(b) 0
(c) $3 G m^{2} / L^{2}$
(d) $(12 / 5) G m^{2} / L^{2}$
(e) $G m^{2} / L^{2}$
24. What is the minimum energy required to make the distance between the system $S$ and mass $m_{1}$ infinite (initial velocities are $0)$.
(a) $6 m^{2} G / L$
(b) $10 m^{2} G / L$
(c) $2 m^{2} G / L$
(d) $10 m^{2} G / L^{2}$
(e) $-10 m^{2} G / L \mathrm{~N}$
25. Initial velocity of $m_{1}$ is 0 .Initial velocity of $m_{2}$ and $m_{3}$ is $v_{0} \vec{i}$. What must be the minimum value of $v_{0}$ so that the distance between $m_{1}$ and system $S$ increases for ever? (Hint: for an observer sitting at centre of mass of masses $m_{1}, m_{2}, m_{3}$, the minimum kinetic energy must be equal to the absolute value of the energy found in problem 24)
(a) $3 \sqrt{m G / L}$
(b) $\sqrt{(27 / 2) m G / L}$
(c) $\sqrt{15 m G / L}$
(d) $\sqrt{3 m G} / L$
(e) $\sqrt{(15 / 2) m G / L}$

