Errors

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1 Introduction

While making numerical calculations errors are inevitable whether due to finite precision of floating point algebra or truncation of the algorithm (or series for that matter) at some level [1]. *Error* is defined as the difference between the true value and the approximated value (1). On the other hand as per its literal definition absolute error is nothing but the absolute value of the error itself (2).

$$\mathbf{Error} = \mathbf{TrueValue} - \mathbf{ApproximateValue} \tag{1}$$

$$\mathbf{AbsoluteError} = |\mathbf{TrueValue} - \mathbf{ApproximateValue}| \tag{2}$$

Errors are often quantified in terms of percentages in order to non-dimensionalize them and therefore allow a non-biased comparison. Equations (1) and (2) can simply be transformed into their percentage forms as per the following equations respectively.

$$\%\mathbf{Error} = \frac{\mathbf{TrueValue} - \mathbf{ApproximateValue}}{\mathbf{TrueValue}} \times 100 \tag{3}$$

$$\%\mathbf{AbsoluteError} = |\frac{\mathbf{TrueValue} - \mathbf{ApproximateValue}}{\mathbf{TrueValue}}| \times 100 \tag{4}$$

Most of time the "true value" is unbeknownst to the engineer, since it is often the quantity that is being solved for. In this case in order to quantify the error (hence to decide on the stopping criterion for the solution algorithm) engineer resorts to the definition of the so called "approximate error". Approximate error is the difference between the current estimate of the actual value and its previous estimate (5). Approximate error is easy to compute and is quite useful regarding that almost all numerical solution schemes are of iterative nature. Therefore there is always a current and a previous value. Hopefully through iteration the value of the current estimate converges to the true value.

$$\mathbf{ApproximateError} = \mathbf{CurrentEstimate} - \mathbf{PreviousEstimate} \tag{5}$$

$$\%\mathbf{ApproximateError} = \frac{\mathbf{CurrentEstimate} - \mathbf{PreviousEstimate}}{\mathbf{CurrentEstimate}} \times 100 \tag{6}$$

The sign of (6) can either be negative or positive. On the other hand while making computations the sign of the error is of no importance but its absolute value is used as a termination criterion. When the absolute value of the error is smaller than a prescribed tolerance ϵ the calculation is stopped (7).

$$|\%\mathbf{ApproximateError}| < \epsilon \tag{7}$$

2 Sources of Error

In numerical calculations total error has two constituents within, namely the round-off and truncation errors (8).

$$\mathbf{TotalError} = \mathbf{RoundoffError} + \mathbf{TruncationError} \tag{8}$$

2.1 Round-Off Errors

Since numbers are stored in the computer memory using floating point representation, and since floating point representation has only so many bytes allocated to itself (depending on the datatype), at some point the number has to be rounded off to its closest floating point representation. This inherently introduces an error source. Important thing about the round-off errors is the fact that they accumulate as the calculation progresses. Therefore round-off errors not only impact the final digit but can have a large impact on the final output value.

2.2 Truncation Errors

Truncation errors happen as its very name implies due to the truncation of the algorithm at a finite level. Whether it is a root-finding algorithm or a Taylor series approximation of a fuction or some other sort of computation it is impossible to search for the root unstoppingly or similiarly it is impossible to keep on adding terms to the Taylor series infinitely. Calculation has to stop somewhere. This arbitrary truncation is obviously an error source.

3 Precision vs. Accuracy

Precision refers to the closeness of individual measurements to one another. On the other hand *accuracy* refers to the closeness of individual measurements to the actual value. Similarly *inaccuracy*, also referred to as *bias*, is a systematic deviation from the true value. *Imprecision* (also termed *uncertainty*) is the magnitude of the scatter wit respect to the true value.

References

[1] R. Canale S. C. Chapra. *Numerical Methods for Engineers.* Mc Graw Hill Inc., 2008.