Direct current circuits

Goals for Chapter 26

- Analyze circuits with resistors in series & parallel

- Apply Kirchhoff's rules to multiloop circuits

- Use Ammeters & Voltmeters in a circuit

Goals for Chapter 26

- Analyze "RC" circuits containing capacitors and resistors, where time now plays a role.

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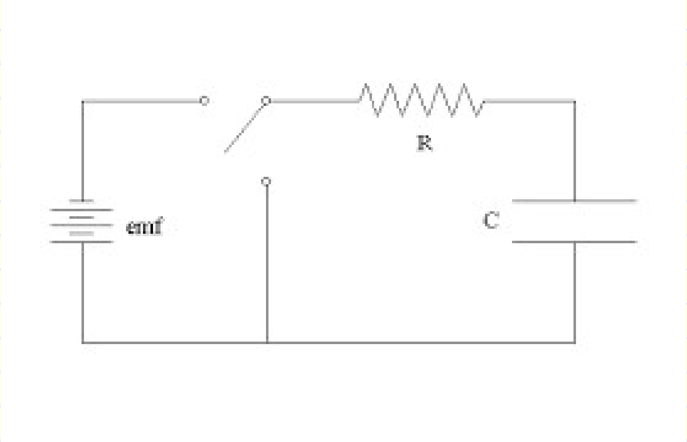
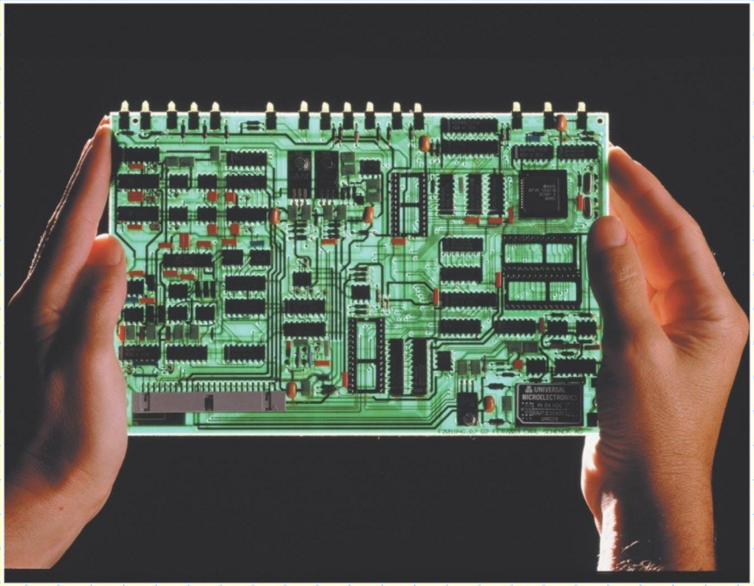
- Analyze "RC" circuits containing capacitors and resistors, where time now plays a role.



- Study power distribution in the home

Introduction

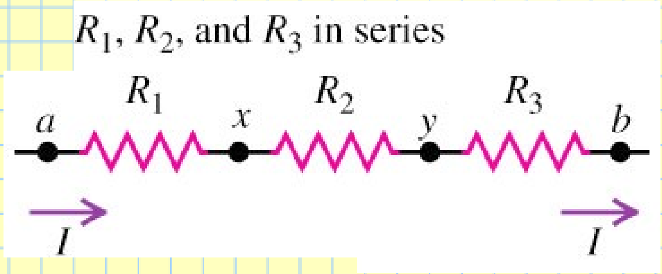
- How to apply *series*/*parallel* combinations of resistors to complex circuit board?
- Learn general methods for analyzing complex networks.
- Look at various instruments for measuring electrical quantities in circuits.



Resistors in series and parallel

- Resistors are in ***series*** if they are connected one after the other so the current is the same in all of them.

- The *equivalent resistance* of a series combination is the *sum* of the individual resistances: $R_{eq} = R_1 + R_2 + R_3 + \ldots$



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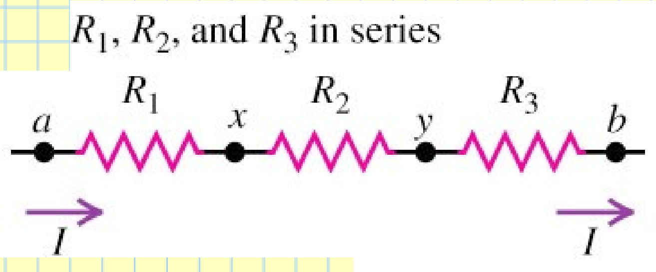
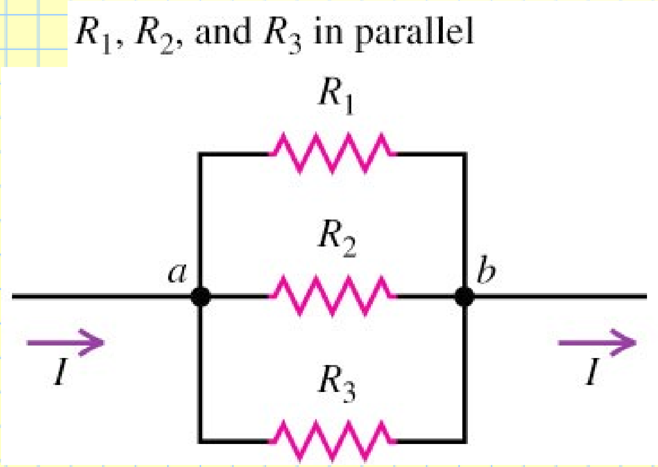


Series Resistors have resistance LARGER than the largest value present.

Resistors in series and parallel

- Resistors are in ***parallel*** if they are connected so that the potential difference must be the same across all of them.
- The *equivalent resistance* of a parallel combinaton is given by

$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$



Resistors in series and parallel

- Resistors are in ***parallel*** if they are connected so that the potential difference must be the same across all of them.
- The *equivalent resistance* of a parallel combinaton is given by

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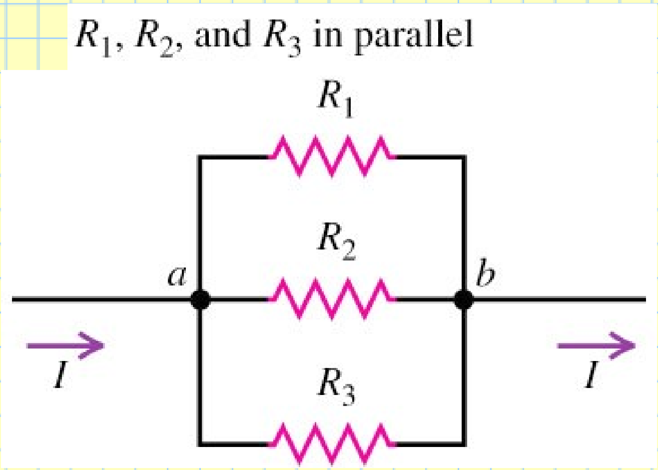
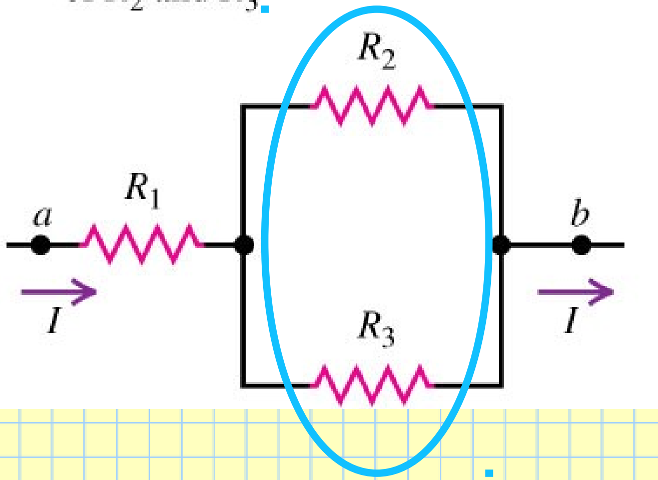


Parallel Resistors have resistance SMALLER than the smallest value present.

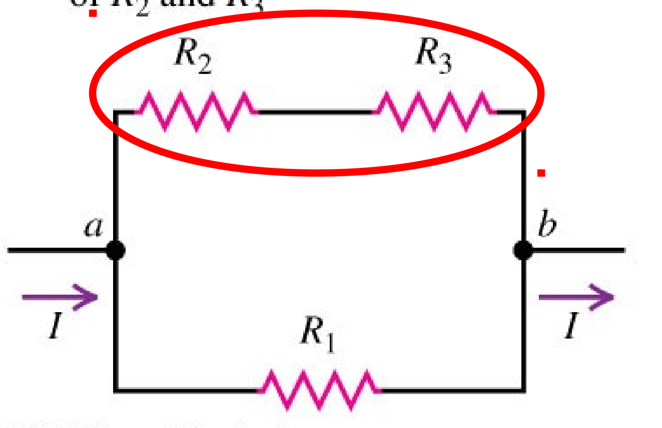
Series and parallel combinations

- Resistors can be connected in combinations of series and parallel

(c) R_1 in series with parallel combination of R_2 and R_3.

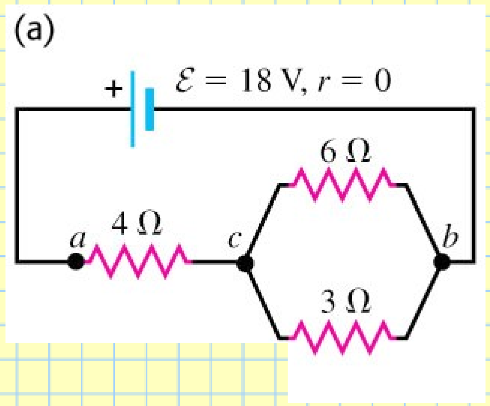


(d) R_1 in parallel with series combination of R_2 and R_3



Equivalent resistance

- Consider this ideal circuit (internal r of battery = 0)

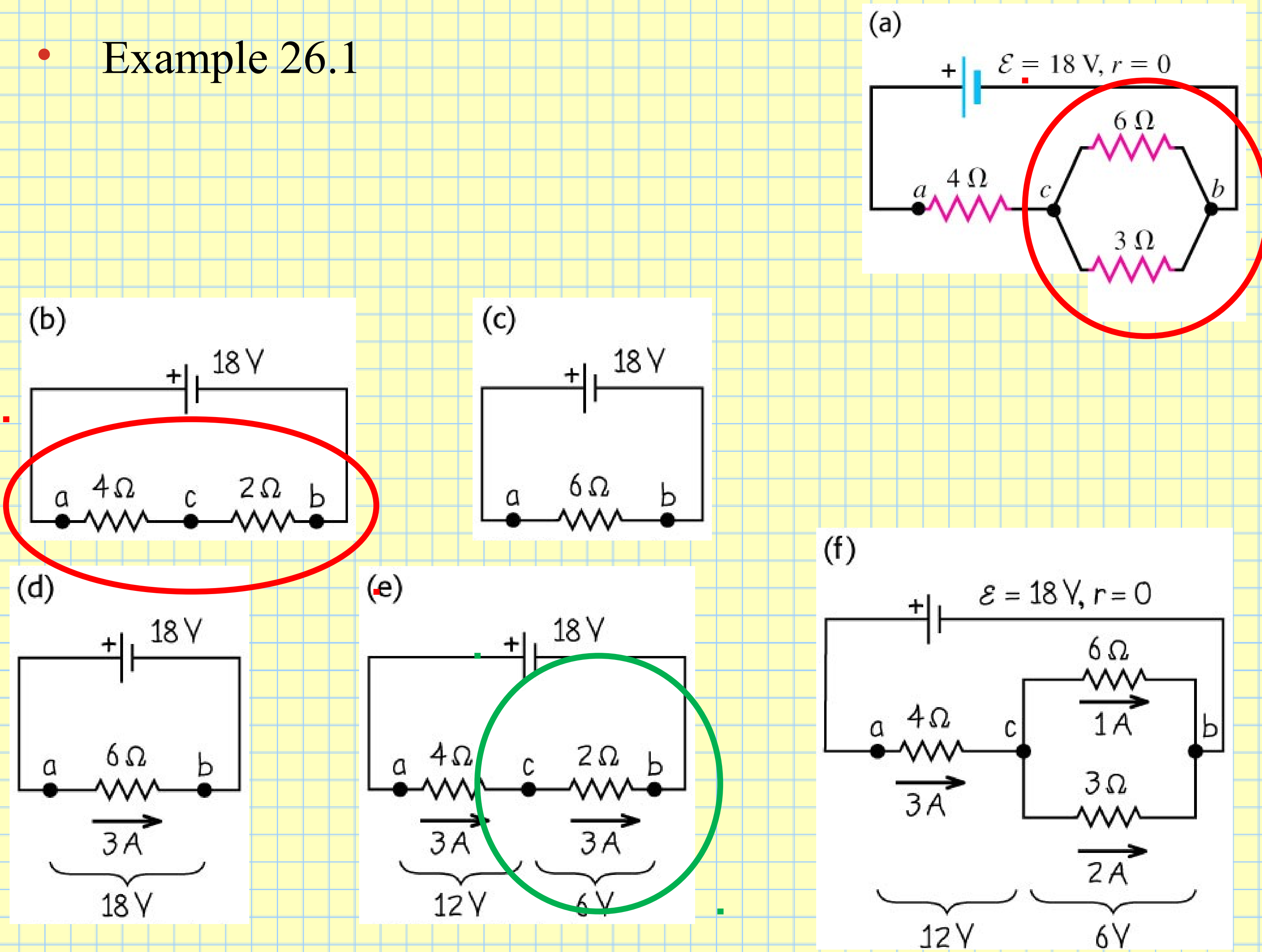


- How do you analyze its equivalent resistance & current through each resistor?

- Start by identifying series and parallel components.

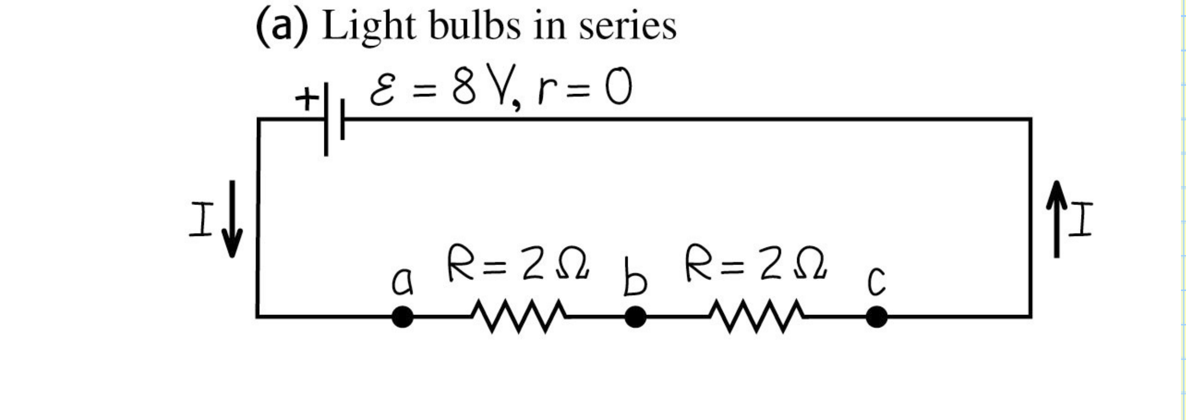
Equivalent resistance

- Example 26.1



Series versus parallel combinations

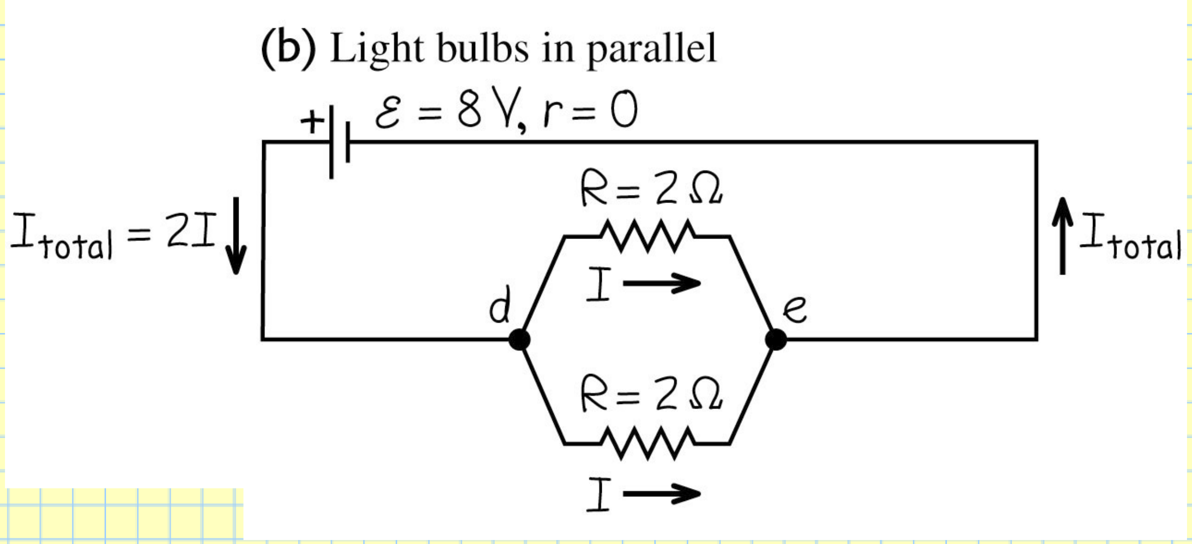
- Ex 26.2: Current through each R & Power dissipated?



- $R_{\text{equivalent (series!)}} = 2 + 2 = 4$ Ohms
- $I = 8V / 4\,\Omega = 2$ A
- Power = i^2R = 16 Watts total (8 Watts for each bulb)

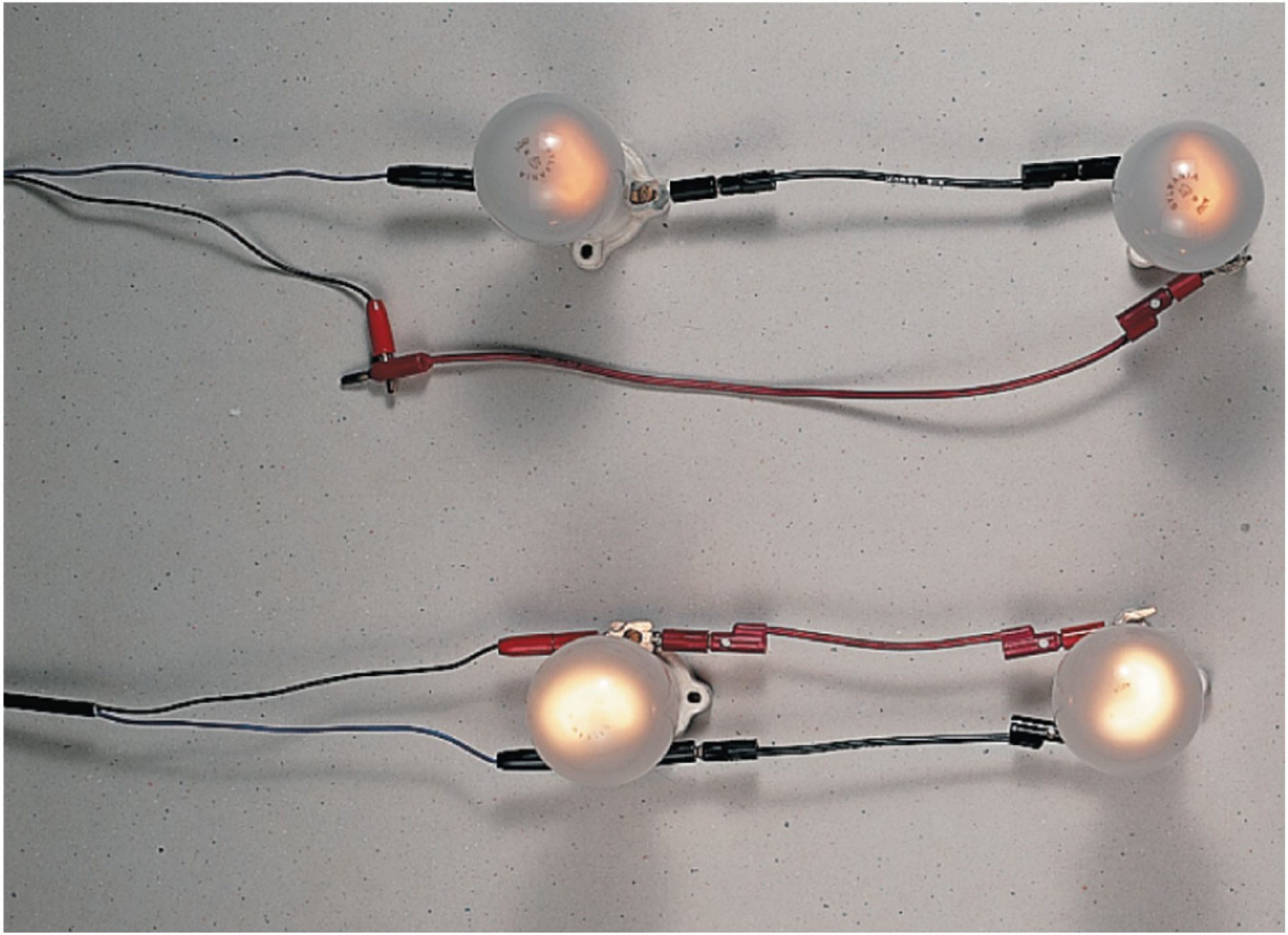
Series versus parallel combinations

- Ex 26.2: Current through each R & Power dissipated?



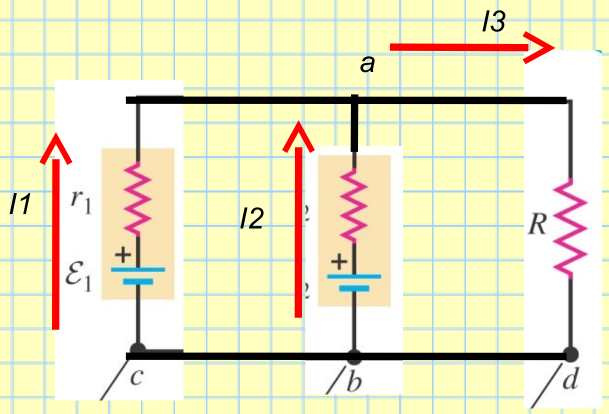
- $R_{\text{equivalent (parallel!)}} = (½ + ½)^{-1} = 1$ Ohm
- $I = 8V / 1\,\Omega = 8$ A
- Power = i^2R = 64 Watts total (32 Watts for each bulb)

Figure 26.5



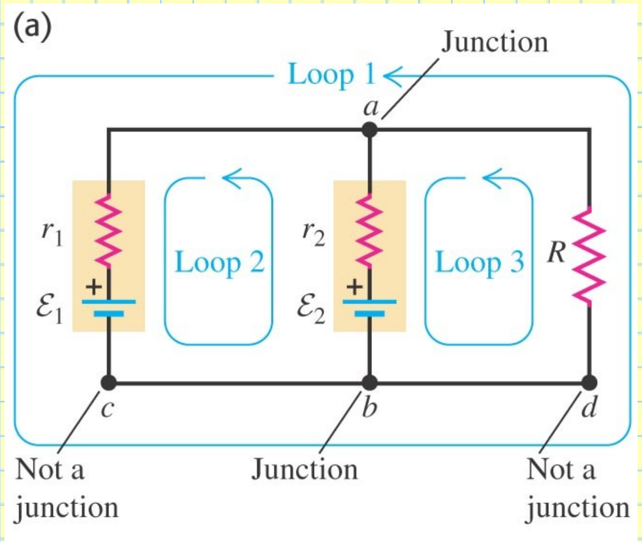
Circuit Analysis Step 1

- Identify & label currents in each segment of a circuit!
- Establish directions for those currents!
- No worries if you are wrong! The analysis will show “i” as negative!



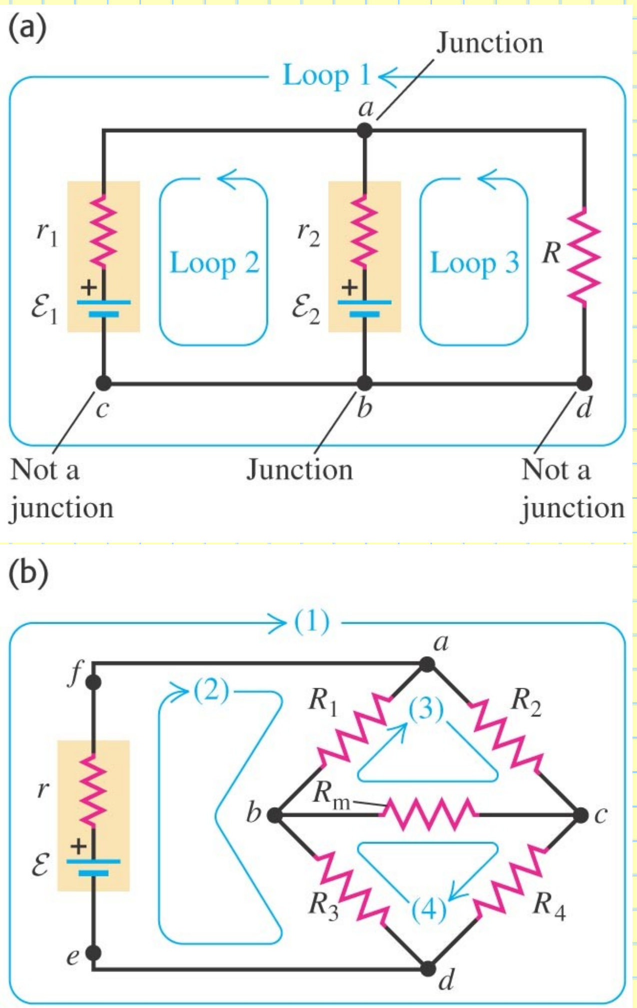
Circuit Analysis Step 2

- Create closed LOOPS around the circuit.
- Keep track of DIRECTIONS as you travel each loop.
- No worries if you are wrong! Algebra will catch sign errors!



Kirchhoff's Rules

- A *junction* is point where <u>*three*</u> or more conductors meet.
- A *loop* is any closed conducting path.
- Loops start & end at same point.



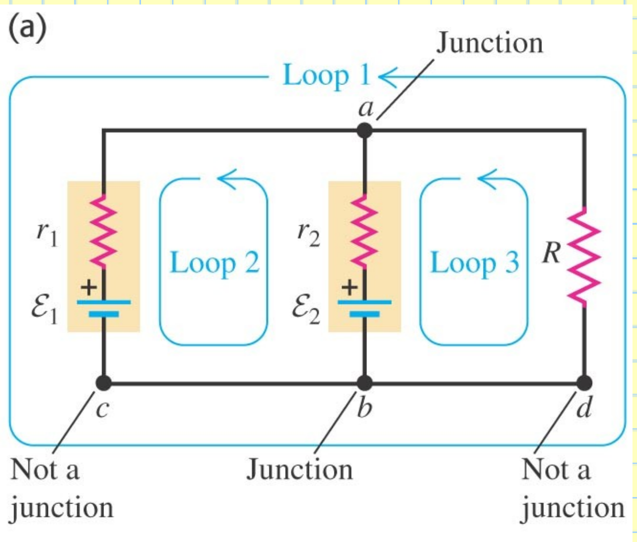
Kirchhoff's Rules I

- A *junction* is a point where three or more conductors meet.

- Kirchhoff's *junction rule:*

  The *algebraic* sum of the currents into or out of any junction is zero:

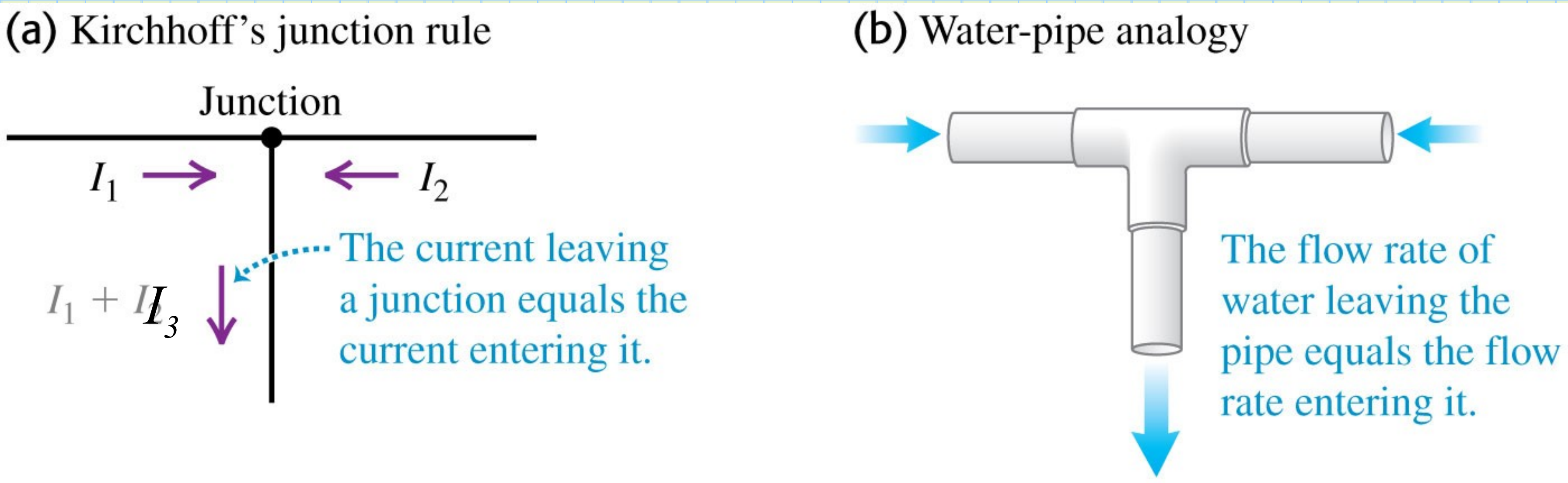
$$\Sigma I = 0$$



Kirchoff's Rules I

- Kirchhoff's *junction rule*: The algebraic sum of the currents into any junction is zero: $\Sigma I = 0$.

- Conservation of Charge in time (steady state currents)



$$I_1 + I_2 = I_3$$

Kirchoff's Rules I

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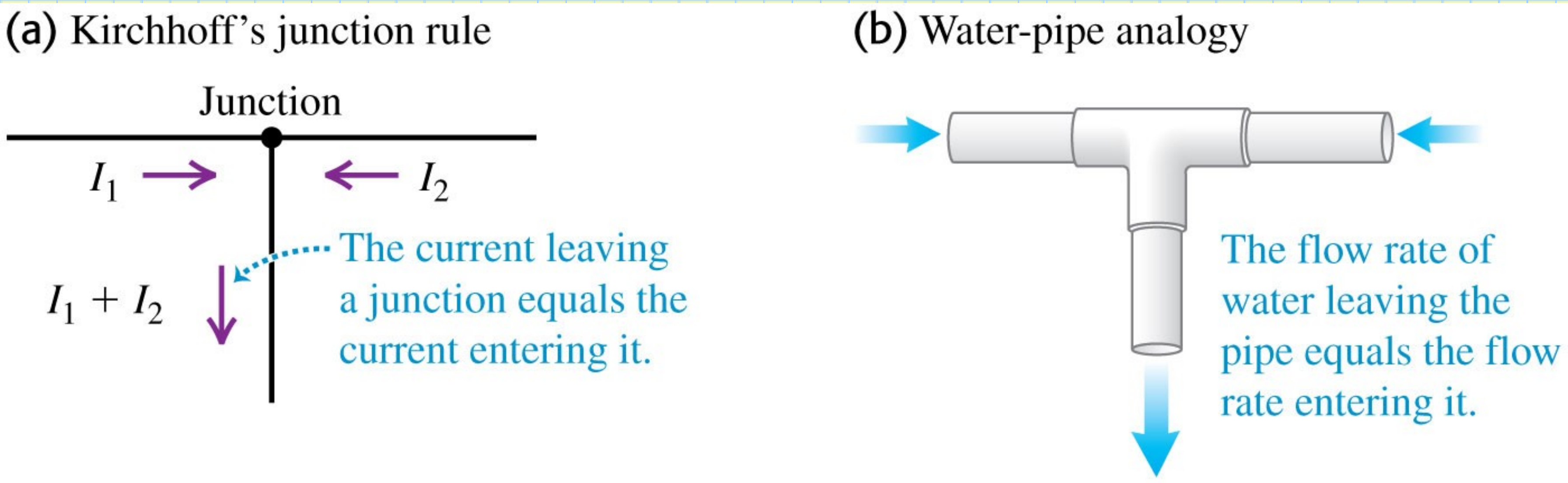


$$I_1 + I_2 = I_3$$

Kirchhoff's Rules I

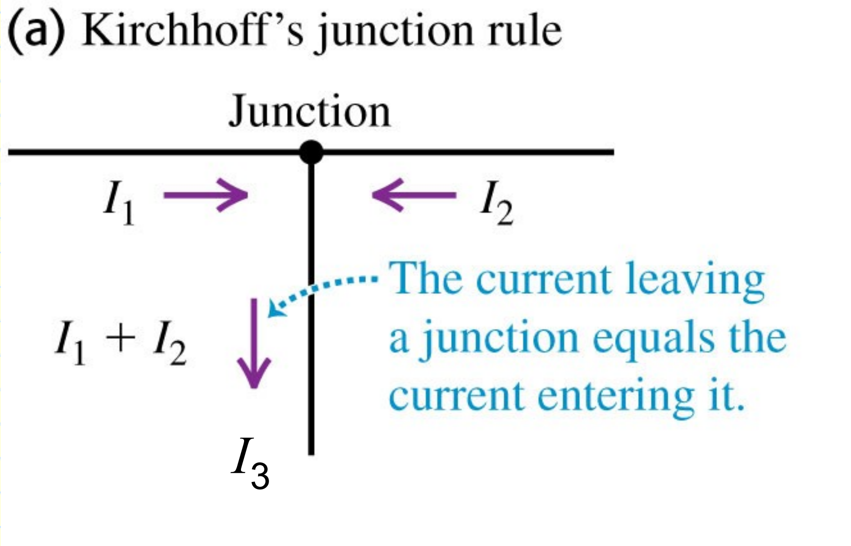
- Kirchhoff's *junction rule:*

  The *algebraic* sum of the currents into or out of any junction is zero:

  $\Sigma I = 0$ means

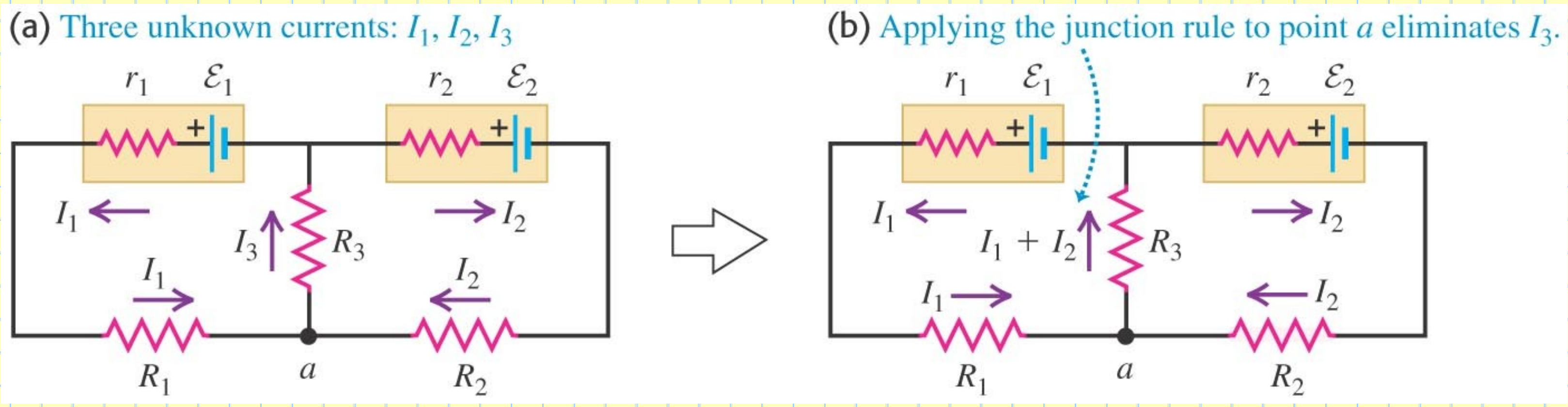
  $I_1 + I_2 - I_3 = 0$

OR $I_1 + I_2 = I_3$



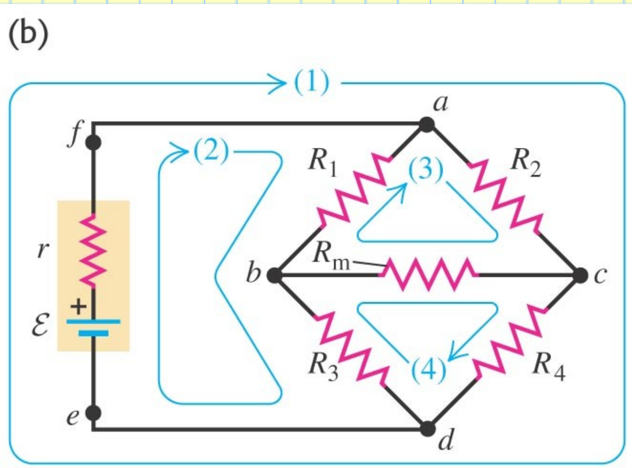
Reducing the number of unknown currents

- How to use the junction rule to reduce the number of unknown currents.



Kirchhoff's Rules II

- A *loop* is any closed conducting path.
- YOU choose them!
- You can't be wrong…
- *…yet!*

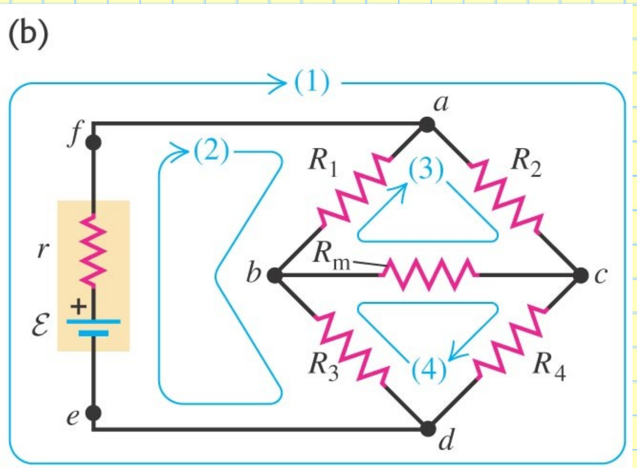


Kirchhoff's Rules II

- Kirchhoff's ***loop rule***:

  The *algebraic* sum of the **potential differences** in any loop must equal zero:

  $$\Sigma V = 0$$



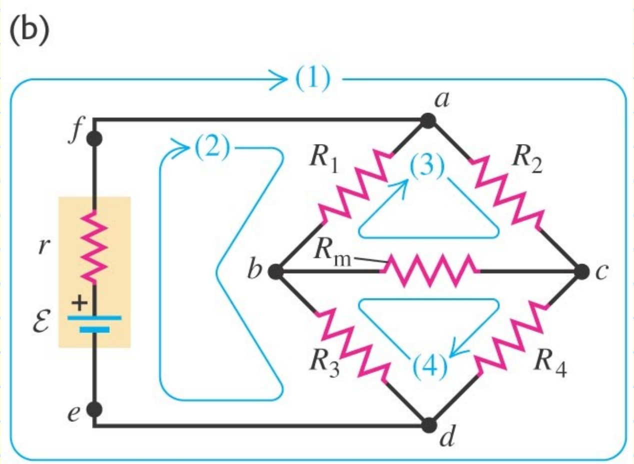
- *Loop 1*: e => f=> a=> c=> d=> e
- *Loop 2:* e => f=> a=> b=> d=> e
- *Loop 3*: a=> c=> b=> a
- *Loop 4*: d=> b=> c=> d

Kirchoff's Rules II

- Kirchhoff's ***loop rule***: The algebraic sum of the **potential differences** in any loop must equal zero: $\Sigma V = 0$.

- Conservation of Energy!

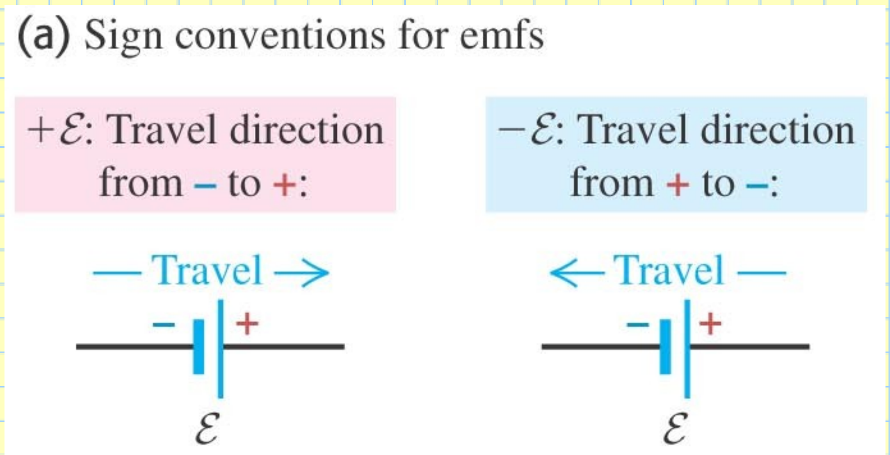
Gain PE going through battery (EMF) ↑



↓ Lose PE going across resistors in direction of + current (Voltage drops)

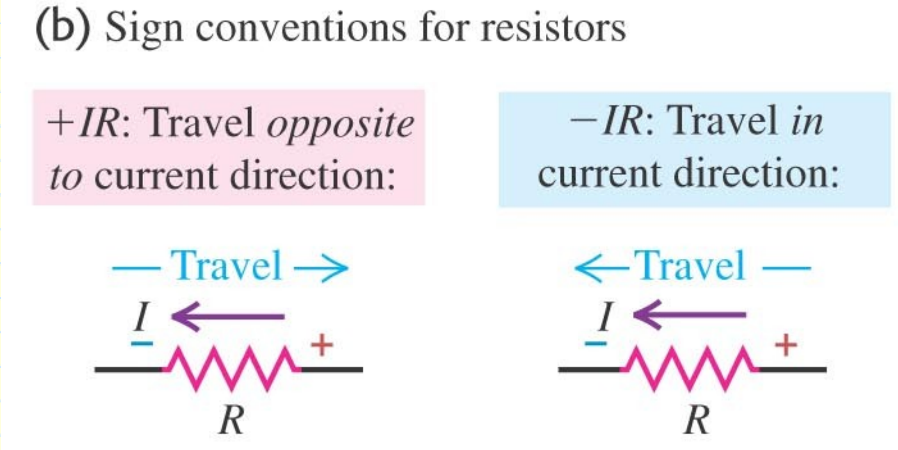
If you end up where you start in a circuit, you have to be back at the same potential! So $\Delta V = 0$

Sign convention for the loop rule



Lose potential as you move *in direction of current* across resistor

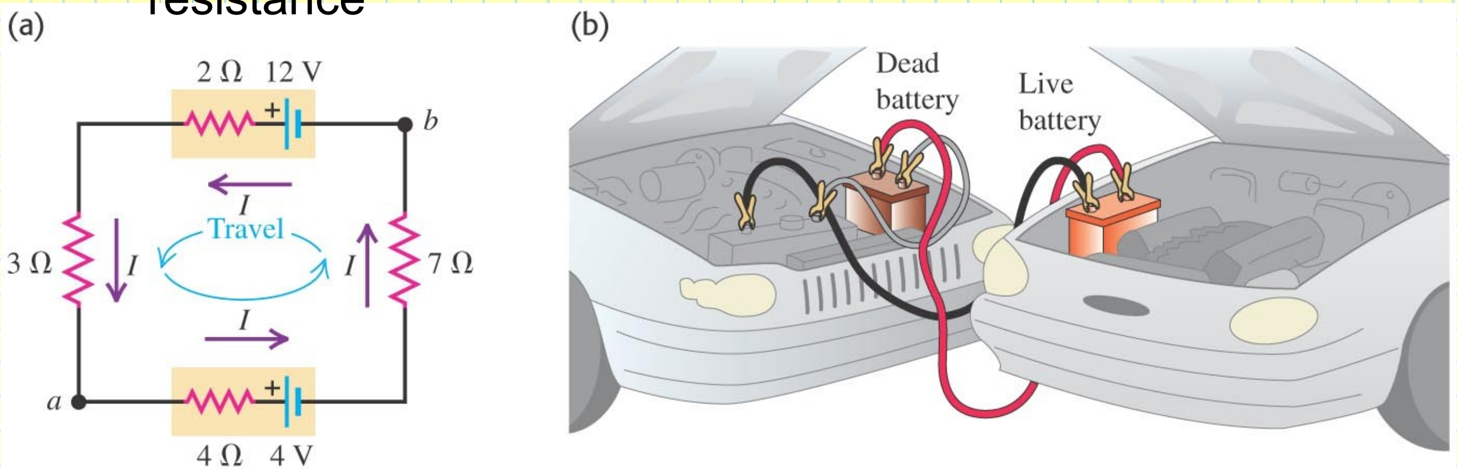
Gain potential as you move *in direction of EMF*



A single-loop circuit

- Find Current in circuit, Vab, & Power of emf in each battery!

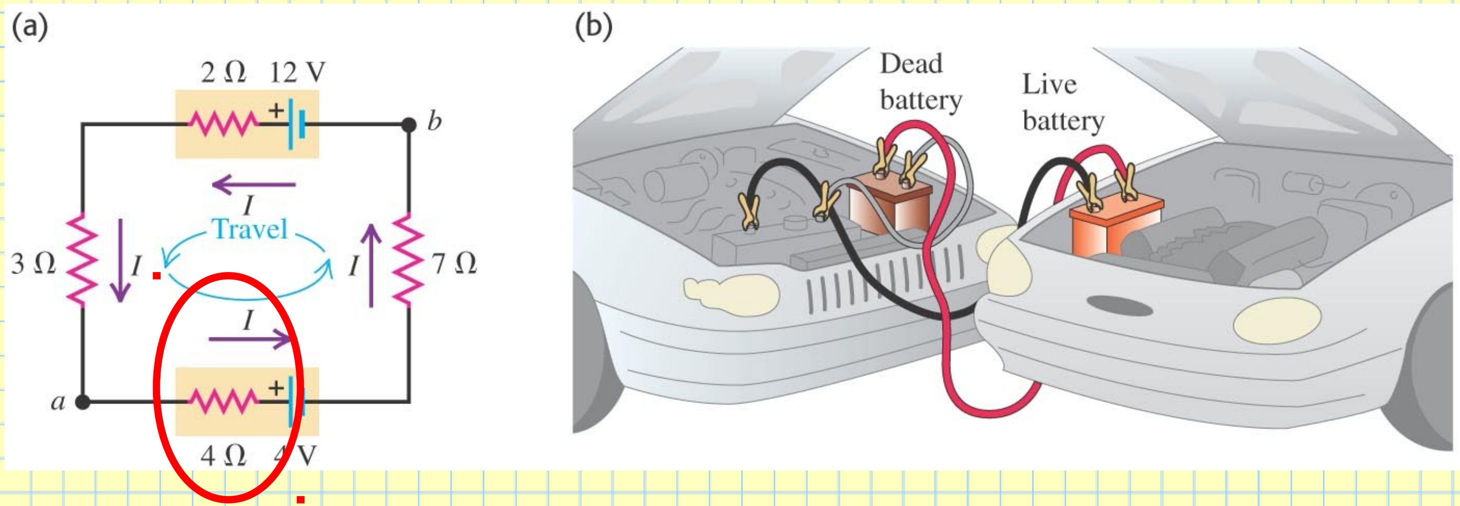
Good battery (not much internal resistance



Dead battery (old, lots of internal resistance)

A single-loop circuit

- Find Current in circuit, Vab, and Power of emf in each battery!



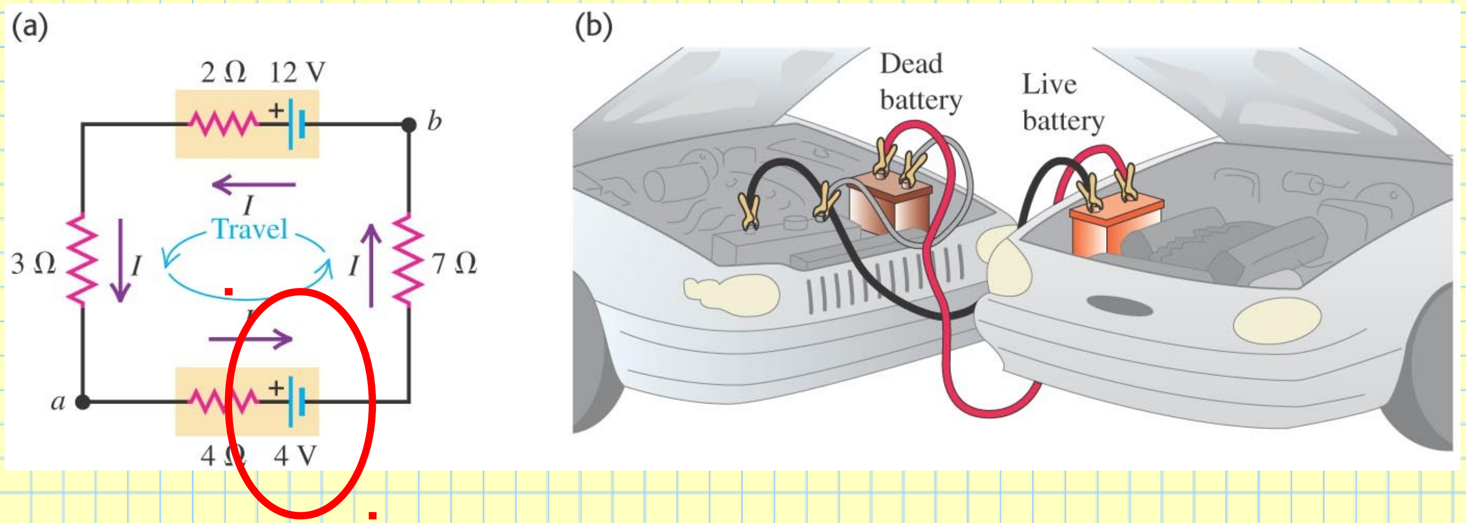
Start Loop at point “a”: *-4I*

Voltage *drop* across 4Ω: (V = IR) Current x Resistance =

- (I) x (4)

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

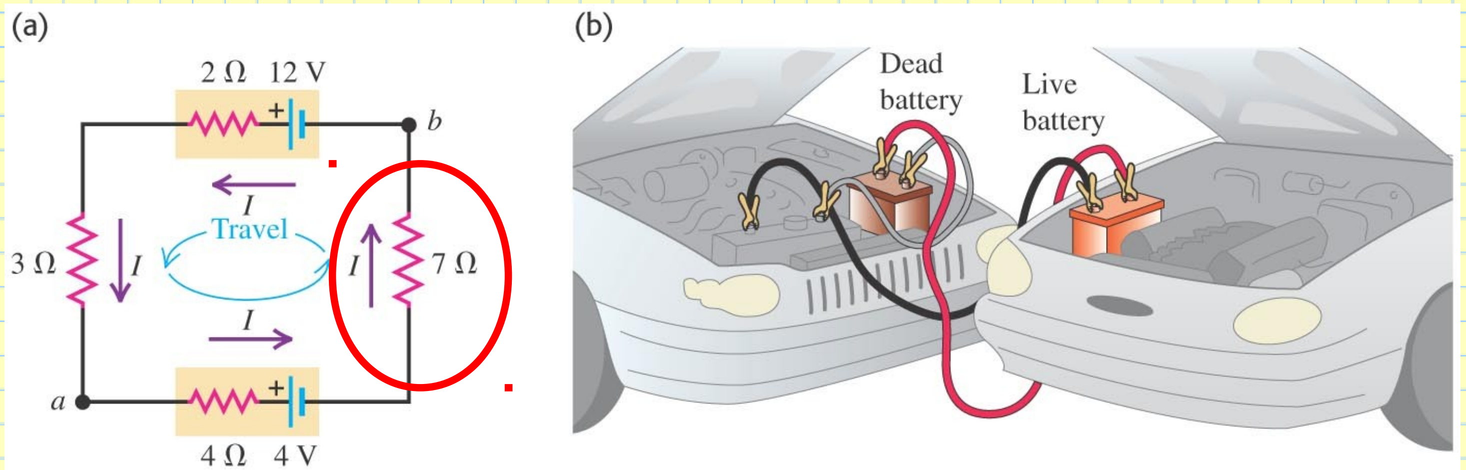


Drop across EMF source:

$$-4I - 4V$$

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

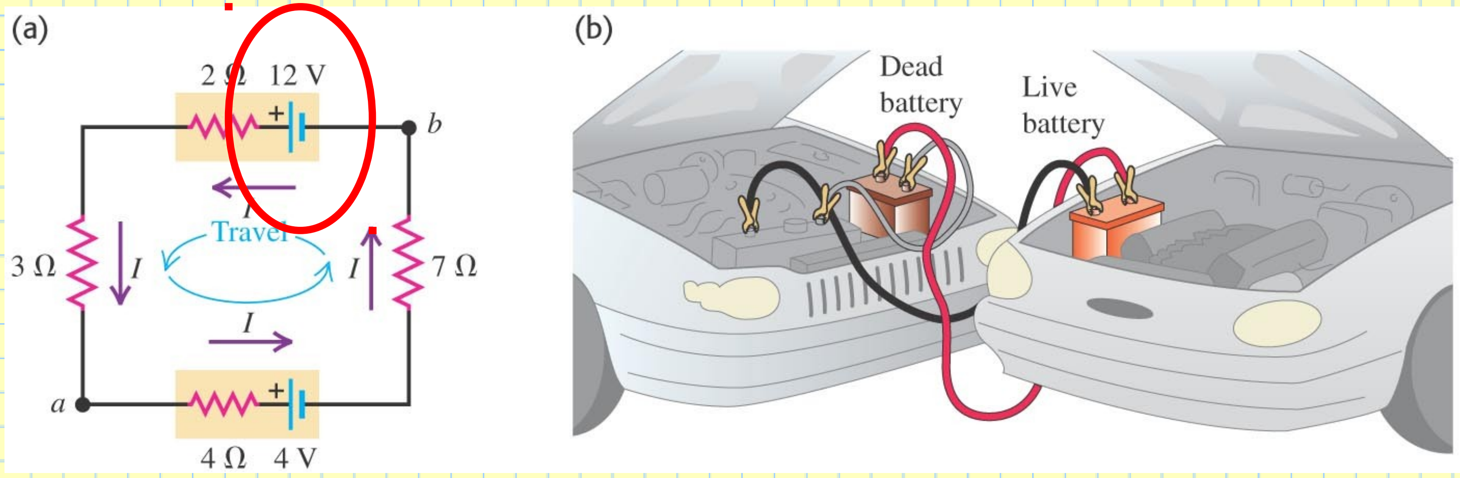


Drop across 7Ω resistor:

$$-4I - 4V - 7I$$

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

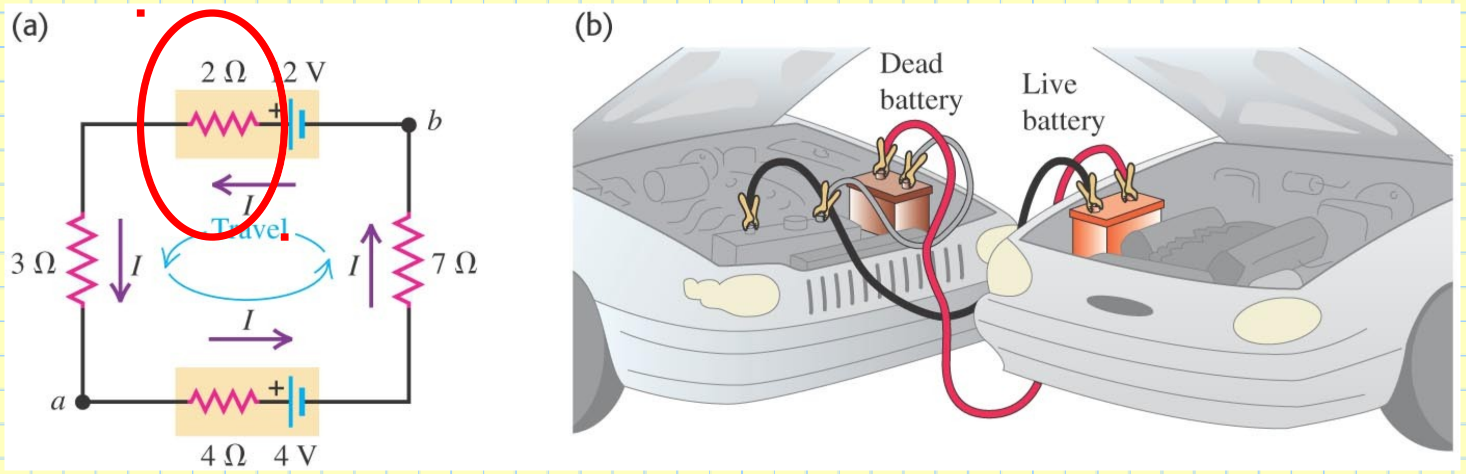


Gain going “upstream” in EMF:

$$-4I - 4V - 7I + 12V$$

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

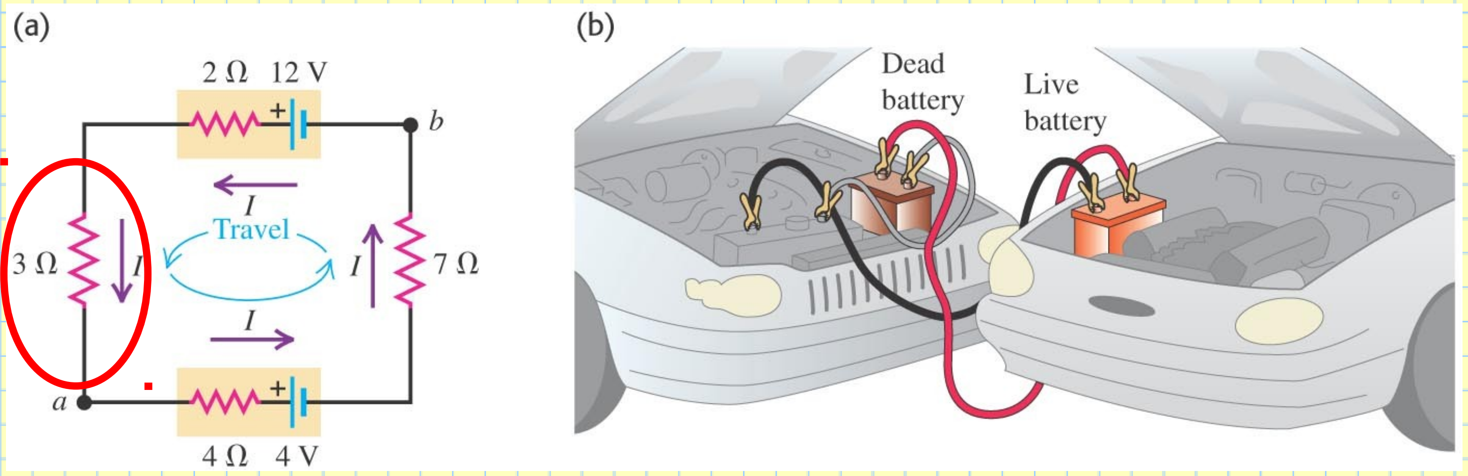


Drop across 2Ω:

$$-4I - 4V - 7I + 12V - 2I$$

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

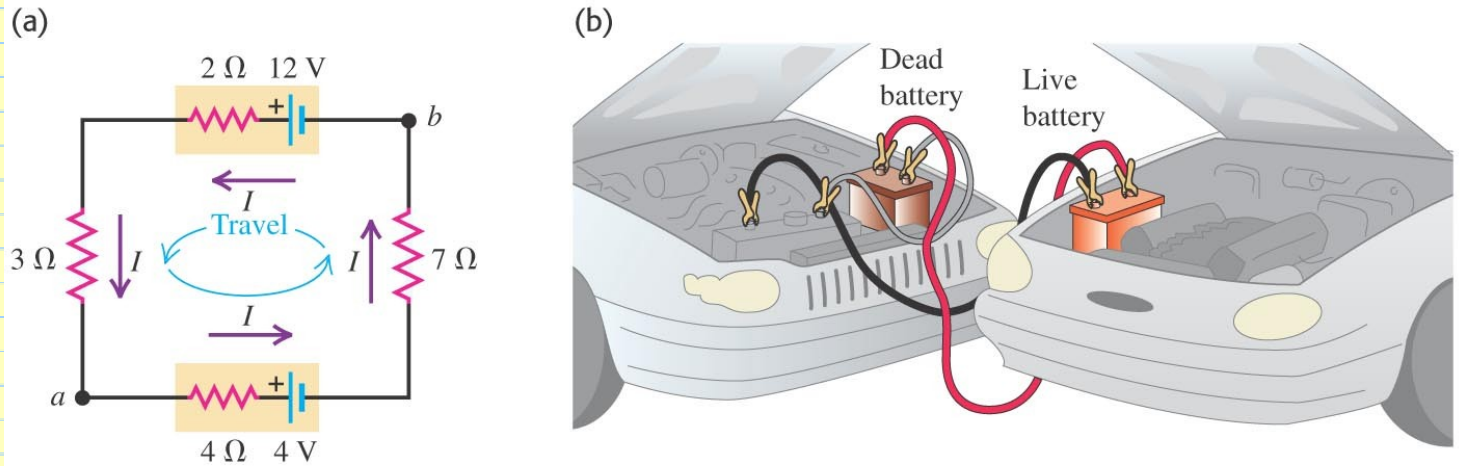


Finish back at “a”:

$$-4I - 4V - 7I + 12V - 2I - 3I = 0$$

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

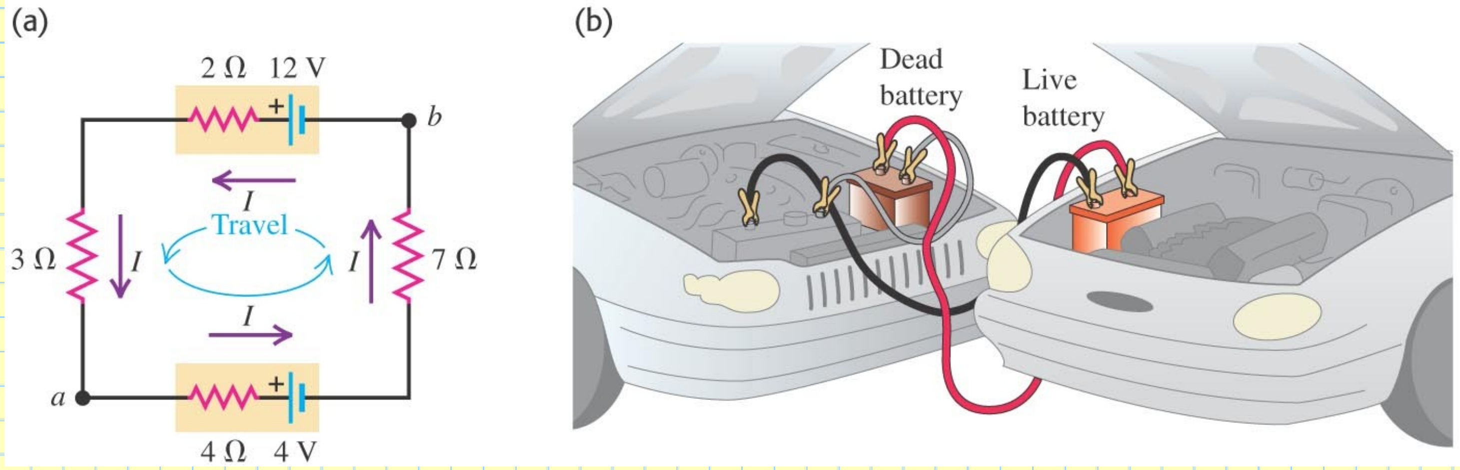


Complete Loop: $-4I - 4V - 7I + 12V - 2I - 3I = 0$

$8\ V = 16\ I$ so $I = 0.5\ Amps$

A single-loop circuit

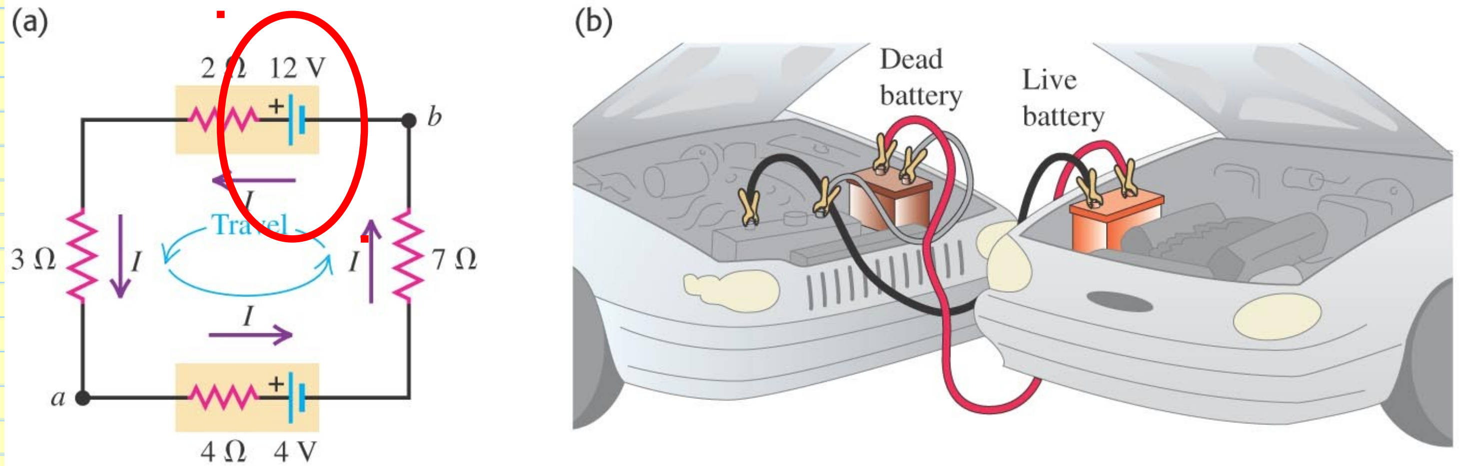
- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!



V_{ab}? Potential of a relative to b? Start at b, move to a:

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

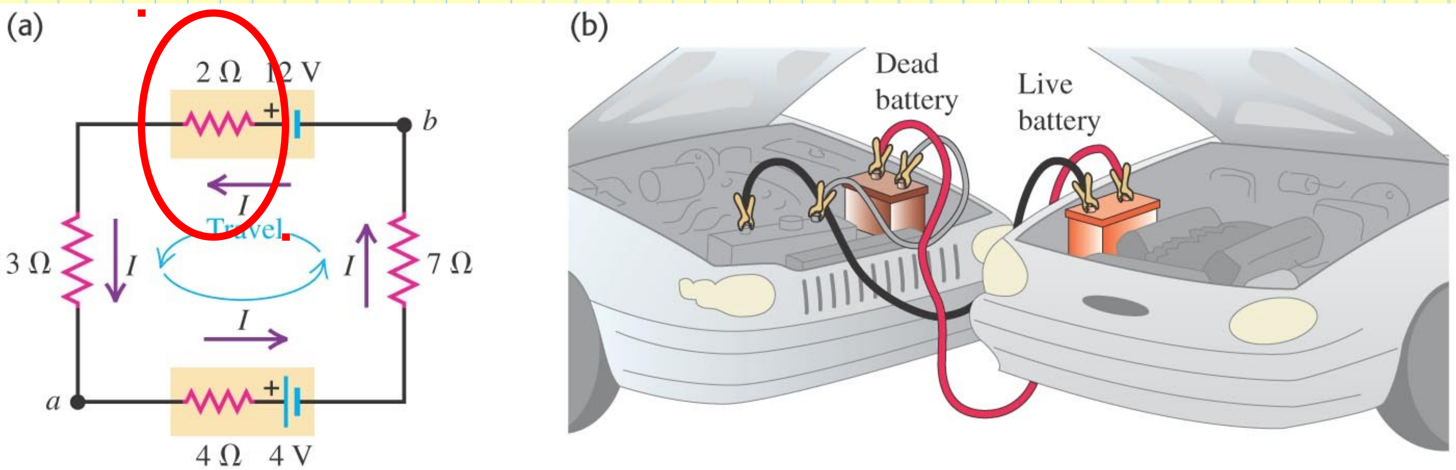


Vab? Potential of a relative to b? Start at b, move to a:

Vab = +12

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

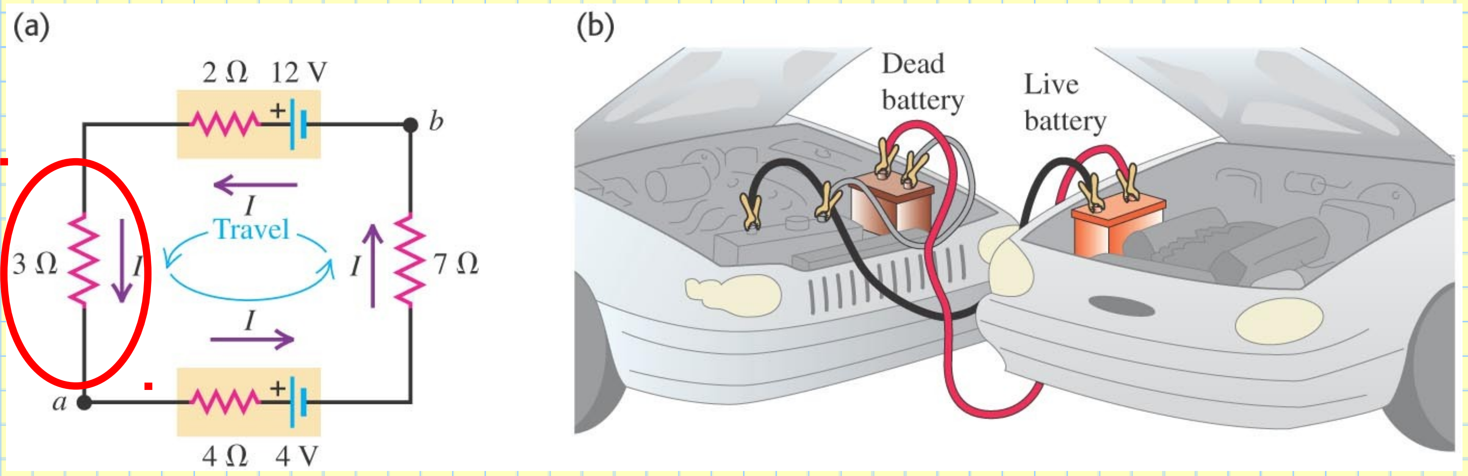


Vab? Potential of a relative to b? Start at b, move to a:

Vab = +12 – 2Ω(0.5 A)

A single-loop circuit

- Ex. 26.3: Find Current in circuit, Vab, and Power of emf in each battery!

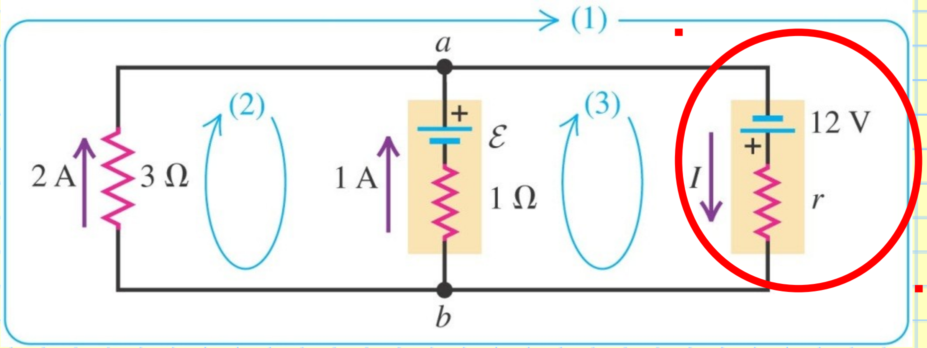


Vab? Potential of a relative to b? Start at b, move to a:

Vab = +12 – 2Ω(0.5 A) - 3Ω(0.5 A) = 9.5 V

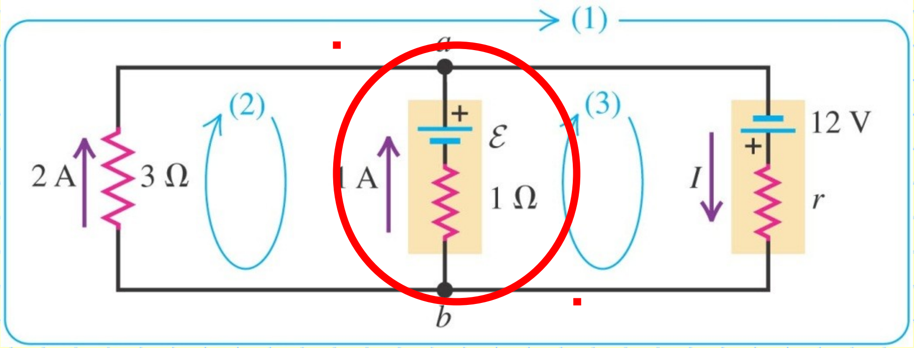
Charging a battery – Example 26.4

- 12V power supply with unknown internal resistance "r"



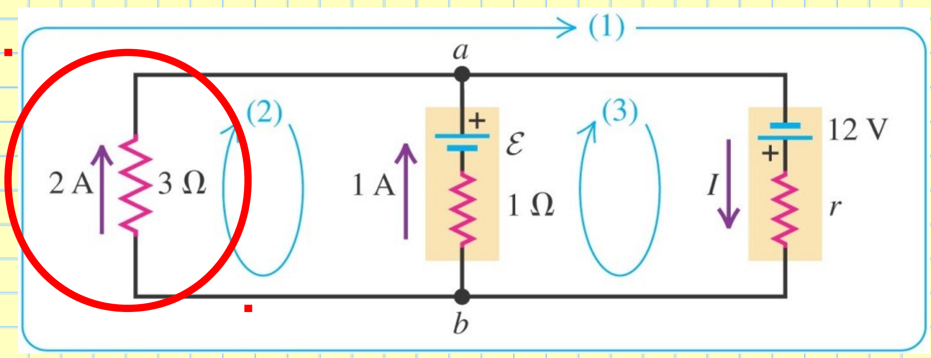
Charging a battery – Example 26.4

- 12V power supply with unknown internal resistance "r"
- Connect to battery w/ unknown EMF and 1Ω internal resistance



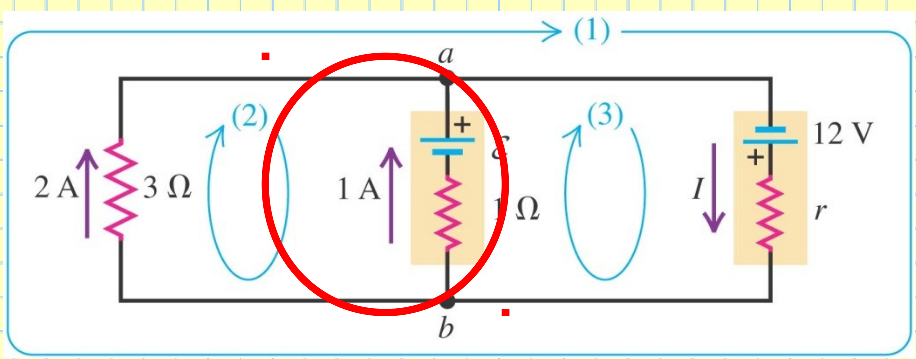
Charging a battery – Example 26.4

- 12V power supply with unknown internal resistance "r"
- Connect to battery w/ unknown EMF and 1Ω internal resistance
- Connect to indicator light of 3Ω carrying current of 2A



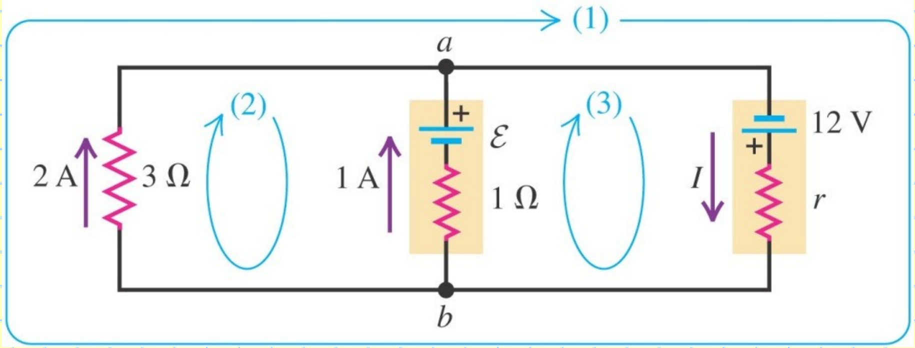
Charging a battery – Example 26.4

- 12V power supply with unknown internal resistance “r”
- Connect to battery w/ unknown EMF and 1Ω internal resistance
- Connect to indicator light of 3Ω carrying current of 2A
- Generate 1A through run-down battery.



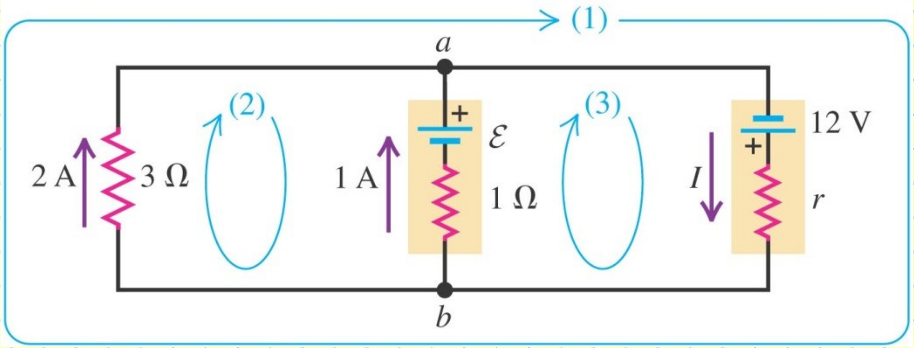
Charging a battery – Example 26.4

- 12V power supply with unknown internal resistance "r"
- Connect to battery w/ unknown EMF and 1Ω internal resistance
- Connect to indicator light of 3Ω carrying current of 2A
- Generate 1A through run-down battery.
- What are r, EMF, and I through power supply?



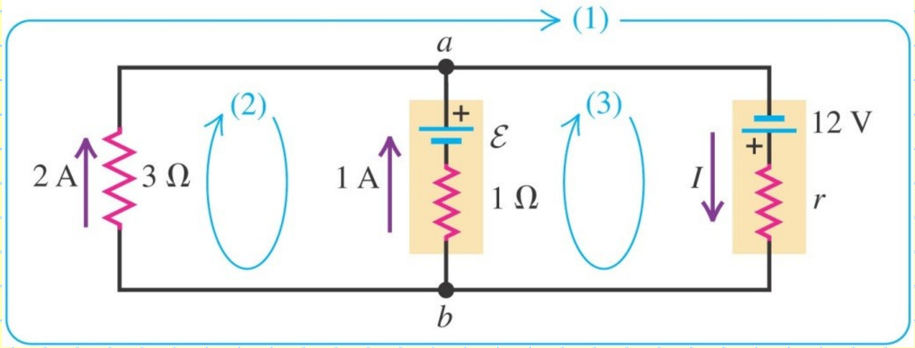
Charging a battery – Example 26.4

- Junction rule at "a":
  - $2A + 1A = I$ *or* $+2 + 1 - I = 0$
  - $I = 3$ Amps
- Loop rule starting at "a" around (1)
  - $+12\text{ V} - 3A(r) - 2A(3\Omega) = 0 \;\; \Rightarrow \;\; r = 2\ \Omega$



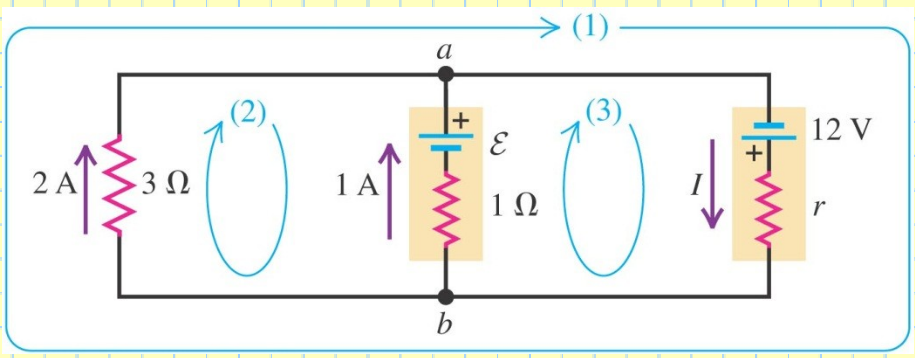
Charging a battery – Example 26.4

- Junction rule at "a":
  - 2A + 1A = I   *or*   +2 + 1 – I = 0
  - I = 3 Amps
- Loop rule starting at "a" around (2)
  - $-\mathcal{E} + 1A(1\Omega) - 2A(3\Omega) = 0$ => EMF ($\mathcal{E}$) = -5V
  - Negative value for EMF => Battery should be "flipped"



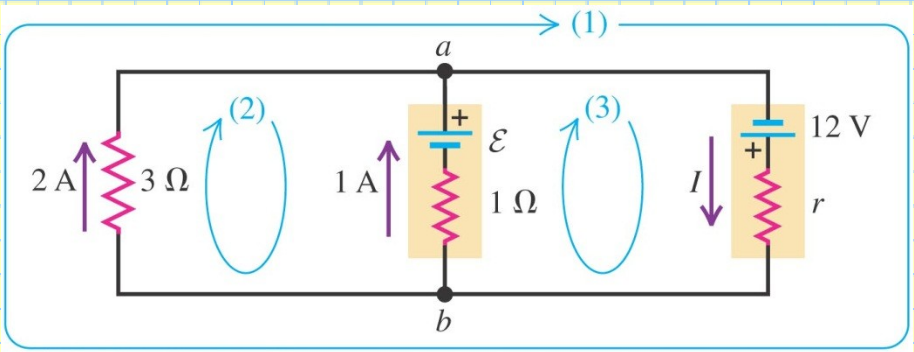
Charging a battery – Example 26.4

- Junction rule at "a":
  - 2A + 1A = I *or* +2 + 1 – I = 0
  - I = 3 Amps
- Loop rule starting at "a" around (3)
  - +12 V – 3A(2Ω) – 1A(1) +E = 0 => E = -5V (again!)
  - *Check your values with third loop!!*



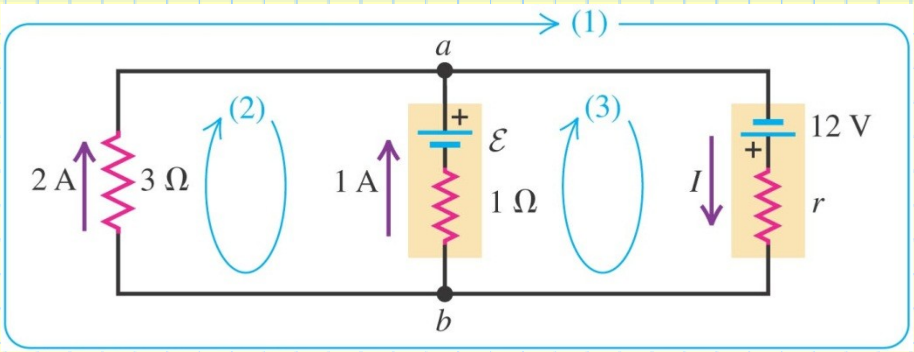
Charging a battery (cont.) – Example 26.5

- What is the power delivered by the 12V power supply, and by the battery being recharged?
- What is power dissipated in each resistor?



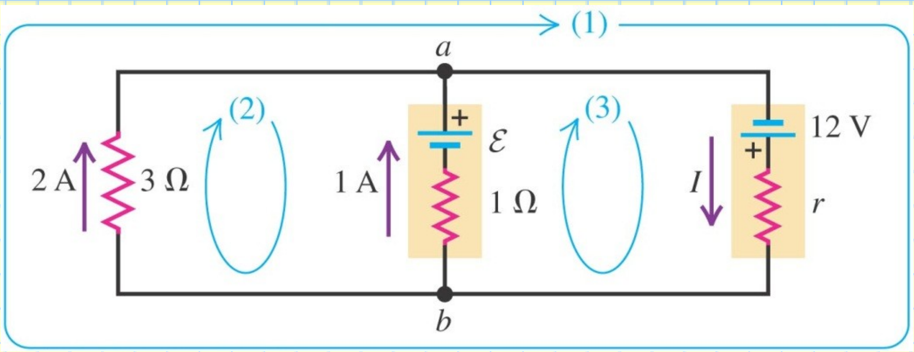
Charging a battery (cont.) – Example 26.5

- What is the power delivered by the 12V power supply, and by the battery being recharged?
- $P_{supplied}$ = EMF x Current = 12 V x 3 Amps = 36 Watts
- $P_{dissipated\ in\ supply} = i^2r = (3Amps)^2 \times 2\Omega = 18W$
- Net Power = 36 – 18 = 18 Watts



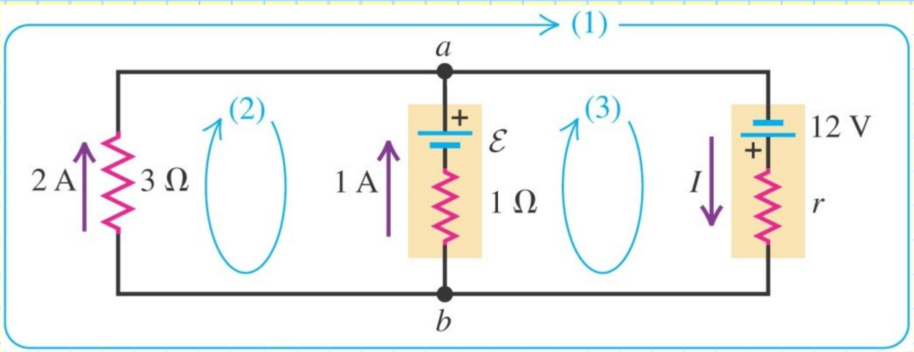
Charging a battery (cont.) – Example 26.5

- What is the power delivered by the 12V power supply, and by the battery being recharged?
- P_{EMF} = E x Current = -5 V x 1 Amps = -5 Watts
- Negative => power not provided – power is being stored!



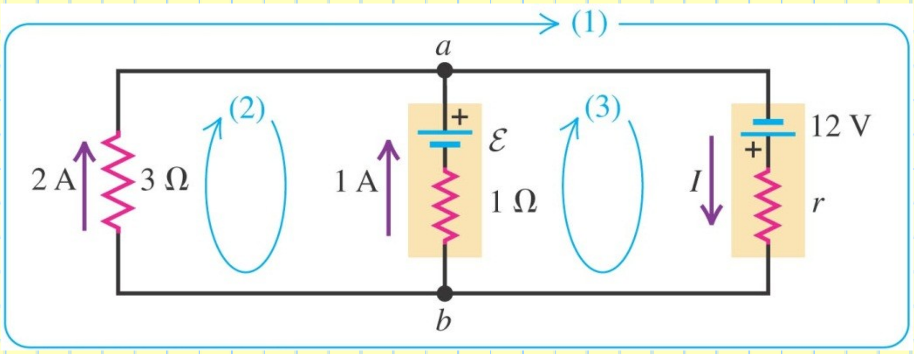
Charging a battery (cont.) – Example 26.5

- What is power dissipated in each resistor?
- $P_{\text{dissipated in battery}} = i^2r = (1\text{Amps})^2 \times 1\Omega = 1\text{W}$
- $P_{\text{dissipated in bulb}} = i^2r = (2\text{Amps})^2 \times 3\Omega = 12\text{W}$



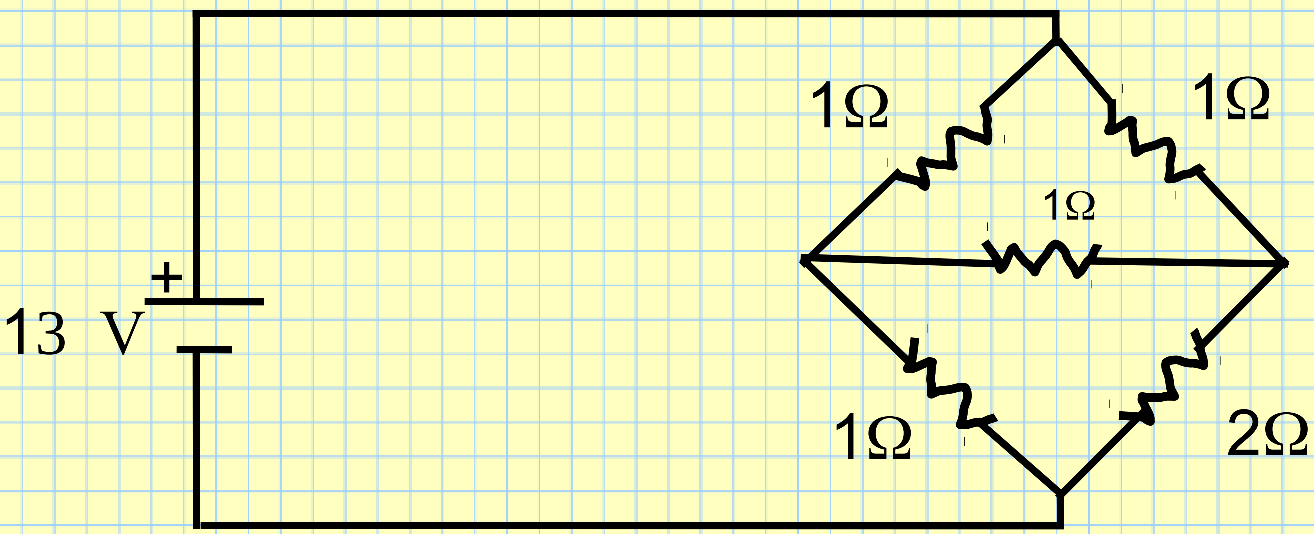
Charging a battery (cont.) – Example 26.5

- Total Power: +36W from supply
- - 18 W to its internal resistance r
- - 5 W to charge dead battery
- - 1 W to dead battery's internal resistance
- - 12 W to indicator light.



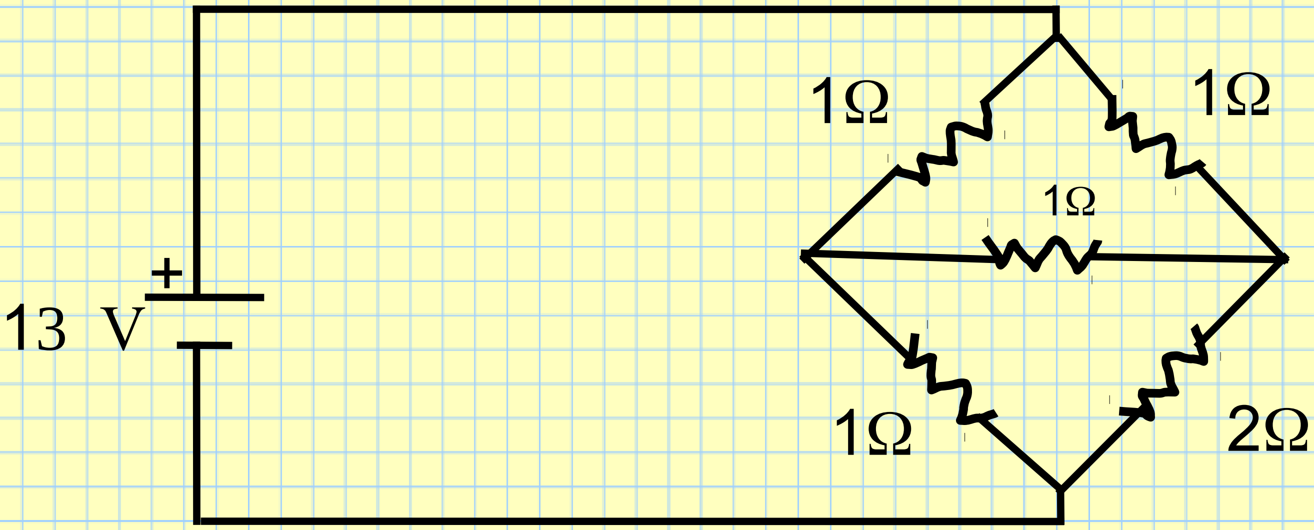
A complex network – Example 26.6

- Find Current in each resistor! Find equivalent R!!



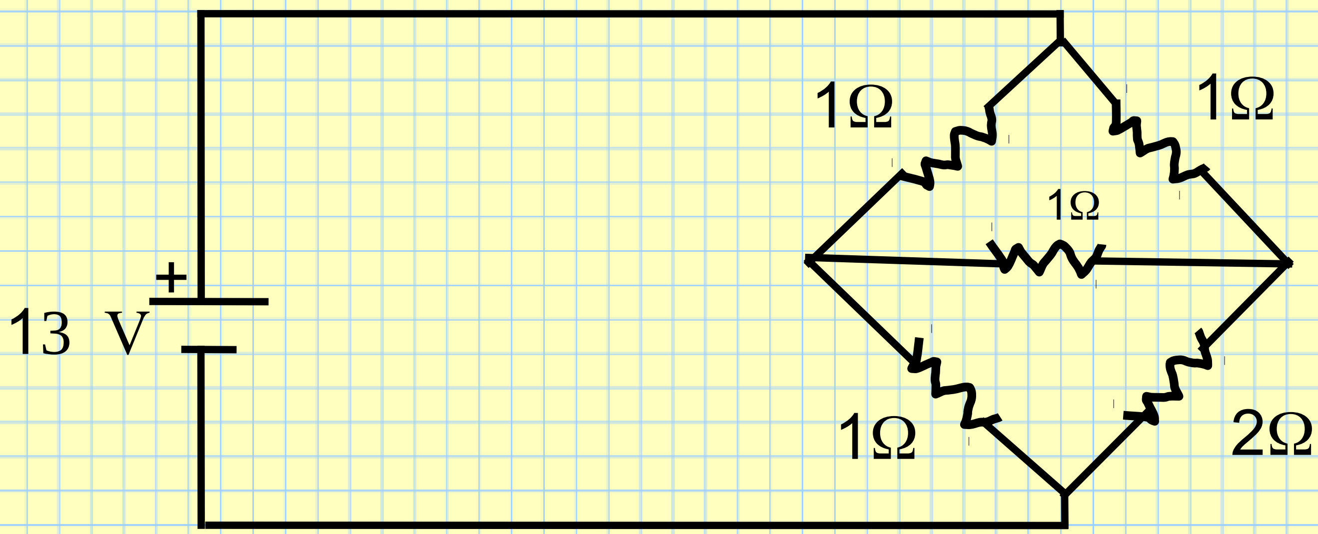
A complex network – Example 26.6

- Step 1: Junction Rule!
  - Define current directions and labels



A complex network – Example 26.6

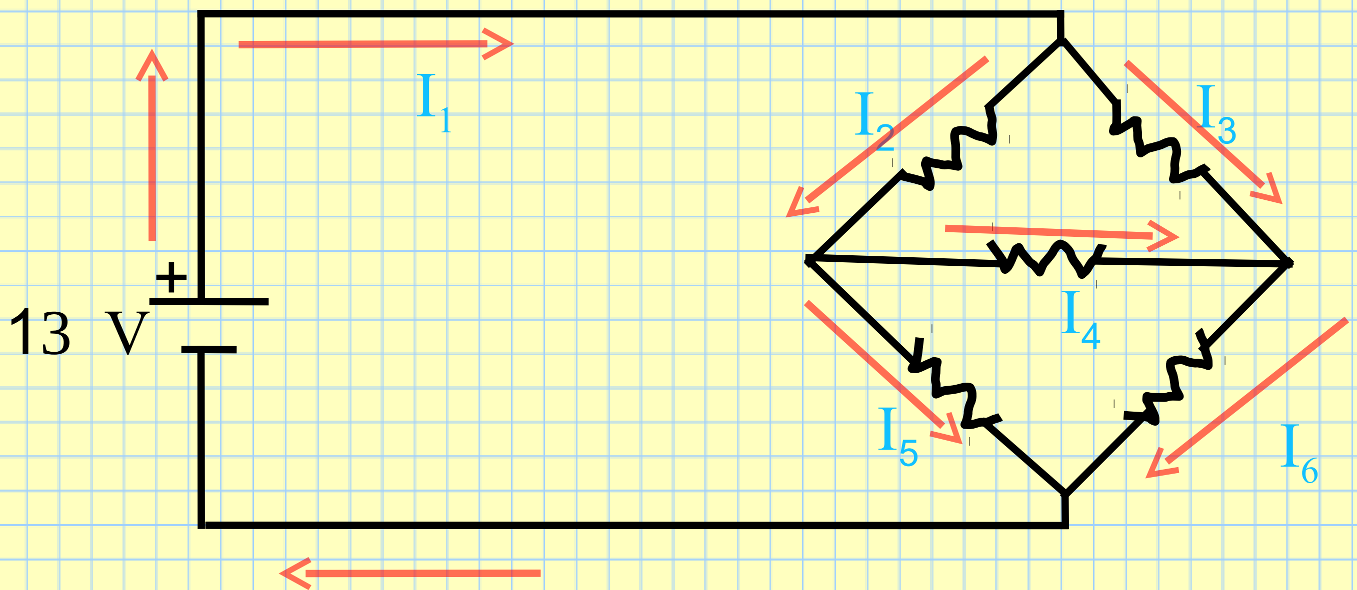
- Step 1: Junction Rule!
  - Define current directions and labels



- *NOTE for Junction Rule! Actual directions of current may differ, but value of current derived is correct!*

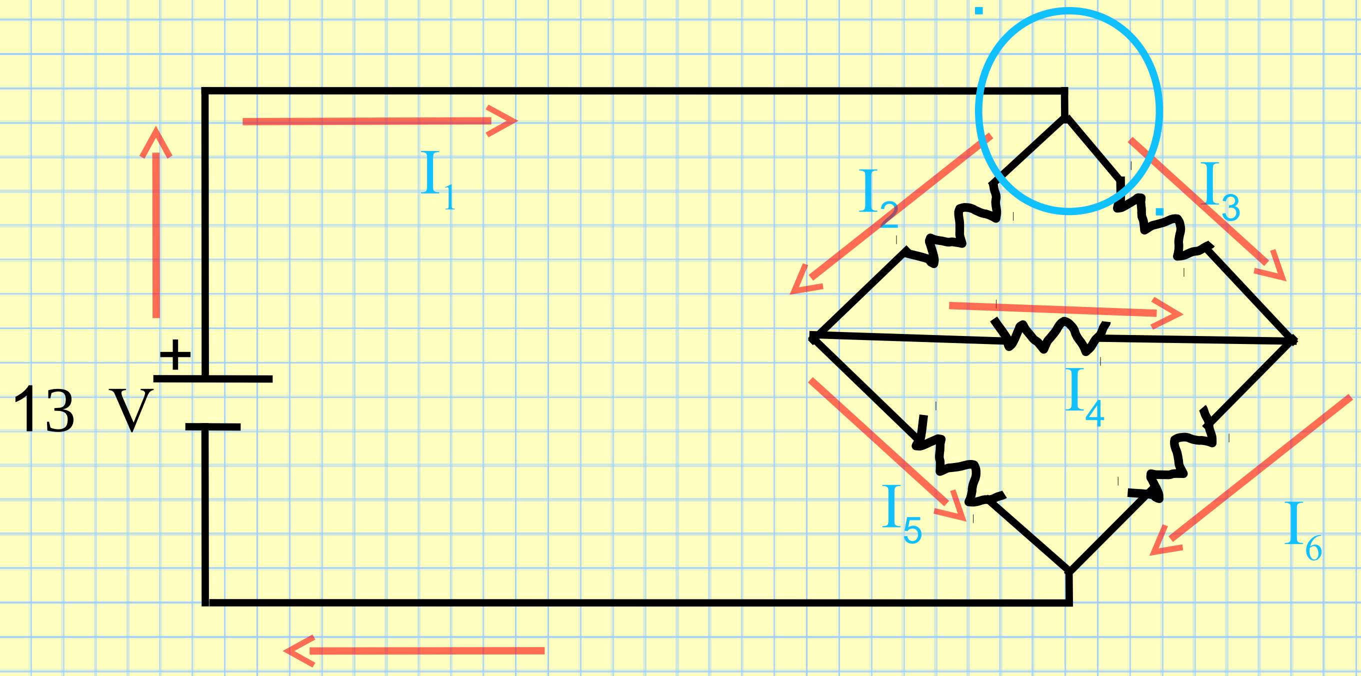
A complex network – Example 26.6

- Step 1: Junction Rule!
  - Define current directions and labels



A complex network – Example 26.6

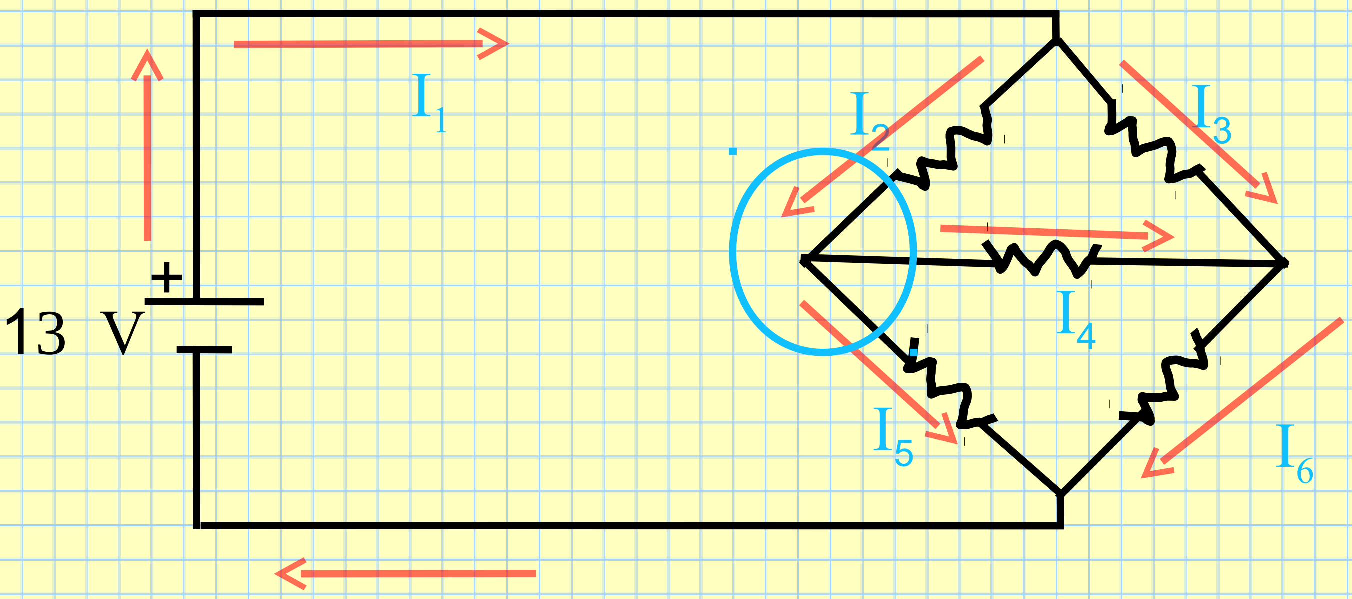
- Step 1: Junction Rule!
  - Define current directions and labels



$I_1 - I_2 - I_3 = 0$ *or* $I_1 = I_2 + I_3$

A complex network – Example 26.6

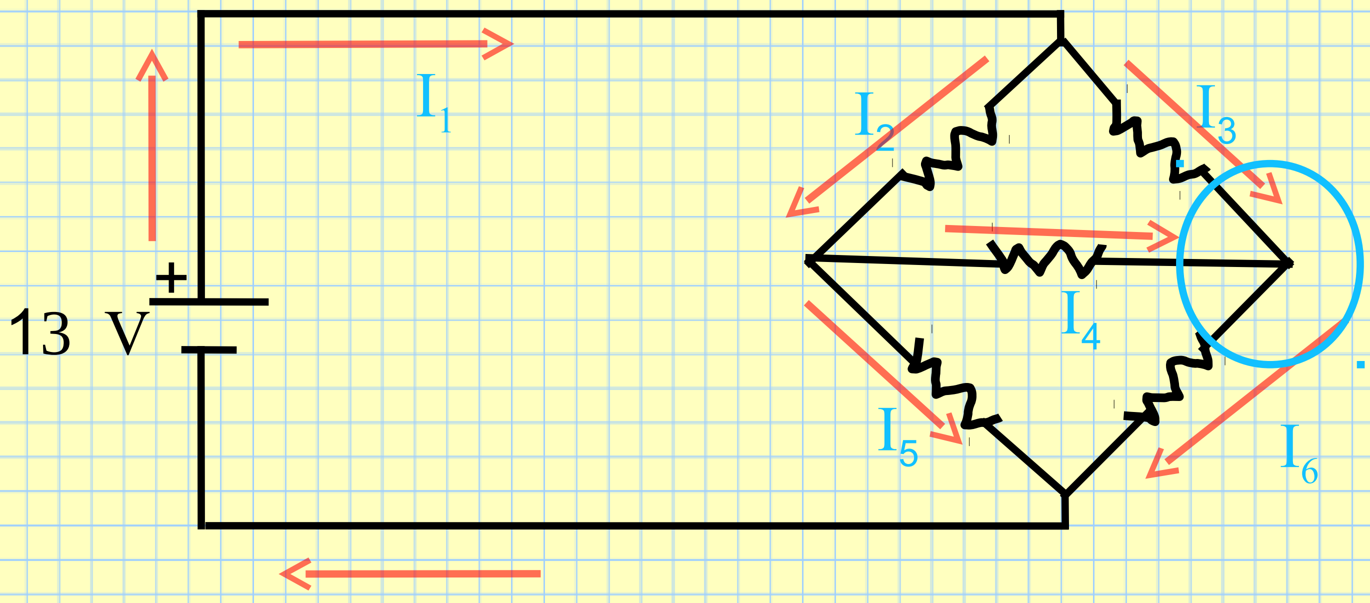
- Step 1: Junction Rule!
  - Define current directions and labels



$$I_2 - I_5 - I_4 = 0 \textit{ or } I_2 = I_4 + I_5$$

A complex network – Example 26.6

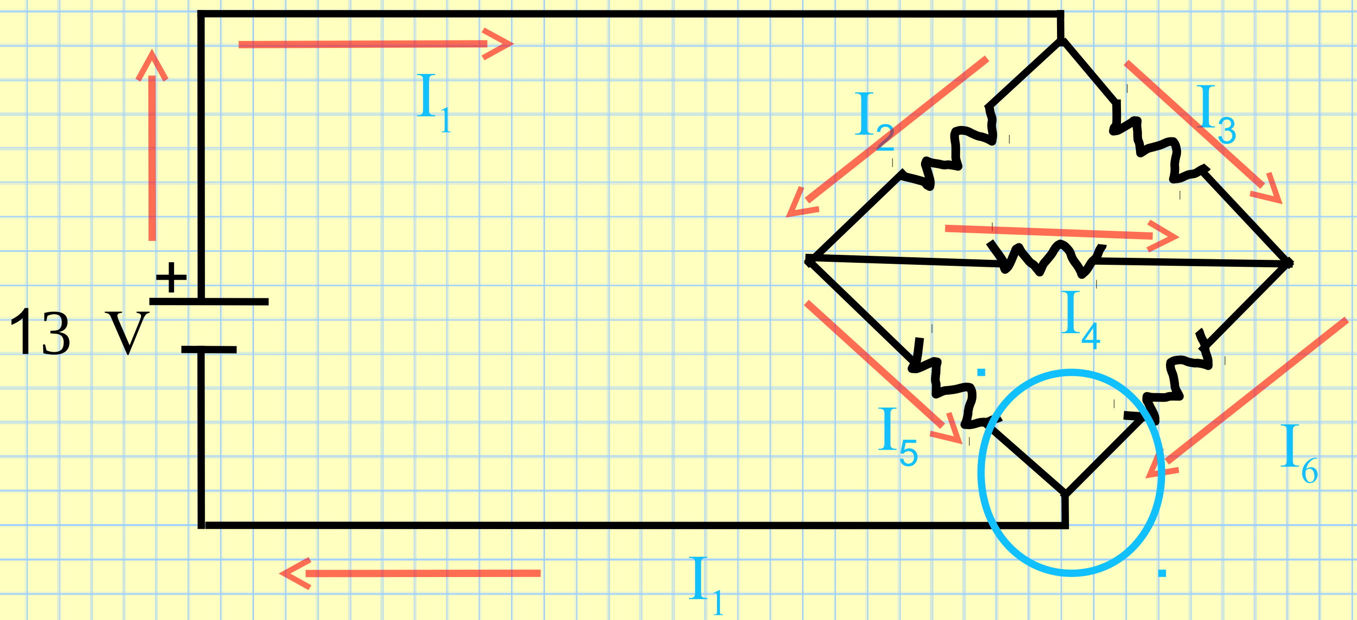
- Step 1: Junction Rule!
  - Define current directions and labels



$$I_4 + I_3 - I_6 = 0 \textit{ or } I_6 = I_4 + I_3$$

A complex network – Example 26.6

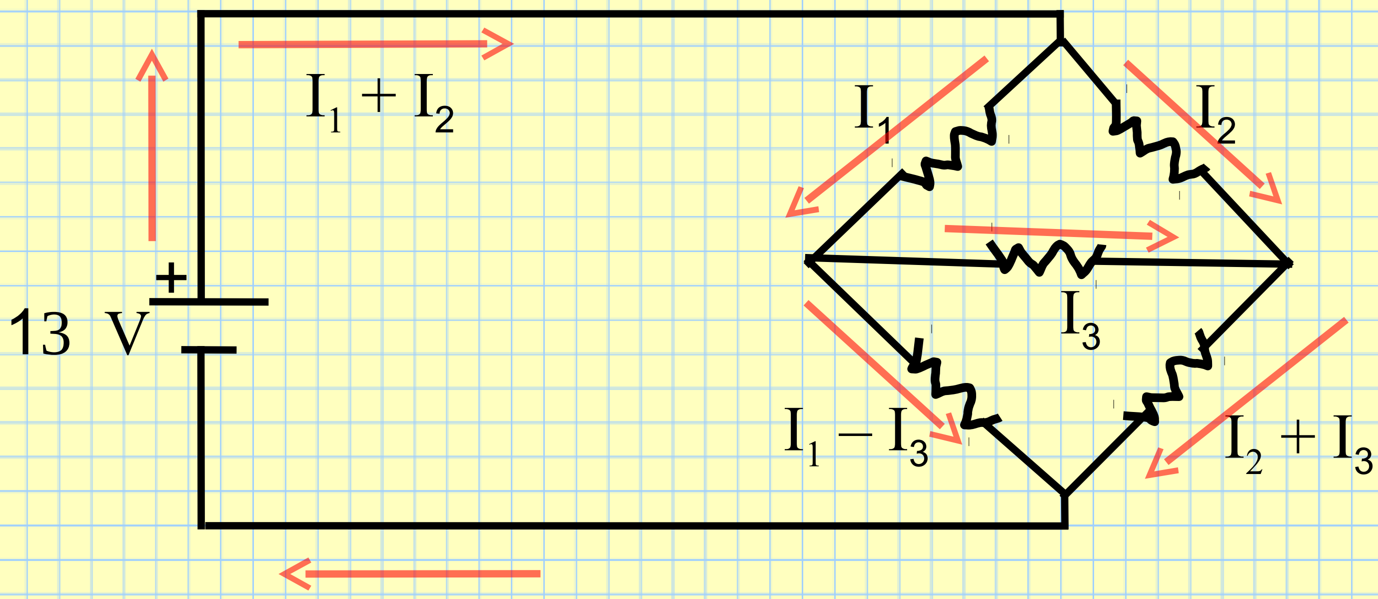
- Step 1: Junction Rule!
  - Define current directions and labels



$I_5 + I_6 - I_1 = 0$ *or* $I_1 = I_5 + I_6$

A complex network – Example 26.6

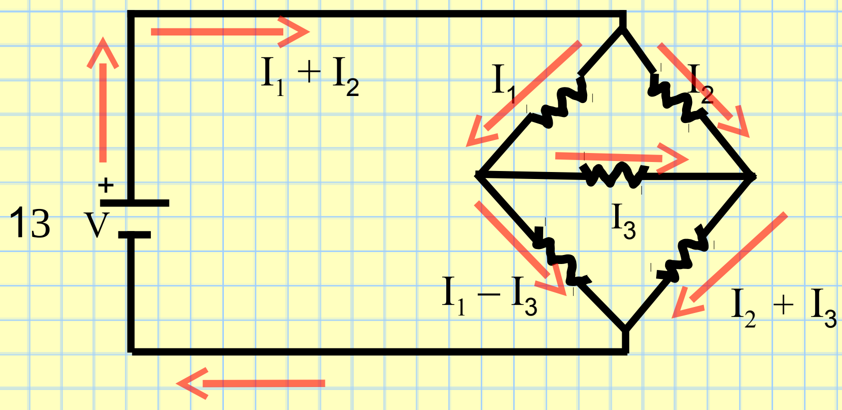
- Step 1: Junction Rule!
  - Define current directions and labels



- *NOTE for Junction Rule! How you divide current doesn't matter, but it can simplify solution steps…*

A complex network – Example 26.6

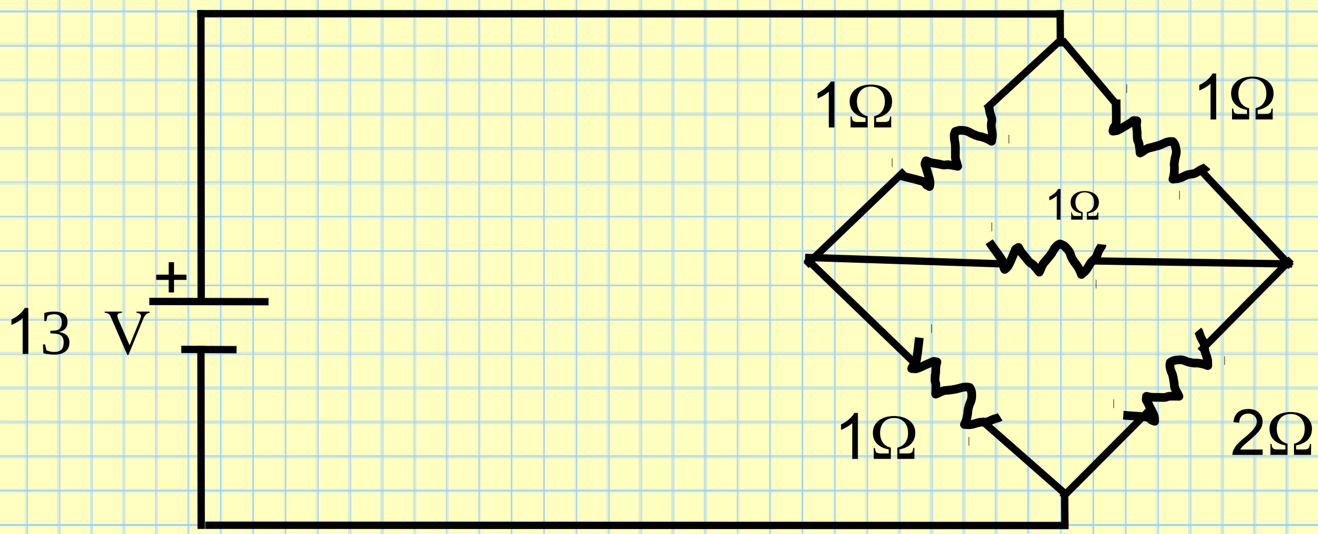
- Step 1: Junction Rule!
  - Define current directions and labels



- Lots of ways to do this – none is necessarily better than another.
- Direction WILL affect final signs in your answer.

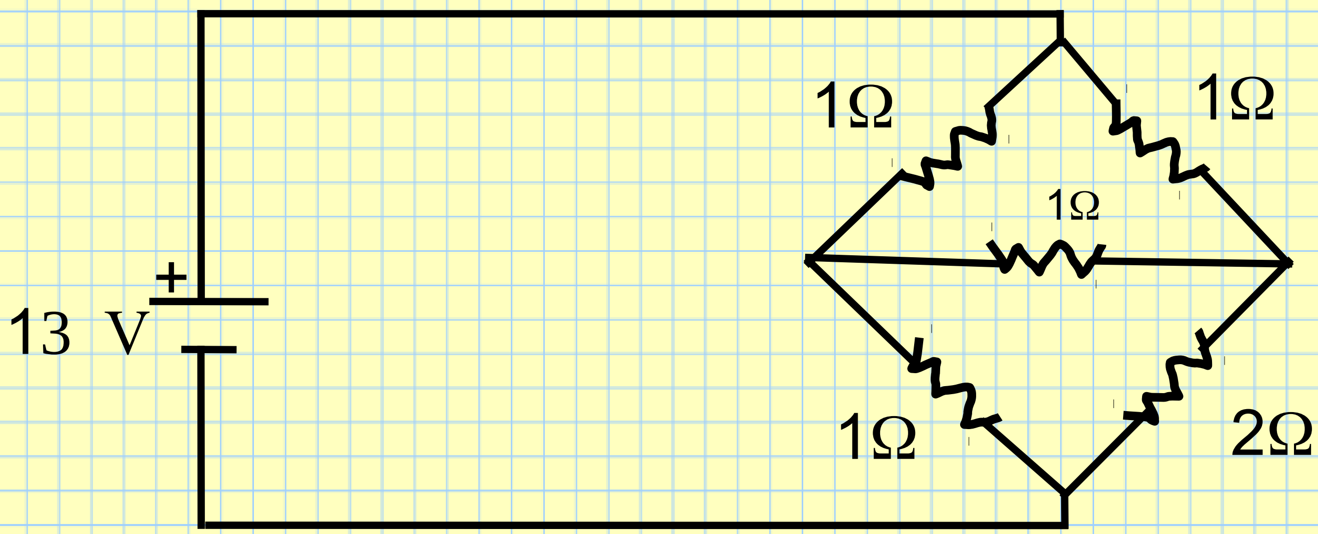
A complex network – Example 26.6

- Step 2: Loop Rule!
    - Define loop directions and labels



A complex network – Example 26.6

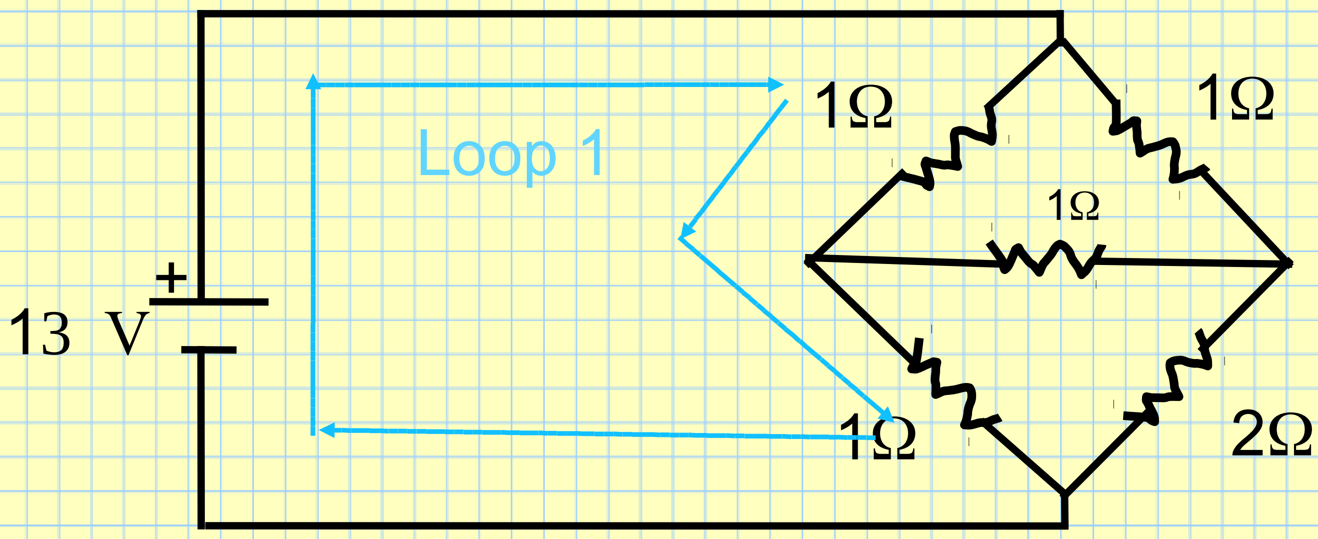
- Step 2: Loop Rule!
  - Define loop directions and labels



- *Note: Loop Rule!*
  - *Loop directions do NOT have to be in any particular direction nor order!*

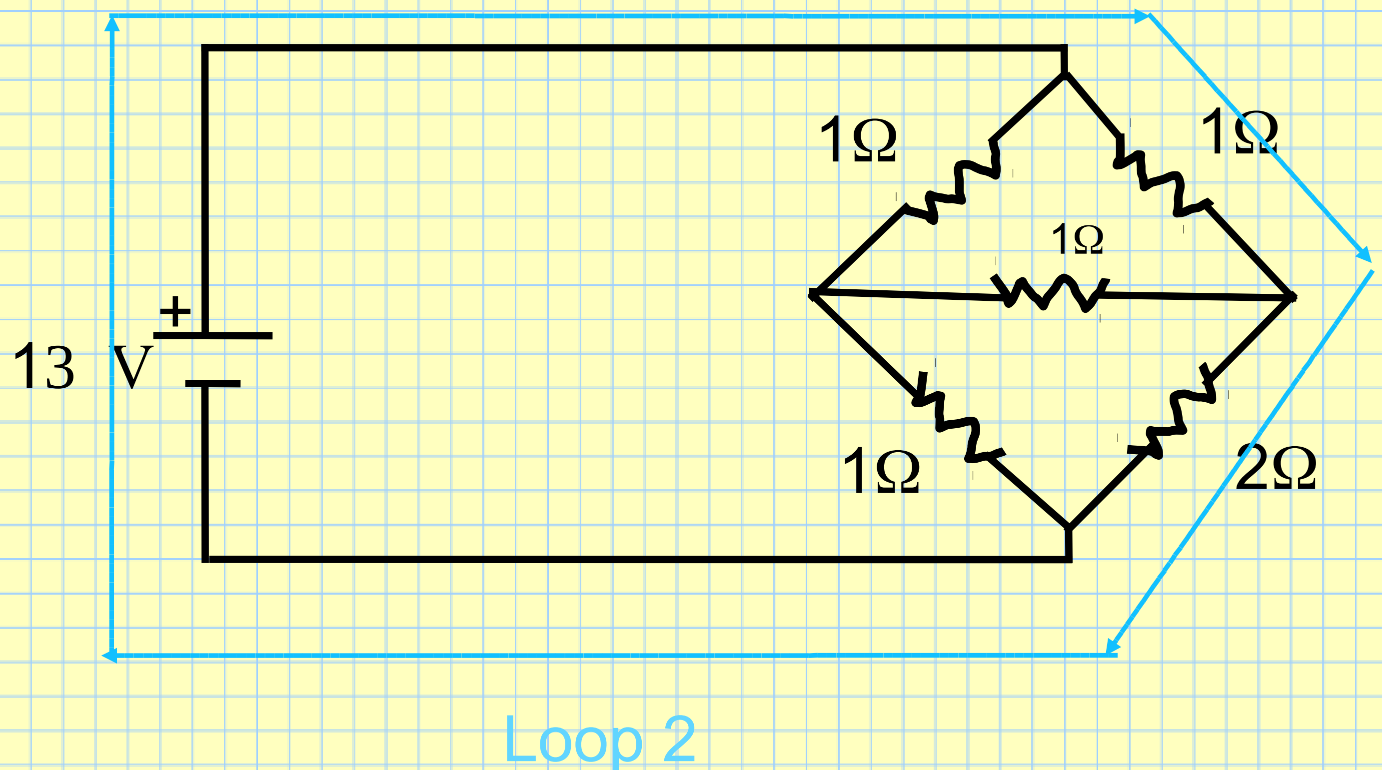
A complex network – Example 26.6

- Step 2: Loop Rule!
  - Define loop directions and labels



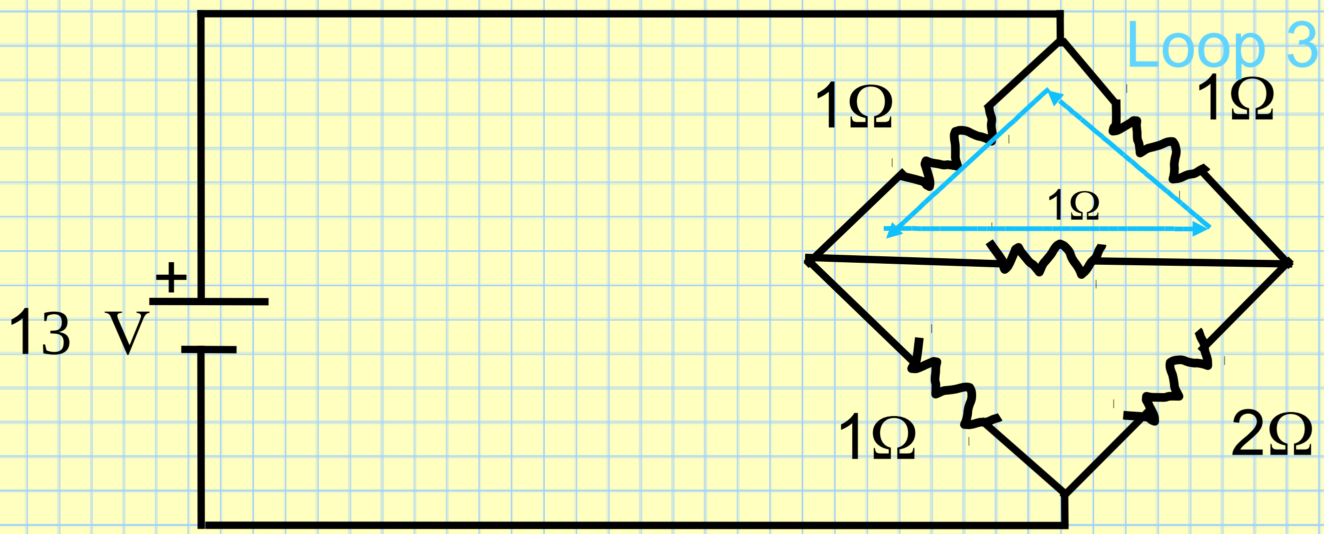
A complex network – Example 26.6

- Step 2: Loop Rule!
  - Define loop directions and labels



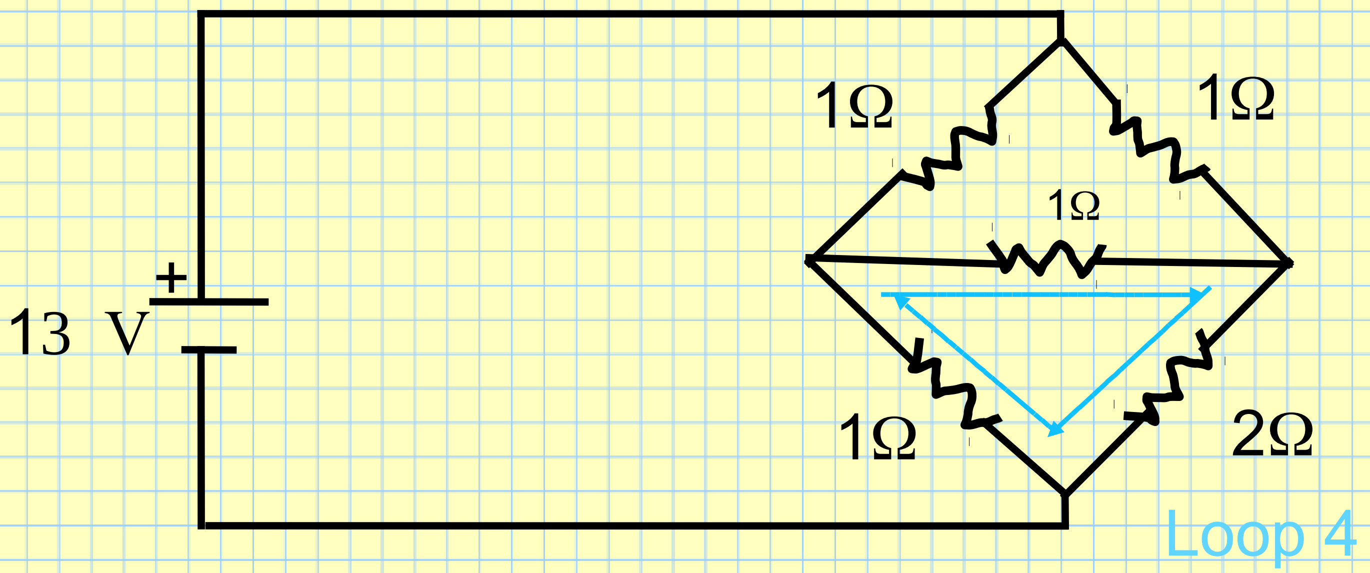
A complex network – Example 26.6

- Step 2: Loop Rule!
  - Define loop directions and labels



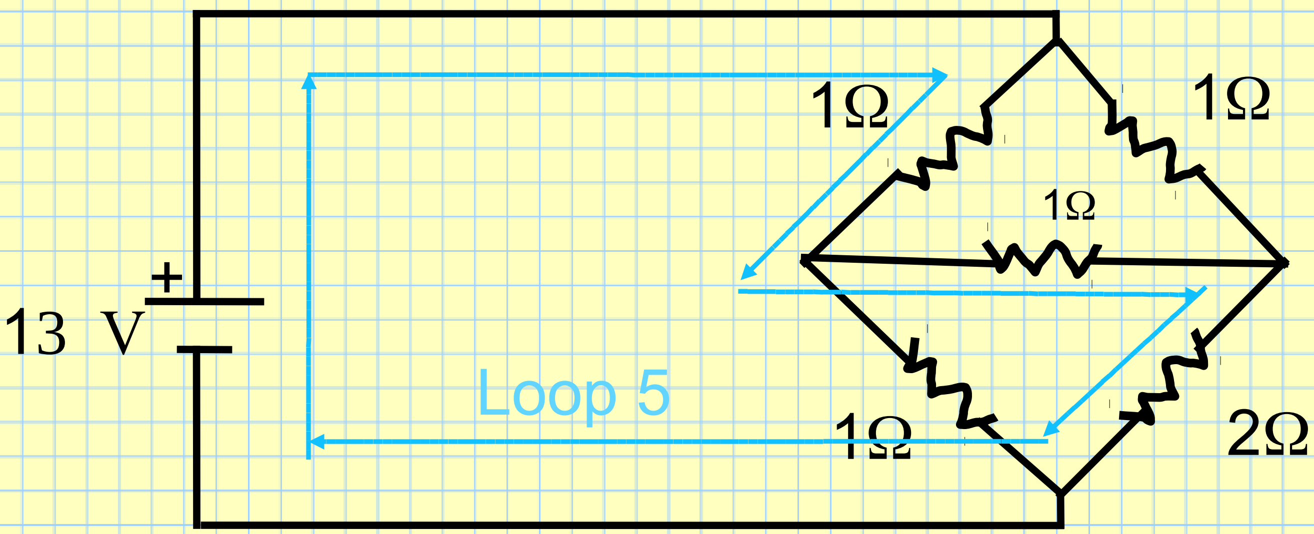
A complex network – Example 26.6

- Step 2: Loop Rule!
    - Additional loops available!



A complex network – Example 26.6

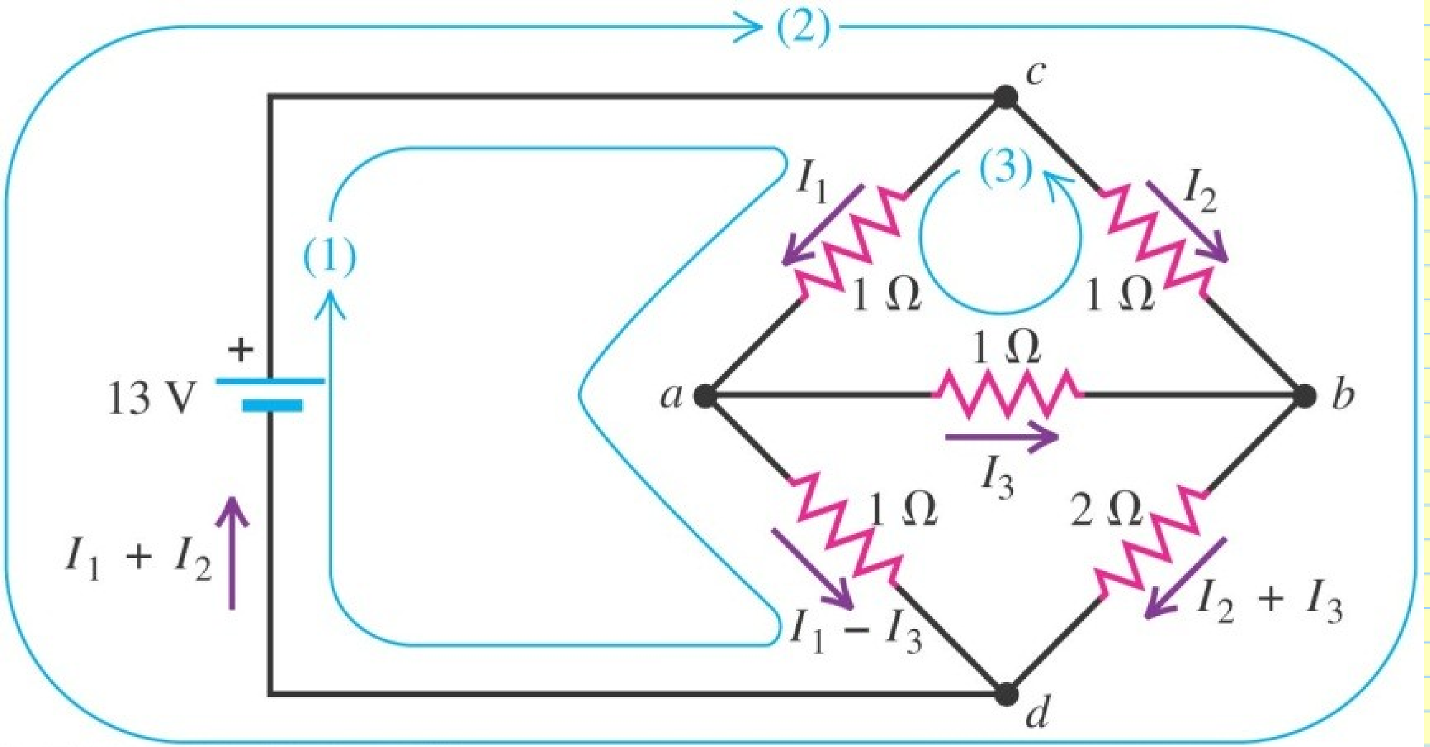
- Step 2: Loop Rule!
  - Additional loops available!



- Any closed path will work.
- Extra loops good for checking

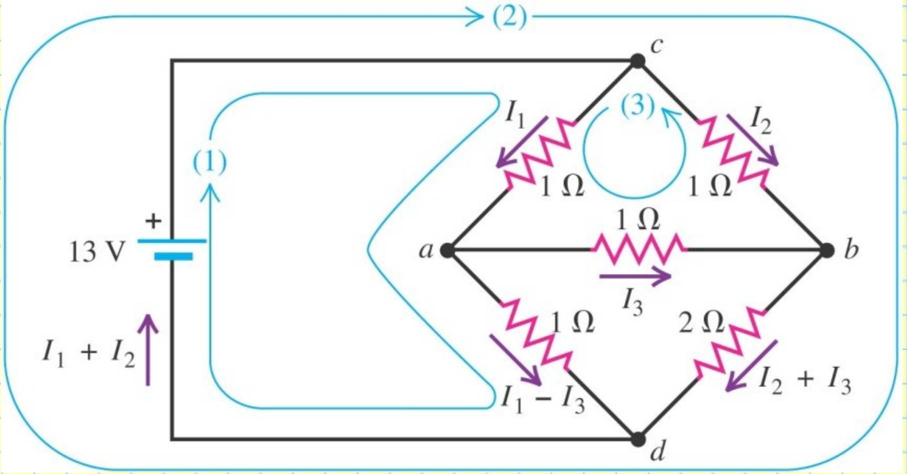
A complex network – Example 26.6

- Step 3: Make Loop Equations!



A complex network – Example 26.6

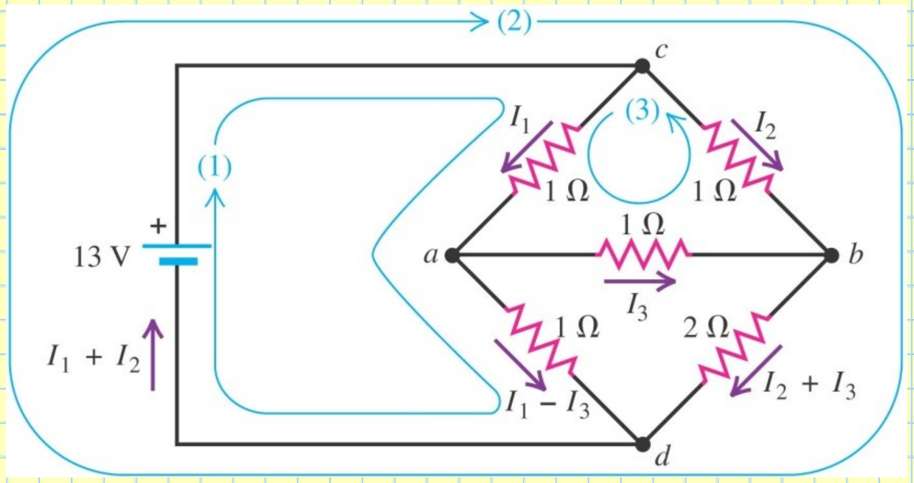
- Step 4: Solve equations (substitution or matrix)



- $+13 - 1I_1 - 1(I_1 - I_3) = 0$
- $+13 - 1I_2 - 2(I_2 + I_3) = 0$
- $-1I_1 - 1I_3 + 1I_2 = 0$

A complex network – Example 26.6

- Step 4: Solve equations (substitution or matrix)

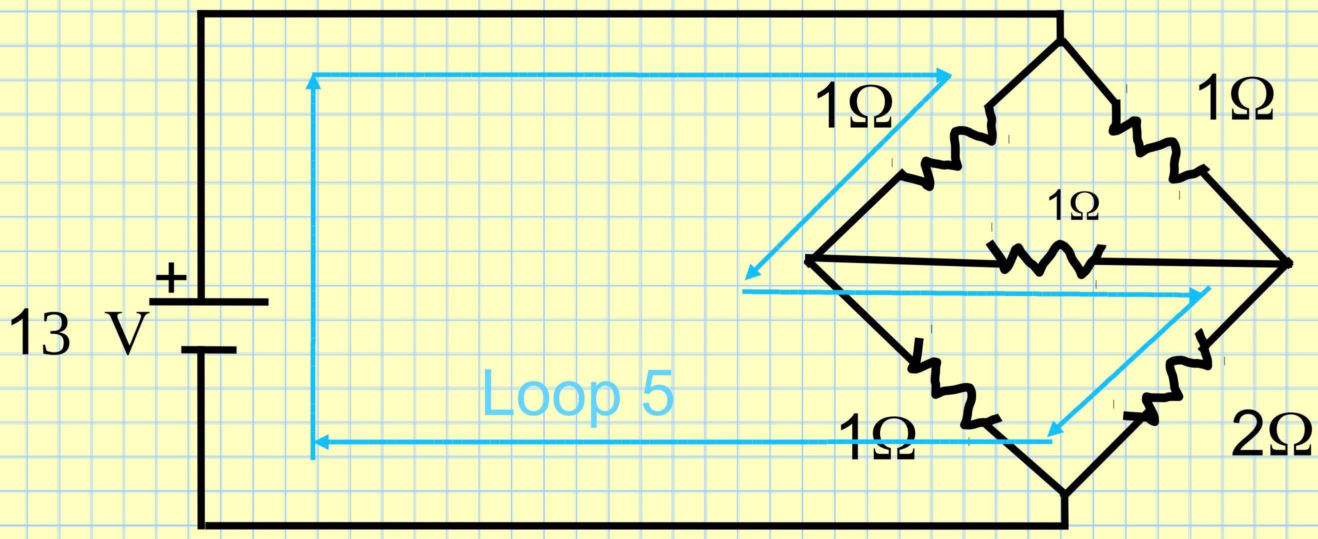


- $I_1 = +6$ A
- $I_2 = 5$ A
- $I_3 = -1$ A

So I_3 is really going from b to c

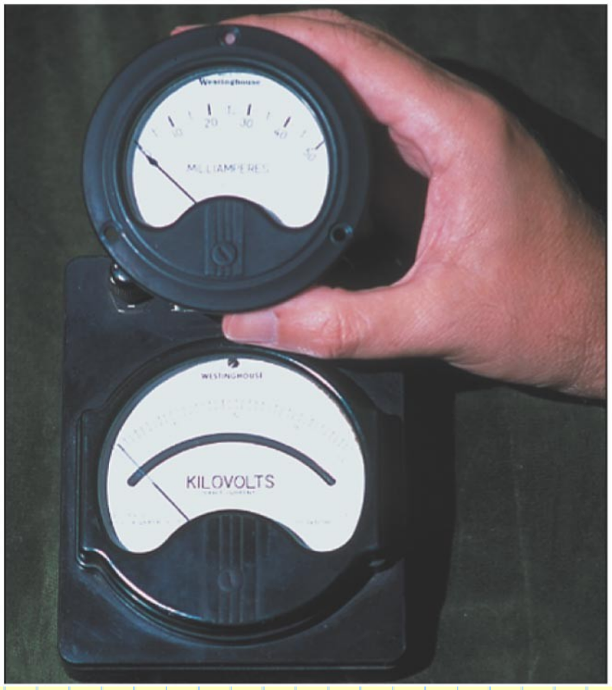
A complex network – Example 26.6

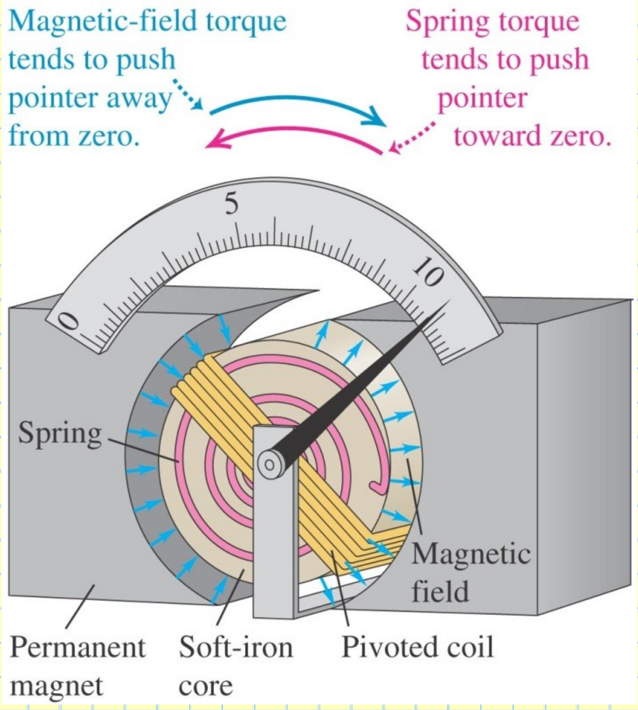
- Step 5: Check with extra loop equations!



D’Arsonval galvanometer

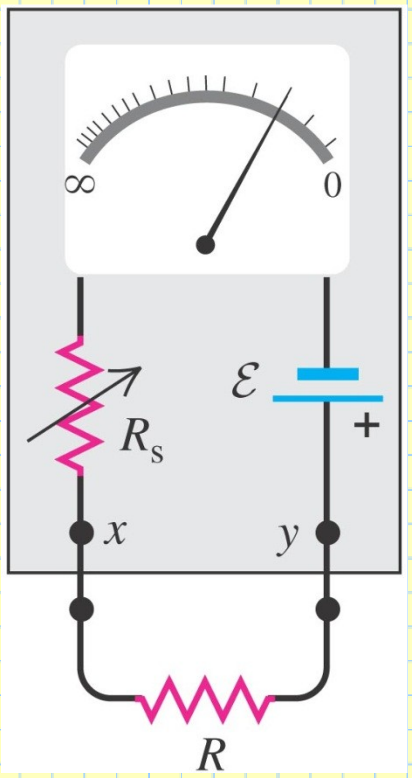
- A *d’Arsonval galvanometer* measures the current through it (see Figures below).
- Many electrical instruments, such as ammeters and voltmeters, use a galvanometer in their design.

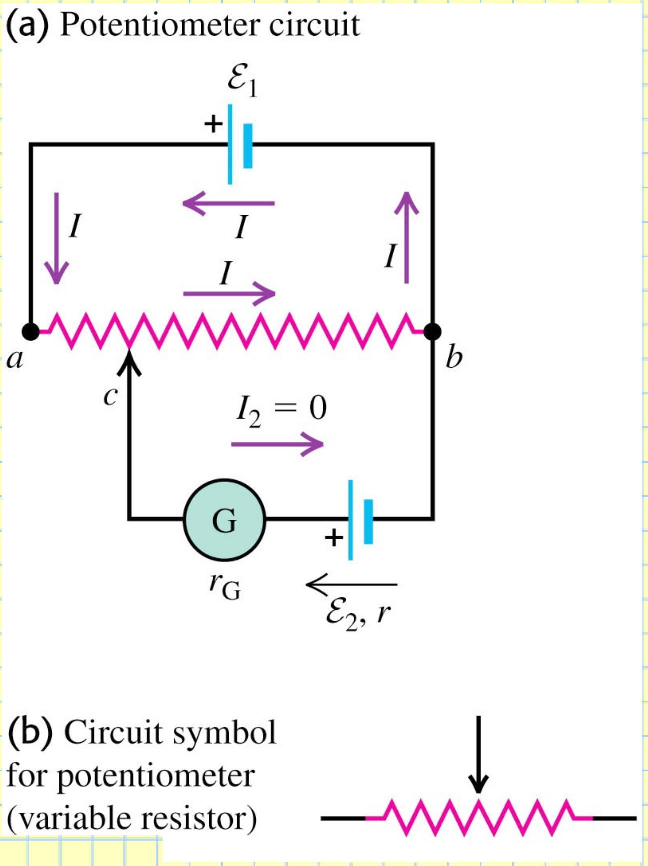




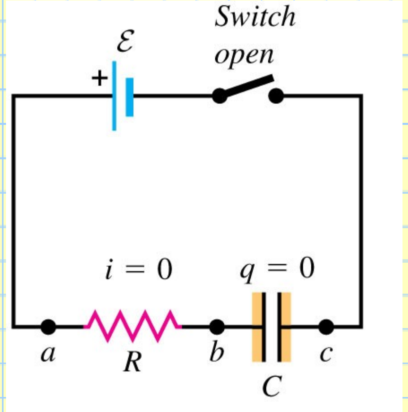
Ohmmeters and potentiometers

- An *ohmmeter* is designed to measure resistance.
- A *potentiometer* measures the emf of a source without drawing any current from the source.





Adding Capacitors to DC circuits!

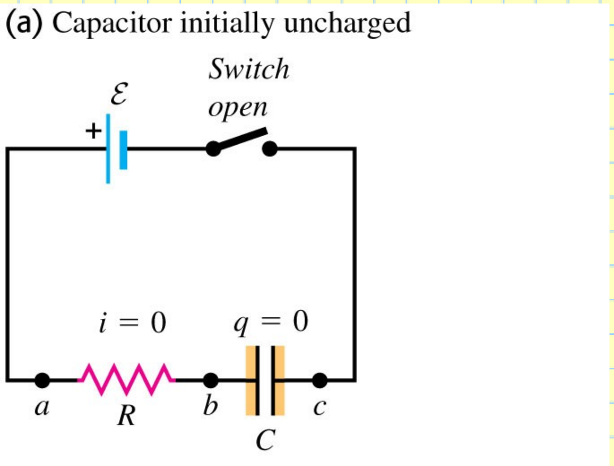


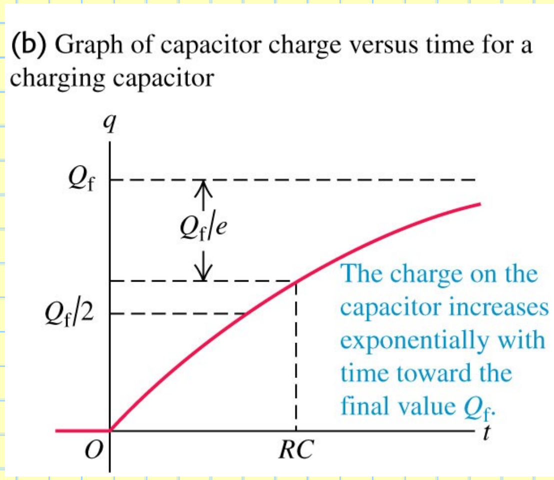
- **RC** circuits include
  - Batteries (Voltage sources!)
  - Resistors
  - Capacitors
  - … and switches!

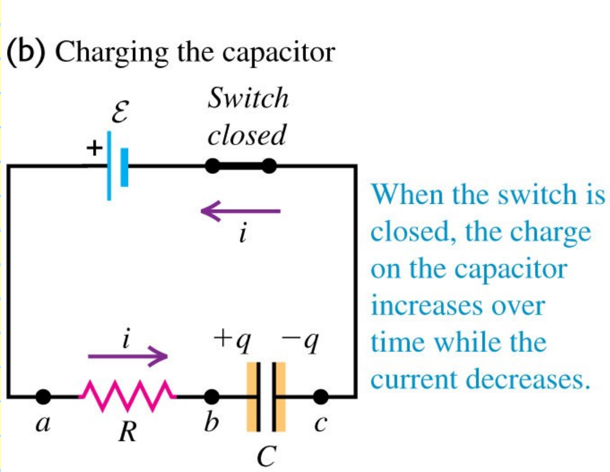
- RC circuits will involve **TIMING** considerations
  - Time to *fill up* a capacitor with charge
  - Time to *drain* a capacitor that is already charged

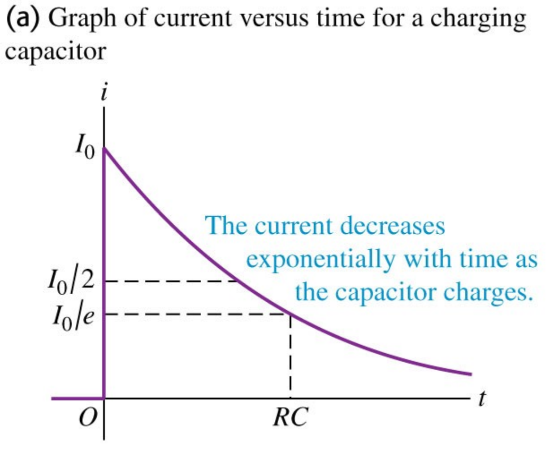
Charging a capacitor

- Start with uncharged capacitor! What happens??



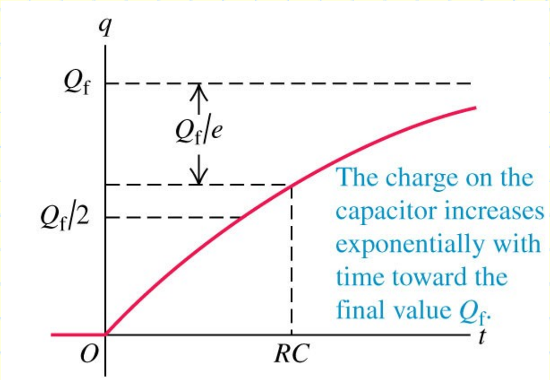






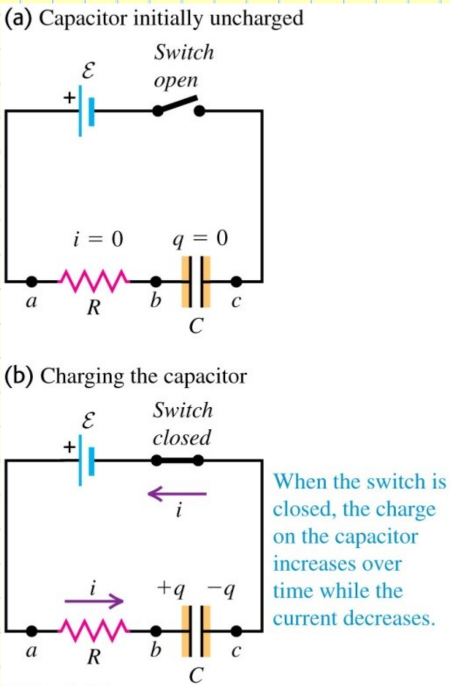
Adding Capacitors to DC circuits!

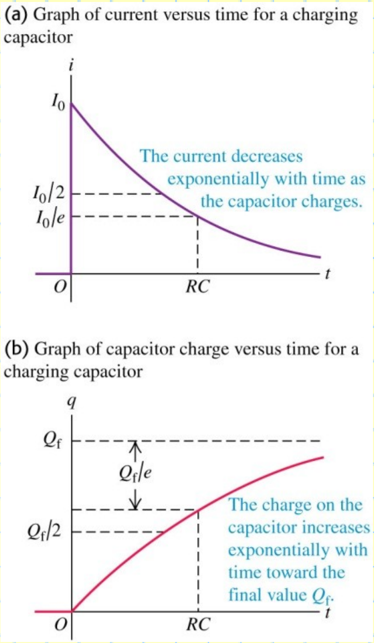
- **In charging RC** circuits the *time constant* is τ
- τ *increases* with R
  - Larger resistors decrease current
  - Less charge/time arrives at capacitor
  - It takes longer to fill up capacitor
- τ *increases* with C
  - Larger capacitors have more capacity!
  - They take longer to fill up!



Charging a capacitor

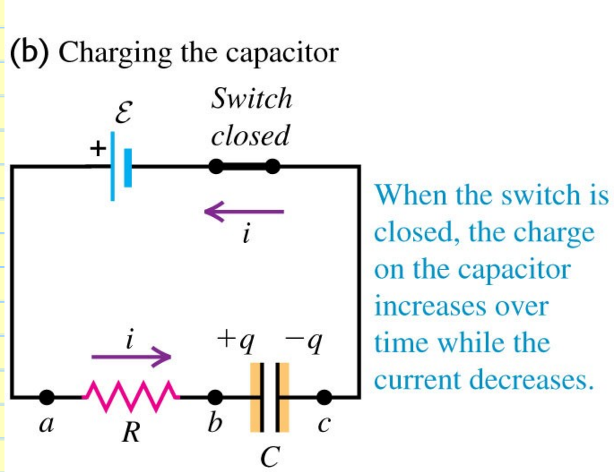
- The *time constant* is $\tau = RC$.
- In ONE time constant:
  - Current drops to 1/e of initial value (about 36%)
  - Charge on capacitor plates rises to ~64% of maximum value





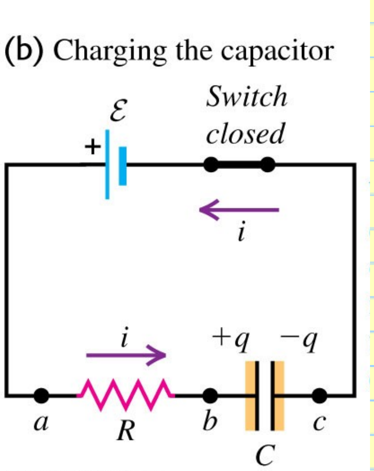
Charging a Capacitor

- $V_{ab} = iR$
- $C = q/V_{bc}$ so $V_{bc} = q/C$
- $E - iR - q/C = 0$
  - Current is a function of time
  - $i = dq/dt$
  - $E - (dq/dt)R - q(t)/C = 0$
  - A *differential equation involving charge q*



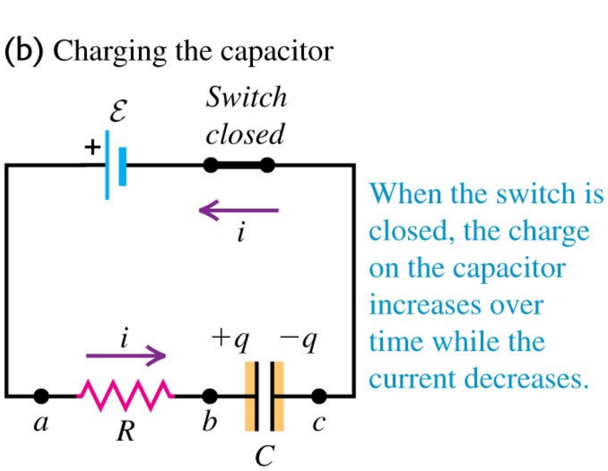
Charging a Capacitor

- $E - (dq/dt)R - q/C = 0$
- *Boundary conditions* relate q and t at key times:
  - q(t) on capacitor = 0 @ t=0
  - q= Qmax, i(t) = 0 @ t = ∞ (when capacitor is full)



Charging a Capacitor

- $E - iR - q/C = 0$
- General Solution
  - $q(t) = Q_{max}\,(1 - e^{-t/RC})$
  - Check?
    - $q(t)$ on capacitor = 0 @ t=0
    - $i(t) = dq/dt = (Q_{max}/RC)\, e^{-t/RC}$
    - $i(0)$ is maximum current, $Qmax/RC = E\ /R = I_0$
    - $i(t) = 0$ when capacitor is full, @ $t = \infty$



Charging a capacitor

- 10 MΩ resistor connected in series with 1.0 μF un-charged capacitor and a battery with Emf of 12.0 V.
  - What is the time constant?
  - What fraction of final charge is on capacitor after 46 seconds?
  - What fraction of initial current I_0 is flowing then?

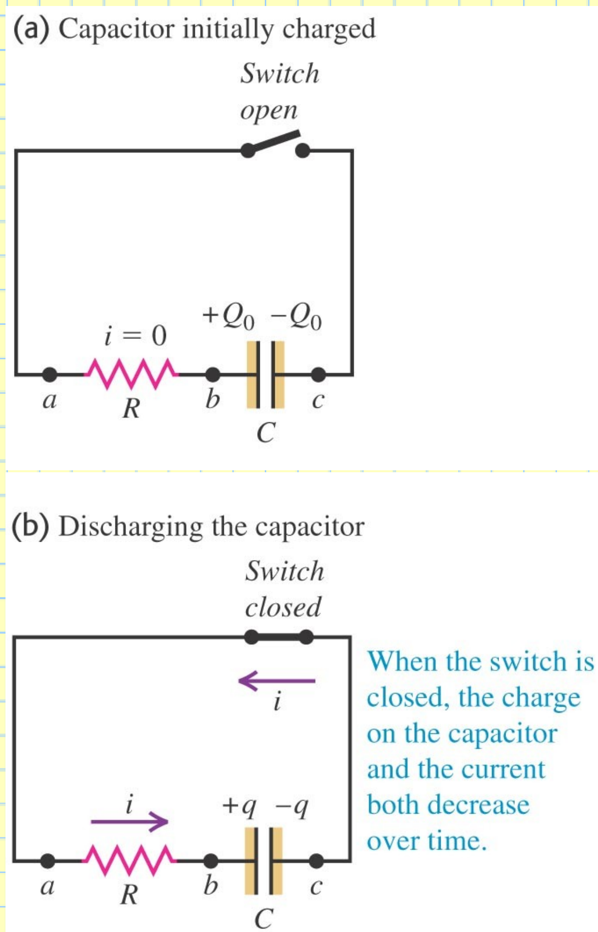
Charging a capacitor

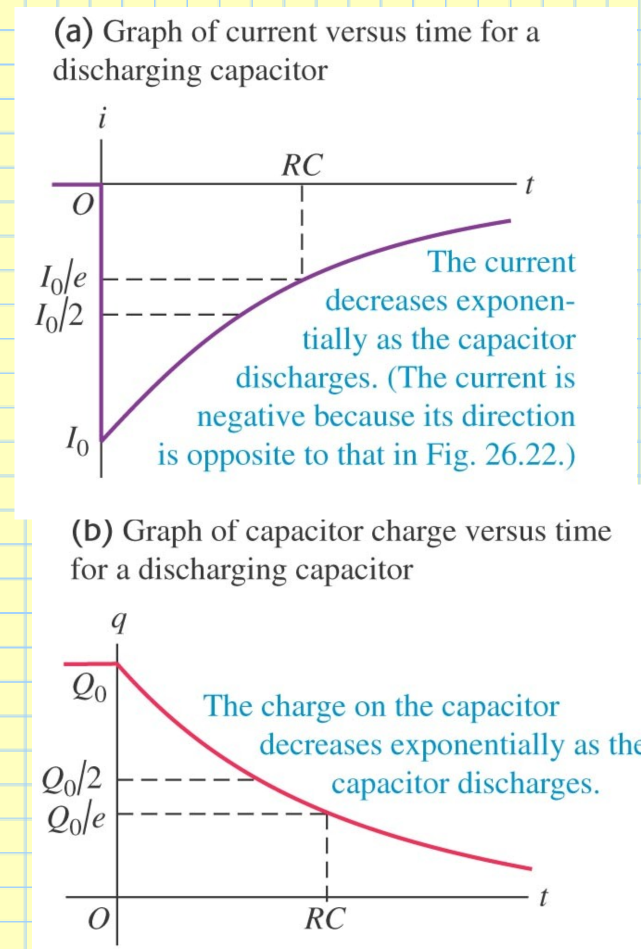
- 10 MΩ resistor connected in series with 1.0 μF un-charged capacitor and a battery with emf of 12.0 V.
  - What is time constant?
  - What fraction of final charge is on capacitor after 46 seconds?
  - What fraction of initial current I_0 is flowing then?

- $\tau = RC = 10$ seconds
- $Q(t) = Q_{max}\,(1 - e^{-t/RC})$ so $Q(_{46\ seconds})/Qmax = 99\%$
- $I(t) = I_0\, e^{-t/RC}$ so $I(_{46\ seconds})/I_0 = 1\%$

Discharging a capacitor

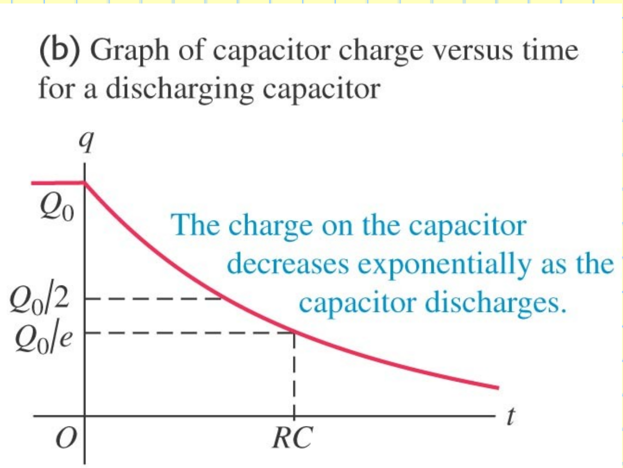
- Disconnect Battery – let Capacitor "drain"





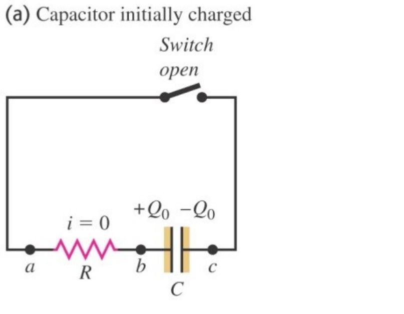
Adding Capacitors to DC circuits!

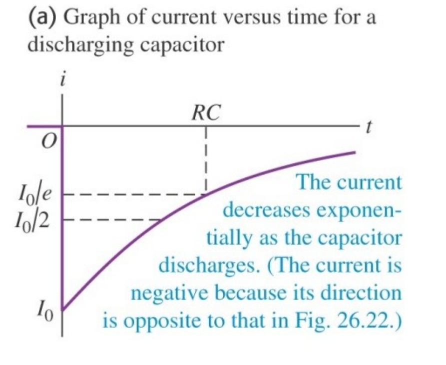
- **In discharging RC** circuits the *time constant* is still τ
- τ *increases* with R
    - Larger resistors decrease current
    - Less charge/time *leaves* capacitor
    - It takes longer to *drain* capacitor
- τ *increases* with C
    - Larger capacitors have more capacity!
    - They take longer to drain!

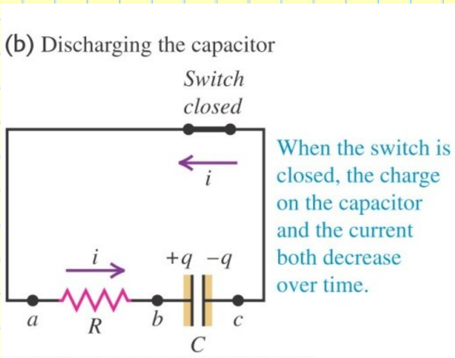


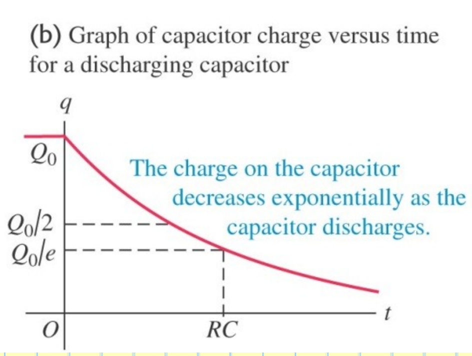
Discharging a capacitor

- Time constant is still RC!
- In one time time constant:
  - Charge on plates DROPS by 64%
  - Current through resistor DROPS by 64% (“negative”)



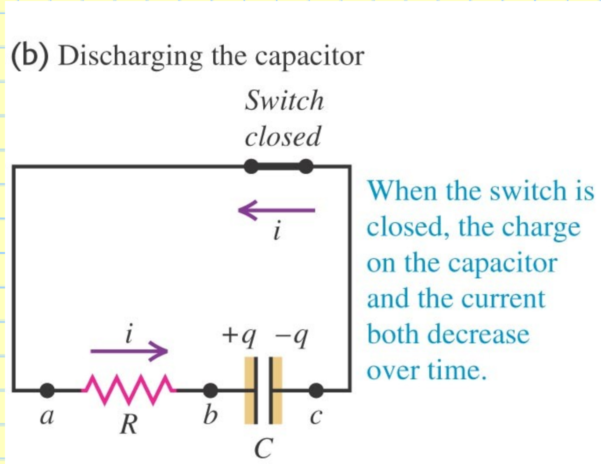






Charging a Capacitor

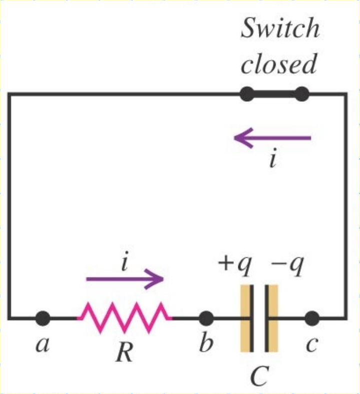
- $V_{ab} = iR$
- $V_{bc} = q/C$



- $iR + q/C = 0$
  - Another differential equation involving charge
  - $i = dq/dt$
  - $(dq/dt)R + q(t)/C = 0$

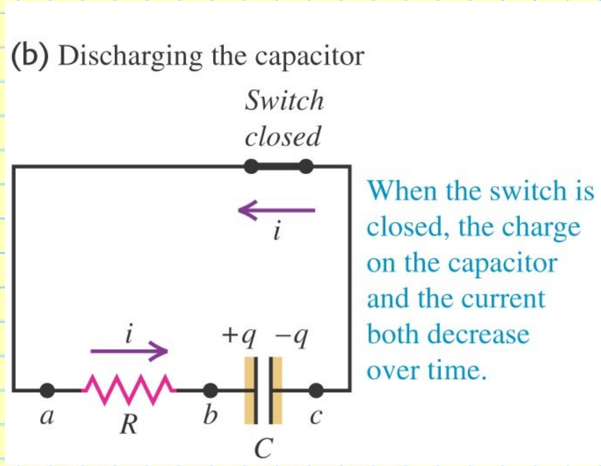
Disharging a Capacitor

- $iR + q/C = 0$
- $(dq/dt)R + q(t)/C = 0$
- Boundary conditions:
  - q(t) on capacitor = Qmax
    @ t=0
  - q= 0, i(t) = 0
    @ t = ∞
    when capacitor is *empty*



Disharging a Capacitor

- $iR + q/C = 0$
- General Solution
  - $q(t) = Q_{max}\, e^{-t/RC}$
    - $i = dq/dt = -\, I_0 e^{-t/RC}$
      (sign changes because direction changes
    - Check
      - q(t) on capacitor = Qmax @ t=0
      - q= 0, i(t) = 0 when capacitor is empty, @ t = ∞



Discharging a capacitor

- Same circuit as before; 10 MΩ resistor connected in series with 1.0 μF capacitor; battery with emf of 12.0 V is disconnected.
- Assume at $t = 0$, $Q(0) = 5.0$ μC.
- When will charge = 0.50 μC?
- What is current then?

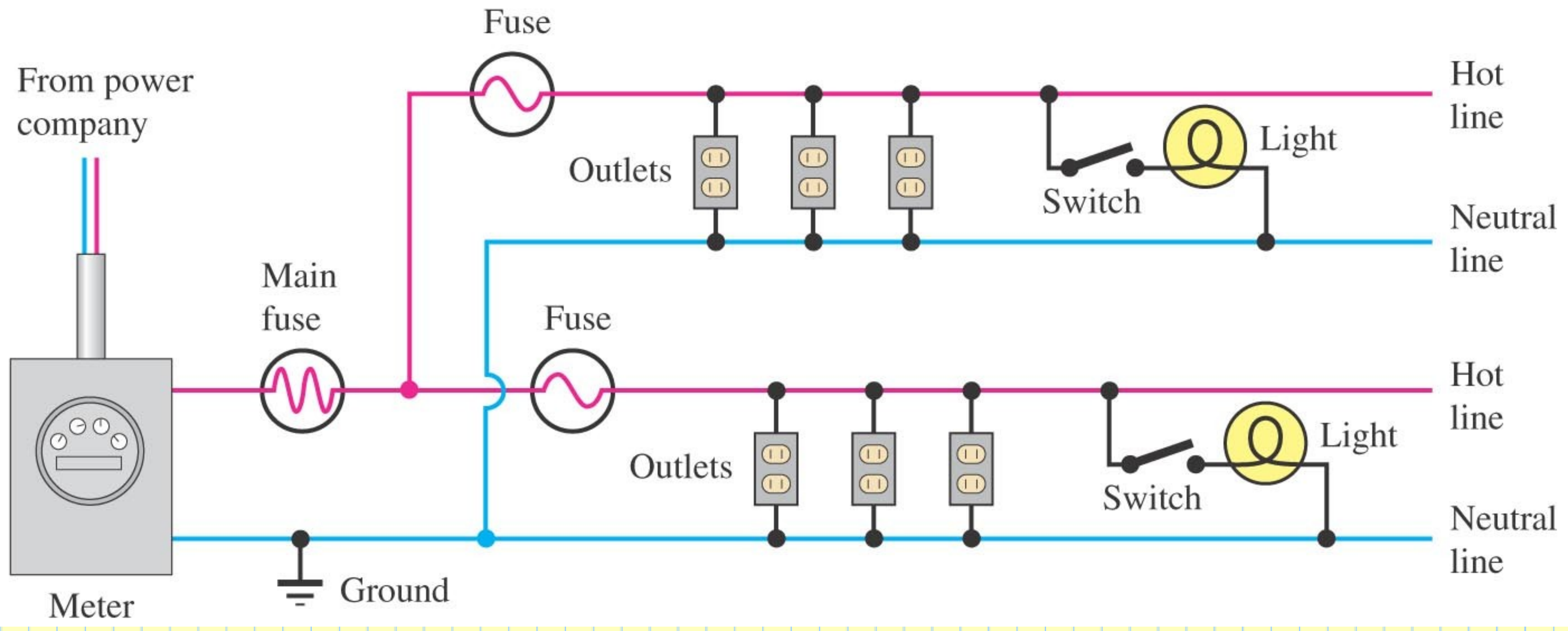
Discharging a capacitor

- Same circuit as before; 10 MΩ resistor connected in series with 1.0 μF capacitor; battery with emf of 12.0 V is disconnected.
- Assume at $t = 0$, $Q(0) = 5.0\ \mu C$.
- When will charge = $0.50\ \mu C$?
- What is current then?

- $\tau = RC = 10$ seconds (still!)
- Q max (initially at $t = 0$) = $5.0\ \mu C$.
- $Q(t) = Q_{max}\, e^{-t/RC}$ so $Q(t) = 0.5\ \mu C = 1/10^{th}$ Q max =>
  $t = RC \ln(Q/Qmax) = 23$ seconds $(2.3\ \tau)$
- $I(t) = -Q_0/RC\, (e^{-t/RC})$ so $I(2.3\ \tau) = -5.0 \times 10^{-8}$ Amps

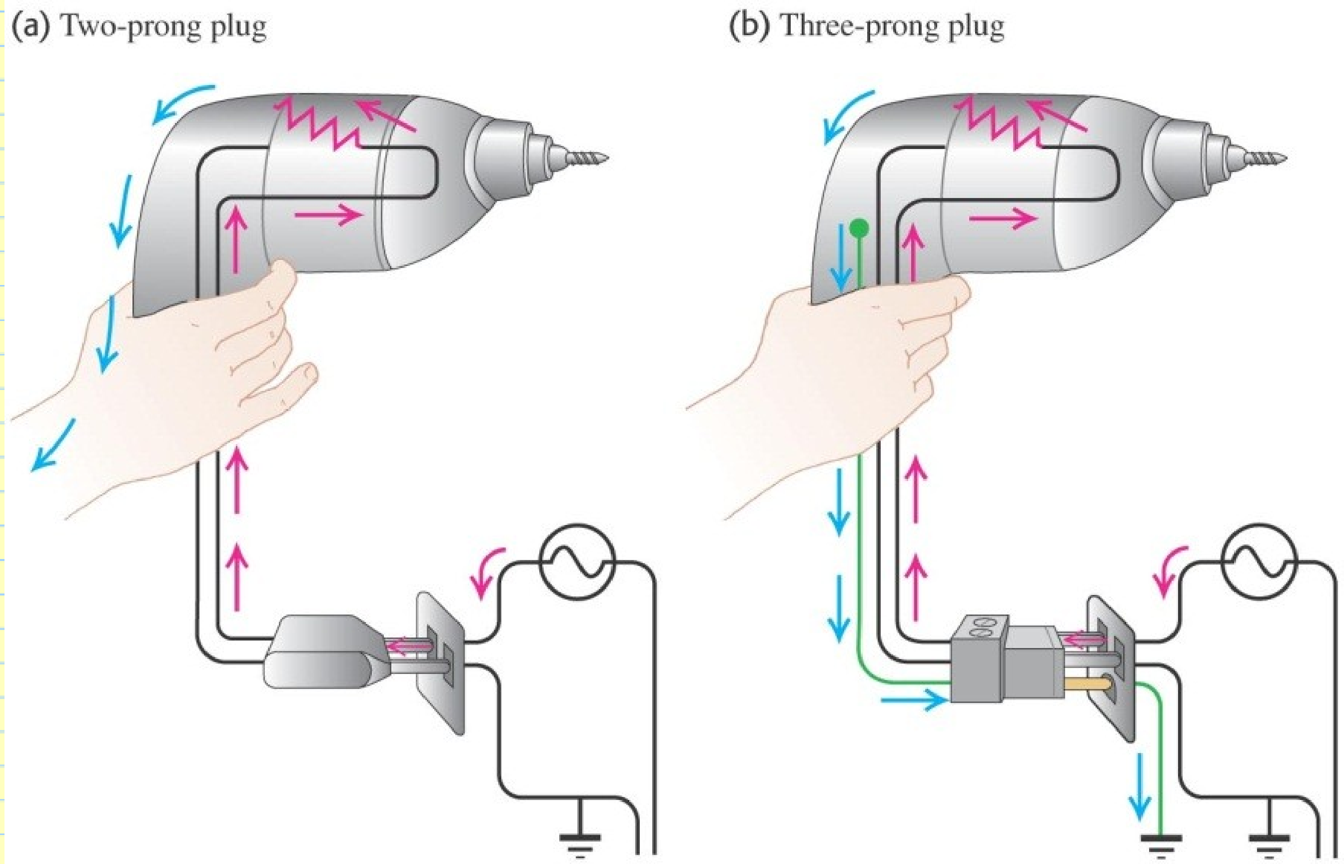
Power distribution systems

Circuits, lines, loads, and fuses…



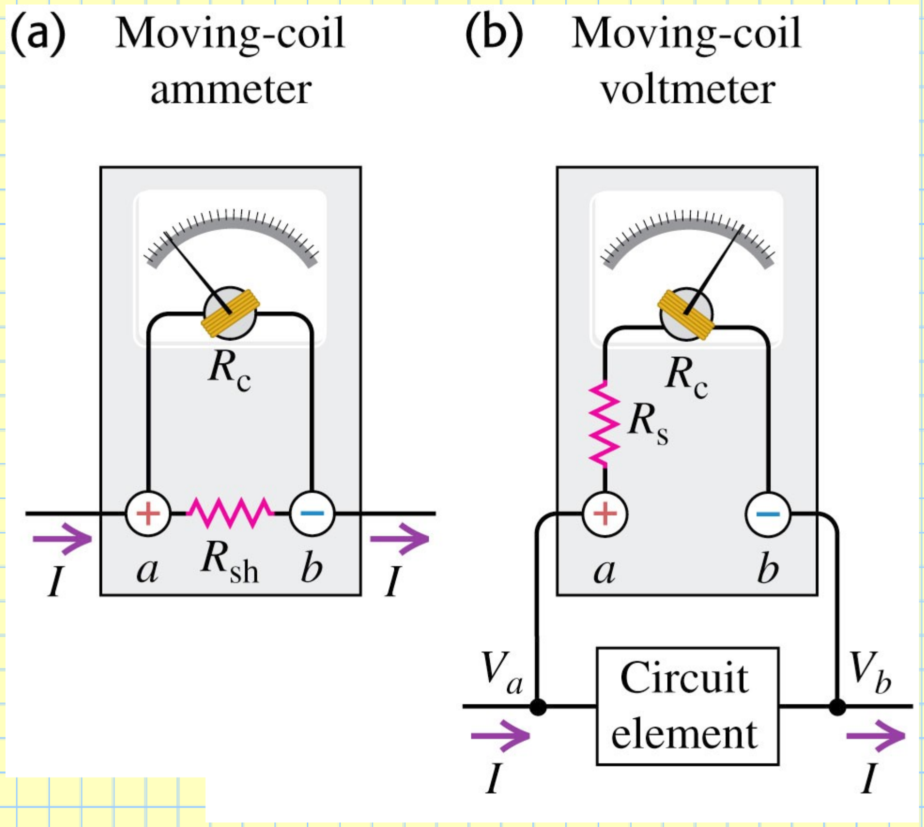
Household wiring

- Why it is safer to use a three-prong plug for electrical appliances…



Ammeters and voltmeters

- An *ammeter* measures the current passing through it.
- A *voltmeter* measures the potential difference between two points.
- Figure 26.15 at the right shows how to use a galvanometer to make an ammeter and a voltmeter.



Ammeters and voltmeters in combination

- An ammeter and a voltmeter may be used together to measure resistance and power.