Electromagnetic induction

FIZ102E: Electricity & Magnetism

Yavuz Ekşi

İTÜ Fizik Mühendisliği Bölümü

Outline

Learning Goals

- The experimental evidence that a changing magnetic field induces an emf.
- How Faraday's law relates the induced emf in a loop to the change in magnetic flux through the loop.
- How to determine the direction of an induced emf.
- How to calculate the emf induced in a conductor moving through a magnetic field.
- How a changing magnetic flux generates an electric field that is very different from that produced by an arrangement of charges.
- The remarkable electric and magnetic properties of superconductors.

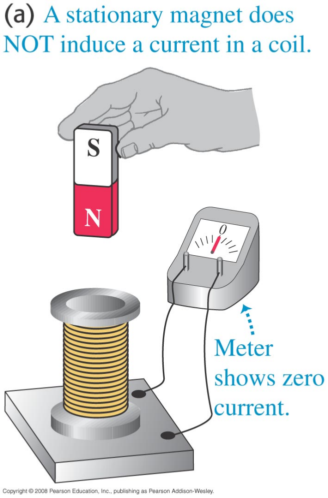
When a credit card is "swiped" through a card reader, the information coded in a magnetic pattern on the back of the card is transmitted to the cardholder's bank. Why is it necessary to swipe the card rather than holding it motionless in the card reader's slot?

EMF that is not like that of a battery

- an electromotive force (emf) is required for a current to flow in a circuit.
- Up to now we almost always took the source of emf to be a battery.
- But for the most of electric devices that are used in industry and in the home (including any device that you plug into a wall socket), the source of emf is not a battery but an electric generating station.
- Electrical energy-conversion devices such as motors, generators, and transformers.
- A time-varying magnetic field can act as a source of electric field.
- A time-varying electric field can act as a source of magnetic field.

Induction experiments

Induction experimets: stationary magnet



- A coil of wire is connected to a galvanometer. When the nearby magnet is stationary, the meter shows no current.
- Not surprising; no source of emf in the circuit.

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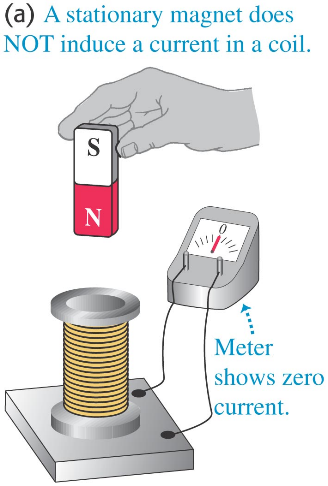

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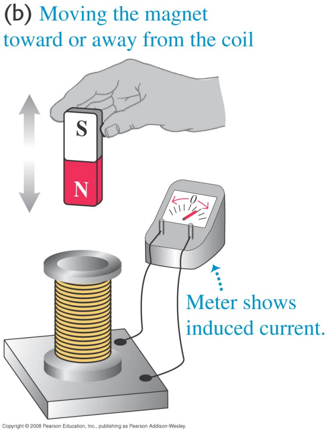
Induction experimets: moving magnet



- But when we move the magnet either toward or away from the coil, the meter shows current in the circuit, but only while the magnet is moving.
- If we keep the magnet stationary and move the coil, we again detect a current during the motion.
- We call this an *induced current*, and the corresponding emf required to cause this current is called an *induced emf*.

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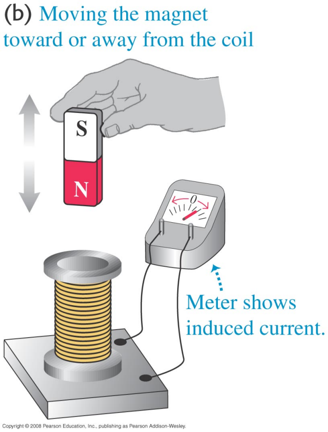

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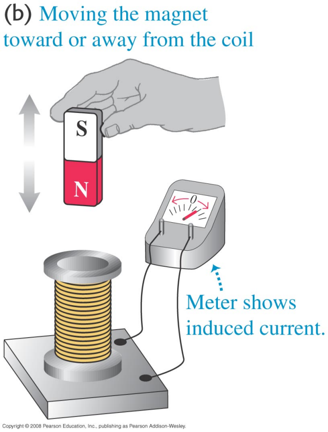

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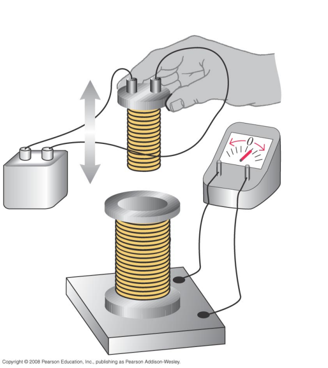
Induction experimets: moving current carrying coil

(c) Moving a second, current-carrying coil toward or away from the coil



- We replace the magnet with a second coil connected to a battery.
- When the second coil is stationary, there is no current in the first coil.
- However, when we move the second coil toward or away from the first or move the first toward or away from the second, there is current in the first coil, but again only while one coil is moving relative to the other.

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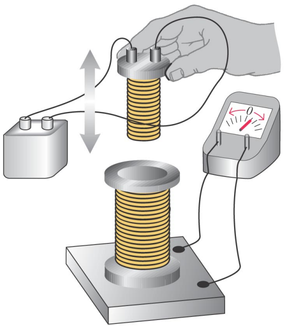

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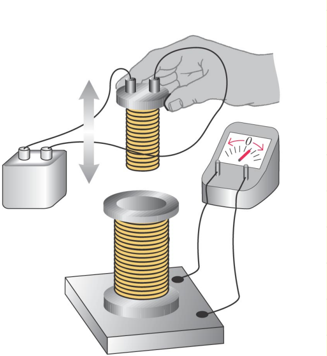

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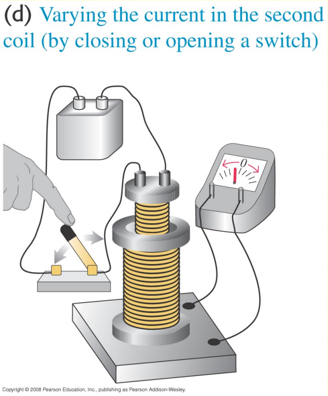
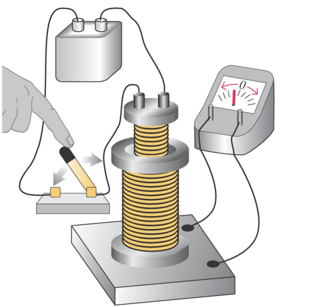
Induction experimets: stationary coil with varying current



- Finally, we keep both coils stationary and vary the current in the second coil, either by opening and closing the switch.
- We find that as we open or close the switch, there is a momentary current pulse in the first circuit.

Induction experimets: stationary coil with varying current

(d) Varying the current in the second coil (by closing or opening a switch)



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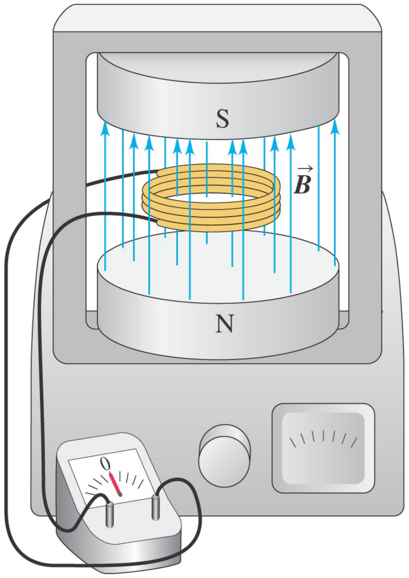
Video

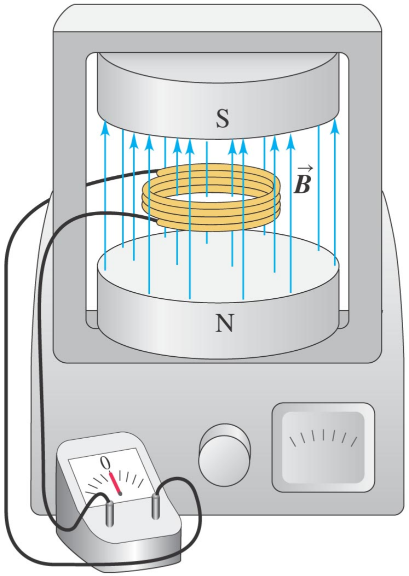
https://www.youtube.com/watch?v=vwIdZjjd8fo&t=46s

A more detailed series of experiments



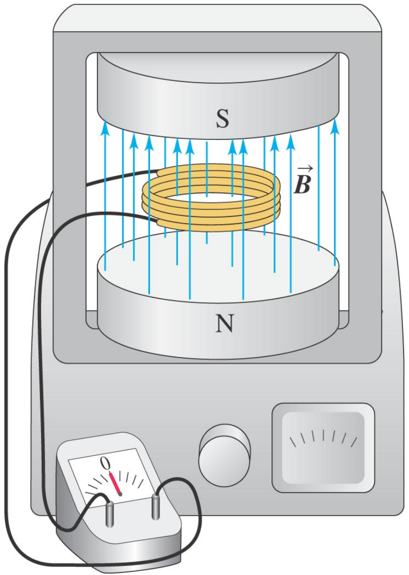
We connect a coil of wire to a galvanometer and then place the coil between the poles of an electromagnet whose magnetic field we can vary.

A more detailed series of experiments



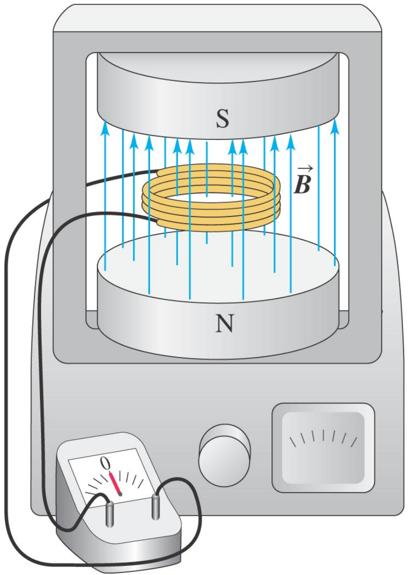
When there is no current in the electromagnet, so that $\vec{\mathbf{B}} = 0$ the galvanometer shows no current.

A more detailed series of experiments



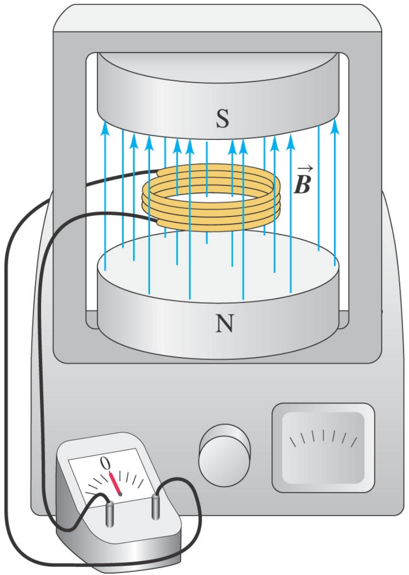
When the electromagnet is turned on, there is a momentary current through the meter as $\vec{\mathbf{B}}$ increases.

A more detailed series of experiments



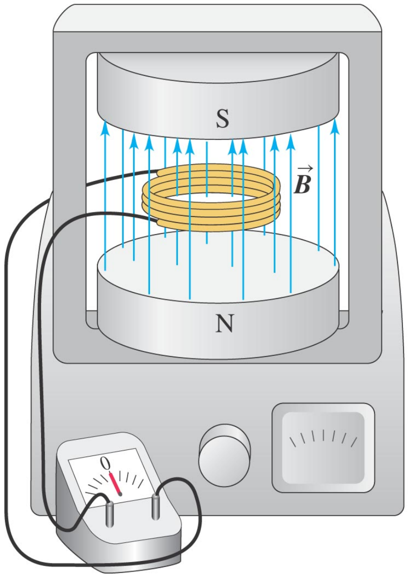
When $\vec{\mathbf{B}}$ levels off at a steady value, the current drops to zero, no matter how large $\vec{\mathbf{B}}$ is.

A more detailed series of experiments



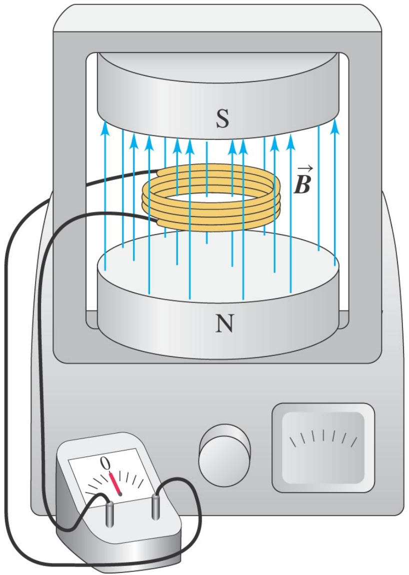
With the coil in a horizontal plane, we squeeze it so as to decrease the cross-sectional area of the coil. The meter detects current only during the deformation, not before or after. When we increase the area to return the coil to its original shape, there is current in the opposite direction, but only while the area of the coil is changing.

A more detailed series of experiments



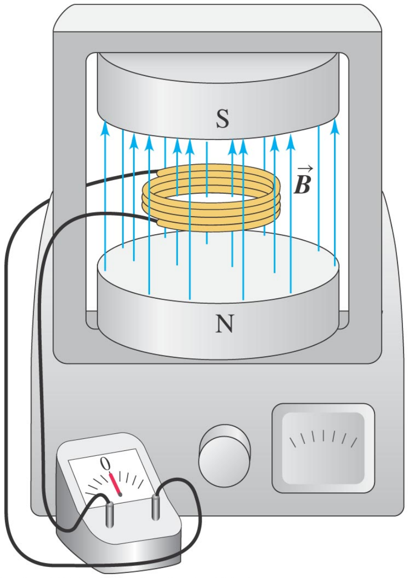
- If we rotate the coil a few degrees about a horizontal axis, the meter detects current during the rotation, in the same direction as when we decreased the area.
- When we rotate the coil back, there is a current in the opposite direction during this rotation.

A more detailed series of experiments



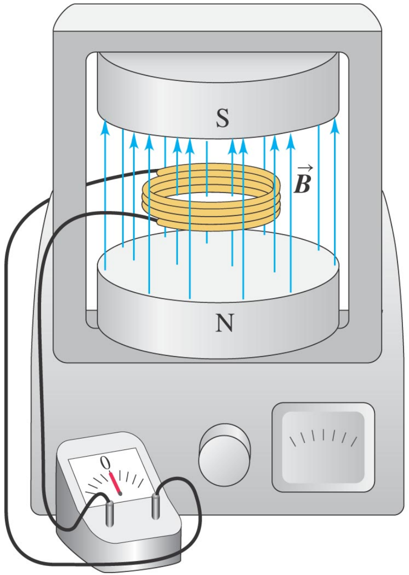
If we jerk the coil out of the magnetic field, there is a current during the motion, in the same direction as when we decreased the area.

A more detailed series of experiments



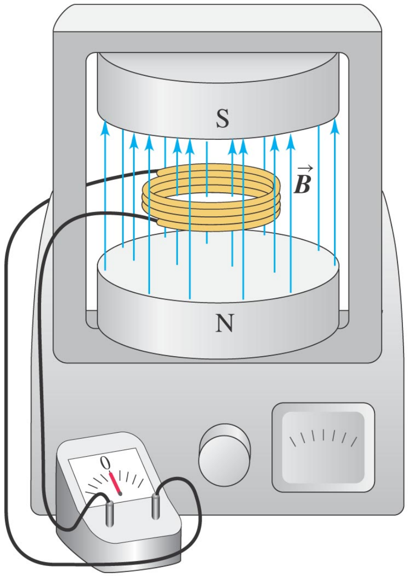
- If we decrease the number of turns in the coil by unwinding one or more turns, there is a current during the unwinding, in the same direction as when we decreased the area.
- If we wind more turns onto the coil, there is a current in the opposite direction during the winding.

A more detailed series of experiments



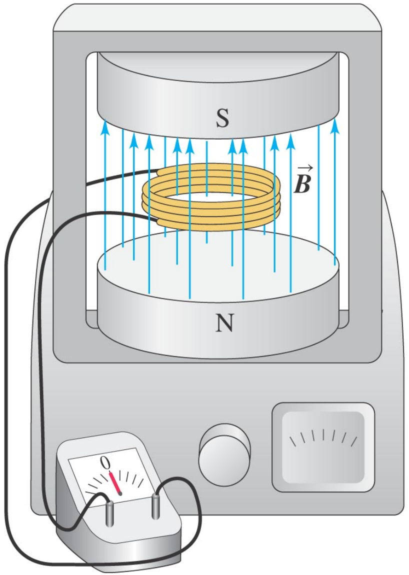
When the magnet is turned off, there is a momentary current in the direction opposite to the current when it was turned on.

A more detailed series of experiments



The faster we carry out any of these changes, the greater the current.

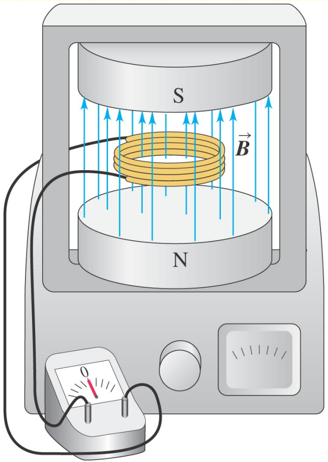
A more detailed series of experiments



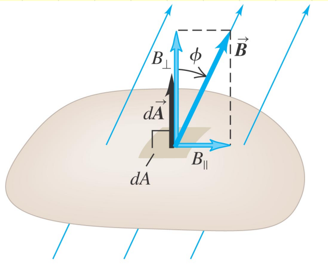
- If all these experiments are repeated with a coil that has the same shape but different material and different resistance, the current in each case is inversely proportional to the total circuit resistance.
- This shows that the induced emfs that are causing the current do not depend on the material of the coil but only on its shape and the magnetic field.

Faraday’s law

Faraday's idea: changing magnetic flux $\Phi_B(t)$



The common element in all these experiments is changing magnetic flux Φ_B through the coil connected to the galvanometer.



Magnetic flux through element of area $d\vec{A}$:
$d\Phi_B = \vec{B} \cdot d\vec{A} = B_\perp dA = B\, dA \cos\phi$

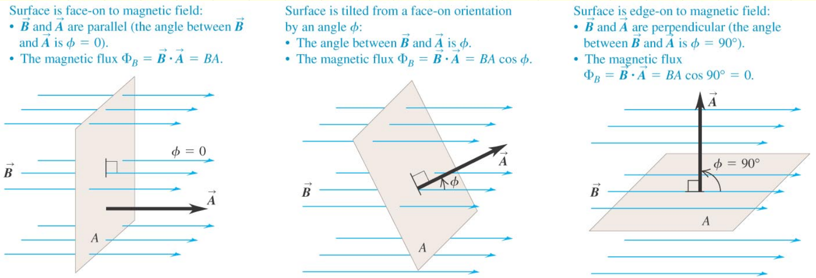
The magnetic flux is defined as

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

The unit of magnetic flux is the weber (Wb)

$$1\mathrm{Tm}^2 = 1\,\mathrm{Wb}$$

Magnetic Flux



Changing magnetic flux

The magnetic flux

$$\Phi_B = \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = \int B \, \mathrm{d}A \cos\phi$$

can change in 3 ways:

- The field itself changes.
- The area changes.
- The angle, ϕ, between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ changes.

What does the changing magnetic flux do?

Maxwell's Equations

- Gauss's law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0}$$

- Faraday's law

$$\boxed{\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}}$$

- Gauss's law for magnetism

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

- Generalized Ampere's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_{\mathrm{enc}} + \mu_0 \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

Faraday's law of induction

Faraday's law

$$\boxed{\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}}$$

states that the induced emf,

$$\mathcal{E} \equiv \oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}}$$

in a closed loop equals the negative of the time rate of change of magnetic flux

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

through the loop. Hence

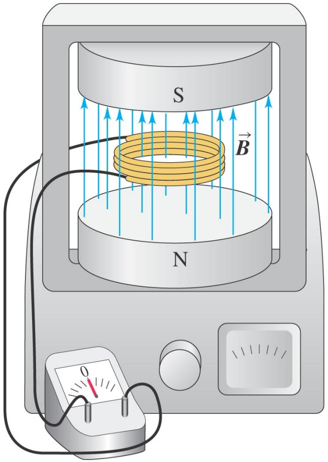
$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

Faraday's law of induction

If we have a coil with N identical turns, and if the flux varies at the same rate through each turn, the total rate of change through all the turns is N times as large as for a single turn. If Φ_B is the flux through each turn, the total emf in a coil with N turns is

$$\mathcal{E} = -N \frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

EMF and current induced in a loop



If the magnetic flux through a circuit changes, an emf and a current are induced in the circuit according to Faraday's law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

Changing magnetic flux

The magnetic flux

$$\Phi_B = \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = \int B \, \mathrm{d}A \cos\phi$$

can change in 3 ways:

- The magnitude of the field, B, changes.
- The area, A, changes.
- The angle, ϕ, between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ changes.

Changing magnetic flux

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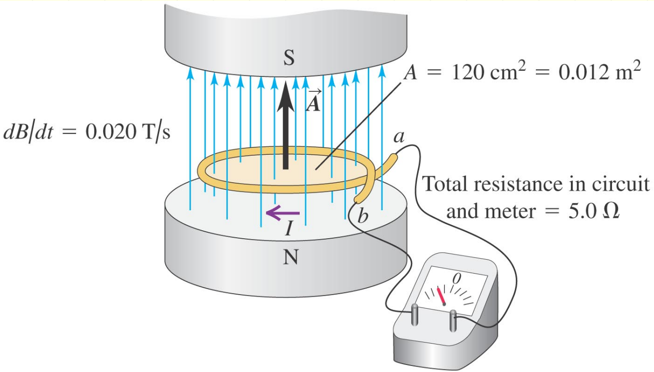
What does the changing magnetic flux do?

Ex: Changing flux by changing the magnitude of the field

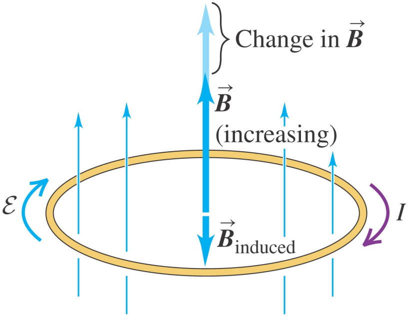


- $\mathrm{d}\Phi_B/\mathrm{d}t = \mathrm{d}(BA)/\mathrm{d}t = (\mathrm{d}B/\mathrm{d}t)A = 0.020\,\mathrm{T/s} \times 0.012\,\mathrm{m}^2$
  $= 0.24\,\mathrm{mV}$
- $I = \mathcal{E}/R = 0.24\,\mathrm{mV}/5.0\,\Omega = 0.048\,\mathrm{mA}$

Direction of the induced emf

- Define a $+$ direction for $\vec{\mathbf{A}}$.
- From the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, determine the sign of Φ_B and its rate of change $\mathrm{d}\Phi_B/\mathrm{d}t$.
- Determine the sign of the $\mathcal{E}$ or I. If the Φ_B is increasing, so $\mathrm{d}\Phi_B/\mathrm{d}t > 0$, then $\mathcal{E}$ or I is $-$; if the flux is decreasing, $\mathrm{d}\Phi_B/\mathrm{d}t < 0$ and $\mathcal{E}$ or I is $+$.
- Finally, use your right hand to determine the direction of $\mathcal{E}$ or I. Curl the fingers of your right hand around $\vec{\mathbf{A}}$, with your right thumb in the direction of $\vec{\mathbf{A}}$. If $\mathcal{E}$ or I in the circuit is $+$, it is in the same direction as your curled fingers; if $\mathcal{E}$ or I is $-$, it is in the opposite direction.

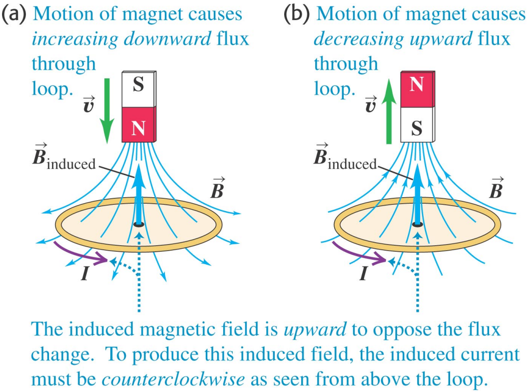
Direction of induced current from EMF creates B field to KEEP original flux constant!



Direction of the induced emf

Direction of induced current from EMF creates B field to KEEP original flux constant!

The magnetic flux is becoming (a) more positive $d\Phi_B/dt > 0$, (b) less positive $d\Phi_B/dt < 0$. The magnetic flux is becoming (c) more negative $d\Phi_B/dt < 0$, and (d) less negative $d\Phi_B/dt > 0$.



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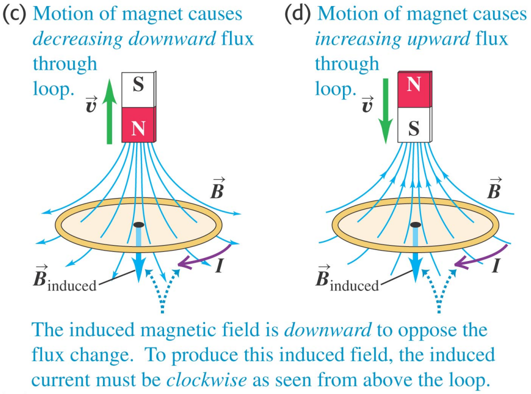
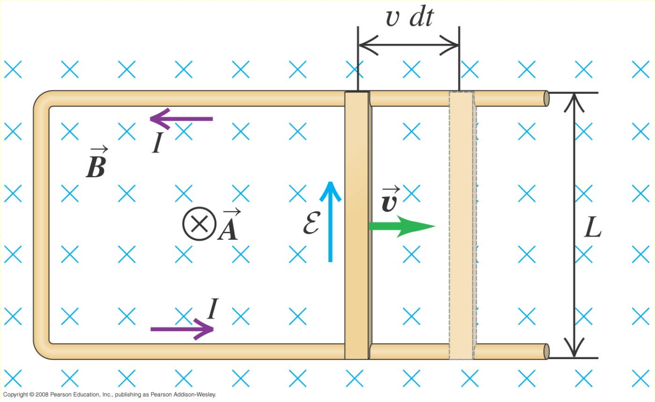
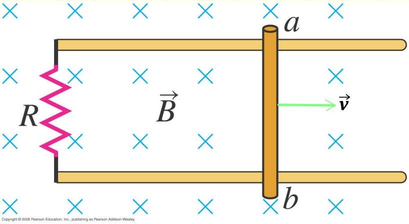


Ex: Changing flux by changing area



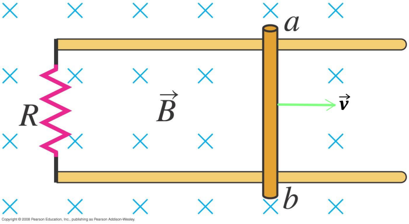
- $A(t) = Lx(t)$
- $\mathrm{d}\Phi_B/\mathrm{d}t = \mathrm{d}(BA)/\mathrm{d}t = BL\,\mathrm{d}x/\mathrm{d}t = BLv$
- $\mathcal{E} = -BLv$



Question

A 0.50 m-long metal bar is pulled to the right at a steady 8.0 m/s perpendicular to a uniform, 0.25 T magnetic field. The bar rides on parallel metal rails connected through a 5.0 Ohm, resistor so the apparatus makes a complete circuit. Ignore the resistance of the bar and the rails. Calculate the magnitude & direction of the emf induced in the circuit, and the current.

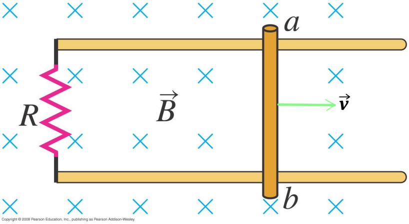
Ex: Motional emf



Answer

- Givens are $L = 0.5\,\mathrm{m}$, $v = 8.0\,\mathrm{m/s}$, $B = 0.25\,\mathrm{T}$, $R = 5.0\,\mathrm{Ohm}$.
- Thus $|\mathcal{E}| = BLv = 0.25\,\mathrm{T} \times 0.50\,\mathrm{m} \times 8.0\,\mathrm{m/s} = 1.0\,\mathrm{V}$.
- $I = |\mathcal{E}|/R = 1.0\,\mathrm{V}/5\,\mathrm{Ohm} = 0.20\,\mathrm{A}$.
- The flux into the page is increasing. The induced current be in such a direction to decrease it:

Ex: Motional emf



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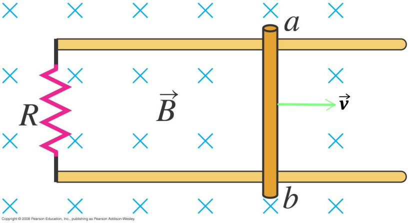

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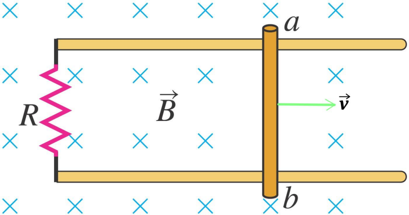
- The flux into the page is increasing. The induced current be in such a direction to decrease it:

Ex: Motional emf



Answer

- Givens are $L = 0.5\,\text{m}$, $v = 8.0\,\text{m/s}$, $B = 0.25\,\text{T}$, $R = 5.0\,\text{Ohm}$.
- Thus $|\mathcal{E}| = BLv = 0.25\,\text{T} \times 0.50\,\text{m} \times 8.0\,\text{m/s} = 1.0\,\text{V}$.
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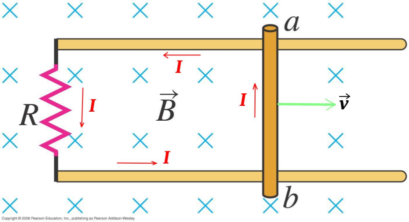

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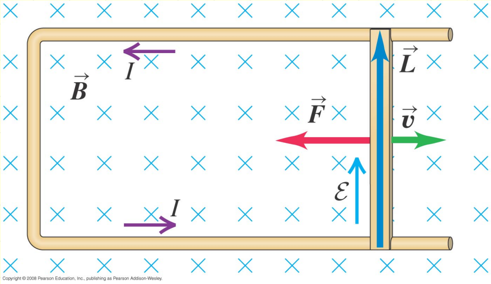
Ex: energy in the slidewire generator



- EMF: $\mathcal{E} = -BLv$, the current $I = |\mathcal{E}|/R = BLv/R$
- $\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$, $F = ILB = (BL)^2 v/R$
- This force does work at the rate $P_{\text{applied}} = Fv = (BLv)^2/R$.
- $P_{\text{dissipated}} = I^2 R = (BLv/R)^2 R = (BLv)^2/R$.
- The rate at which work is done is exactly equal to the rate at which energy is dissipated in the resistance!

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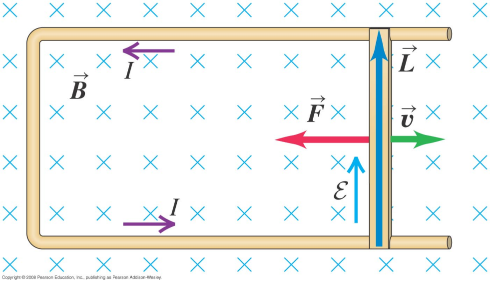

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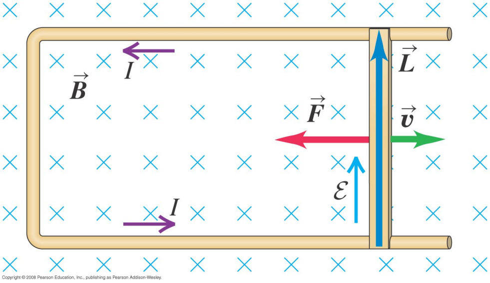

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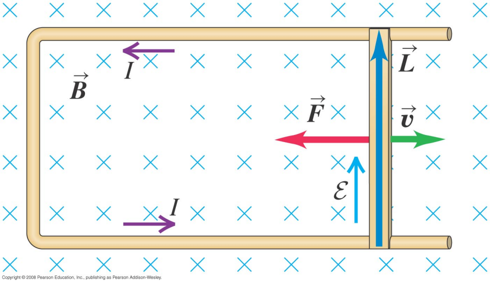

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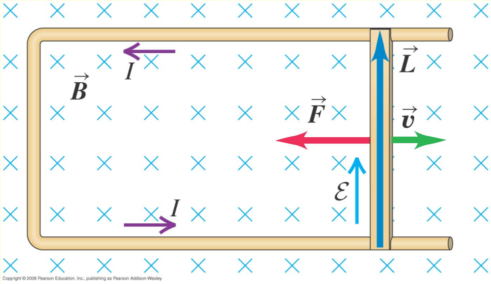

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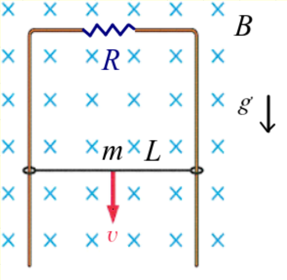
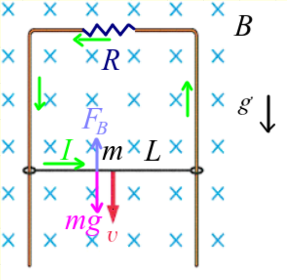


Question

A metal bar of length L and mass m is released through a U-shaped wire in a magnetic field B in a gravitational field g.

(a) What is the terminal speed, v_T of the bar?

(b) What is the velocity of the bar at any time.



Solution (a)

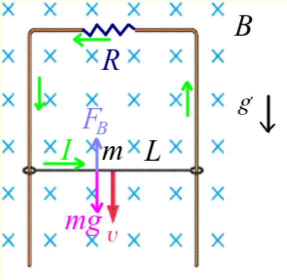
- At the terminal speed the gravitational force mg is balanced by the magnetic force $F_B = ILB$.
- Here $I = \mathcal{E}/R$ and $\mathcal{E} = BLv$. Thus

$$I = \frac{BLv}{R}, \qquad F_B = \frac{(BL)^2}{R} v$$

- Thus

$$\frac{(BL)^2}{R} v_T = mg \Rightarrow \boxed{v_T = \frac{mgR}{(BL)^2}}$$

Ex:



Solution (b)

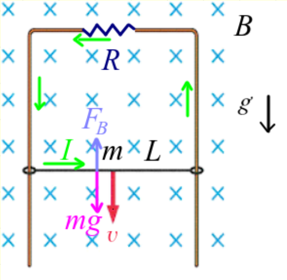
- $F_{\text{net}} = ma = m\frac{\mathrm{d}v}{\mathrm{d}t}$

$$mg - \frac{(BL)^2}{R}v = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

- Initial condition $v(t=0) = 0$.
- Define $u \equiv v/v_T$ and $\tau \equiv v_T/g$ and $t^* = t/\tau$

$$1 - u = \frac{\mathrm{d}u}{\mathrm{d}t^*}$$

Ex:



Solution (b)

- The solution of the equation is

$$u = 1 - \mathrm{e}^{-t^*}$$

- Thus

$$v = v_T \left(1 - \mathrm{e}^{-t/\tau}\right)$$

where $v_T = \frac{mgR}{(BL)^2}$ and $\tau = v_T/g$.

Ex: changing the orientation

Start with

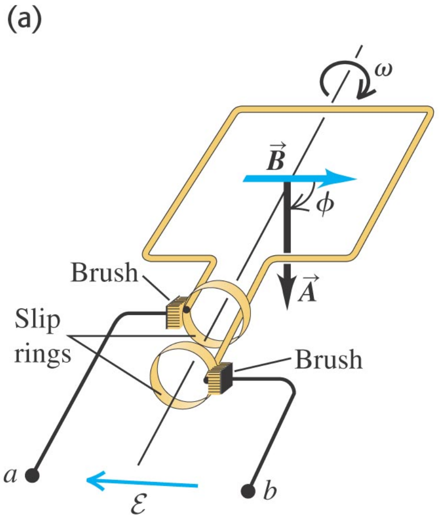
$$\mathcal{E} = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

where

$$\begin{aligned}\Phi_B(t) &= BA\cos\phi(t) \\ &= BA\cos(\omega t)\end{aligned}$$

We thus obtain

$$\mathcal{E} = NBA\omega\sin(\omega t)$$



Ex: Changing flux by changing the orientation

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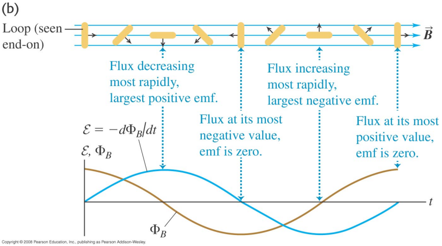

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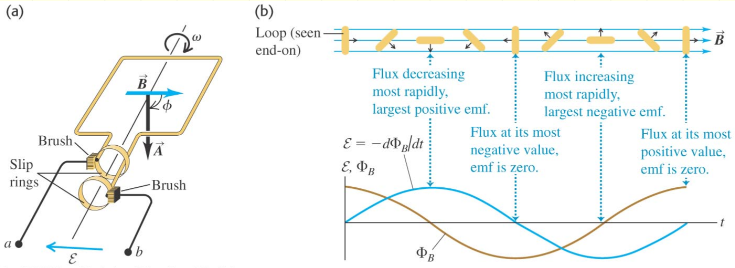
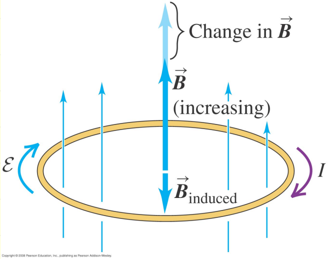


Lenz's law

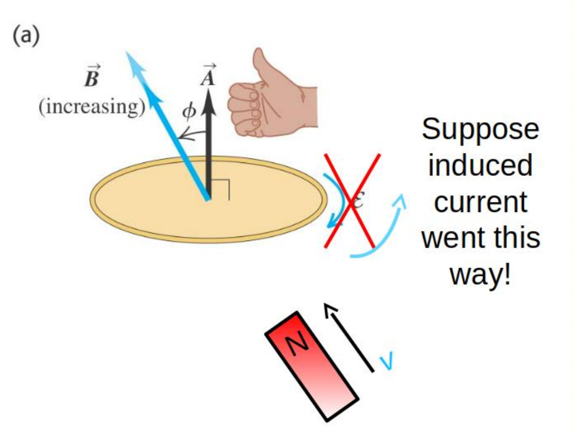
Lenz's law



- Recall: Direction of induced current from EMF creates B field to KEEP original flux constant!
- Induced current generates B field whose flux is opposing change.
- The direction of any magnetic induction effect is such as to oppose the cause of the effect.

Why Lenz's Law?

Suppose the opposite was true - increasing flux generated supportive current ...



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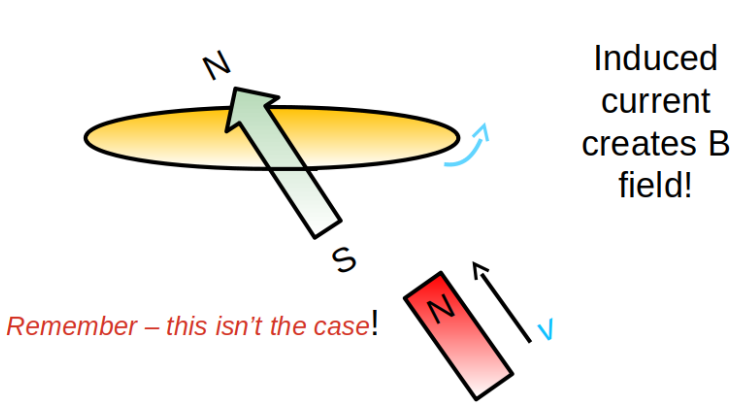

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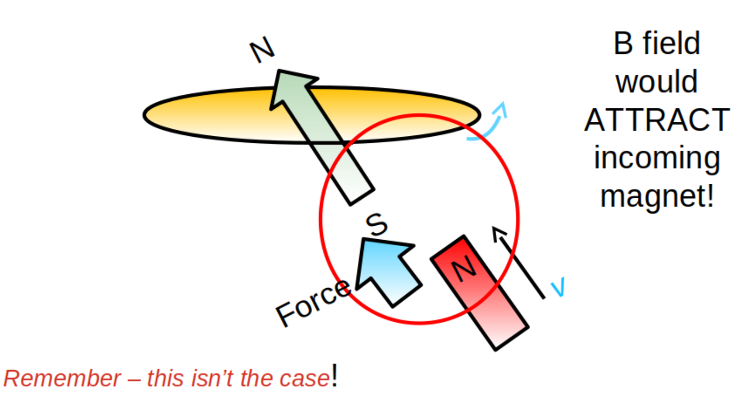

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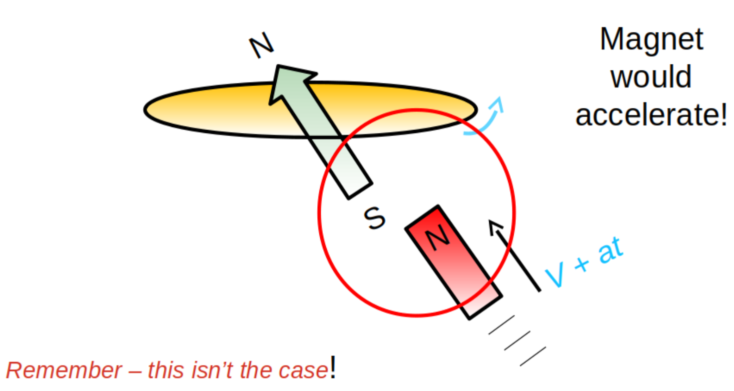

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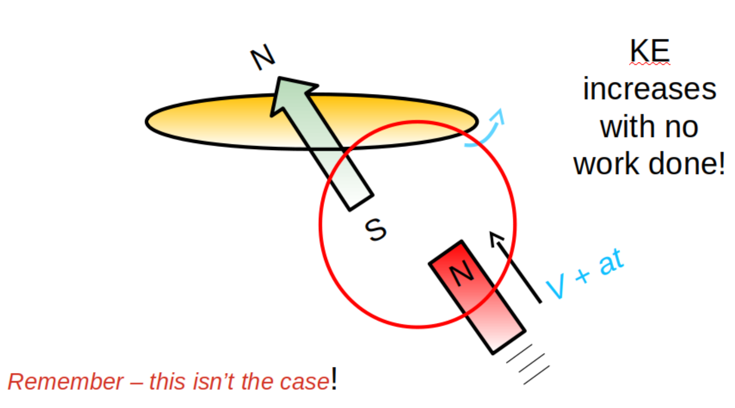
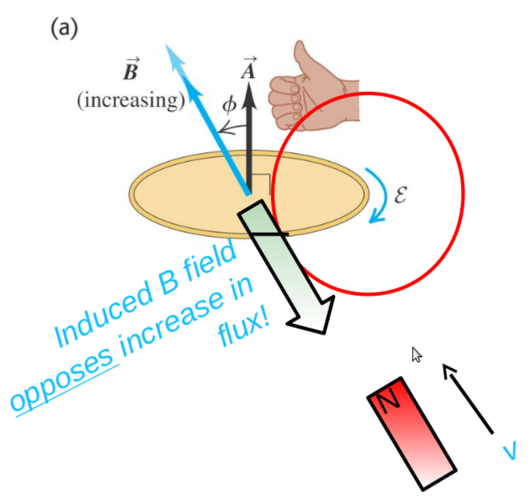


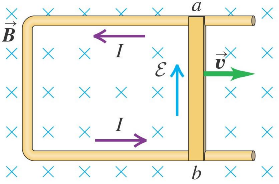
Why Lenz's Law?

Lenz's Law is like a conservation of energy relation!



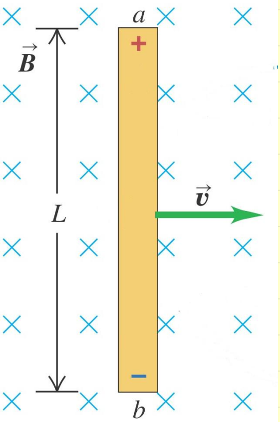
Motional EMF

Motional electromotive force



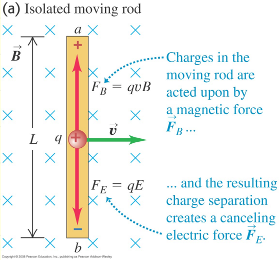
We have seen that an EMF $|\mathcal{E}| = BvL$ is induced in the loop because as the area changes, the flux of magnetic field changes:

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -BL\frac{\mathrm{d}x}{\mathrm{d}t} = -BLv$$



What if I told you that the same EMF, $|\mathcal{E}| = BvL$, is induced at the tips of the same conductor even there was no U-shaped wire?!

Motional electromotive force



- Assume for simplicity that the charge carriers in the conductor are $+$.
- A magnetic force $F_B = qvB$ acts on the q because it is moving in a B.
- As $+$ charge accumulates on one tip the other tip becomes $-$ and hence an E is induced which acts $F_E = qE$ to balance F_B.
- The equilibrium is achieved when $F_E = F_B$ i.e. $\cancel{q}E = \cancel{q}vB \Rightarrow E = vB$
- Recall $V_a - V_b = -\int_b^a \vec{\mathbf{E}} \cdot d\vec{\ell}$ which gives

$$\Delta V = EL = BLv$$

Ex: Plane moving in the B-field of the Earth

Question

A plane is moving with a speed of $v = 720\,\mathrm{km/hour}$. The wings of the plane span $30\,\mathrm{m}$. The magnetic field of Earth is $B = 0.5\,\mathrm{G}$. What is the EMF induced between the tips.

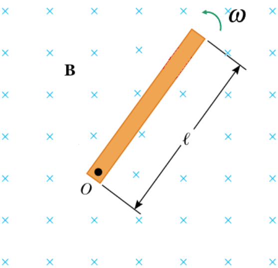
Ex: Plane moving in the B-field of the Earth

Solution

Convert to SI:

$v = 720\,\mathrm{km/hour} = 200\,\mathrm{m/s}$. The wings of the plane span $30\,\mathrm{m}$. The magnetic field of Earth is $B = 0.5\,\mathrm{G} = 0.5 \times 10^{-4}\,\mathrm{T}$.

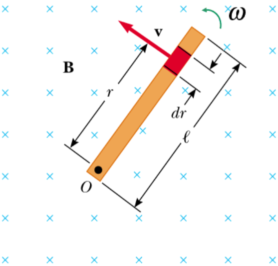
$$\begin{aligned} \mathcal{E} &= BLv \\ &= 0.5 \times 10^{-4}\,\mathrm{T} \times 30\,\mathrm{m} \times 200\,\mathrm{m/s} \\ &= 0.3\,\mathrm{V} \end{aligned}$$



Question

A rod of length $\ell = 0.5\text{m}$ is rotating, about a pivot (hinge) at one of its tips, with angular velocity $\omega = 60\,\text{rad/s}$ in a magneic field of $B = 2 \times 10^{-3}\,\text{T}$. What is the induced EMF between the tips?

Ex: Rod rotating in a B field



Solution

- Every point on the rod is moving with a different velocity $v = \omega r$ where r is the radial distance form the pivot.
- The EMF induced between r and $r + \mathrm{d}r$ is $\mathrm{d}\mathcal{E} = Bv\,\mathrm{d}r$. Hence

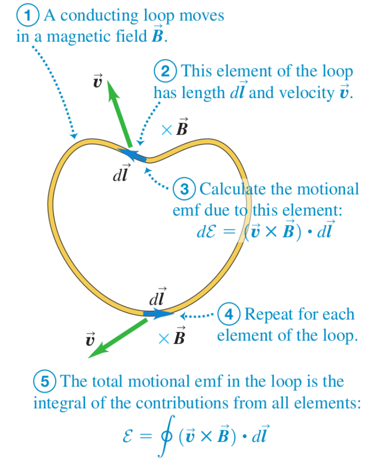
$$\begin{aligned}
\mathrm{d}\mathcal{E} &= B\omega r\,\mathrm{d}r\,, \\
\mathcal{E} &= B\omega \int_0^{\ell} r\,\mathrm{d}r \\
&= \frac{1}{2} B\omega\ell^2 \\
&= 0.5 \times 2\,10^{-3}\,\mathrm{T} \times 60\,\mathrm{rad/s} \times (0.5\,\mathrm{m})^2 \\
&= 1.5 \times 10^{-2}\,\mathrm{V}
\end{aligned}$$

Motional EMF: general form

The concept of motional emf for a conductor with *any* shape, moving in any *time-independent* B, uniform or not.

For an element $\mathrm{d}\vec{\mathbf{l}}$ of the conductor, the contribution $\mathrm{d}\mathcal{E}$ to the emf is the magnitude $\mathrm{d}l$ multiplied by the component of $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ (F_B per unit q) parallel to $\mathrm{d}\vec{\mathbf{l}}$; that is $\mathrm{d}\mathcal{E} = \vec{\mathbf{v}} \times \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}}$:

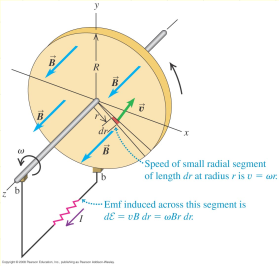
$$\boxed{\mathcal{E} = \oint \left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) \cdot \mathrm{d}\vec{\mathbf{l}}}$$



Although it looks different than $\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$ they are equivalent.

This alternative form is convenient for moving conductors.

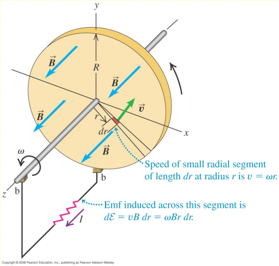
Ex: Faraday disk dynamo



Question

A conducting disk with radius R that lies in the xy-plane and rotates with constant angular velocity ω about the z-axis. The disk is in a uniform, constant $\vec{\mathbf{B}}$ field in the z-direction. Find the induced emf between the center and the rim of the disk.

Ex: Faraday disk dynamo



Solution

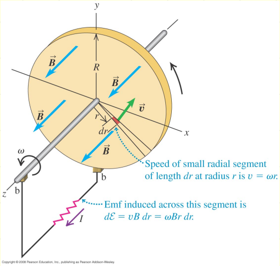
- A point at a radial distance r has velocity $v = \omega r$
- The EMF induced between r and $r + \mathrm{d}r$ is $\mathrm{d}\mathcal{E} = Bv\,\mathrm{d}r$. Hence

$$\mathrm{d}\mathcal{E} = B\omega r\,\mathrm{d}r\,,$$

$$\mathcal{E} = B\omega \int_0^R r\,\mathrm{d}r$$

$$= \frac{1}{2} B\omega R^2$$

Ex: Faraday disk dynamo



Note

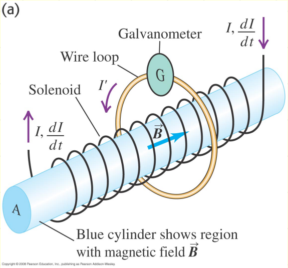
- We can use this device as a source of emf in a circuit by completing the circuit through two stationary brushes (labeled b) that contact the disk and its conducting shaft.
- Faraday disk dynamo is also called a a *homopolar generator*.

Induced electrical fields

Induced electric fields

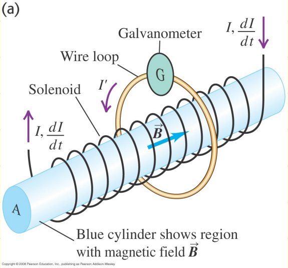
- When a conductor moves in a magnetic field, we can understand the induced emf on the basis of magnetic forces on charges in the conductor.
- But an induced emf also occurs when there is a changing flux through a stationary conductor.
- What is it that pushes the charges around the circuit in this type of situation?

Induced electric fields



- A long, thin solenoid with cross-sectional area A and n turns per unit length is encircled at its center by a circular conducting loop.
- The galvanometer G measures the current in the loop.
- A current I in the winding of the solenoid sets up a magnetic field $B = \mu_0 n I$.
- If we ignore the small field outside the solenoid then $\Phi_B = BA = \mu_0 n I A$.

Induced electric fields

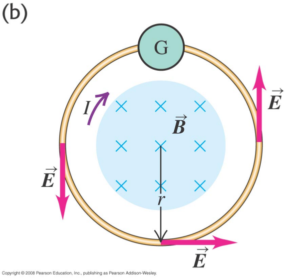


- When the solenoid current I changes with t, the magnetic flux Φ_B also changes, and according to Faraday's law the induced emf in the loop is

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = \mu_0 n A \frac{\mathrm{d}I}{\mathrm{d}t}$$

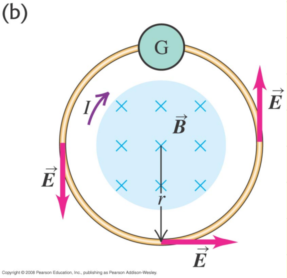
- If the total resistance of the loop is R, the induced current is $I' = \mathcal{E}/R$.
- But what force makes the charges move around the wire loop?
- It can't be a F_B because the loop isn't even in a B.

Induced electric fields



- There is an **induced electric field** in the wire *caused by the changing magnetic flux.*
- Induced electric fields are *very* different from the electric fields caused by charges.
- when a charge q goes once around the loop, $W = q\mathcal{E}$.
- The $\vec{\mathbf{E}}$ in the loop is *not conservative* whereas $\vec{\mathbf{E}}$ produced by stationary charges was conservative.

$$\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

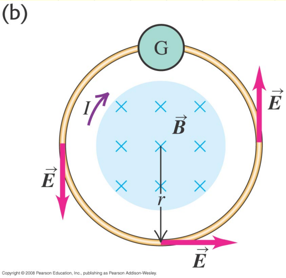


- From Faraday's law, $\mathcal{E} = -\,\mathrm{d}\Phi_B/\mathrm{d}t$ the emf is also the negative of the rate of change of magnetic flux through the loop. Thus for this case we can restate Faraday's law as

$$\boxed{\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}}$$

- Faraday's law is always true in the form $\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$; the form given in above is valid only if the path around which we integrate is stationary.

Induced electric fields

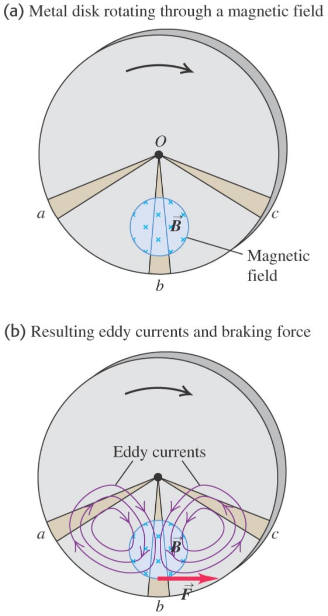


- Let's apply $\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$ to the loop with radius r.
- Because of cylindrical symmetry, $\vec{\mathbf{E}}$ has the same magnitude at every point on the circle and is tangent to it at each point: $\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = 2\pi r E$. Thus

$$E = \frac{1}{2\pi r}\left|\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}\right|$$

Eddy currents

Eddy currents



- Many pieces of electrical equipment contain masses of metal moving in B or located in changing magnetic fields.
- They induce currents that circulate throughout the volume of a material.
- The flow of these currents resemble swirling eddies in a river, we call these *eddy currents*.

Applications of eddy currents: induction owen

- Used for heating but it is not hot if you touch it.
- It can heat a metal saucepan (pot) but not a glass one. Why?
- The changing magnetic flux induces an emf in the pot. The currents driven by emf heat the pot.

Applications of eddy currents: induction owen

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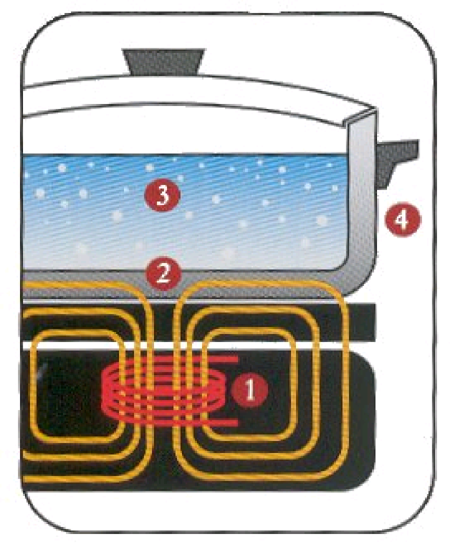

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Metal detector

When a metal is crossed the magnetic flux in the coil changes; this induces an emf. The current induced is then converted to sound signals to warn the operator.

Other applications of Faraday's law

Applications of Faraday's law induction is everywhere...

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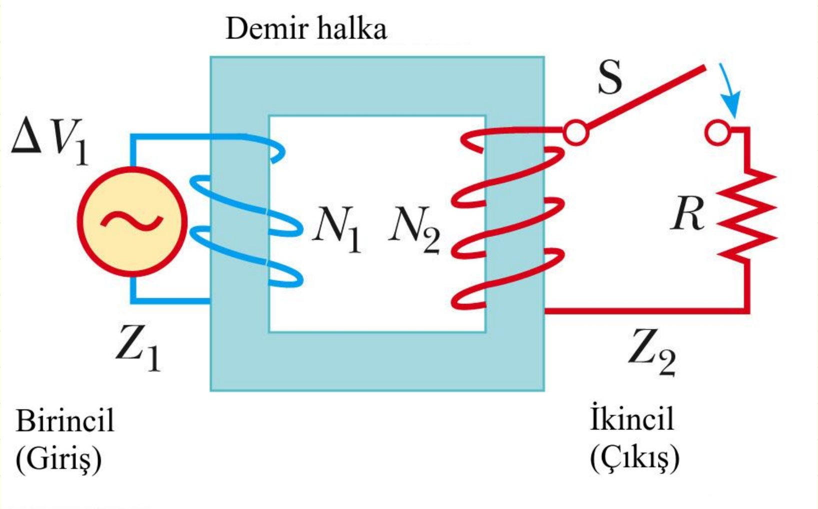
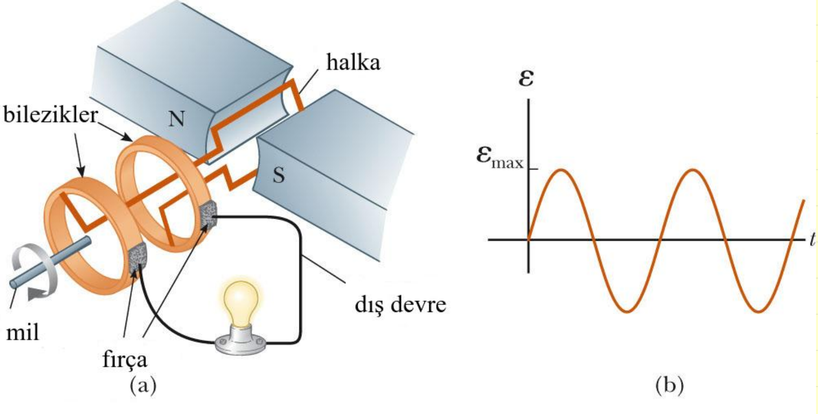
Transformator

Used for transforming the potential to higher or lower values.

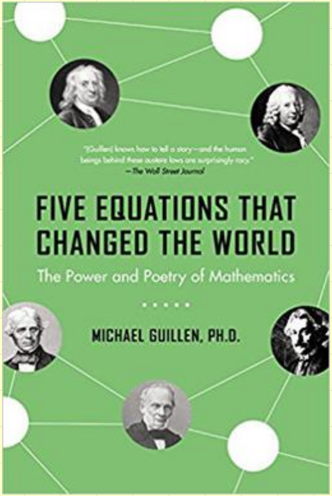


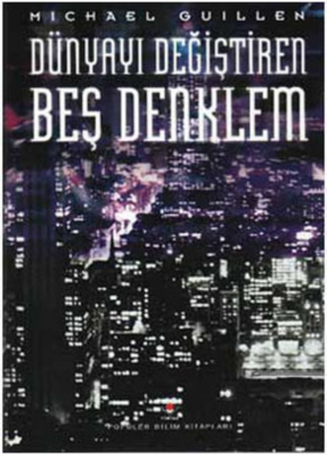
Generator

Electric generation by Faraday's law



A book to look at:

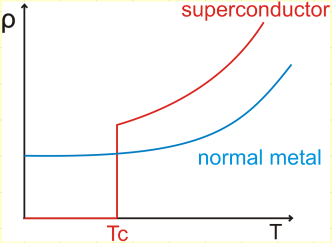




https://www.wikiwand.com/en/Five_Equations_That_Changed_the_World

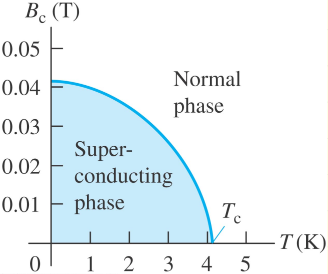
Superconductivity

Superconductivity



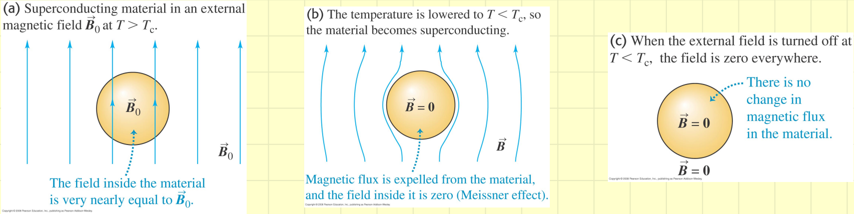
- The most familiar property of a superconductor is the sudden disappearance of all electrical resistance when the material is cooled below a temperature called the critical temperature, denoted by T_{c}.
- But superconductivity is far more than just the absence of measurable resistance as they also have extraordinary magnetic properties.

Magnetic properties



- The critical temperature changes when the material is placed in an externally produced magnetic field.
- As the external field magnitude B_0 increases, the superconducting transition occurs at lower and lower temperature.
- The minimum B needed to eliminate superconductivity at a temperature below T_c is called the critical field, denoted by B_c .

The Meissner Effect



During a superconducting transition in the presence of the field B_0, all of the magnetic flux is expelled from the bulk of the sphere, and the magnetic flux Φ_B through the coil becomes zero. This expulsion of magnetic flux is called the *Meissner effect*.

Superconductor Levitation

https://www.youtube.com/watch?v=PXHczjOg06w

https://www.youtube.com/watch?v=M1ppmUKW7Lc

Walter Lewin lecture of Faraday's law

https://www.youtube.com/watch?v=nGQbA2jwkWI