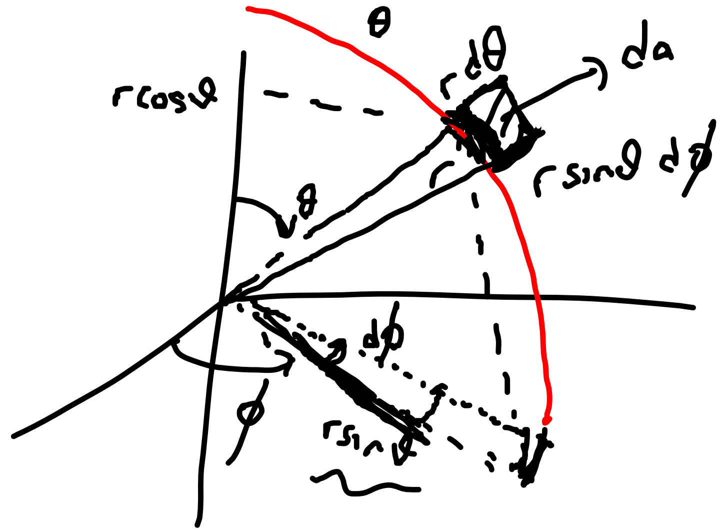
1.4 Curvilinear coordinates

Küresel koordinatler



$\hat{x}, \hat{y}, \hat{z}$

$\hat{r}, \hat{\theta}, \hat{\phi}$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$
$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \sin\phi\cos\theta\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

$$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$d\vec{l}$

$$d\vec{l} = \underset{m}{dr}\,\hat{r} + \underline{r\,d\theta}\,\hat{\theta} + \underline{r\sin\theta\,d\phi}\,\hat{\phi}$$

$$da = (r\sin\theta\,d\phi)\cdot(r\,d\theta)$$

$$dV = d\tau = \underbrace{dl_r}_{dr}\cdot\underbrace{dl_\theta}_{r\,d\theta}\cdot\underbrace{dl_\phi}_{r\sin\theta\,d\phi} = r^2\sin\theta\,dr\,d\phi\,d\theta$$

$r = -\infty, \infty$, $\theta = 0 \to \pi$, $\phi ; 0 \to 2\pi$

$x = s\cos\phi$; $y = s\sin\phi$; $z = z$

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$T \to T + \Delta T(x,y,z)$

$$\Delta T = \hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z} =$$

$$\vec{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}$$

$\frac{dT}{d\vec{l}}$ dr $r\,d\theta$ $r\sin\theta\,d\phi$

$$\frac{1}{1+x} = 1 - x + x^2$$

$x \ll 1$

$$1 = (1 - x + x^2)(1+x) = 1 + \cancel{x} - \cancel{x}$$

$\cancel{+x^2} - \cancel{x^2}$

$$\left[\sqrt{1+x}\right]^2 = \left[1 + ax +\right]^2 = 1 + \frac{x}{2} +$$

$$\underbrace{1}+\underbrace{x} = \underbrace{1}+\underbrace{2ax}+a^2x^2$$

$$a=\frac{1}{2}$$

Dirac Delta fnc:

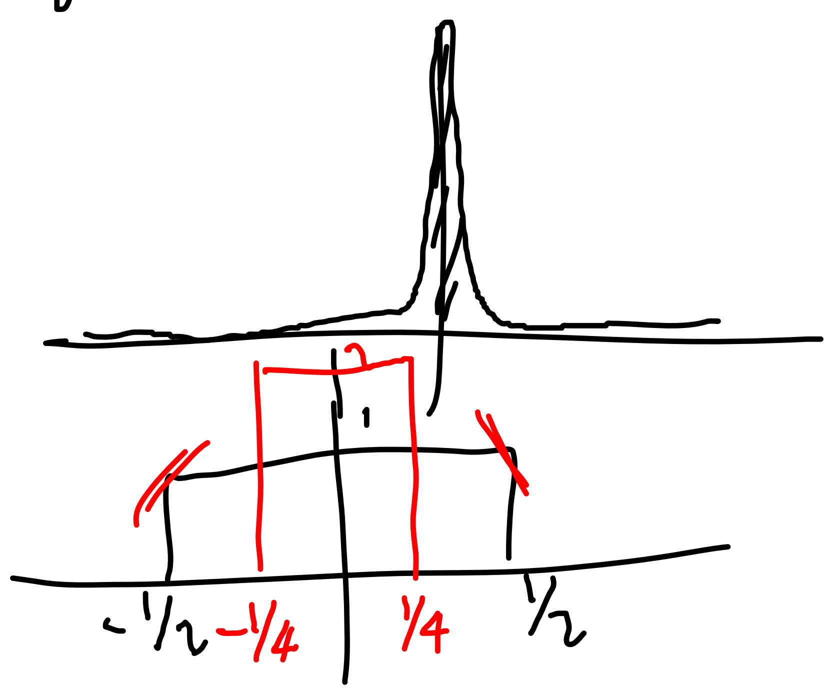
$$\underbrace{\vec{v}=\frac{\hat{r}}{r^2}} \Rightarrow \vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{1}{r^2}\right)$$

$$\int_V (\vec{\nabla}\cdot\vec{v})\,d\tau = \oint_S \underbrace{\vec{v}\cdot d\vec{a}}$$

$$\left(\frac{1}{\cancel{R^2}}\hat{r}\right)\cdot\left(\cancel{R^2}\sin\theta\,d\theta\,d\phi\,\hat{r}\right)$$

$$\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\phi = 4\pi$$

$$\delta(x) = \begin{cases} 0 & x\neq 0 \\ \infty & x=0 \end{cases}$$



$$\int_{-\infty}^{\infty}\delta(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} \delta(x-a)\, dx = 1$$

$$\int f(x)\, \delta(x-a)\, dx = \int f(a)\, \delta(x-a)\, dx \qquad \int \delta(x)\, dx = 1$$

$$\delta(x) =$$

$$\delta(kx) = \frac{1}{|k|}\, \delta(x) \qquad \delta(p) = \frac{1}{p}$$

$$\int f(x)\, \delta(kx)\, dx = \quad kx = y \quad dy = k\, dx$$

$$\int_{-\infty}^{\infty} f\left(\frac{y}{k}\right) \delta(y)\, \frac{dy}{k} = \frac{f(0)}{k}$$

$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = \int 4\pi\, \delta^3(\hat{r})\, d^3 r$$

$$4\pi$$

$$\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2} \qquad \nabla^2\left(\frac{1}{r}\right) = -4\pi\, \delta^3(r)$$