Ship Hydrodynamics

Resistance

1. A SHORT GENERAL INTRODUCTION TO SHIP HYDRODYNAMICS

Ship Hydrodynamics (**GEM341E**) consists of two main topics, namely *Ship Resistance* and *Ship Propulsion* which are closely related to each other. The ultimate aim of predicting ship propulsive power requires both the resistance and the propulsion knowledge. This requirement, in other words, implies a correct match between the installed power and the resistance of the ship hull form and its propulsive capability. Both quantities (resistance & propulsion) are complicated problems of marine hydrodynamics.

There were no serious scientific basis until the mid of the second half of 19th century in the investigation of ship resistance and in the propeller design. It was first W. Froude (1870) who modelled the total resistance as the sum of two components namely frictional and residuary components depending on his experimental observations. First mathematical/theoretical attempt to calculate the wave resistance - which is the primary part of residual resistance - is due to Michell (1898). Though W. Froude derived estimates of frictional resistance on planks by experimentation, it was L. Prandtl (1904) who first pointed out that viscous flows can be separated into a thin viscous layer (boundary layer) near solid surfaces and outer layer (region) nearly inviscid. One can find many flat-plate formulas for frictional resistance thereafter.

Early propeller theories started with Rankine (1865) who employed momentum theory in propeller design process. Wake effect in propeller design was introduced and understood at the beginning of 1900's by Luke.

The period of the first 70 years of 20th century may be regarded as golden age of experimental and analytical hydrodynamics. After 1970's computational studies for potential and viscous flows gradually have been gaining importance and turning out to be versatile design tools.

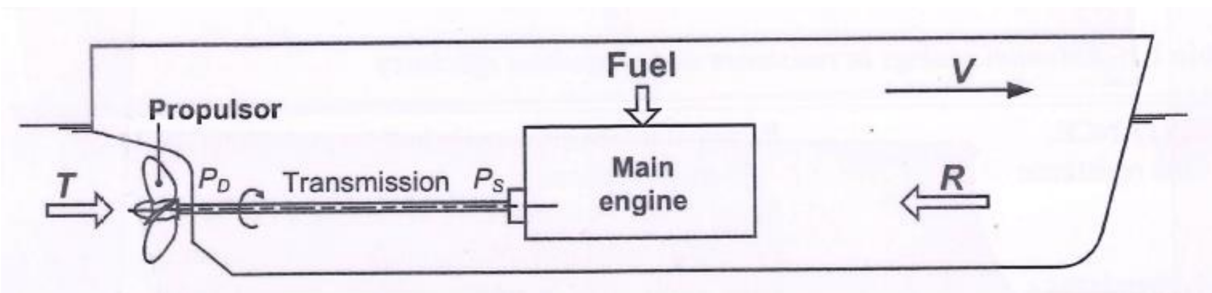


Fig. 1.1: Basic elements in the powering of ships (Molland et. al. 2011)

Taking the above sketch into account gives us the basic items of powering of ships on which the Naval Architect and Marine Engineer should gain insight and skill to improve their efficiency by:

i) calculate the resistance and corresponding effective power ($P_E = R\ V$),

ii) reduce the resistance by means of hull form optimization and by careful design of appendages, sophisticated coatings etc.,

iii) optimum choice of propeller dimensions (propeller design),

iv) study of the propeller-hull interaction, determine the wake, thrust deduction and hull efficiency, which results in overall propulsive efficiency which yields delivered power $P_D=P_E/\eta_D$

v) right choice of suitable machinery with low emissions qualities.

2. DIMENSIONAL ANALYSIS AND MODEL TESTING

W. Froude, in his quest to understand the resistance problem of ships, made systematic model tests and observe that similar models with different lengths (scales) have similar wave patterns corresponding equivalent velocities. He found that corresponding speeds vary proportionally to the square root of the model length: $V_2 \propto V_1\sqrt{L_2}$. This is indeed a great finding, but one requires more precise relationship between resistance and speed in model testing which is not affected by the choice of units of measurements. Thus predicting full-scale quantities by model testing requires non-dimensionalization process which is based on *Dimensional Analysis* and *Similarity Laws*.

The physical problems in hydrodynamics have always three fundamental units: mass (M), length (L) and time (T). Let Q be the unknown quantity which depends on n-1 significant parameters. In this case there will be n dimensional quantities including Q. According to *Buckingham's Pi Theorem*, by taking three fundamental units (M,L,T) into account; non-dimensionalization reduce the number n to a total of n-3 non-dimensional quantities interrelated. We introduce two different approaches; mathematical and intuitive approaches. Let's first study them by considering a simple problem in mechnics.

2.1 Free fall of a body in a vacuum

Take the unknown quantity Q as the vertical distance y travelled by the body. The unknown quantity y might be related to time t, body mass m and gravitational acceleration g. We do not need to take the shape of the body into account, since the body moves in a vacuum.

i) Intuitive approach: according to the above descriptions, the unknown quantity y should be a function of t, g and m:

$$y = f(t, g, m)$$

There are four interrelated parameters (y, t, g, m) and according to the Pi theorem there should be 4-3=1 interrelated non-dimensional quantity. Since y can be non-dimensionalized as y/gt^2 and g and t do not contain any mass unit (mass m is discarded in this case), thus:

$$y/gt^2=f(\cancel{m})=c$$

As is seen, there is one non-dimensional quantity (relation) which is a constant.

ii) Mathematical approach: as g, t, m are significant parameters related to y, it is usual in dimensional analysis to relate the parameters as in the following

$$y = C \cdot g^a \cdot t^b \cdot m^c$$

Expressing the quantities by fundamental units (M,L,T):

$$L \equiv C(LT^{-2})^a T^b M^c$$

Same units have to be equal powers:

L:	1=a	
T:	0=-2a+b	b=2
M:	0=c	

$$y = Cgt^2 \Rightarrow \frac{y}{gt^2} = C$$

where C is a constant for non-dimensional quantity y/gt^2. We can determine C from an experiment which is valid for all other possible values of g and t.

2.2 Resistance of a moving body on the free surface: Drag on a ship hull

Consider a ship hull (or a rigid body) moving with constant velocity V on the free surface. Imagine that, in this case, the moving body generates a pattern of waves with the effect of gravity. One normally expects friction between the surface of the body and the fluid due to viscosity. Under these circumstances the unknown quantity Resistance (Drag) R may be related to gravity g, characteristic length of the body L, velocity V, water density ρ and kinematic viscosity $\upsilon=\mu/\rho$.

i) Intuitive approach: The six dimensional quantities given above is interrelated with a functional relation, f:

$$R_T = f(\rho, \mu, L, V, g)$$

This relation and the parameters in it can be non-dimensionalized to give (6 – 3 = 3) three non-dimensional ratios within the relation:

$$\frac{R_T}{\rho L^2 V^2} = f\left(\frac{V}{\sqrt{gL}}, \frac{VL\rho}{\mu}\right) = f(Fr, \mathrm{Re})$$

where $Fr = \dfrac{V}{\sqrt{gL}}$ is the Froude number and $\mathrm{Re} = \dfrac{VL}{\upsilon}$ is the Reynolds number.

ii) Mathematical approach: In this approach we can use the following expression

$R_T = C\rho^a L^b V^c \mu^d g^e$, and with fundamental units:

$$(MLT^{-2}) \equiv C(ML^{-3})^a L^b (LT^{-1})^c (ML^{-1}T^{-1})^d (LT^{-2})^e$$

Equating the same powers:

M: 1=a+d

L: 1=-3a+b+c+d+e

T: -2=-c-d-2e

Solving the parameters in terms of d and e;

a=1-d

b=2-d+e

c=2-d-2e Thus;

$R_T = C\rho^{1-d} L^{2-d+e} V^{2-d-2e} \mu^d g^e$, which gives

$$R_T = C\rho L^2 V^2 \left(\frac{VL\rho}{\mu}\right)^{-d} \left(\frac{V^2}{gL}\right)^{-e}$$

$$\frac{R_T}{\rho L^2 V^2} = C\left(\frac{VL}{\upsilon}\right)^{-d} \left(\frac{V^2}{gL}\right)^{-e} = f\left(\frac{VL}{\upsilon}, \frac{V}{\sqrt{gL}}\right)$$

This expression is usually written in a conventional way by taking $S=L^2$ as the wetted surface of the body and (ρ/2) instead of ρ as:

$$\frac{R_T}{\frac{\rho}{2} S V^2} = C_T\left(\frac{VL}{\upsilon}, \frac{V}{\sqrt{gL}}\right)$$

Now the question is: can we satisfy the scaling of Reynolds (VL/υ) and Froude ($V/\sqrt{gL}$) numbers simultaneously? Let's take two model (length) scales L_1 and L_2. Scaling with regard to Reynolds number requires

$\left(\dfrac{VL}{\upsilon}\right)_2 = \left(\dfrac{VL}{\upsilon}\right)_1$ which gives (assuming constant υ):

$$V_2 = V_1\left(\frac{L_1}{L_2}\right).$$

On the other hand, scaling with regard to Froude number yields;

$$\left(\frac{V}{\sqrt{gL}}\right)_2 = \left(\frac{V}{\sqrt{gL}}\right)_1, \text{ which gives;}$$

$$V_2 = V_1\sqrt{\frac{L_2}{L_1}}.$$

Since it is impossible to use the same corresponding speeds with the same scale, thus it is impossible to scale simultaneously both the Reynolds and Froude numbers. Meantime it might be useful to relate Reynolds and Froude numbers to the three principal forces experienced in hydrodynamics, namely inertial, gravitational and viscous forces which are proportional to:

Inertial $\sim \rho V^2 L^2$

Gravitational $\sim \rho g L^3$

Viscous $\sim \mu V L$

In this case, the ratios of pair of forces give:

$$\frac{Inertial\ force}{Gravitational\ force} = \frac{\rho V^2 L^2}{\rho g L^3} = \frac{V^2}{gL} = Fr$$

$$\frac{Inertial\ force}{Viscous\ force} = \frac{\rho V^2 L^2}{\mu V L} = \frac{\rho V L}{\mu} = \mathrm{Re}$$

With this background, we can now proceed to analyze total resistance (drag) coefficient C_T which is a function of Froude and Reynolds numbers as understood from dimensional analysis. Depending on this understanding, Froude put forward a hypothesis that C_T may be split into two parts as;

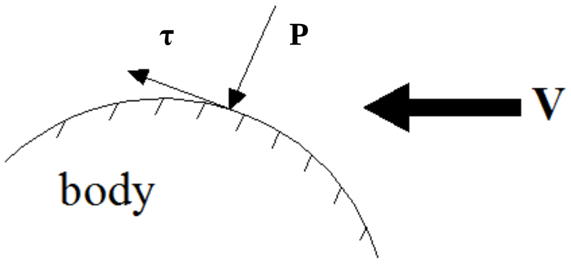
$$C_T(Re, Fr) \cong C_F(Re) + C_R(Fr)$$

where C_F is called equivalent flat-plate frictional drag coefficient and C_R (Fr) is the residual drag coefficient in which wave resistance plays the primary role. But one should bear in mind that part of the residual drag is due to viscous form drag (or viscous pressure drag) which depends on Reynolds number. The above discussion points out that a more elaborate investigation of the components of the total resistance is required.

3. COMPONENTS OF SHIP RESISTANCE

Dimensional analysis and Froude's hypothesis pinpoints the basic components of the total ship resistance as described in the previous section. But one should be careful in splitting the total resistance into components in that each component may include residuals and also the interaction between these components makes the scaling a very complicated matter. Accordingly, physical flow phenomena causing the drag on the ship hull should be understood clearly.

First, if we ignore the free surface effect (there is no wave formation in this case), then the resultant drag for a body moving with a constant speed in an infinite fluid comes from viscous effects. The total viscous force is due to the tangential shear stress (τ) and normal (p) stress (viscous pressure):



The tangential shear stress ends up with frictional resistance on the body. This force component may be approximated by 2-dimensional flat-plate friction. But one should be aware of the fact that 3-dimensional effect is excluded in this case. Integration of normal stresses due to viscosity over the wetted surface of the body gives the viscous pressure resistance. Cross-flows, vortex shedding and flow separation are the main causes of the viscous pressure drag.

When a free surface exists on (or in) which the body moves with constant velocity, as a consequence; gravity waves are generated by the body. Wave generation dissipates energy from the body and results in wave making resistance which contains also wave-breaking and spray resistances. Wave pattern resistance, which does not contain the effects of wave breaking and spray, can be determined by the integration of normal pressure over the wetted surface. The schematic diagram given below shows the basic resistance components and the relation between them. The following figure explains resistance components in a detailed manner which covers also appendage resistance, roughness resistance, air resistance etc.

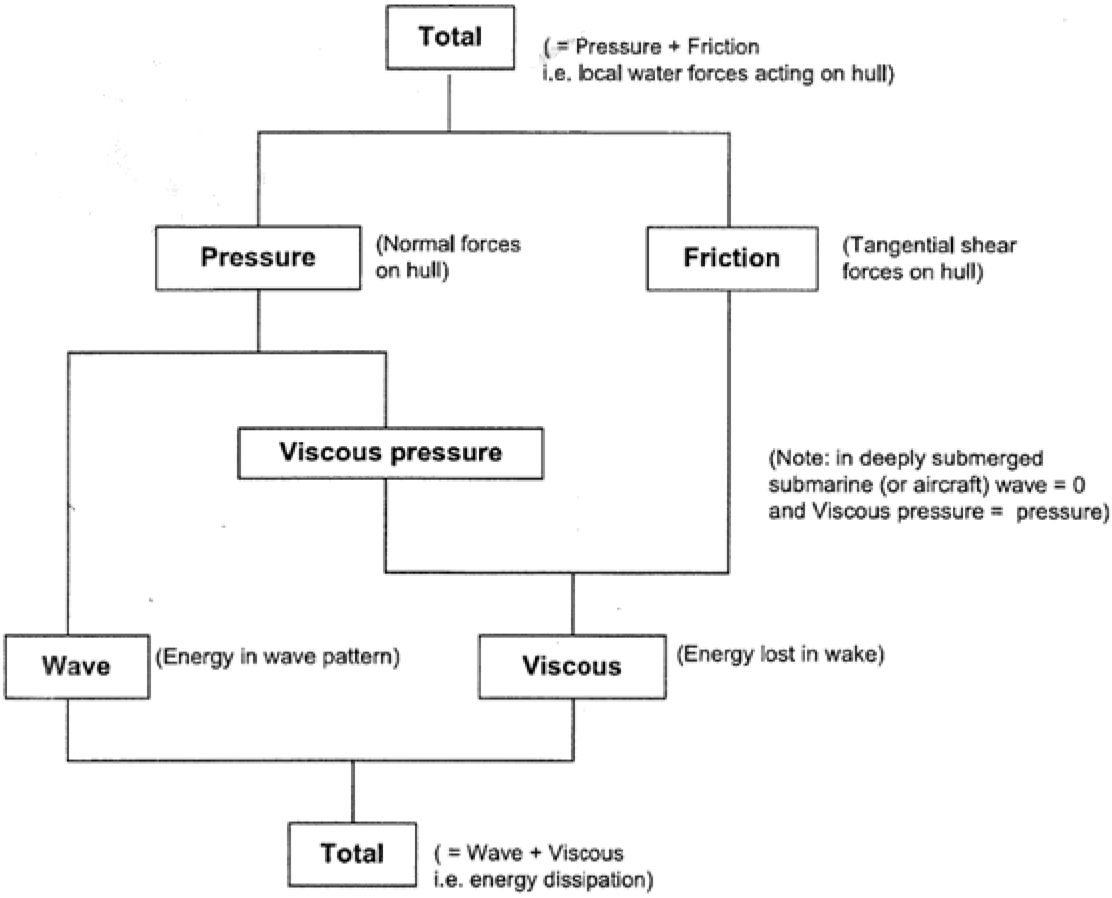


Fig. 3.1: Basic resistance components (Molland et. al. 2011)

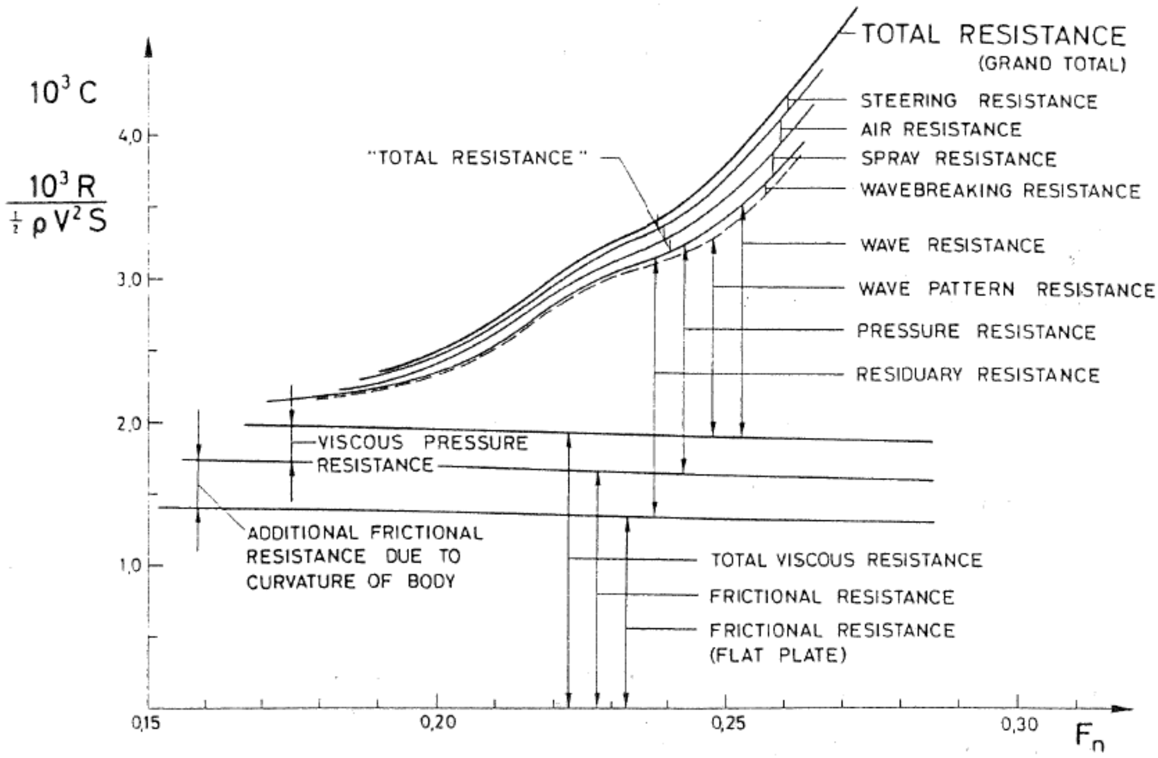


Fig. 3.2: Reduction of total resistance into components (Harvald, 1991)