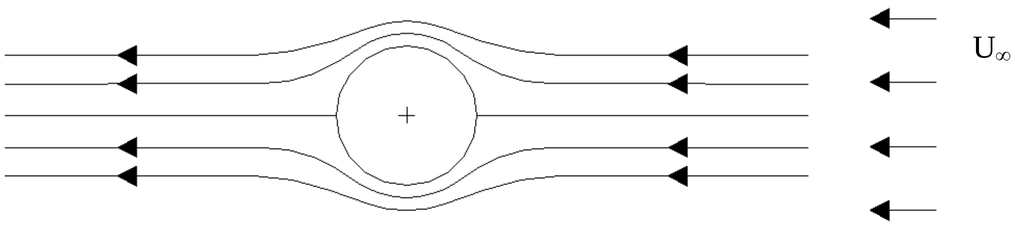
3.2 Wave Resistance

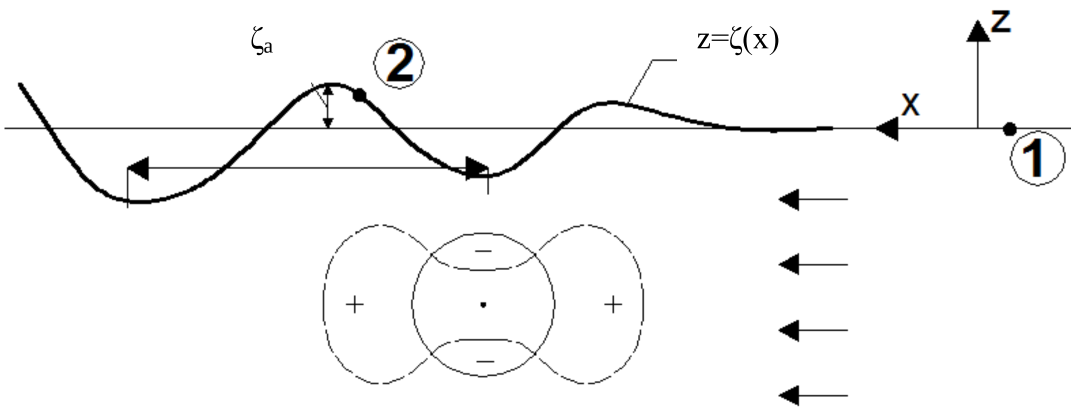
As mentioned previously, wave resistance is expressed as the sum of wave-pattern resistance and wave breaking resistance. Since the effects of viscosity and vorticity on the wave formation are negligible, the waves generated by bodies in motion -and also the prediction of wave resistance for a moving ship- can be investigated by assuming ideal fluid flow which makes the problem and its solution simpler as compared to viscous flow. First, we will direct our attention to wave-making resistance which is the dominant component.

3.2.1 The nature of the wave-making problem

Consider a 2D circular cylinder in an infinite, ideal, uniform fluid flow, and let's imagine the streamlines around it:



according to *Bernoulli's equation*: the velocity speeds up where the pressure drops and vice versa. Now, if we think this 2D circular cylinder is placed under the free surface, the pressure distribution on the cylinder perturbes the free surface (which in turn affects the pressure distribution on the body to some extend) and causes the generation of a wave system down the stream:



Due to the effect of free surface on the pressure distribution on the body, the result of the pressure integration over the body is other than zero and gives the wave pattern resistance in this case. Note that *Bernoulli's equation* on the free surface (streamline) gives:

$$\left(\frac{P_a}{\rho}+\frac{1}{2}U_\infty^2\right)_1=\left(\frac{P_a}{\rho}+\frac{1}{2}(\vec{V}\cdot\vec{V})+g\xi\right)_2$$

where the pressure can be taken as atmospheric pressure and the velocity field can be derived from a scalar potential function $\vec{V} = \nabla\Phi$ where Φ should satisfy Laplace's equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

if the flow is assumed to be ideal. (This present attempt is just for giving an introductory understanding of the 2-D resistance problem within the context of fluid mechanics). Accordingly, wave elevation due to the presence of the body in the flow is obtained by

$$\xi = \frac{1}{2g}\left[U_\infty^2 - (\nabla\Phi \cdot \nabla\Phi)\right]$$

The magnitude of the wave elevation is important, since energy of the wave is important, since energy of the wave is proportional to the square of the wave amplitude generated:

$$\bar{E} = \frac{1}{2}\rho g \xi_a^2 \lambda$$

Here λ is the wavelength which can be related to the wave (or body) velocity U_∞ as;

$$\lambda = \frac{2\pi U_\infty}{g}$$

But wave-making problem of a ship is indeed a 3D problem. It was first Lord Kelvin (William Thompson) in 1887formulated the formation of a wave system by considering a moving point pressure in the free surface. Today, the wave system obtained by his earlier analyses is still used as a good example of describing the wave system due to a moving ship with constant velocity in the same direction.

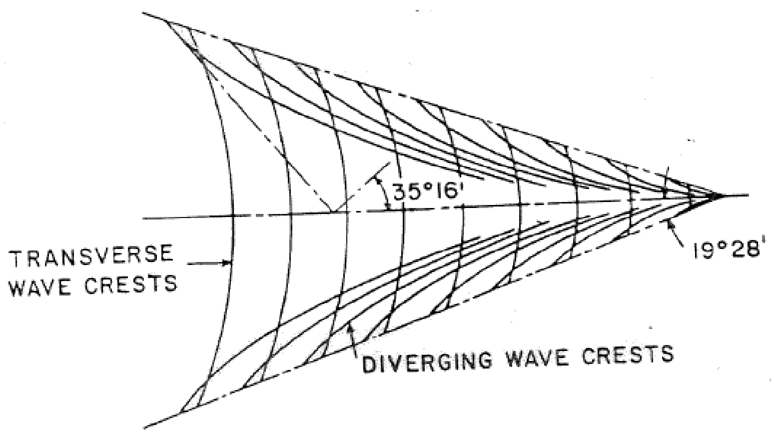


Fig. 3.10: Ship-wave system as obtained from Kelvin's analysis (Newman, 1980).

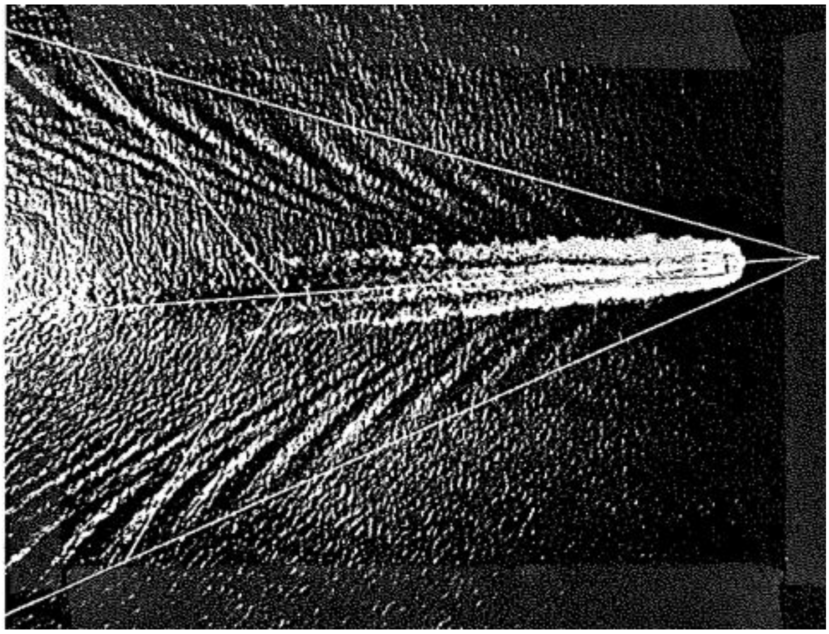


Fig. 3.11: Observation from above of ship waves confirming Kelvin wave system (Newman, 1980).

Following a simplified approach in reducing ship-wave system into components, gives us the four basic components which can be explained by the pressure distribution on the hull near the free surface:

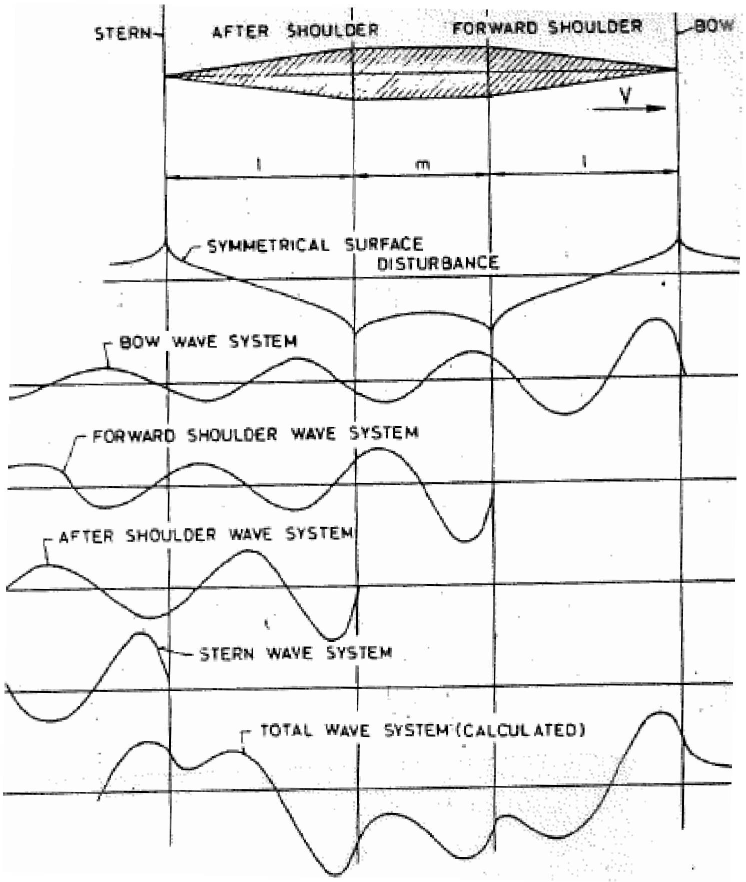


Fig. 3.12: Sources of wave systems on the hull and their interference (Harvald, 1991).

Since there is a rise in pressure at the bow so that an increase in the wave elevation is expected and since there is a pressure drop around the fore-shoulder and accordingly a through around fore-shoulder occurs and so on. This approach of reduction total wave system into components also is able to explain the interference of wave systems which is important in the design phase. Because if the designer has computational tools at hand, he/she is then able to search for an optimal position of ship length and location of shoulders for the given design speed, in order to obtain the least wave resistance.

Now, let's try to rationalize the relation between wave components and wave resistance by superimposing the bow and aft wave systems. Consider $\xi_1 = \xi_{1a} \cos \frac{2\pi x}{\lambda}$ as the bow wave and $\xi_2 = \xi_{2a} \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi l}{\lambda} \right)$ as the aft wave system where l is the distance between the wave troughs (or crests) of the two systems. Thus, superposing the two wave system gives the resultant wave amplitude as:

$$\xi_{12a} = \sqrt{\xi_{1a}^2 + \xi_{2a}^2 + 2\xi_{1a}\xi_{2a} \cos \frac{2\pi l}{\lambda}}$$

If l is represented by $l = \beta L + \frac{\lambda}{2}$ (where β is an empirical parameter) then ξ_{12a} ;

$$\xi_{12a} = \sqrt{\xi_{1a}^2 + \xi_{2a}^2 + 2\xi_{1a}\xi_{2a} \cos \frac{\beta}{(V^2 / gL)}}$$

where $\lambda = \frac{2\pi V^2}{g}$. From *Bernoulli's equation*; ξ's are in the order of V^2 or are proportional with V^2. Thus; $\xi_{1a} = AV^2$, $\xi_{2a} = BV^2$ may be written with empirical coefficients A and B. Recall that energy flux in regular waves is proportional to ξ_a^2, which is closely related to the wave resistance. Therefore,

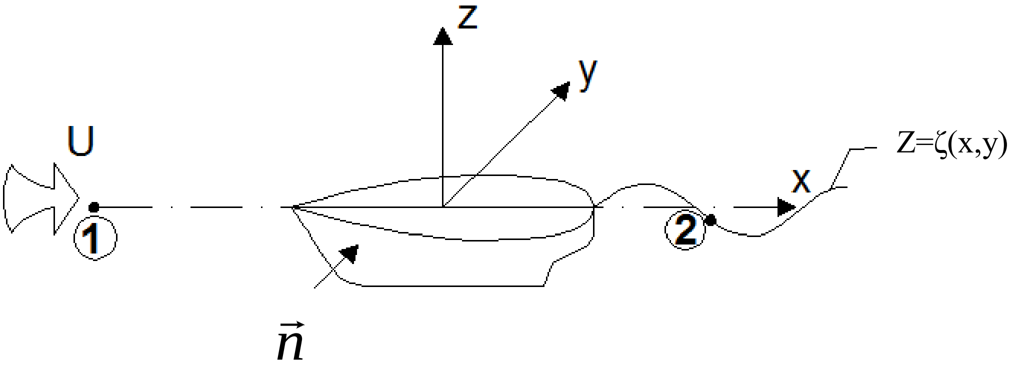
$$R_w \propto V^4 \left[A^2 + B^2 + 2AB \cos \frac{\beta}{(V^2 / gL)} \right]$$

Note that the least term in parenthesis represents the interference effect which may reduce or increase the wave resistance depending on the position of aft and bow points at a given Fr. This semi-empirical/semi-theoretical modelling of wave resistance paved the way to Havelock's modelling of wave resistance by moving bow and aft pressure distributions and in turn yields the sophisticated empirical methods of Oortmerssen and Holtrop-Mennen.

3.2.2 Theoretical wave resistance

Since Lord Kelvin (1887), theoretical analysis of wave resistance is one of the primary concerns of naval hydrodynamicists. Negligence of viscosity in wave motion makes the problem analytically solvable on the one hand, but the related non-linear boundary conditions turn the problem out a challenging one on the other.

We can start with the mathematical definition of the full/exact problem. Assume that the body/ship is fixed in an onset flow with velocity U on the free surface.



Under the ideal fluid flow assumptions the velocity field is derived from a potential function; $\vec{V} = \nabla\phi$; which satisfies *Laplace's equation:*

$$\nabla^2\phi(x, y, z) = 0$$

Note that the flow is steady so that Bernoulli's equation gives by considering points 1 and 2 on the free surface streamline:

$$\left(\frac{P_a}{\rho} + \frac{1}{2}U^2\right)_1 = \left(\frac{P_a}{\rho} + \frac{1}{2}(\nabla\phi\cdot\nabla\phi) + g\xi\right)_2 ;(z = \xi)$$

which is called as dynamic boundary condition on the free surface. The kinematic free surface condition is obtained by making the material derivative of the free surface equation $F = z - \xi(x, y)$ equal to zero $\frac{DF}{Dt} = \left(\frac{\partial}{\partial t} + \vec{V}\cdot\nabla\right)F = 0$, which gives:

$$\phi_x\xi_x + \phi_y\xi_y + \phi_z = 0 \;;\; (z = \xi).$$

ξ can be discarded from the free surface equations by taking the derivatives of $\xi(x, y)$ in the dynamic boundary condition, and then substituting them is the kinematic boundary condition to give:

$$g\phi_z + \nabla\phi\cdot\nabla\left[\frac{1}{2}(\nabla\phi\cdot\nabla\phi)\right] = 0 \;;\; (z = \xi)$$

which is the resultant nonlinear free surface condition.

The boundary condition on the wetted hull surface implies $(\vec{n})$ normal fluid velocities to the surface should be zero:

$$\frac{\partial \phi}{\partial n} = 0 \text{ (on the hull surface S).}$$

Normal velocities to the flat sea bottom at $z \to -\infty$:

$$\frac{\partial \phi}{\partial z} = 0 .$$

Additionally, there is also a radiation condition at infinity (as $r = \sqrt{x^2 + y^2} \to \infty$):

$$\phi = \begin{cases} Ux + o\left(\frac{1}{r}\right); & x < 0 \\ Ux + O\left(\frac{1}{r}\right); & x > 0 \end{cases}$$

This is indeed a challenging problem due to its nonlinear boundary conditions, but solvable with sum assumptions. Once the potential solution is obtained, wave resistance is calculated by integrating the x-component pressure distribution on the body surface S:

$$R_w = \iint_S p n_x dS$$

where n_x is the x-component of the normal vector to the body and hydrodynamic pressure is given by means of the *Bernoulli's equation*:

$$p = -\frac{\rho}{2}\left[\left(\nabla\phi \cdot \nabla\phi\right) - U^2\right]$$

The boundary value problem given above is very difficult to solve by analytical means, so that some simplifications/assumptions are made. For example total velocity potential is assumed to be the sum of: $\phi = U_x + \varphi(x, y, z)$, where φ is the perturbation potential due to the body in uniform flow U and due to the free surface effects. In this case, nonlinear free surface condition may be linearized in terms of φ to give the following Newman-Kelvin (N-K) problem:

$$\nabla^2 \varphi = 0$$

$$U^2 \varphi_{xx}(x, y, 0) + g\varphi_z(x, y, 0) = 0 \quad ; \quad \text{(on the free surface z=0)}$$

$$\frac{\partial \varphi}{\partial n} = -Un_x \quad ; \quad \text{(on hull surface S)}$$

$$\frac{\partial \varphi}{\partial z} = 0 \quad ; \quad \text{(at the bottom; } z \to -\infty \text{)}$$

and radiation conditions at infinity.

Note that boundary condition $\varphi_n = -Un_x$ on the body is indeed a nonlinear expression.

Thin-ship Theory – Michell's Integral

If one assumes a very thin ship (as L/B >>1), boundary condition on the hull surface can also be linearized to give:

$$\frac{\partial \varphi}{\partial y}(x, \pm 0, z) = \pm U \frac{\partial f}{\partial x}(x, z); \text{ (on the centerplane)}$$

where y=f(x,z) denotes half-breadths and subsequently the hull surface; S=y-f(x,z)=0. With this linearization, (N-K) problem, it turns out to be the *thin-ship theory*. This problem was first solved solved by Michell (1898) by distribution of sources/sinks on the centerplane of ship and then Havelock (1923) made important contributions to the solution of this problem. There are various versions of Michell's integral currently used in today's applications. Widely used version is given by:

$$R_w = 4\frac{\rho}{\pi}\frac{g^2}{U^2}\int_1^{\infty}\frac{\lambda^2}{\sqrt{\lambda^2 - 1}}\left[P^2(\lambda) + Q^2(\lambda)\right]d\lambda$$

$$\text{where } \left.\begin{matrix} P \\ Q \end{matrix}\right\} = \int_0^T\int_0^L f_x(x, y)\exp\left[\frac{g\lambda^2}{U^2}y\right]\begin{matrix}\cos \\ \sin\end{matrix}\left\{\frac{g\lambda}{U^2}x\right\}dxdy .$$

The ship geometry is introduced by y=f(x,z), Although, Michell's integral gives exaggerated humps and hollows in wave resistance the positions of the local minimums show the correct optimum points of wave interference:

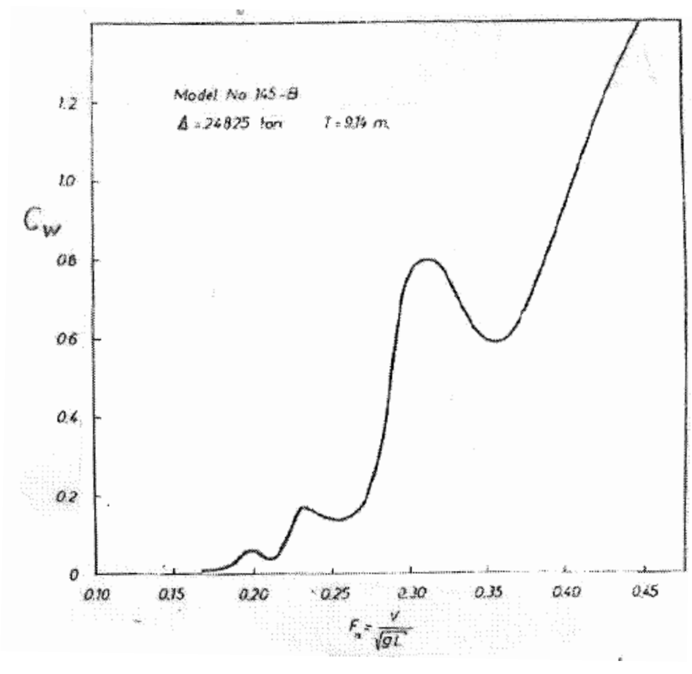


Fig. 3.13: Basic humps and hollows as obtained from Michell's integral.

(Gören & Sabuncu, 1986)

With today's standards Michell integral may not be regarded as a reliable, high-precision code, but it can still be used to compare the wave-resistances of the hull forms.

Low Froude Number Theory

If total velocity potential in N-K problem is taken as:

$$\phi = \Phi_D + \varphi \text{ (instead of } \phi = U_x + \varphi \text{ as in thin-ship theory)}$$

where Φ_D is called double-model potential of a double-model, obtained by taking the mirror image of the hull form under the still water level, assumed to be in an infinite fluid flow with constant speed. In this exact free surface condition is linearized in terms of the perturbation potential φ. *Low Froude number theory* satisfies kinematic boundary condition on the body $\frac{\partial \varphi}{\partial n} = 0$ exactly. Today most of the research centers, well-established design offices utilize wave resistance analysis codes based on *Low Froude number theory*.

The other well-known wave resistance theory is the *Slender Body Theory* which assumes both the beam and the draft are very small as compared to the length of the ship. One can imagine a ship which fits slender body assumption roughly as a pencil with both ends sharpened. According the *slender ship theory* the ships having the same sectional area curves have the same wave-making characteristics.

3.2.3 Computational wave resistance

Computational applications regarding with *Michell's integral* and the *Low Froude number theory* are given in the following.

Michell's integral (ITU- Michell)

In this code (see http://160.75.46.2/staff/devrim/michell.rar) Michell's integral is evaluated by considering a mesh depicted in the figure below:

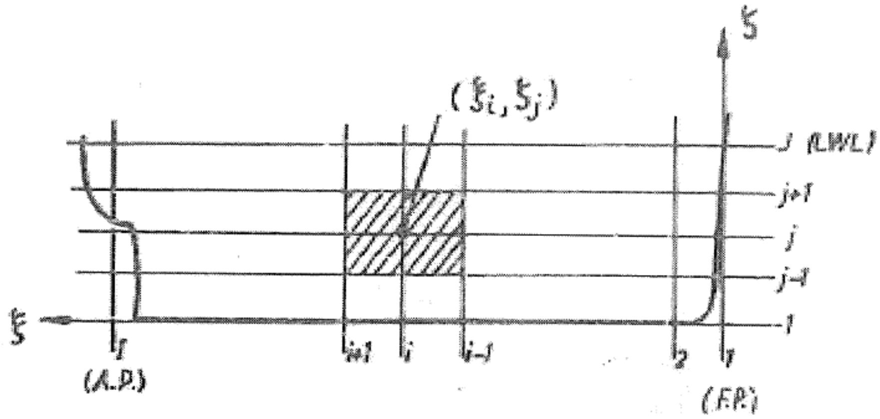


Fig. 3.14: Meshing of centerplane for ITU-Michell.

The hull geometry is represented by tent functions. This representation of the geometry by tent functions approximates the surface of the hull by a kind of linear spline surface such as:

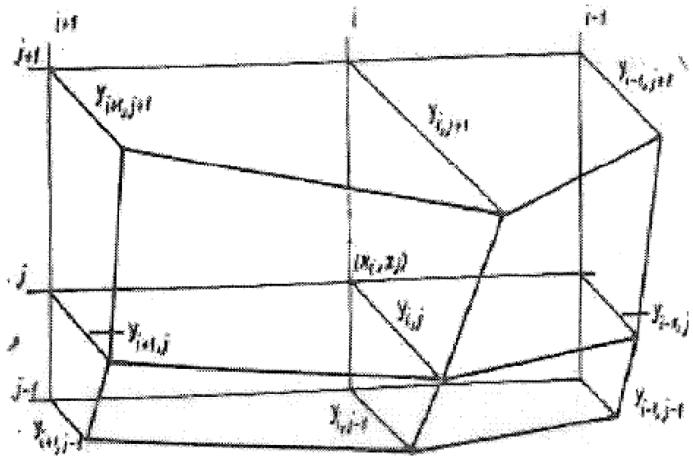


Fig. 3.15: Approximation of the hull surface by a family of tent functions.

(For details of this geometrical formulation required in Michell integral: Sabuncu, T. and Gören, Ö. "Çadir Fonksiyonlari Yardimiyla Gemi Direnci Hesabi", İTÜ Dergisi, Cilt 44, Sayı 3-4, 1986, pp. 32-41)

As mentioned before, Michell integral is able to give reasonable results for very fine and thin ships, but not for fuller forms and for relatively high Froude number. An example of such a study for a bi-Wigley catamaran hull (which is indeed a very fine form with high L/B ratio) is given in the following figure. Thin-ship results are due to ITU-Michell. (The curve with squares on it represents Low Froude number theory).

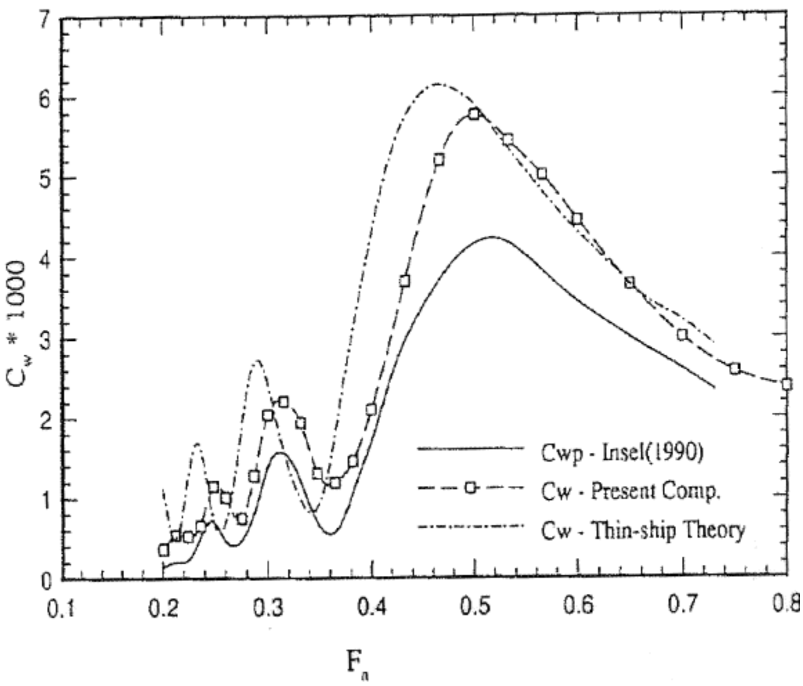


Fig. 3.16: Comparison of wave resistance coefficients.

In order to run ITU-Michell given in the folder that you can find via the web address given above the following input files should be prepared your ship geometry.

input_waterline.dat *(height of the waterlines from the baseline [m])*:

0.000 1.000 2.000 3.000 4.000 5.000

input_section.dat (*distance of the sections/stations from the bow [m]) :*

0.000 2.000 4.000 8.000 12.000 16.000 24.000 32.000 40.000 48.000 56.000 64.000 68.000 72.000 76.000 80.000

input_parameter.dat *(# of stations, # of waterlines, length between the first and the last section [m], beam (max) [m], draft [m], speed [m/s], respectively)* :

16 6 80.000 8.000 5.000 8.824

input_halfbreadth.dat (half-breadths starting from the 1st station and from the baseline [m]) :

```
0.0   0.000 0.000 0.000 0.000 0.000
0.000 0.140 0.250 0.328 0.374 0.390
0.000 0.274 0.486 0.638 0.730 0.760
0.000 0.518 0.922 1.210 1.382 1.440
0.000 0.734 1.306 1.714 1.958 2.040
0.000 0.922 1.638 2.150 2.458 2.560
0.000 1.210 2.150 2.822 3.226 3.360
0.000 1.382 2.458 3.226 3.686 3.840
0.000 1.440 2.560 3.360 3.840 4.000
0.000 1.382 2.458 3.226 3.686 3.840
0.000 1.210 2.150 2.822 3.226 3.360
0.000 0.922 1.638 2.150 2.458 2.560
0.000 0.734 1.306 1.714 1.958 2.040
0.000 0.518 0.922 1.210 1.382 1.440
0.000 0.274 0.486 0.638 0.730 0.760
0.000 0.000 0.000 0.000 0.000 0.000
```

Please do not exceed the limits 16 for stations number and 6 for waterlines number. Consequently, double clicking of exe file Michell gives the following output:

input_output.out :

```
96
  PARTICULARS OF THE SHIP :

  L= 80.000 M
  B=  8.000 M
  T=  5.000 M
  V=  8.824 M/SEC


  WETTED SURFACE AREA=   980.478 (SQUARE METERS)
  CF=  .1239804D+00    RF=  .6351723D+05 (NEWTONS)

  CW=  .1344914D+00    RW=  .6890221D+05 (NEWTONS)
```

Low-Froude Number Theory (ITU- Dawson)

The code developed at ITU by Goren (1987,1990) namely ITU-Dawson is based on *Low Froude number theory* and uses, in combination , Hess & Smith's (1967) source-panel method and Dawson's (1977) algorithm for free surface condition. A sample of geometrical discretization of a ship hull is given in the following.

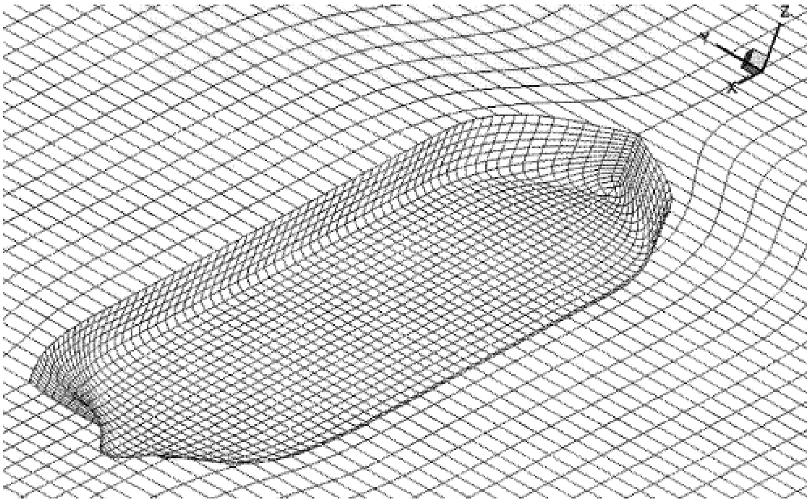


Fig. 3.17: A sample of panelling of the free surface and the body surface for ITU-Dawson.

As seen from the figure of "Comparison of wave resistance coefficients, the code based on *Low Froude number theory* (Present Comp. In figure) gives more realistic and high accuracy results for moderate speeds. The resulting wave system around the hulls as obtained by ITU-Dawson could be seen in the following figure.

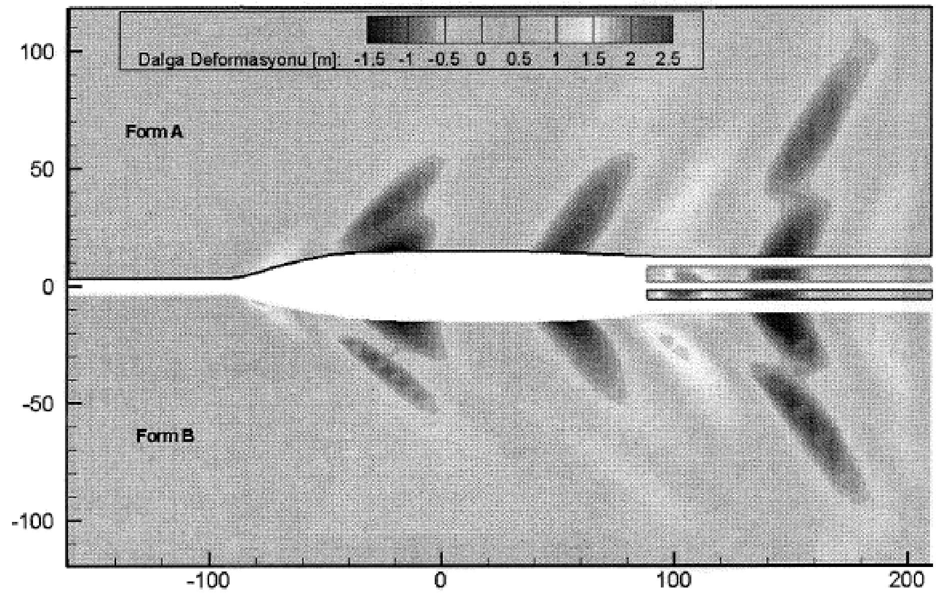


Fig. 3.18: Wave systems of two hull forms as computed by ITU-Dawson