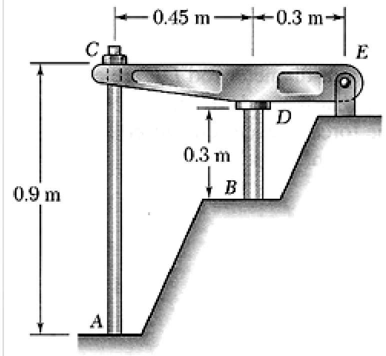
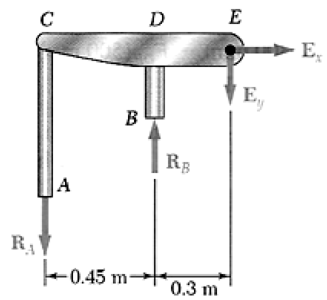


QUESTION 1

The rigid bar *CDE* is attached to a pin support at *E* and rests on the 30-mm-diameter brass cylinder *BD*. A 22-mm-diameter steel rod *AC* passes through a hole in the bar and is secured by a nut which is snugly fitted when the temperature of the entire assembly is 20°C. The temperature of the brass cylinder is then raised to 50°C while the steel rod remains at 20°C. Assuming that no stresses were present before the temperature change, determine the stress in the cylinder.

Rod *AC*: Steel	Cylinder *BD*: Brass
$E = 200$ GPa	$E = 105$ GPa
$\alpha = 11.7 \times 10^{-6}/°C$	$\alpha = 20.9 \times 10^{-6}/°C$

SOLUTION 1



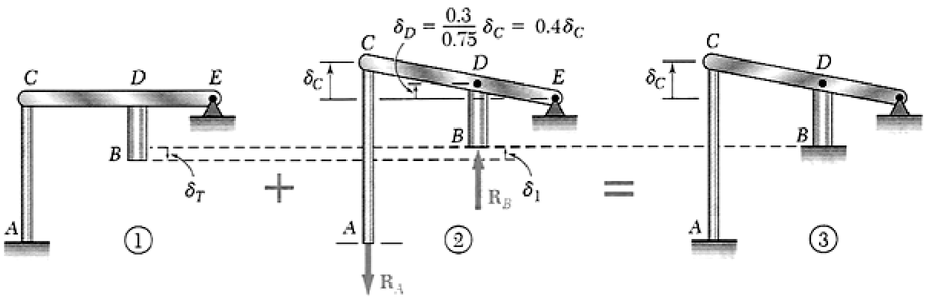
Statics. Considering the free body of the entire assembly, we write

$$+\circlearrowleft \Sigma M_E = 0: \quad R_A(0.75 \text{ m}) - R_B(0.3 \text{ m}) = 0 \qquad R_A = 0.4R_B \tag{1}$$

Deformations. We use the method of superposition, considering $\mathbf{R}_B$ as redundant. With the support at *B* removed, the temperature rise of the cylinder causes point *B* to move down through δ_T. The reaction $\mathbf{R}_B$ must cause a deflection δ_1 equal to δ_T so that the final deflection of *B* will be zero (Fig. 3).

Deflection δ_T. Because of a temperature rise of $50° - 20° = 30°C$, the length of the brass cylinder increases by δ_T.

$$\delta_T = L(\Delta T)\alpha = (0.3 \text{ m})(30°C)(20.9 \times 10^{-6}/°C) = 188.1 \times 10^{-6} \text{ m} \downarrow$$



Deflection δ_1. We note that $\delta_D = 0.4\delta_C$ and $\delta_1 = \delta_D + \delta_{B/D}$.

$$\delta_C = \frac{R_A L}{AE} = \frac{R_A(0.9 \text{ m})}{\frac{1}{4}\pi(0.022 \text{ m})^2(200 \text{ GPa})} = 11.84 \times 10^{-9} R_A \uparrow$$

$$\delta_D = 0.40\delta_C = 0.4(11.84 \times 10^{-9} R_A) = 4.74 \times 10^{-9} R_A \uparrow$$

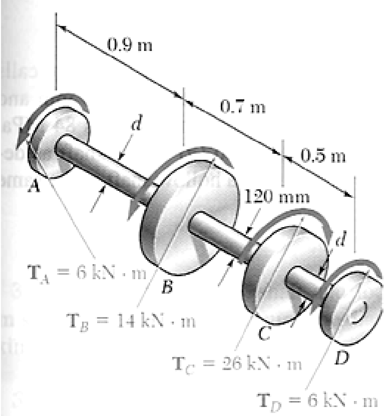
$$\delta_{B/D} = \frac{R_B L}{AE} = \frac{R_B(0.3 \text{ m})}{\frac{1}{4}\pi(0.03 \text{ m})^2(105 \text{ GPa})} = 4.04 \times 10^{-9} R_B \uparrow$$

We recall from (1) that $R_A = 0.4R_B$ and write

$$\delta_1 = \delta_D + \delta_{B/D} = [4.74(0.4R_B) + 4.04R_B]10^{-9} = 5.94 \times 10^{-9} R_B \uparrow$$

But $\delta_T = \delta_1$: $\quad 188.1 \times 10^{-6} \text{ m} = 5.94 \times 10^{-9} R_B \qquad R_B = 31.7 \text{ kN}$

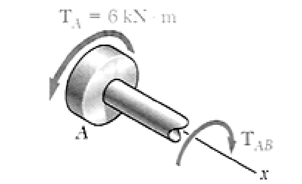
Stress in Cylinder: $\quad \sigma_B = \dfrac{R_B}{A} = \dfrac{31.7 \text{ kN}}{\frac{1}{4}\pi(0.03)^2} \qquad \sigma_B = 44.8 \text{ MPa}$ ◄



QUESTION 2

Shaft BC is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts AB and CD are solid and of diameter d. For the loading shown, determine (*a*) the maximum and minimum shearing stress in shaft BC, (*b*) the required diameter d of shafts AB and CD if the allowable shearing stress in these shafts is 65 MPa.

SOLUTION 2

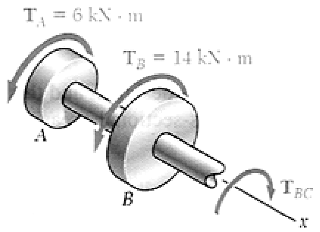


Equations of Statics. Denoting by $\mathbf{T}_{AB}$ the torque in shaft AB, we pass a section through shaft AB and, for the free body shown, we write

$$\Sigma M_x = 0: \qquad (6\ \text{kN} \cdot \text{m}) - T_{AB} = 0 \qquad T_{AB} = 6\ \text{kN} \cdot \text{m}$$

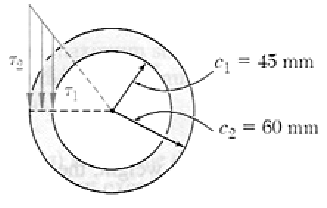
We now pass a section through shaft BC and, for the free body shown, we have

$$\Sigma M_x = 0: \qquad (6\ \text{kN} \cdot \text{m}) + (14\ \text{kN} \cdot \text{m}) - T_{BC} = 0 \qquad T_{BC} = 20\ \text{kN} \cdot \text{m}$$



a. Shaft BC. For this hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.060)^4 - (0.045)^4] = 13.92 \times 10^{-6}\ \text{m}^4$$

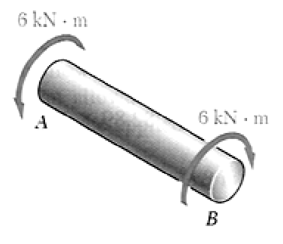


Maximum Shearing Stress. On the outer surface, we have

$$\tau_{max} = \tau_2 = \frac{T_{BC}c_2}{J} = \frac{(20\ \text{kN} \cdot \text{m})(0.060\ \text{m})}{13.92 \times 10^{-6}\ \text{m}^4} \qquad \tau_{max} = 86.2\ \text{MPa} \blacktriangleleft$$

Minimum Shearing Stress. We write that the stresses are proportional to the distance from the axis of the shaft.

$$\frac{\tau_{min}}{\tau_{max}} = \frac{c_1}{c_2} \qquad \frac{\tau_{min}}{86.2\ \text{MPa}} = \frac{45\ \text{mm}}{60\ \text{mm}} \qquad \tau_{min} = 64.7\ \text{MPa} \blacktriangleleft$$



b. Shafts AB and CD. We note that in both of these shafts the magnitude of the torque is $T = 6\ \text{kN} \cdot \text{m}$ and $\tau_{all} = 65$ MPa. Denoting by c the radius of the shafts, we write

$$\tau = \frac{Tc}{J} \qquad 65\ \text{MPa} = \frac{(6\ \text{kN} \cdot \text{m})c}{\frac{\pi}{2}c^4}$$

$$c^3 = 58.8 \times 10^{-6}\ \text{m}^3 \qquad c = 38.9 \times 10^{-3}\ \text{m}$$

$$d = 2c = 2(38.9\ \text{mm}) \qquad d = 77.8\ \text{mm} \blacktriangleleft$$