STRENGTH OF MATERIALS I

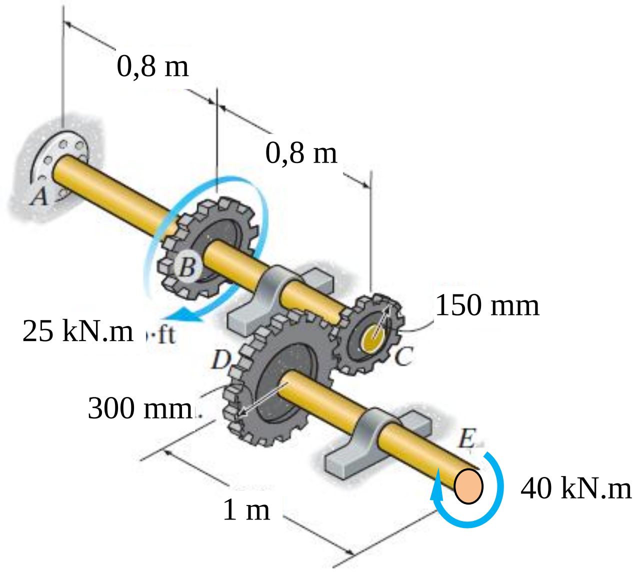
Homework 3, Deadline: 21st December 2012

QUESTION 1. The shafts AC and DE are made of A-36 steel and both have the diameter of 100 mm. Determine

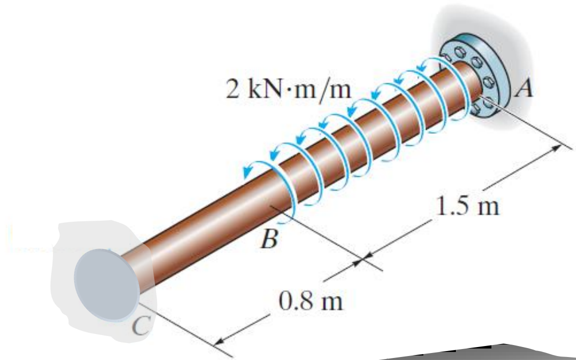
a) The twist angle of point A.

b) The maximum shear stress that developes in each shaft.

c) The rotation angle of the end E.



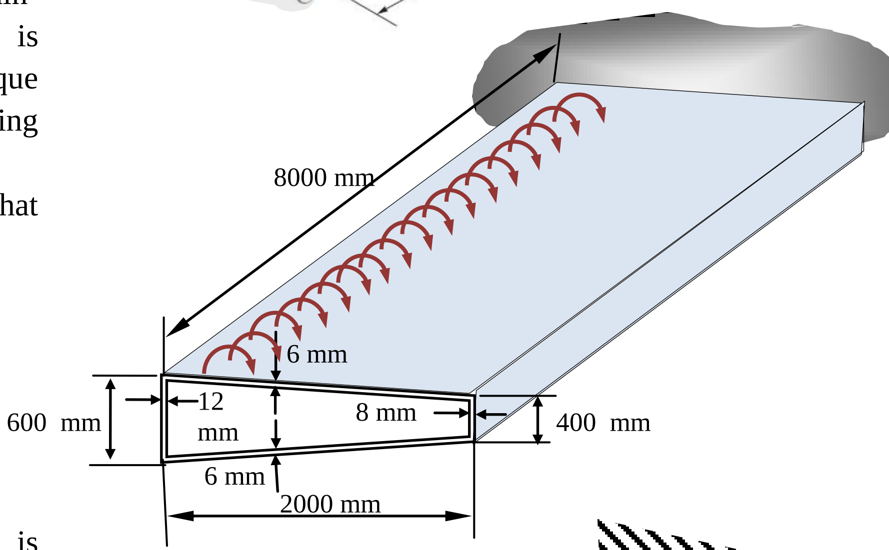
QUESTION 2. For the shaft shown in the figure, determine the support reactions that appear at the ends, A and C.



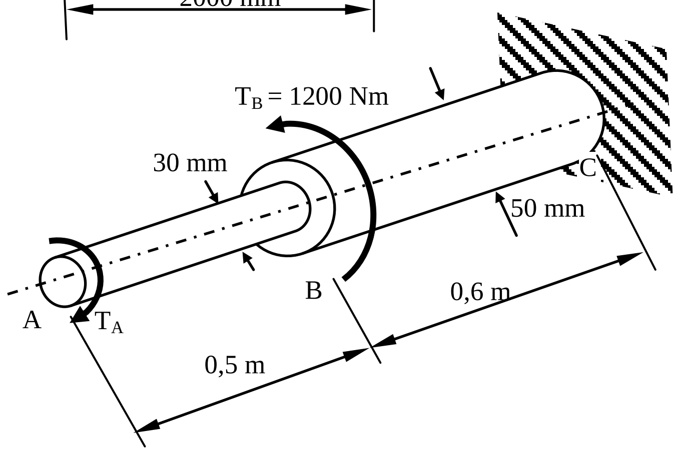
QUESTION 3. An aircraft wing that is made of 2014-T6 aluminum is modeled as a thin-walled tube as shown in the figure. It is assumed that a uniformly distributed torque of 500 kN.m/m is applied along the wing span. Determine

a) The maximum shear stress that developes in the wing.

b) The twist angle of the wing tip.

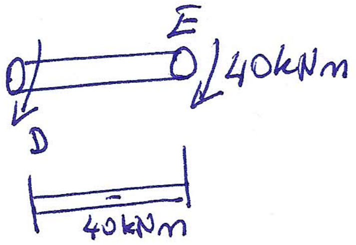


QUESTION 4. A torque of 1200 N.m is applied to the steel shaft AC that has a stepped cross-section as shown in the figure. The shafts AB and BC are connected to each other by a fillet of radius 6mm. If the allowable shear stress of the material is τ_{allow}=120 MPa and if the twist angle of the end A must not exceed 0,02 radian, then determine the maximum torque value, T_A that can be applied to end A.



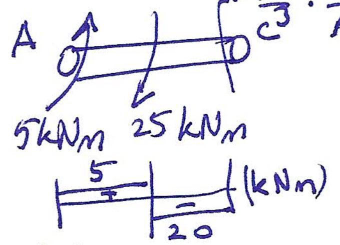
Homework 3

1)



$40\,10^3\,kNmm = Fr = F.300 \qquad F = \frac{400}{3}\,kN$

$\frac{400}{3} \cdot \frac{150}{1000} = 20\,kNm$



$G = 75\,GPa$

$J = \frac{\pi}{2}\,50^4 = 9817477\ mm^4$

$GJ = 7.36\,7\,10^{11}\ Nmm^2$

$\tau = \frac{T}{J}r = -\frac{40\,10^6}{\frac{\pi}{2}r^4}r = -203.72\,MPa$

$\tau = -\frac{20\,10^6}{\frac{\pi}{2}r^4}r = -101.86\,MPa$

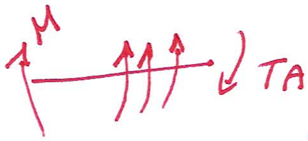
$\phi_E = \frac{5\,10^6}{GJ}800 - \frac{20\,10^6}{GJ}800 - \frac{40\,10^6\,10^3}{GJ}$

$= -0.0706\,rad = -4.046^\circ$

$x = r\tan\phi = 50.\tan 4.046 = 3.54\,mm$

$\tan\gamma = \frac{3.54\,10^{-3}}{2(0.8)+1} = 1.369\,10^{-3}\,rad = 0.078^\circ$

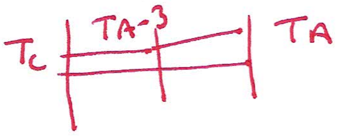
2)



$M + 2x - T_A = 0 \qquad T_A - 2x = M$

$-T_A + 3 + T_C = 0$

$T_A = 3 + T_C$



$0 = \frac{1}{GJ}\left[\int_0^{1.5}(T_A - 2x)\,dx + (T_A - 3)\,0.8\right]$

$T_A(1.5) - 2.25 + T_A\,0.8 - 2.4 = 0$

$T_A = 2.02\,kNm \qquad T_C = T_A - 3 = -0.98\,kNm$

3) $T = 500.8 = 4\,10^3\,kNm$

$\tau_{avg} = \frac{T}{2tA_m} \qquad A_m = \frac{400+600}{2}\,2\,10^3 = 10^6\,mm^2 \qquad G = 27\,GPa$

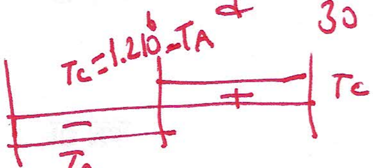
$\tau_{avg} = \frac{4\,10^9\,Nmm}{2.6.10^6} = 333.3\ MPa \qquad \phi = \frac{TL}{4A_m^2G}\oint\frac{ds}{t}$

$= \frac{4\,10^9\,8\,10^3}{4(10^6)^2\,27\,10^3}\left[\frac{600}{12} + 2\frac{\sqrt{100^2+200^2}}{6} + \frac{400}{8}\right] = 0.2274\,rad$

$= 13^\circ$

4) from Fig 5.32 $\quad \frac{D}{d} = \frac{50}{30} = 1.67 \qquad \frac{r}{d} = \frac{6}{30} = 0.2 \qquad \frac{1.3 - 1.2}{4} = \frac{1}{40} \qquad K = 1.2 + \frac{1}{40}$

$K = 1.225$



$G = 75\,GPa$

$\phi = 0.02 = -\frac{T_A\,0.5\,10^3}{75\,10^3\,\frac{\pi}{2}\,15^4} + \frac{(1.2\,10^6 - T_A)\,0.6\,10^3}{75\,10^3\,\frac{\pi}{2}\,25^4}$

$T_A = -62737\,Nmm$

$\tau = K\frac{T}{J}c = 1.225\frac{62737}{\frac{\pi}{2}15^4}.15 \qquad \tau = 14.5\,MPa$