INTERNAL GEODYNAMICS

(ENDOGENOUS PROCESSES OF THE EARTH)

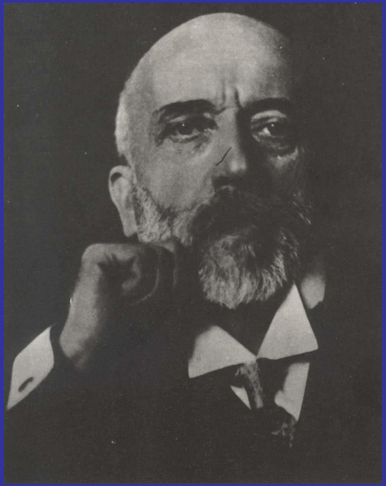
İÇ JEODİNAMİK

(DÜNYANIN ENDOJEN OLAYLARI)

A. M. C. Şengör

İTÜ Avrasya Yerbilimleri Enstitüsü

2005 Kış yarıyılı



To the memory of Andrija Mohorovicic

Lesson 2:

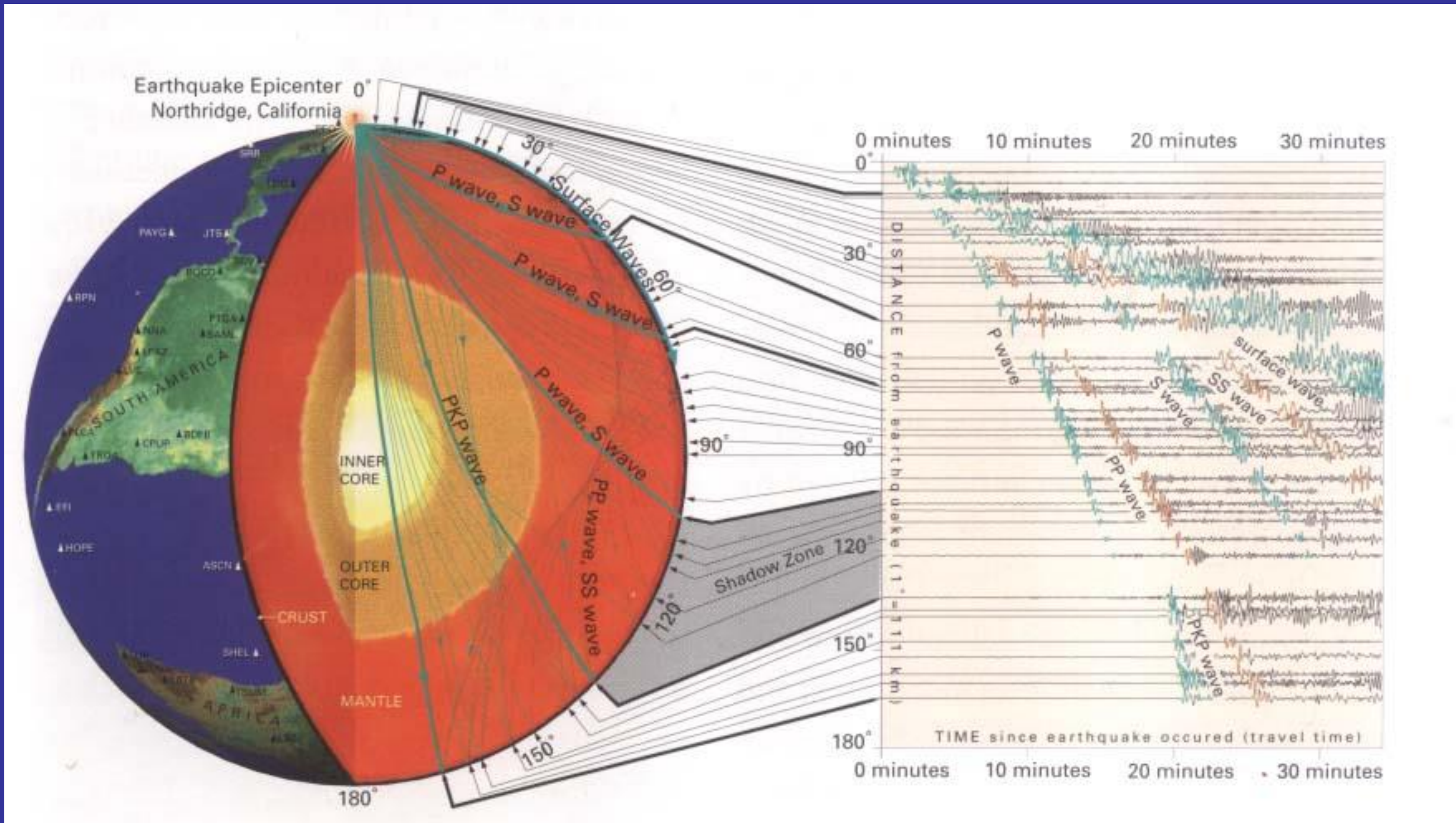
The Earth As A Planet

Part II: The structure of the earth —the contribution of seismology

To summarize: When the twentieth century opened, we still had only a very vague idea of the internal structure of the globe. It was in the first three decades of the twentieth century that the picture was greatly clarified owing to the developments in seismology.

Seismology is the science of earthquakes (and similar phenomena). Today we can perhaps define it as the science of elastic waves in rocky planets (as we now also do seismology on the Moon!)

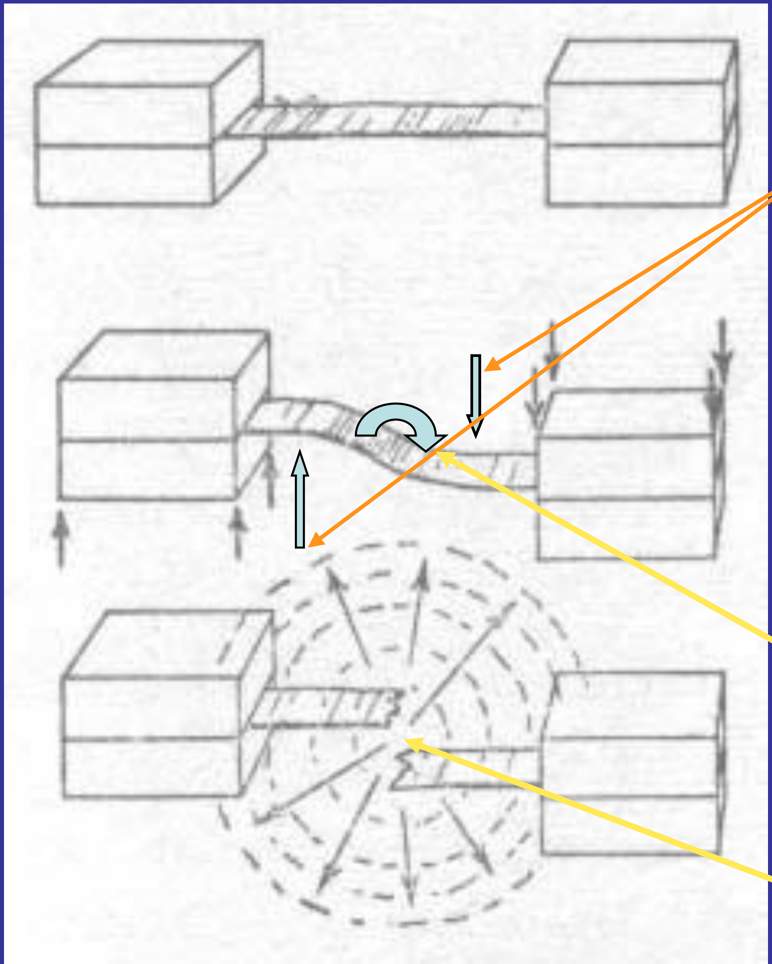
How has seismology helped us to understand the interior of the earth?



Earth interior as understood on the basis of the paths and travel times of earthquake waves.

What are these waves? How do we recognise them? How do we know what they encounter on their parths?

In 1849 Sir George Gabriel Stokes (1819-1903) showed that sudden disturbance of a solid creates two kinds of body waves (In gases and liquids only one kind is generated).

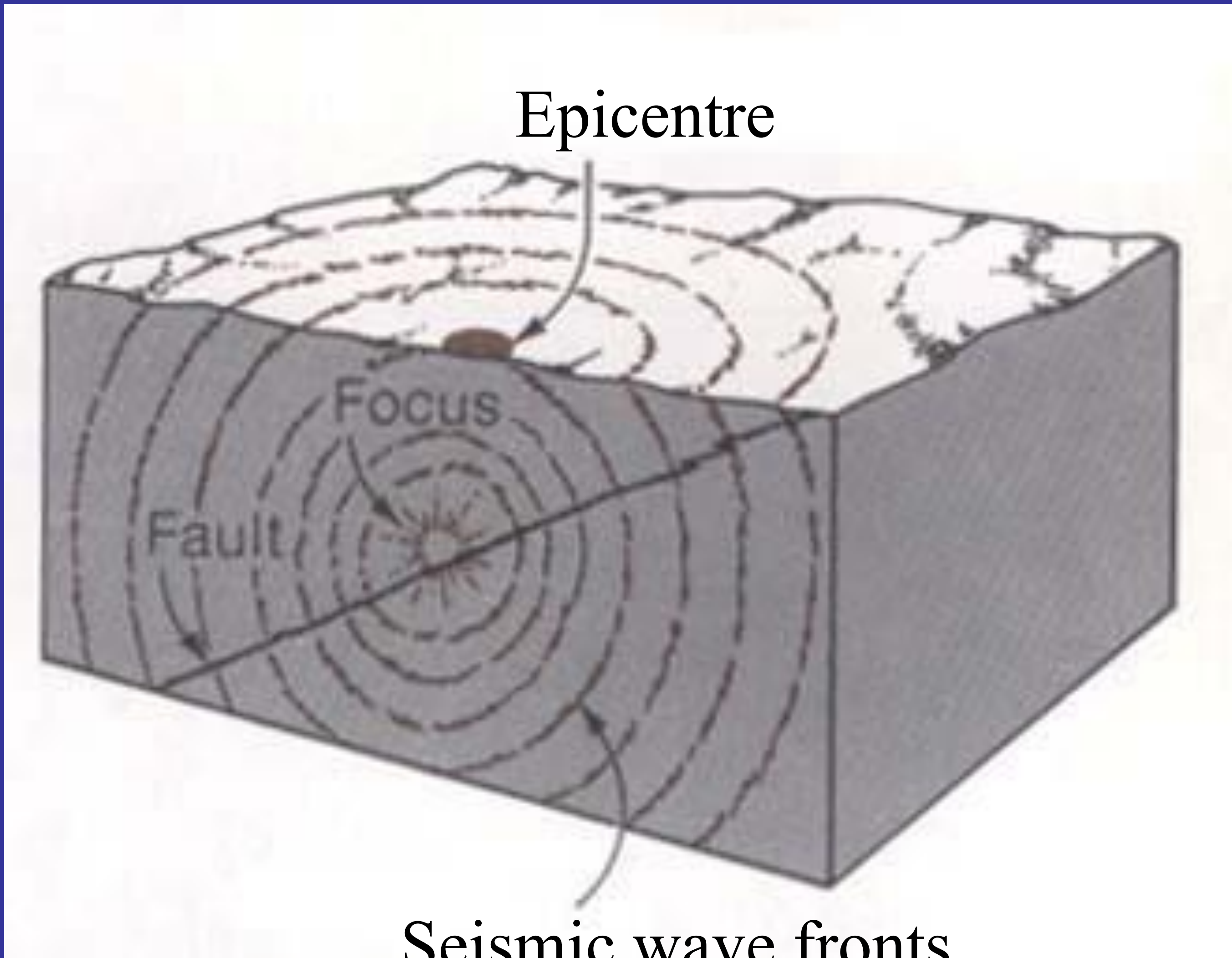


Longitudinal waves are created thus

Transverse waves are created thus

EARTHQUAKE FOCUS

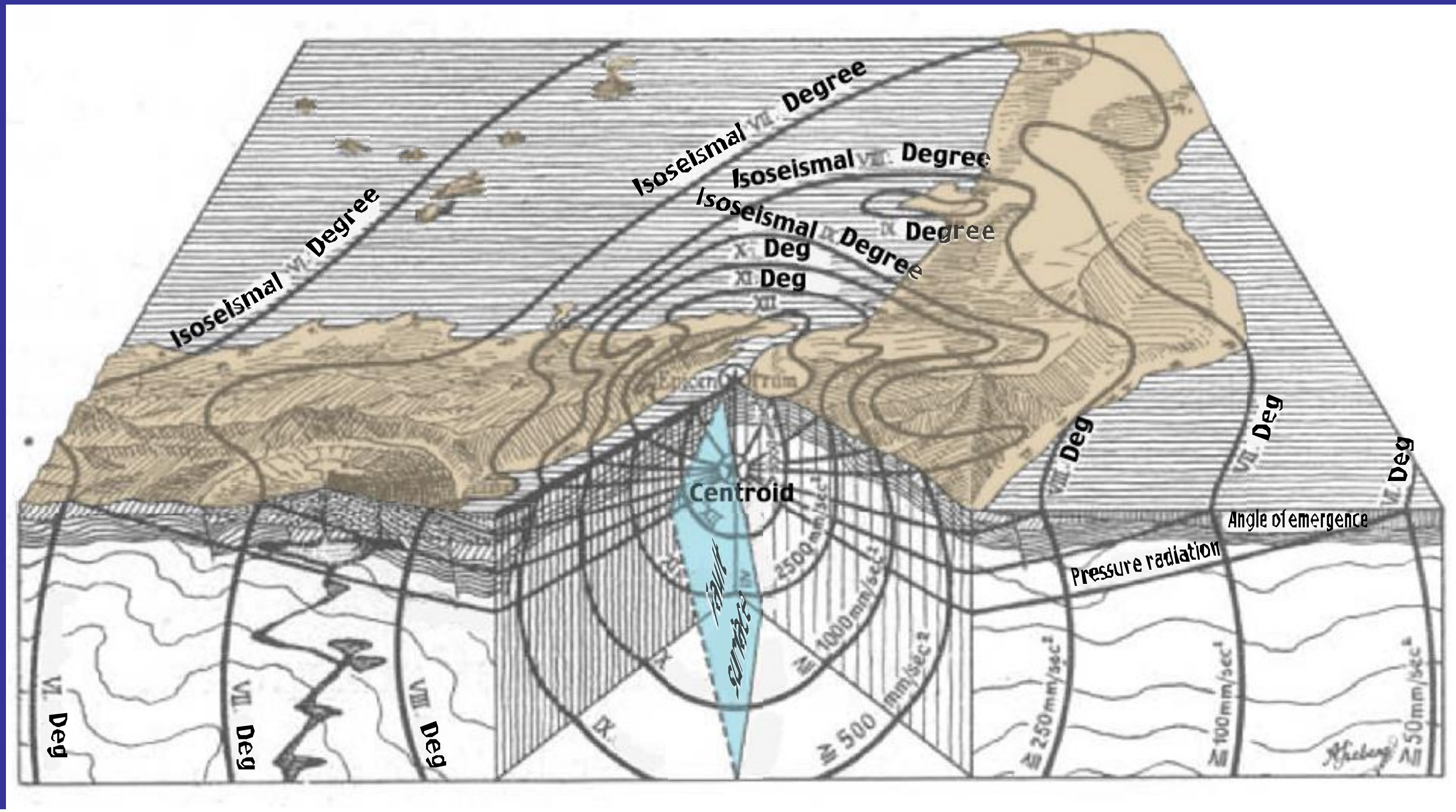
(From Erinç, 1968, after Strahler, 1965)



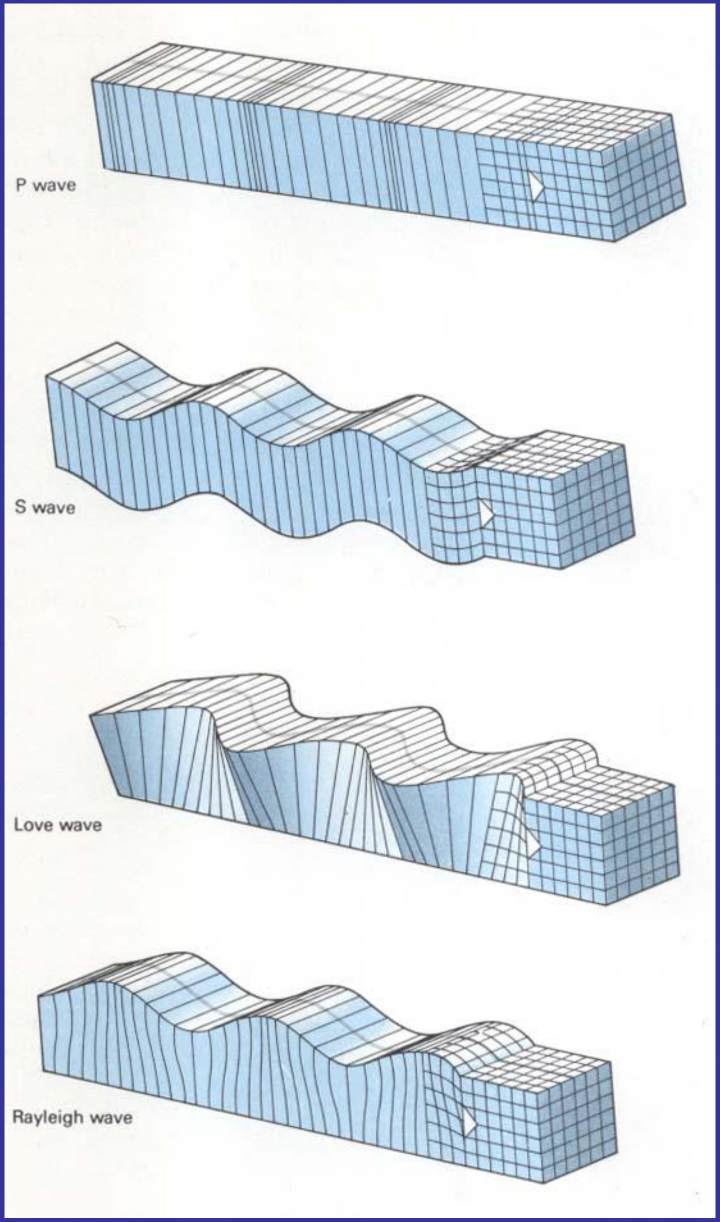
**Prof.
Herbert Hall
Turner**

Focus (=***hypocentre***) and ***epicentre*** and how siesmic waves travel away from the focus (from Press and Siever, 1974). The point at the antipodes from the epicentre is called the ***anticentre*** by the British astronomer and seismologist H. H. Turner (1861-1930) of Oxford.

Centroid: Center of a polygon defined as its centre of gravity (see Appendix I for the calculation of the centroid of any surface)



The concepts of centroid, isoseismal lines (or simply isoseismals) and angle of emergence illustrated on the example of the great Messina earthquake of 28th December 1908 (from Sieberg, 1927, Fig. 213)



From Smith 1981

BODY WAVES

Longitudinal waves cause transient changes in volume and in shape

Transverse waves cause transient changes only in shape

SURFACE WAVES

Earthquakes also cause surface waves that are of two distinct kinds: Love waves are transverse, whereas Rayleigh waves are of longitudinal type. Love waves are only possible in an inhomogeneous medium, in the layers of which the velocity of S waves are different.

The British geologist Richard Dixon Oldham (1858-1936) first emphasised in 1900 that there were three distinct wave phases corresponding with the P, the S and the L waves (he simply termed them as the first, second and the third phases). The first phase he interpreted an "compressional" waves and the second phase as "distortional" waves, interpretations that remain valid today.

However, already in von Reuber-Paschwitz's 1889 report on the 18^{th} April 1889 Tokyo earthquake, one can discern the three distinct phases. Appendix II to this lesson reproduces the historical 1889 paper by von Reuber-Paschwitz.

What determines the velocity of a wave?

For waves in general, the velocity increases as the restoring force from a given deformation increases (as the medium is elastic, so there has to be restoration), but it decreases as the mass increases. So the velocity is proportional to the restoring force and inversely proportional to the mass displaced.

$v \propto F_r$ and $v \propto 1/m$, but force per unit area is stress and mass per unit volume is density, so we can generate an equation:

$v_{wave} = \sqrt{\sigma/\rho}$ (here the square root operation simply stands for a constant to replace the $\propto$ sign with = sign)

The S-waves only change shape, so they only depend on what is called the shear modulus (μ), which is simply force per area times change of shear angle; by contrast the longitudinal waves also involve a compressibility modulus (κ), which is density/decrease of volume, i.e. $\rho/(dV/V)$ (from Musset and Khan, 2000)

The longitudinal bodywaves travel faster than the transverse bodywaves, because the velocity of longitudinal body waves depends both on the shear modulus (that determines the degree of change of shape) and the modulus of incompressibility (that determines the degree of change of volume).

Thus, the velocity (v_l) of a longitudinal body wave is calculated as follows:

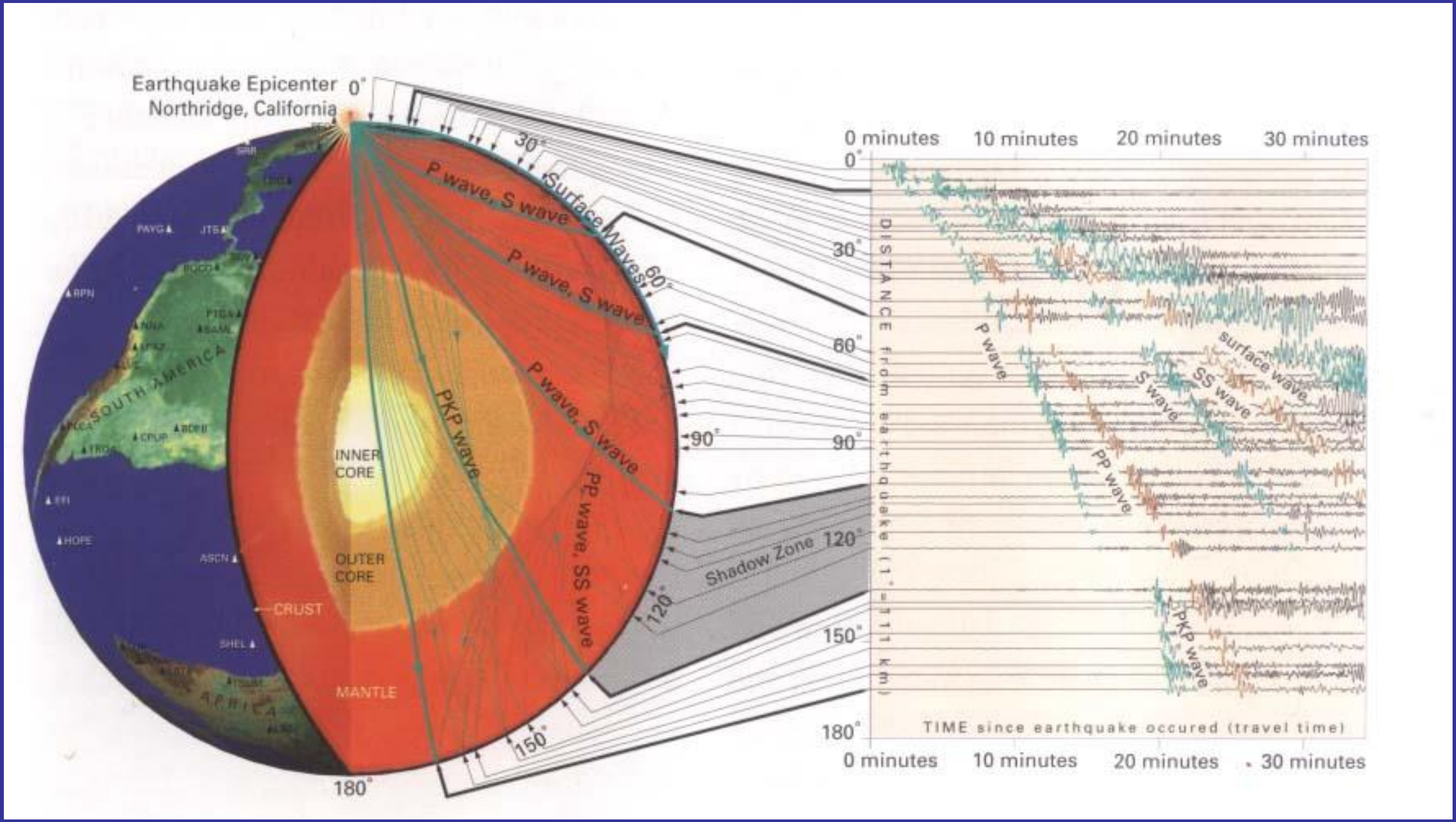
$$v_l = \sqrt{(\kappa + 1.33\mu)/\rho}$$

Where k is the modulus of incompressibility, μ is the shear modulus and ρ is density

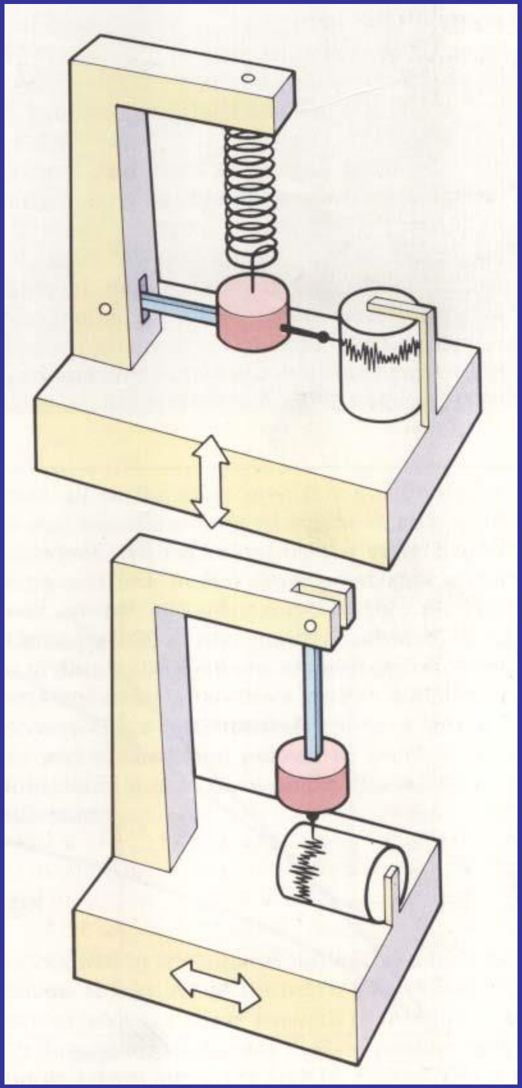
By contrast, the velocity of the transverse waves, v_t, depends only on the density and on the shear modulus:

$$v_t = \sqrt{\mu/\rho}$$

Thus, when an earthquake happens, the longitudinal and the transverse waves travel with different velocities. This has wonderful consequences for locating earthquakes.



When an earthquake happens somewhere on earth, waves radiate radially from it and reach seismometers on various parts of the earth.

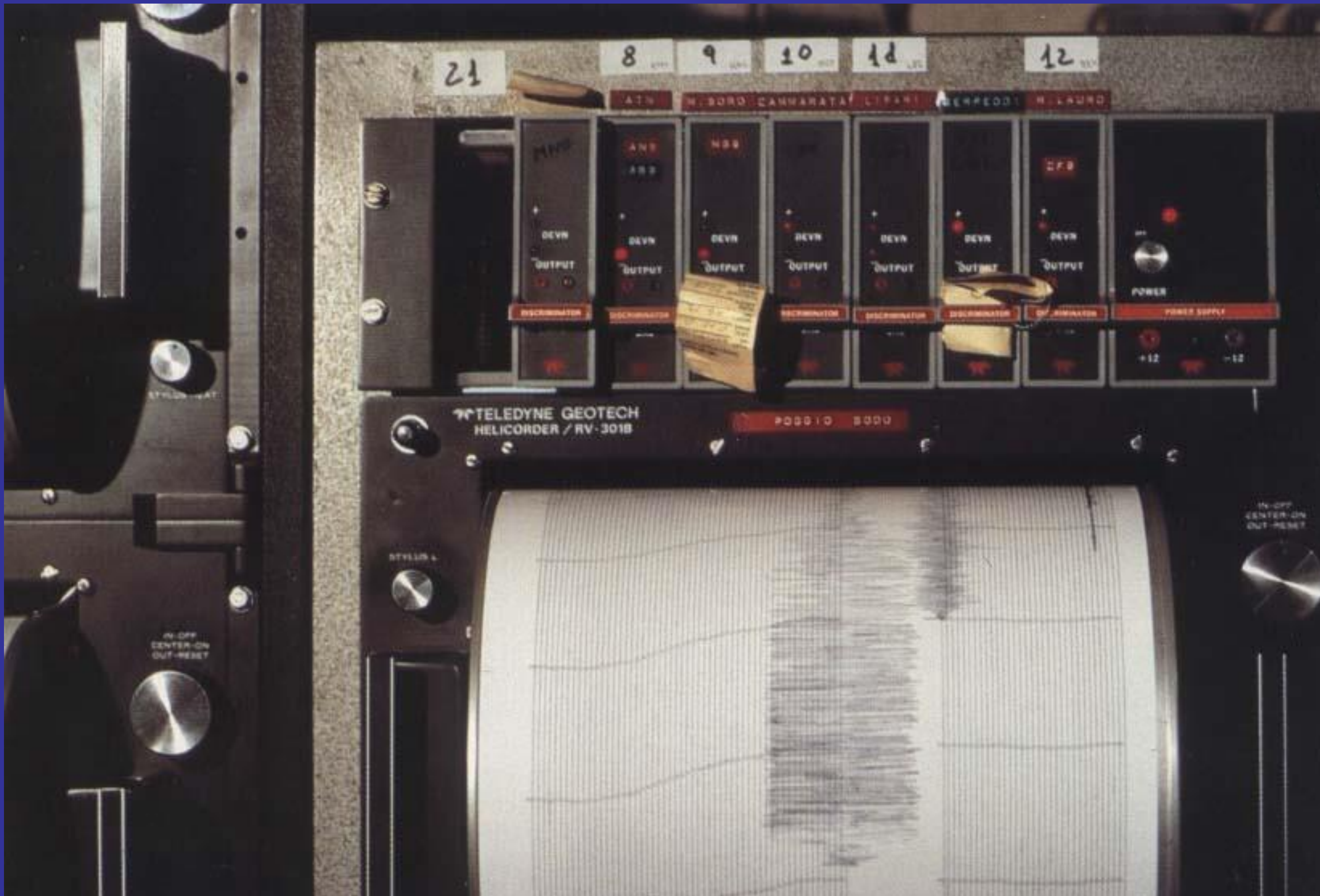


Horizontal (above) and vertical (below) seismometers (from Smith, 1981)

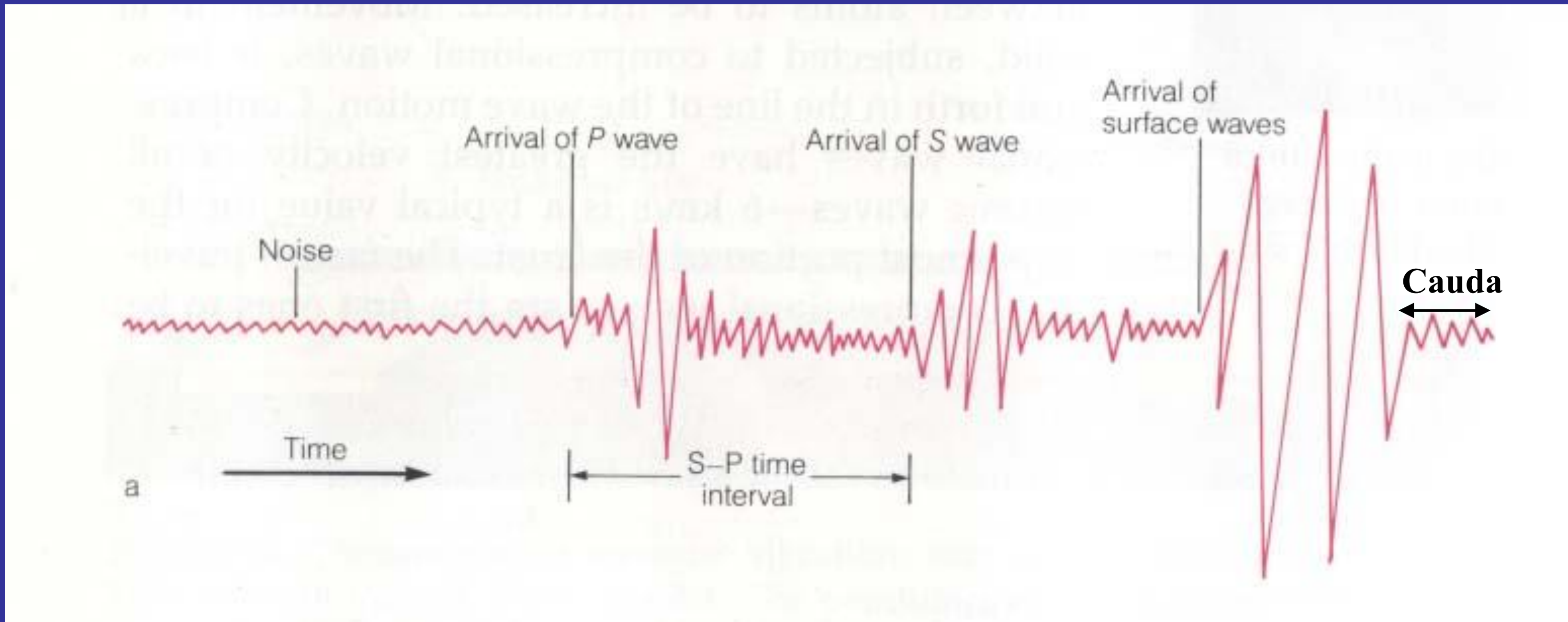
Earthquake waves are sensed and recorded by instruments called seismometers.

Ernst von Rebeur-Paschwitz first noted in 18th April 1889 that certain disturbances of horizontal seismometers at the Telegrafenberg in Podtsdam near Berlin could be explained by the occurrence of an earthquake in Tokyo 9000 km away. This was the birth moment of tele-seismology!

Von Rebeur-Paschwitz, E., 1889, The earthquake of Tokio, April 18, 1889: *Nature*, v. 40, pp. 294-295.

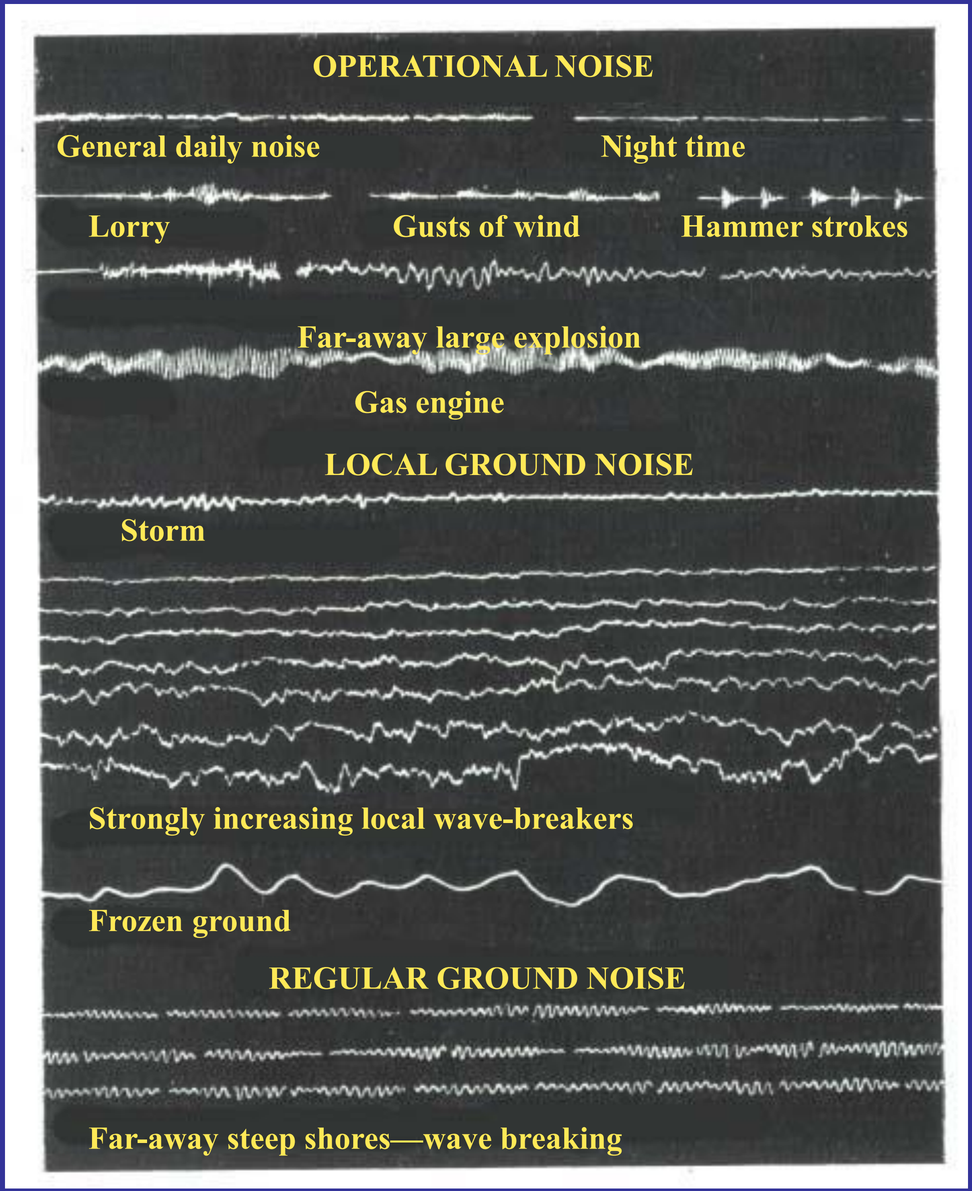


A modern seismometer recording an earthquake of magnitude 4.6 near Potenza, Italy recorded in the Ettore Majorana, Erice, Italy



This is what a seismogram looks like. There is always some disturbance recorded. This is called "noise." When an earthquake happens somewhere on earth, first the longitudinal body waves arrive and that is why they are called the *primae undae* (primary waves or just P-waves). After a time of quiescence, the *secundae undae* (secondary waves or S-waves) arrive. Then the surface waves with large amplitudes and periods come, which are collectively called *longae undae* (long waves or L-waves). Finally the *cauda* (=tail in Latin) or *coda* (=tail in Italian) waves terminate the seismogram (first used by Sir Harold Jeffreys in 1929).

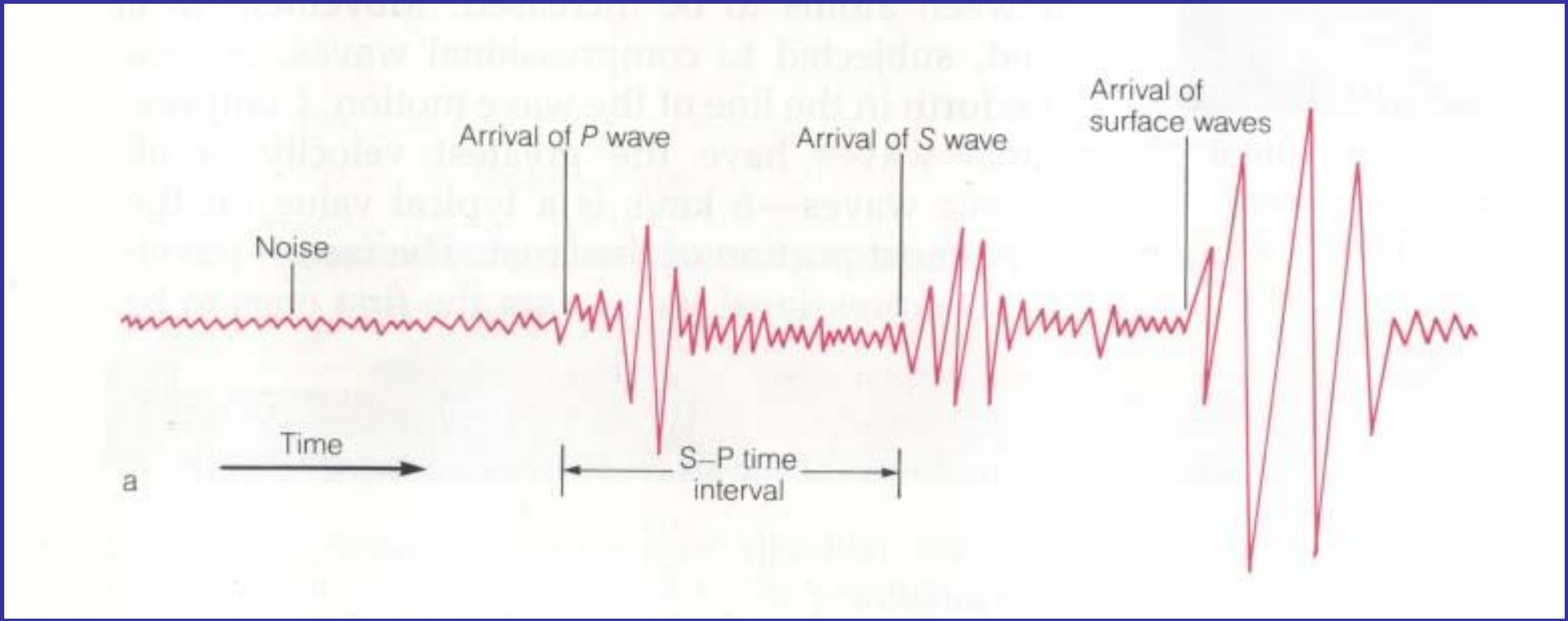
First, let us look at the nature of the "noise"



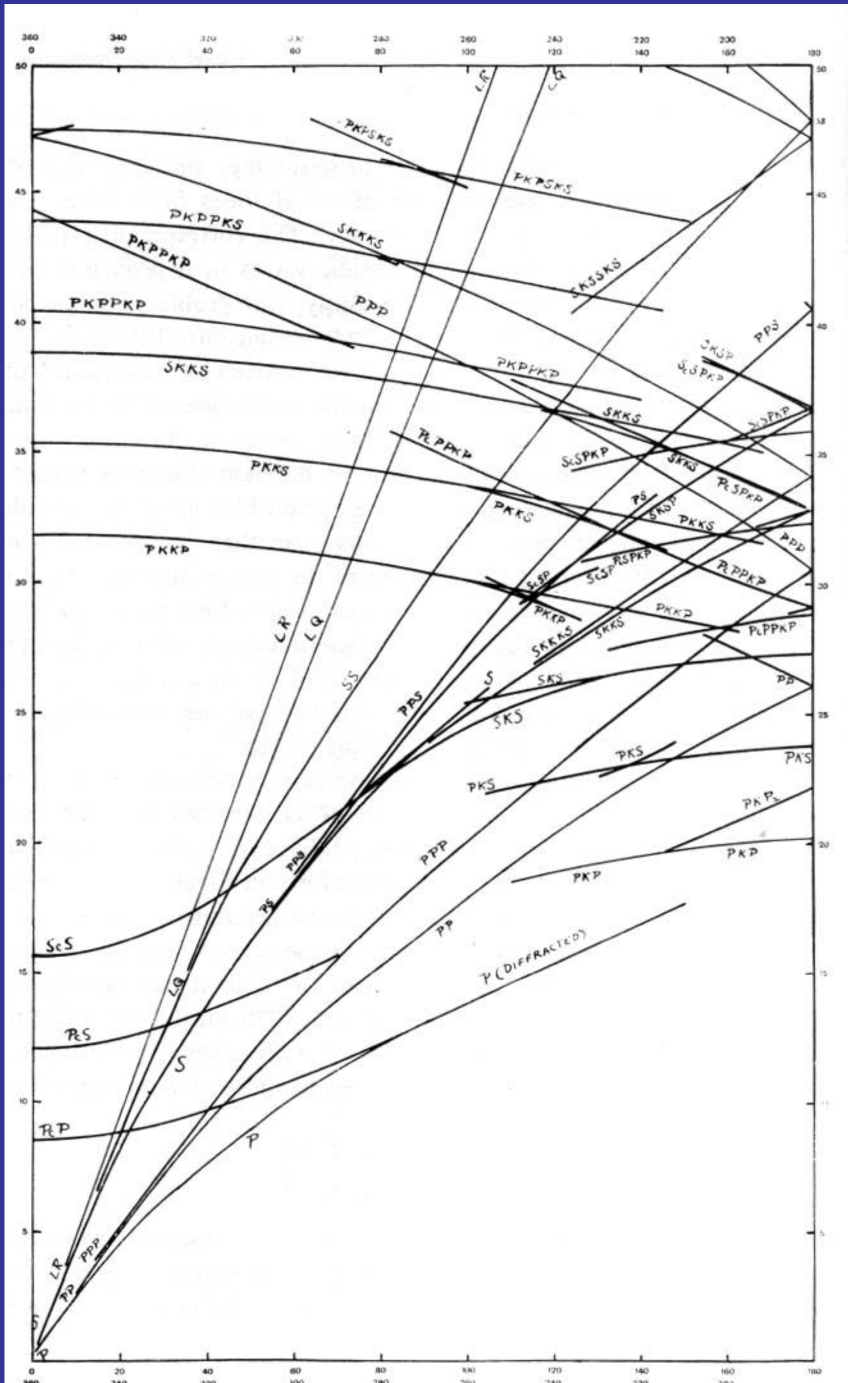
From Sieberg, 1927, Fig. 214

During his doctoral studies, for example, Beno Gutenberg found that the surf in Norway, caused the microseisms in Göttingen! Microseisms are also important in measuring gravity. In Holland, in the reclaimed areas, it was not possible to measure the gravity because of the microseisms until Vening Meinesz developed a suitable gravimeter with two pendulums swinging at 90˚ to one another.

So don't think that the "noise" you see in seismographs is unimportant! It carries a great deal of information.



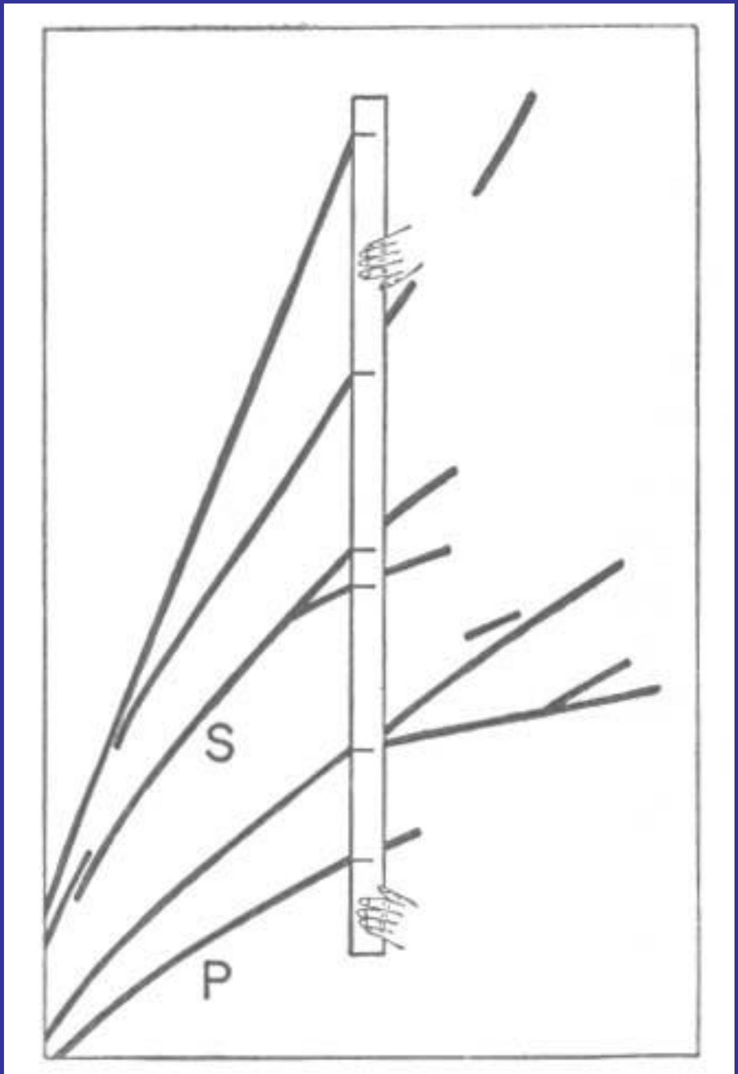
Because of their different velocities, the P and the S waves arrive on our seismographs at different times. The farther away our seismometer is from the epicentral area, the greater is the time gap between the P and S wave arrivals. If we knew the velocity of the waves, we can calculate exactly how far the earthquake was or, conversely, if knew how far the earthquake was, we can calculate the velocity of the waves.



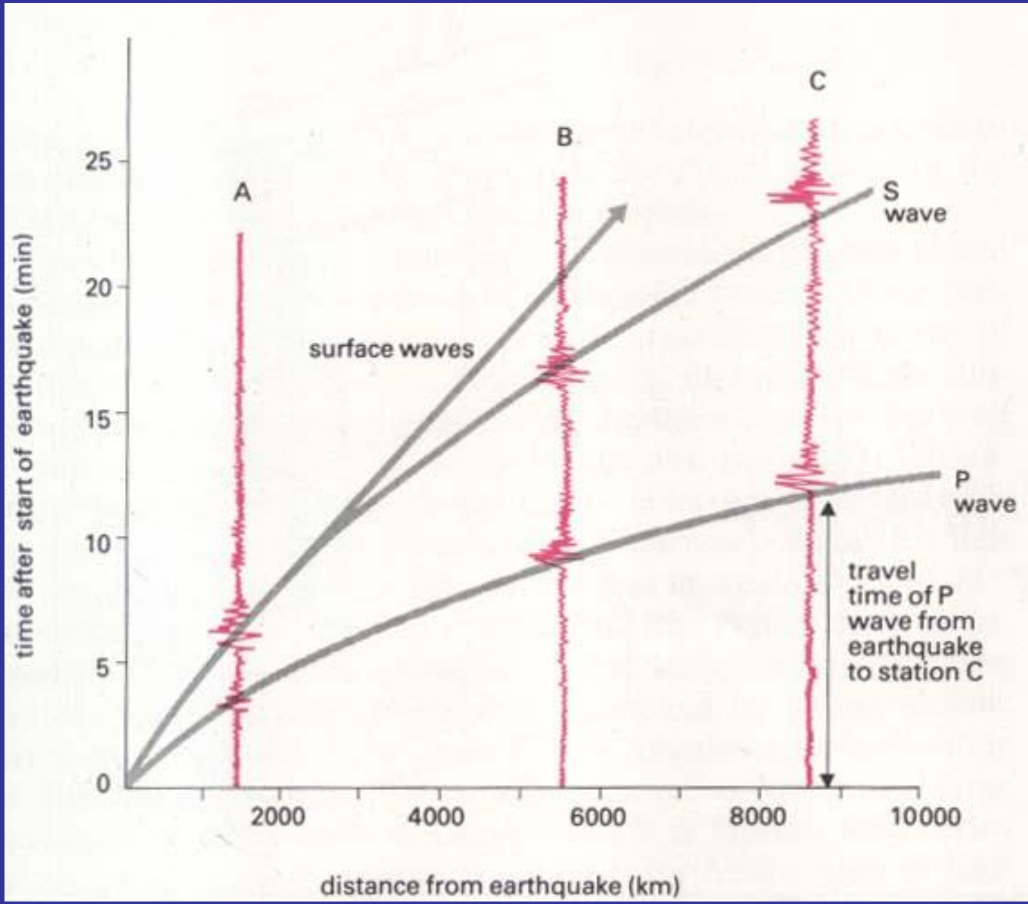
To begin with seismologists certainly did not know the velocities of the seismic waves. So they started with very well-located earthquakes and tried to construct travel time curves (or “transit time” curves) for various wave groups.

They started with an assumed source and an assumed time and checked the records of seismometers at known distances and computed travel times. Then using these they located other sources and went back and forth until the travel time curves did not change much.

Travel time curves for surface focus by Jeffreys and Bullen (from Båth, 1979, Fig. 39)



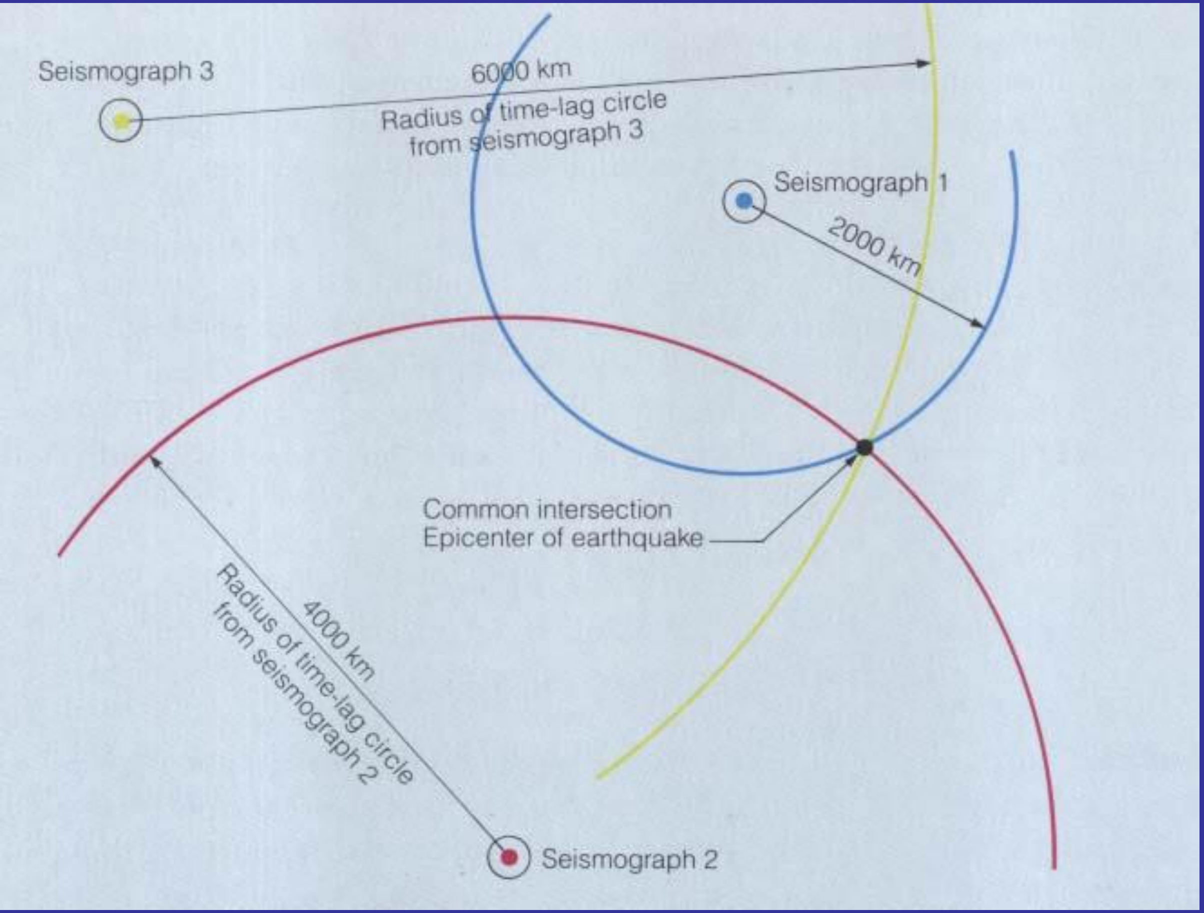
After reading the seismogram and identifying our wave trains, we fit them to the travel-time curves to obtain the distance to the focus.



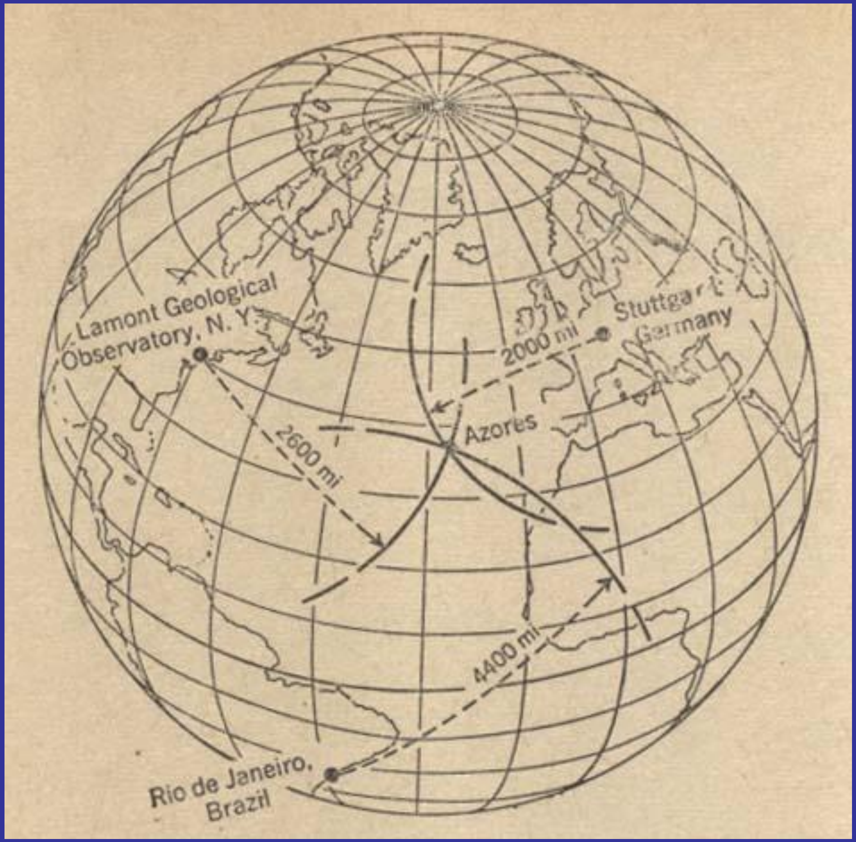
Three seismograms fitted to travel-time curves and the distance read off from them.

From Richter, 1958 and Smith, 1981

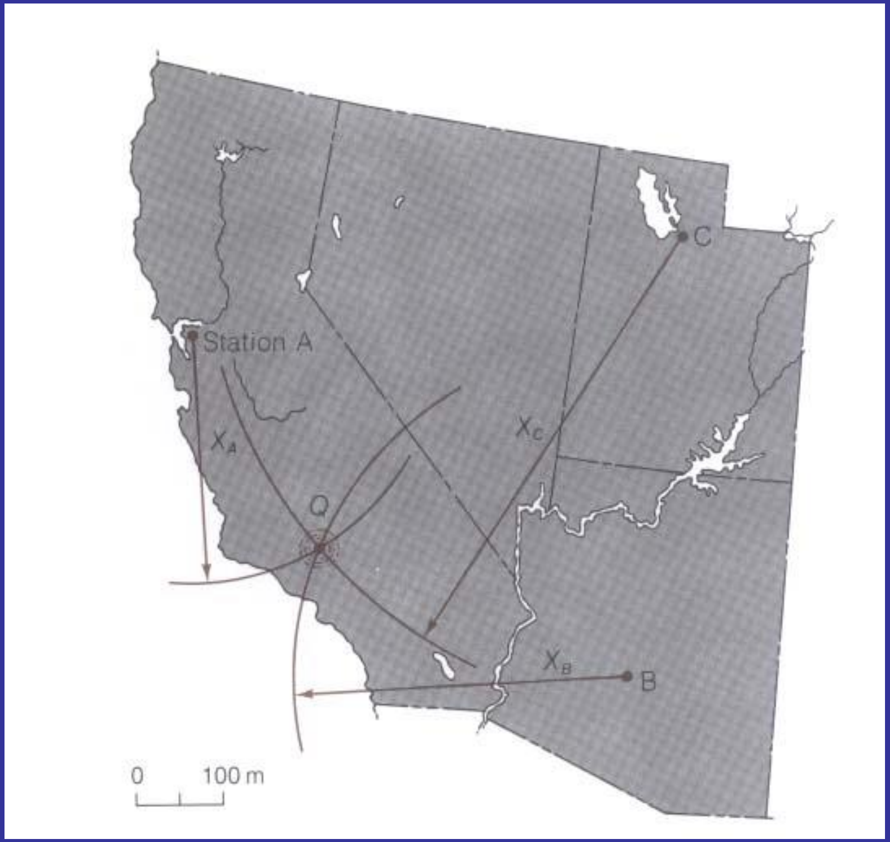
Knowing how far an earthquake is, is not sufficient to localise it. All we can say is that it is located on the periphery of a circle, at the centre of which our seismometer is located. It is thus clear that to know the point location of an earthquake we need at least three seismometer stations:



From Skinner and Porter 1987

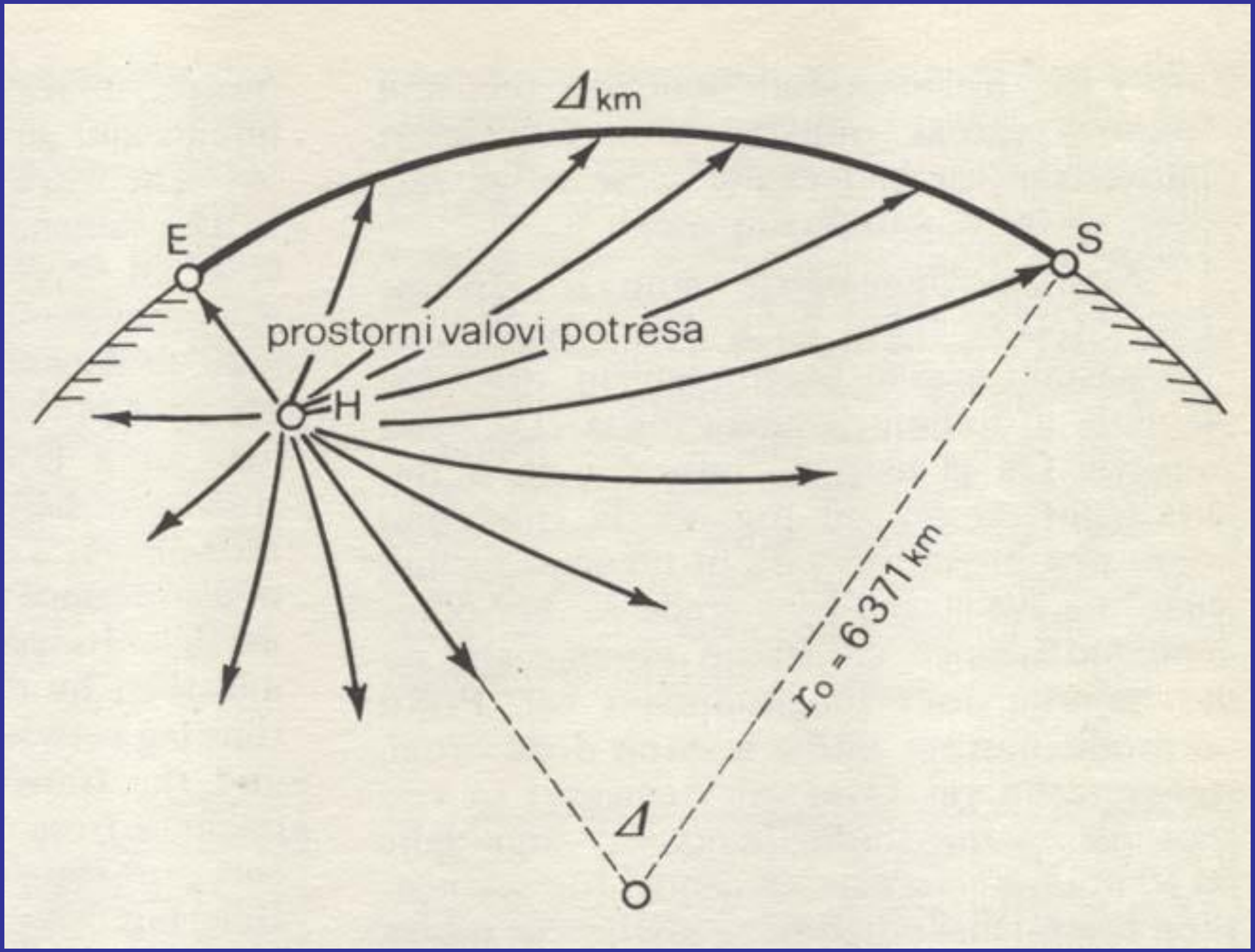


Global



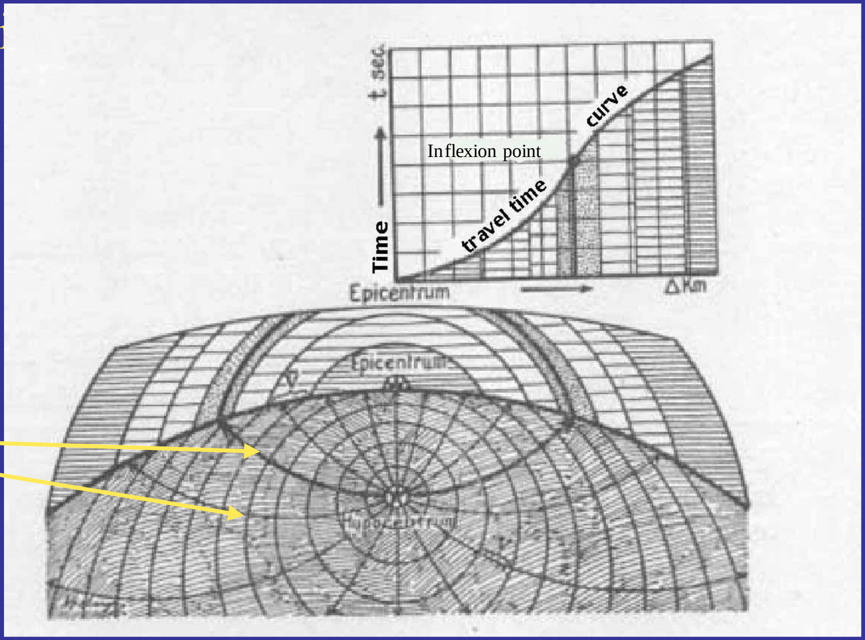
Local

There is no difference between locating earthquakes globally and locally



In the earth, density and elasticity (the constants μ, the shear modulus, and k the modulus of incompressibility or bulk modulus) both increase with depth. As elasticity increases faster than density, the wave velocities increase also and they curve upwards (this is known as *Schmidt's Law* after the German seismologist August Schmidt). This gives them a 'convex-down' aspect as seen above. If the earth had no concentric layers of different density and elasticity, all seismic ray paths would have been like the ones depicted above. (H=hypocentre)

According to Schmidt, the earthquake area is divided into two zones. In the zone near the epicentre the apparent surface speed decreases, while in the zone away from it the speed infinitely increases, but the intensity decreases next to zero. According to Schmidt's Law, the lowest surface speed occurring at the borderline between the two zones should be a "benchmark for measuring the propagation speed of the earthquake waves in the dark depth of the epicentre". The turning points on both sides of the conchoidal earthquake "hodographs" converge in the same degree as the earthquake centre is near to the surface and the smaller the inner zone of the earthquake area i

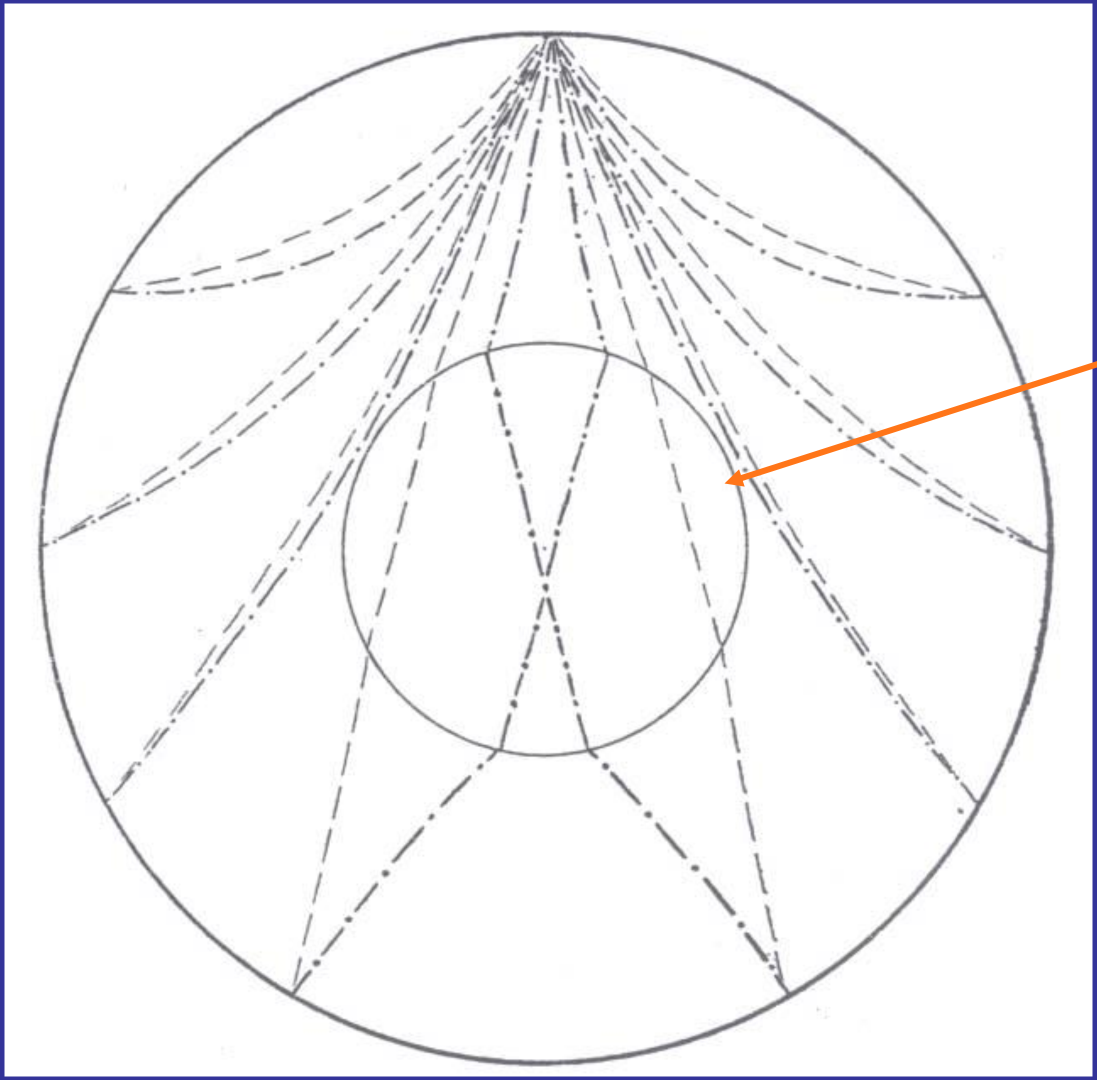


Hodographs

From Sieberg, 1927

SCHMIDT, A., 1888, Wellenbewegung und Erdbeben. Ein Beitrag zur Dynamik der Erdbeben: *Jahresheft 1888 des Vereins für vaterländische Naturkunde in Württemberg*, pp. 249-270

In 1906 the British geologist Richard Dixon Oldham (1858-1936) noticed that what he interpreted as both P- and S-waves arriving near the anticentre of earthquakes invariably arrived substantially later than expected from the velocities in travel-time tables that he first generated. He thus concluded that the waves must be slowed down during transit. He interpreted this in terms of a core. He noticed that the waves emerging 120˚ away from the epicentre do not touch it and those emerging at 150˚ penetrate it deeply. So he fixed the radius of the core at 0.4 of the earth's radius (which comes to 2548.4 km) But his idea of the core was no more precise than that of Wiechert as he could not locate a sharp boundary (although he repeatedly emphasised that the transition occurred quickly). He simply knew that "something" substantially slowed down the waves and he thought that that "something" had a ratio of μ/κ 1/3 of that of the outer layer (which we now call the mantle).

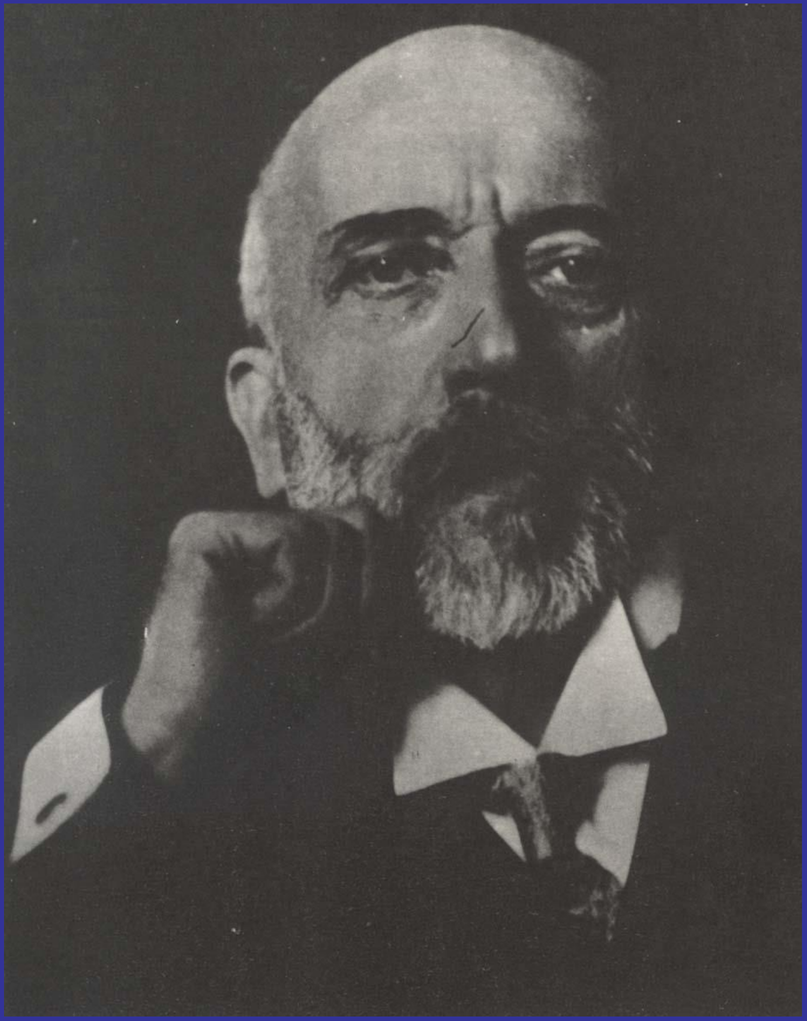


Oldham did not think this was a sharp boundary, but he thought that the transition across it was quick.

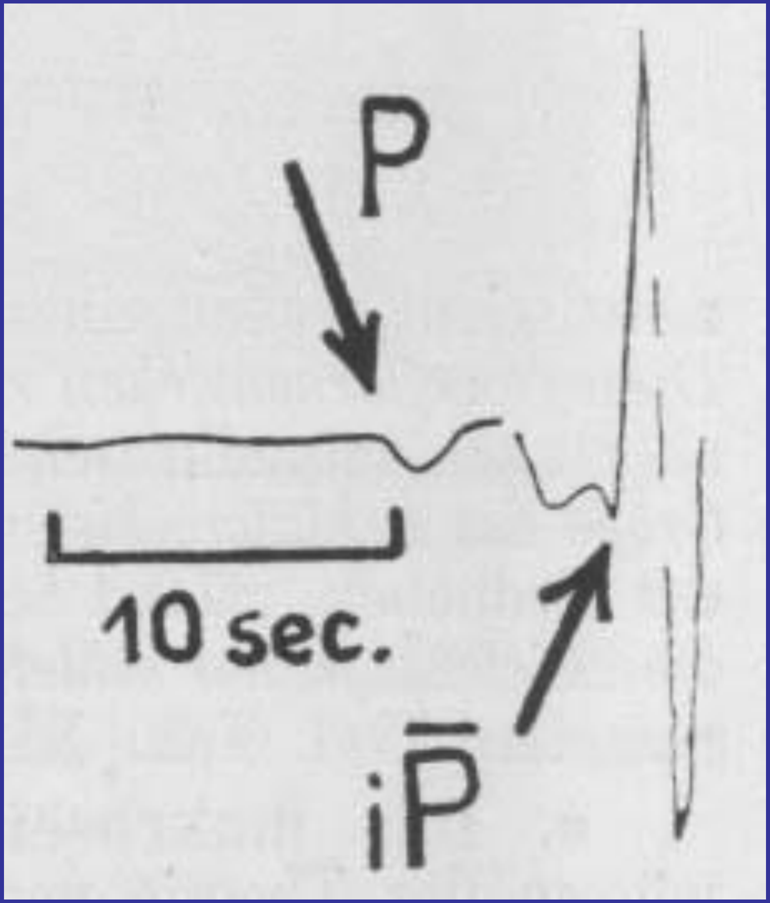
Neither did he speculate as to its nature and composition (at the time ideas ranged from a gas core to an iron core!)

Oldham's core. From Oldham, R. D., 1906, The constitution of the interior of the earth, as revealed by earthquakes: *Quarterly Journal of the Geological Society of London*, v. 62, pp. 456-475.

The first sharp boundary in the earth was identified by the Croatian meteorologist and seismologist Andrija Mohorovicic (1857-1936) an almost exact contemporary of Oldham.



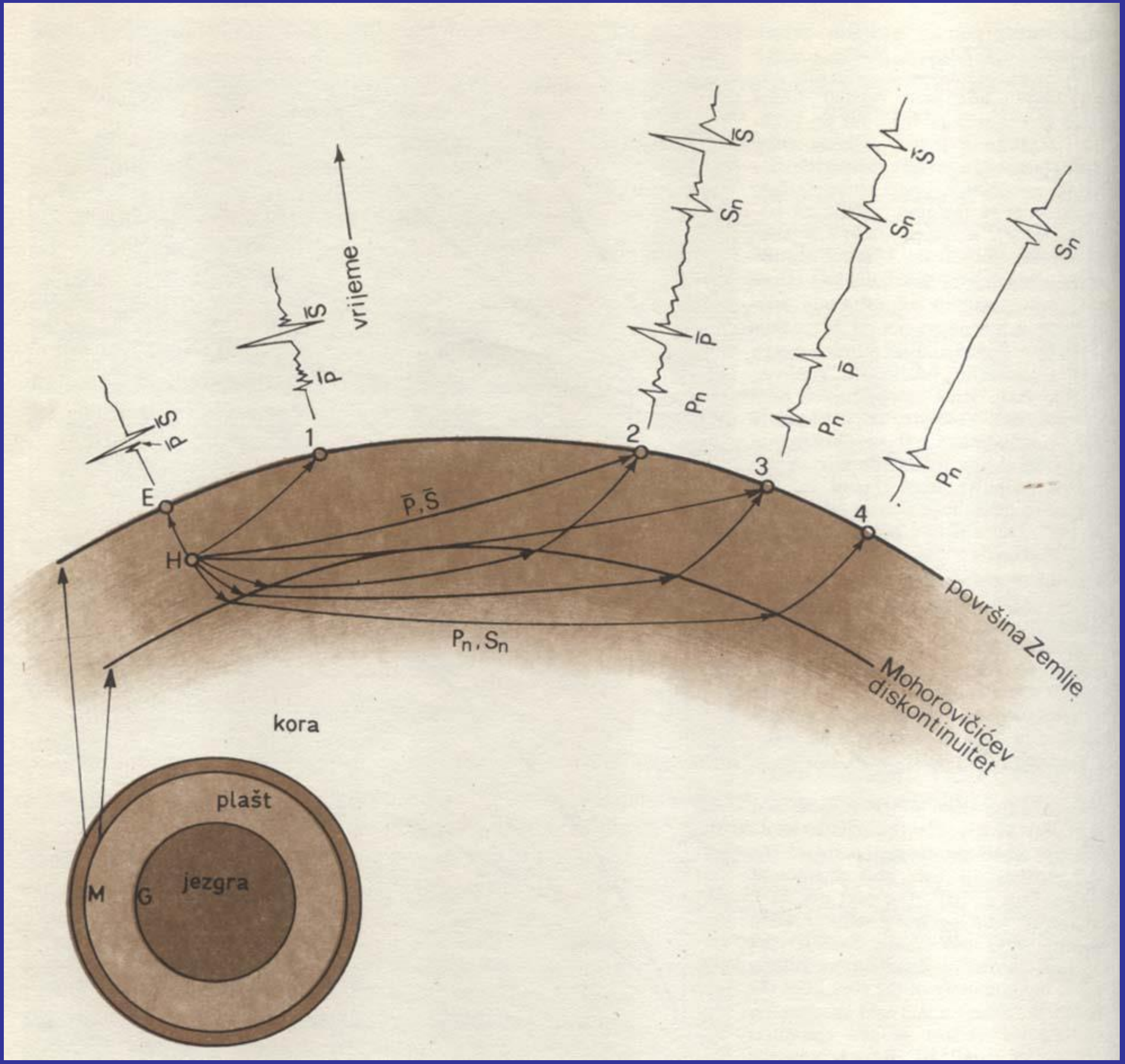
Andrija Mohorovicic
(1857-1936)



P and i$\bar{P}$ (now also written as Pg) as observed by Mohorovicic after the 8th October 1909 Kupa Valley earthquake.

Here we see clearly two P-wave arrivals in the seismograph. Yet we know that only one wave front left the focus. How should the two P-wave signatures be interpreted?

If there are sharp density and elasticity boundaries in the earth, we expect sharp refraction of seismic waves. So Mohorovicic thought that a part of the wave front propagating into the earth's interior must have hit a boundary and got refracted in such a way that a little further it got refracted again and came back to the surface. The velocity below the refracting boundary being higher, one P phase arrived earlier than the other thus giving the impression of having two P waves.

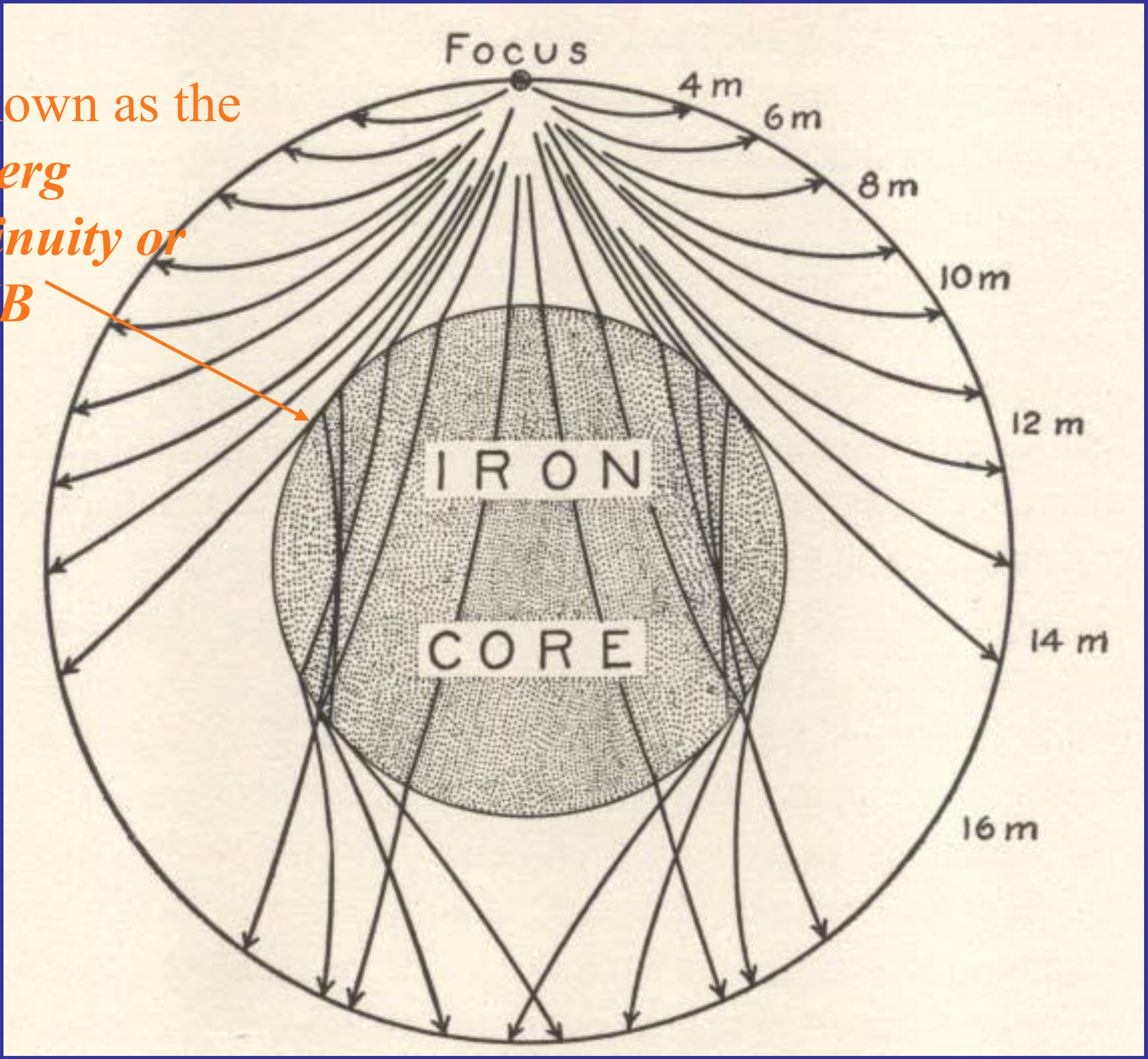


The discontinuity postulated by Mohorovicic prevents the $i\overline{P}$'s arriving beyond station 3 and also enables us to know exactly how deep the discontinuity must be.

In 1910, Wiechert and Geiger developed a formulation to give the velocity of seismic waves simply as a function of depth. This is called the Wiechert-Herglotz Theorem given in Appendix III to this lesson from Sir Harold Jeffreys' *The Earth* (6th edition, 1976). Using it, Beno Gutenberg, one of the founders of modern seismology, found in 1912 that just below the Mohorovicic discontinuity (also called the Moho discontinuity or simply the M-discontinuity) the velocity of a P-wave is 7.7 km/sec. This increases to 13 km/sec. at a depth of 2900 km. Below that depth the velocity sharply drops to 8 km/sec. and then gradually increases again to 11 km/sec. towards the centre of the earth. It was also found that the distortional waves of Oldham (i.e.the S-waves) did not appear around the anticentre, indicating that somehow they did ot propagate through the core suggesting that it is liquid. The sharp boundary of the core is now named the Gutenberg discontinuity or the CMB (i.e. core-mantle bondary).

Beno Gutenberg (1889-1960)
a student of Emil Wiechert in Göttingen and one of the founders of modern seismology. He is said to be perhaps the greatest observational seismologist of all times.

Now known as the ***Gutenberg discontinuity or the CMB***



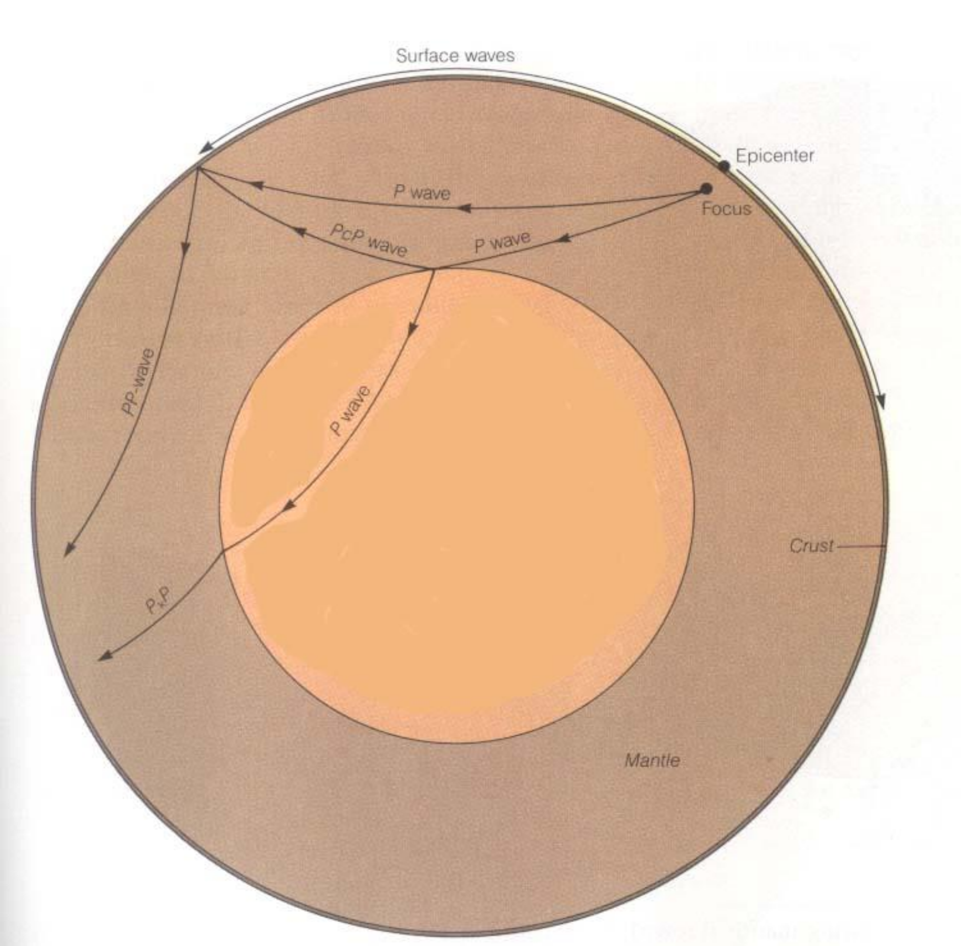
The interior of the earth according to Gutenberg in 1912 (without the crust; the numbers show the travel times of the P-waves in minutes) (after Daly, 1926)

However, one still did not know the physical state of the core. That the S-waves did not propagate through it suggested that it might be liquid, but it was also possible that it was only imperfectly elastic and thus absorbed the S-waves. In 1926, Sir Harold Jeffreys showed, using the earth tides, that the core that Oldham had discovered and Gutenberg had placed at a depth of 2900 km, had to be liquid.

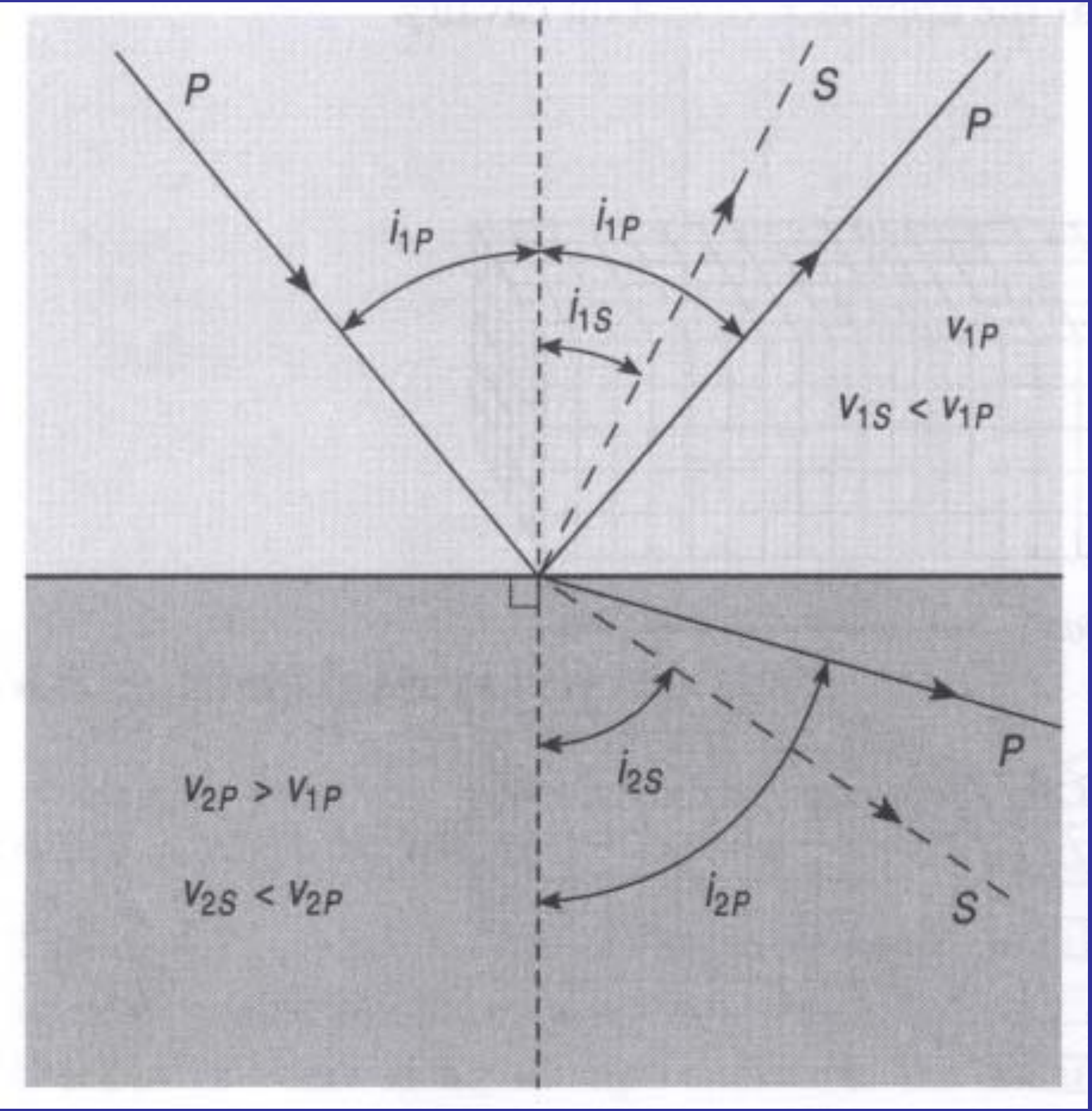
The argument was simple and was taken from an earlier one by Emil Wiechert. Wiechert had shown that the earth tides and the Chandler Wobble showed that the earth could not be as rigid as the S-wave velocities through the mantle suggested. Jeffreys took Weichert's 1897 model with a core with density 8.2 and a radius of about 4970 km and a mantle with a density of 3.2. When he combined these densities with the velocity of the seismic waves he could deduce the rigidiy. He found 5.12×10^{10} Pa for the surface and 15.6×10^{10} Pa for the base of the rocky envelope and 40×10^{10} Pa for the supposed upper part of the metallic core.

These numbers were clearly too high to be reconciled with the deformation of the earth during the tides and the Chandler Wobble (see Appendix IV). He suggested that the core identified by Oldham and Gutenberg had to have a much lower rigidity and that its elastic properties seemed dependent on the periods of the disturbing effects. This in turn agreed with the idea that it must be liquid in view of the inability of the S-waves to go through it!

Jeffreys, H. (Sir), 1926, The rigidity of the earth's central core: *Monthly Notices of the Royal Astronomical Society, Geophysical Supplement*, no.1, pp. 371-383.



With the picture of Gutenberg we are reminded of a whole family of new kinds of seismic body waves, namely the reflected and the refracted waves. They have characteristics and terminologies of their own that we need to know to be able to discuss further the internal constitution of our planet. When we begin to consider the reflected and refracted waves, seismology becomes extremely complicated and very difficult for the observational seismologist.



When P-Waves are reflected and refracted, they generate also S-waves! (From Mussett and Khan, 2000, Fig. 4-18)

When S-waves are reflected at a boundary, they may also generate P-waves. This naturally complicates the picture further.

Terminology of seismic waves in the earth:

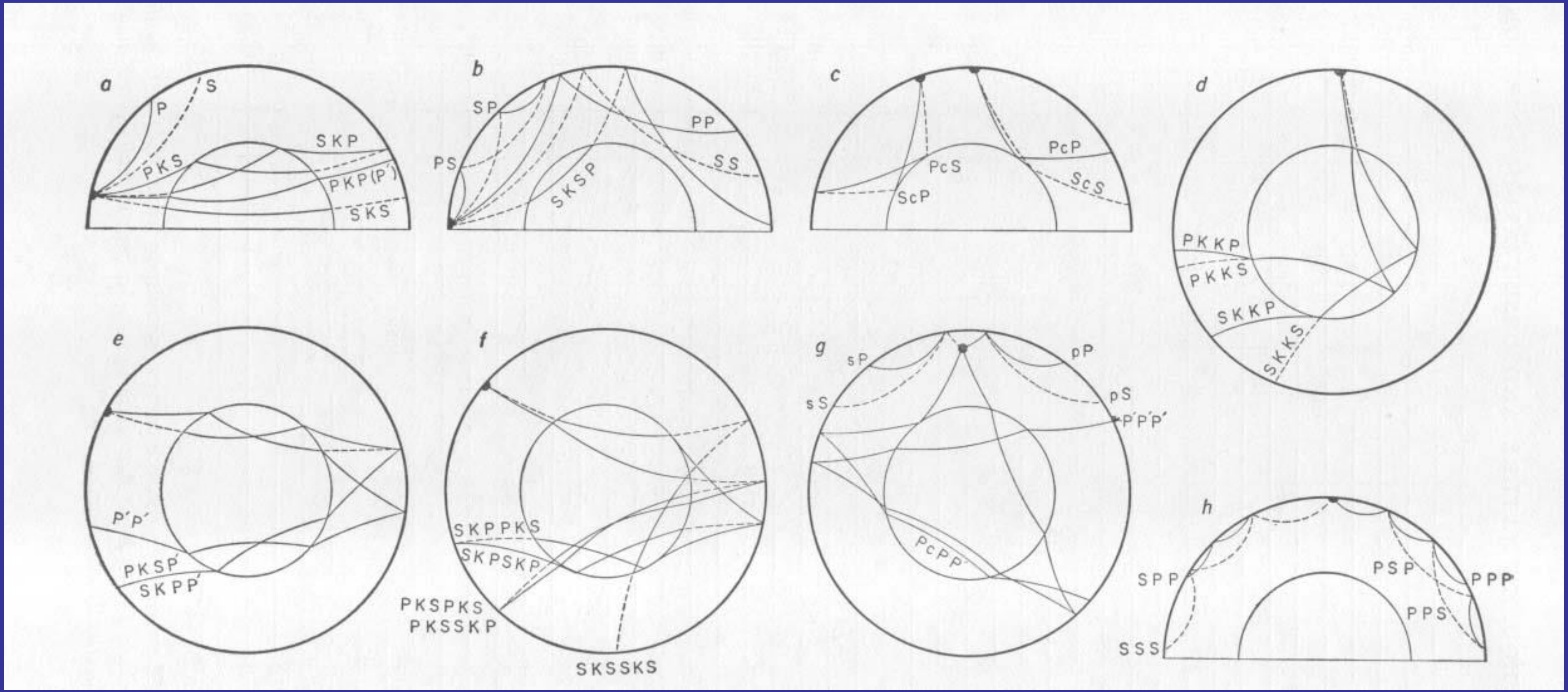
P	a P-wave in the mantle
S	an S-wave in the mantle
K	a P-wave through the outer core (K comes from the German word for the core: *Kern*)
I	a P-wave through the inner core
J	an S-wave through the inner core
c	a reflexion from the mantle/outer core boundary (c from the English word for the *core*)
i	a reflexion from the outer core/inner core boundary (i from *inner* core)
p	a P-wave reflected from the surface of the earth close to the earthquake focus
s	an P-wave reflected from the surface of the earth close to the earthquake focus
LR	a Raleigh wave
LQ	a Love wave

From Coulomb and Jobert 1971 and Fowler 1990

Some examples of how to read the terminology:

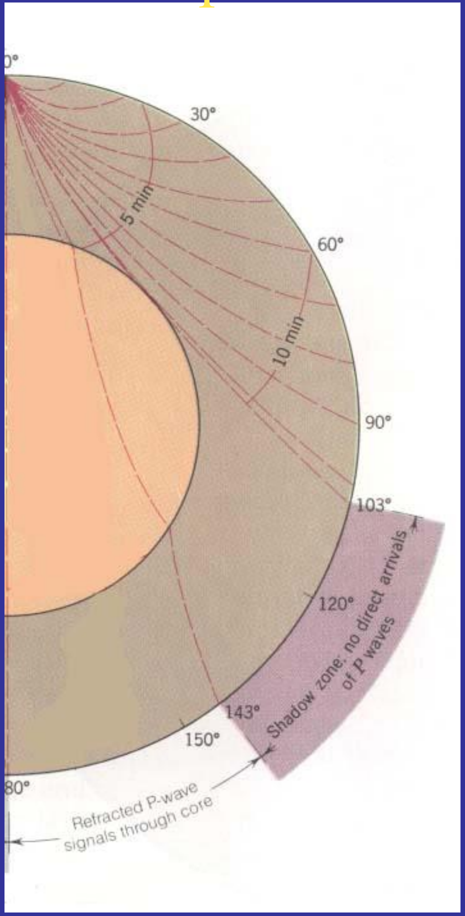
PKP (or P')	a P-wave that passed down through the mantle and the outer core and then up through the mantle again
PKIKP	a P-wave that passed down through the mantle and outer core and then through the inner core and up through the outer core and mantle
PKiKP	a P-wave which travelled down through the mantle and outer core, was reflected at the outer core/inner core boundary and travelled back up through the outer core and mantle
sSP	a wave which travelled from the focus as an S-wave, was reflected at the earth's surface close to the focus, then travelled through the mantle as an S-wave,was reflected for a second time at the earth's surface, converted to a P-wave and travelled as a P-wave through the mantle

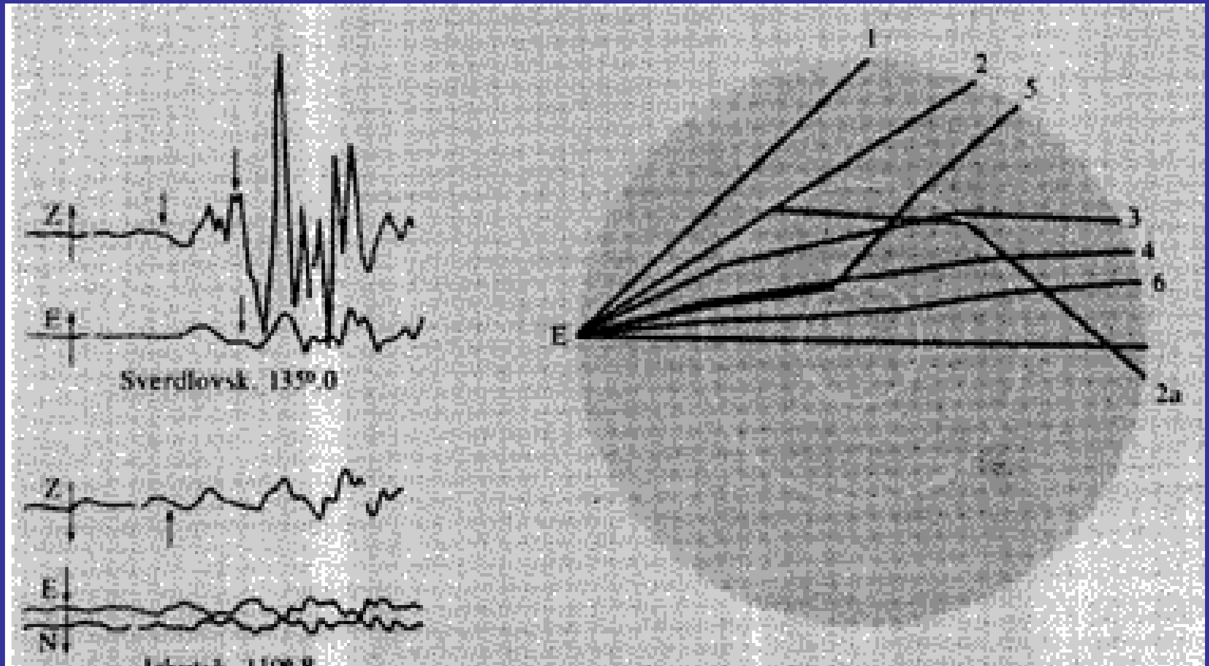
Moreover, when an S-wave encounters the liquid core it gets converted into a P-wave. Combinations of all kinds of multiple refractions and reflections create a vast family of seismic waves.



Paths of bodywaves with letter symbols. Longitudinal wave ray segments are shown as full lines; transverse wave ray segments as dashed. (From Richter, 1958, Fig. 17-5)

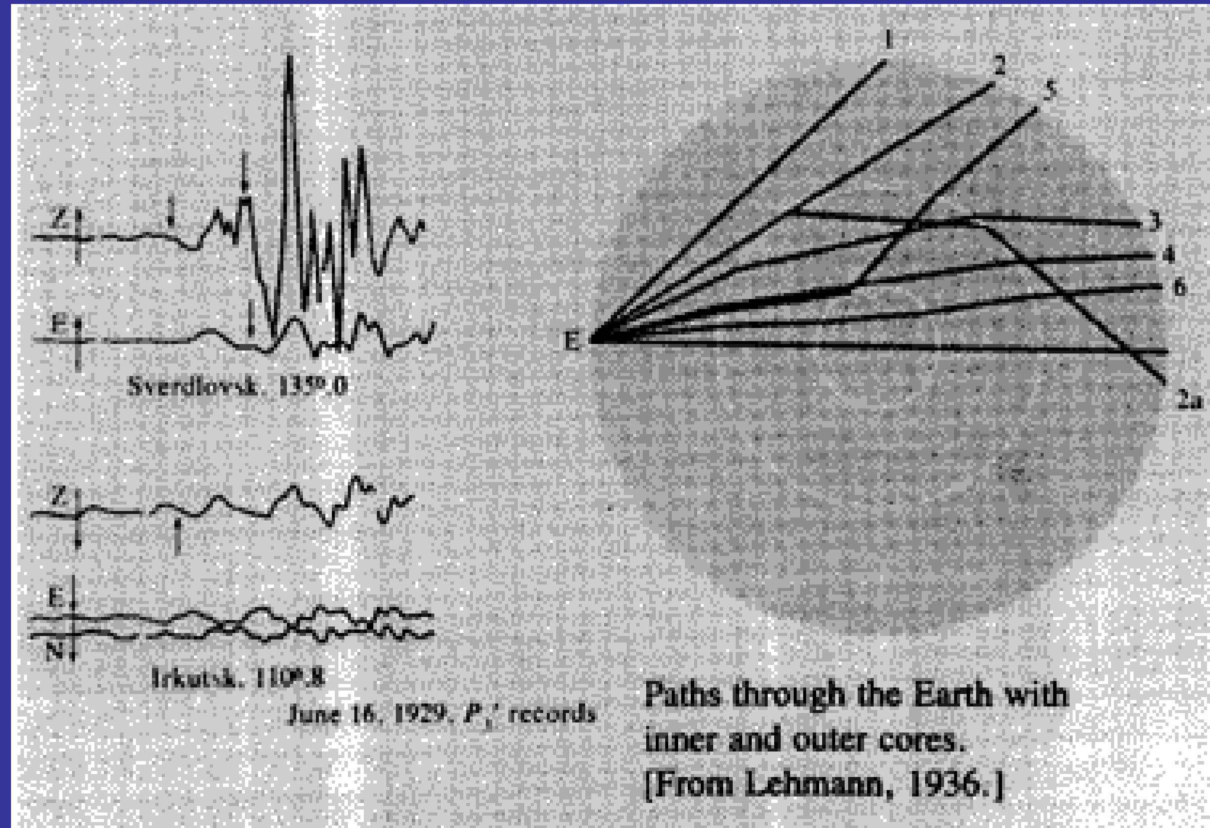
Because the core functions like a lens for the seismic waves, there occurs a "shadow zone" for the longitudinal waves between about 103˚and 143˚ away from the epicentre of any earthquake. What would you do if yo do find longitudinal waves in this shadow zone, as did Dr Inge Lehmann in 1936 when studying the seismograms from an earthquake near Buller on the South Alpine Fault in New Zealand?





Paths through the Earth with inner and outer cores. [From Lehmann, 1936.]

Inge Lehmann
(1888-1993)



Inge Lehmann pointed out that the arrivals in the "shadow" or "forbidden" zone of P-waves could not be due to diffraction. She thought that one plausible explanation was to assume an inner, reflecting core of 1400 km radius and a P-Wave velocity of 10 km/sec withing the core itself. The waves showing up in the shadow zone (no.5 in the fig. above) would be bouncing off it.

Lehmann, I., 1936, P':*Publ. Bur. Cent. Seism. Int*. v. A **14**, pp. 87-115.

In 1938, Beno Gutenberg and Charles Richter showed that the earthquake data could be better satisfied if one assumed an inner core of 1200 km radius and an average P-wave velocity of 11.2 km/sec. They thought that the inner/outer core boundary was not sharp and that there were no reflective waves. A year later Sir Harold Jeffreys pointed out that Lehmann's sharp boundary idea was essentially correct. In his biography of Lehmann Bruce Bolt wrote "Subsequently, independent arguments by Birch (1940) and Bullen (1946) established that the rapid increase in P velocity at the inner core boundary entailed a transition from liquid to solid conditions, with a jump in shear wave velocity from zero to about 3.1 km/sec., if the pressure-induced gradient in incompressibility was to be plausible. It was not until 1962 (Bolt 1962) that direct new evidence supporting Lehmann's sharp boundary was advanced, and not until 1970 that high-angle reflections (PKiKP) of seismic P waves incident on the inner core were observed unequivocally on seismograms (Engdahl *et al.* 1970)." The inner/outer core boundary is today known as the Lehmann discontinuity.

The next important step in understanding the geometry of the interior of the earth came from a New Zealand seismologist, Keith Edward Bullen.

Keith Edward Bullen (1906-1976)

Bullen started his geophysical work with Sir Harold Jeffreys in Cambridge, which consisted in generating travel-time curves (that became the famous Jeffreys-Bullen curves). To generate these curves, Bullen needed to know the density of the earth's interior. For this purpose he used an equation generated by two American geophysicists, E. D.Williamson and Leason H. Adams:

$$d\rho/dr = -Gm\rho/r^2(a^2-\{4/3\}\ B^2),$$

where r is the distance of a point from the earth's centre, ρ is the density at r, G is the universal gravitational constant, m is the mass of the earth within the radius r, and a and B are the velocities of P and S waves respectively at the level r.

Using the Williamson-Adams formula, Bullen calculated the density of the interior at 100-km intervals. He then checked his findings against the moment of inertia of the core. This showed him that the inner core ought to be less dense than the outer core. Thinking this absurd, he readjusted his results for a mantle in which density did not increase uniformly (does this not remind you of Euler almost 200 years earlier than Bullen?) In 1940, Bullen divided up the earth as follows:

A - crust

B - upper mantle

C - middle mantle in which density jump occurred

D - lower mantle

E - outer core

F - transition between outer and inner core

G - inner core

Bullen, K. E., 1940, The Problem of the earth's density variation: *Bulletin of the Seismological Society of America*, v. 30, pp. 235-250

Having come this far, how do we know what is in the earth? What is our planet made up of? How can all that we have learnt of its physical properties help us find out what it consists of?

Since all we really know are pressures and seismic velocities, one way to find out is to compare the seismic velocities of various rocky and metallic substances with the velocities inferred from the earth's interior. We can do this in a laboratory (although it is not always easy, as some of the requisite pressures are obtained by firing a canon in the laboratory!)

This is the subject of the next lesson.

APPENDIX I: How to find the centroid of any surface

The centroid of a lamina with the surface density function σ(x,y)

$$M = \iint \sigma(x, y)\, dA,$$

is located at the coordinates

$$x = \frac{\iint x\, \sigma(x, y)\, dA}{M}$$

$$y = \frac{\iint y\, \sigma(x, y)\, dA}{M}.$$

From: http://mathworld.wolfram.com/GeometricCentroid.html

The graphic method of finding the centroid of any lamina is to hang the lamina from three different points on its periphery and draw the straight lines following a plumb line from those points. Where thepoints meet is the centroid of that lamina.

APPENDIX II: The 1889 *Nature* paper by Ernst von Reuber-Paschwitz

The Earthquake of Tokio, April 18, 1889.

Reading the report on this earthquake in Nature (June 13, p. 162), I was struck by its coincidence in time with a very singular perturbation registered by two delicate horizontal pendulums at the Observatories of Potsdam and Wilhelmshaven. These instruments, which represent, with some modification, Prof. Zöllner's horizontal pendulum, were established in March 1889, for studying the slight movements of the ground. The motion of the pendulum, which is left to oscillate freely whenever its equilibrium is disturbed, is registered by the same photographic method as that employed for magnetic observations. The pendulum is in the plane of the meridian, so that any shock, the direction of which is not in this plane, will produce oscillations of the pendulum, diminishing gradually, if it is left undisturbed after the shock. The pillars supporting the instruments are fixed in a depth of 1 metre below the ground of the cellar which was chosen as a suitable place for the erection of the instrument.

During the three months from April to June, the disturbance of April 17, 18h. G.M.T., was the most remarkable which occurred. The following readings of Greenwich mean time, which are best explained by the accompanying figures, are taken from the original photographs; it must, however, be mentioned that the small scale of 11 millimetres per hour does not allow a very accurate determination of time, and that an error of one minute or two is quite probable.

(1) *Potsdam.*—1889, April 17. From 5h. until 17h. 21m., great steadiness of image.

h. m.	
17 21	First traces of disturbance.
17 39	Beginning of small oscillations.
17 54·3	Motion *suddenly* increases and reaches its maximum at
18 1	Amplitude of oscillation 154 millimetres. The amplitude then suddenly diminishes.
18 43, 18 58, 19 45	Maxima of oscillation.
20 0	Perfect steadiness of image.

(2) *Wilhelmshaven.*—Here, also, the image is perfectly steady until 17h. 30m.

h. m.	
17 30	Beginning of small oscillations.
17 48 – 17 51	A short interval of perfect steadiness.
17 51	The movement *suddenly* increased, and as the light is not strong enough to mark the single oscillations, the image disappears until
18 38	when the principal disturbance reaches its end.
18 51, 19 6, 19 22, 20 2	Maxima of small oscillations.
20 7	Perfect steadiness.

If we compare these dates, it seems most probable that the moment which shows a sudden increase of motion, and is best marked on the curves, may be considered as the beginning of the principal disturbance. We thus have—

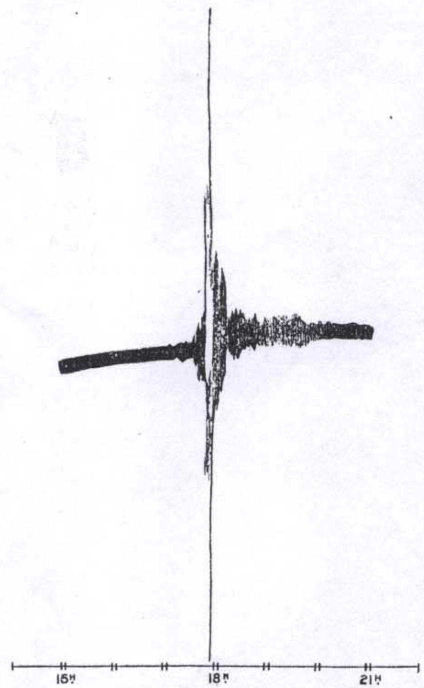
For Potsdam ... ... 17h. 54·3m.
For Wilhelmshaven... 17h. 51m. } Mean, 17h. 52·7m.,

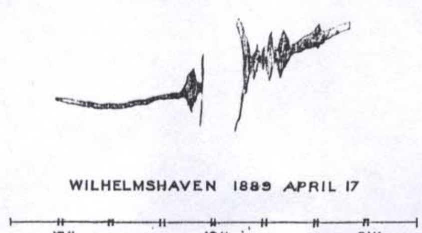
which, considering the error of the readings, may be taken as one and the same moment.

The beginning of the earthquake of Tokio was observed at 2h. 7·7m. Tokio M.T. The difference of longitude (taken from a map) being 9h. 19·3m. E., we find that the shock occurred at 16h. 48·4m. G.M.T. on April 17, and thus it took 1h. 4·3m. to travel across the body of the earth.

Taking the following longitudes and latitudes—

Tokio	139 50 E.,	35 44 N.
Potsdam	13 4 ,,	52 24 ,,
Wilhelmshaven	8 9 ,,	53 32 ,,





and neglecting the ellipticity of the earth, we find the following distances:—

Tokio to Potsdam	8221 kilometres.
Tokio to Wilhelmshaven... ...	8307 ,,

Dividing the mean 8264 by 3858s., we find a velocity of 2142 metres of propagation on the straight line connecting Tokio and a place between Potsdam and Wilhelmshaven, and consequently the shock ought to have been observed at Wilhelmshaven 40s. later than at Potsdam.

The above value of velocity is between the values found by Milne from seismic experiments, viz. 900-1400 metres for different kinds of rock, and by Abbot from the effect of dynamite explosions, viz. 2800 metres. We may therefore safely conclude that the disturbances noticed in Germany were really due to the volcanic action which caused the earthquake of Tokio.

Potsdam, July 5. E. von Rebeur-Paschwitz.

P.S.—I add a list of the most remarkable disturbances noticed during the course of the observations. Unfortunately, the working of the instrument at Wilhelmshaven was often disturbed by the effects of an excessive dampness in the cellar. The time is G.M.T. as above.

1889, April 5.—A day of great steadiness. A small perturbation begins at 9h. (Potsdam) and 9h. 5·4m. (Wilhelmshaven). It is divided by a short time of steadiness, 9h. 11·4m. (Potsdam) and 9h. 16·8m. (Wilhelmshaven).

April 8.—A fine disturbance begins at 16h. 45·6m. (Potsdam) and 16h. 47·4m. (Wilhelmshaven).

April 15.—A day of remarkable unsteadiness; the principal perturbation at both places lasts three hours, and lies between 7h. and 10h. It is impossible to determine a certain phase.

April 25.—A perturbation from 16h. 48m. to 18h. 12m. at Potsdam. No photograph obtained at Wilhelmshaven.

April 28.—An earthquake, consisting of one principal shock, apparently took place at 21½h.; the times noted are 21h. 34·8m. (Potsdam) and 21h. 36·6m. (Wilhelmshaven).

May 21.—A pretty large disturbance at Potsdam, lasting from 10h. 33m. to 11h. 6m., interrupted by a moment of rest at 10h. 42m. No photograph at Wilhelmshaven.

May 25.—Two very remarkable disturbances at Potsdam—7h. 9m. and 10h. 42m.—each lasting 1h. No photograph at Wilhelmshaven.

May 26.—A disturbance noticed at Potsdam, at 9h. 24m. No photograph at Wilhelmshaven.

May 30.—At Wilhelmshaven, two shocks are noticed—8h. 18·6m. and 9h. 24m.—which are probably connected with the English earthquake of this day. Perfect steadiness at Potsdam.

May 31.—A disturbance of earthquake-like appearance. Time of beginning, at Potsdam, 8h. 48m.; at Wilhelmshaven, 8h. 44·4m.; the latter time being rather uncertain, on account of the faintness of the curve.

I hope that one or other of these facts may prove to be of interest to seismologists.

On the Phenomena of the Lightning Discharge, as Illustrated by the Striking of a House in Cossipore, Calcutta.

During a heavy thunderstorm which passed over Calcutta about 5.30 p.m. on Saturday, June 8 last, the house of Conductor W. Viney, at Cossipore (a suburb of the city), was struck by lightning, and I have thought that a description of the phenomena connected with it might perhaps be worth placing on record in the columns of Nature.

I was myself watching the storm from the veranda of my residence about 300 yards distant, and observed that the discharge in question was one of extreme violence. I visited the scene of the accident within a few hours, with Mr. Viney's permission taking the notes from which this account is prepared; and, owing to the exceptional opportunities for observation which obtained in this case, have been able to secure trustworthy statements as to the appearance of the discharge, and further, by inquiry, to satisfy myself upon one or two points which I believe to possess considerable scientific interest.

The house which was struck is large, square, and flat-roofed, and is occupied by three foremen employed in the Government Shell Factory adjacent: it is providedwith a lightning-conductor projecting 8 or 9 feet above the roof-level, and situated near to one end of the building, but apparently unconnected with any other portion of the roof. It is possible that a portion of the discharge passed harmlessly away by the conductor, but of this I have no evidence, positive or negative. The lightning entered Mr. Viney's portion of the house by a corrugated iron covered hatchway standing 6 feet high at the corner diagonally opposite

APPENDIX III: Jeffreys' formulation of the Wiechert-Herglotz Theorem

2·05. Bodily waves in a sphere. We suppose that the velocity c of a pulse, or of a wave of short period, depends only on the distance r from the centre of the sphere. The pulse is supposed at present to originate at the surface, at a point A, and the radius is R. Take polar coordinates r, θ, the initial line being the radius OA. The wave front will at every instant be symmetrical about OA. The time taken to reach a given point P (Fig. 1) is

$$t = \int \frac{ds}{c} = \int \frac{1}{c}\left\{\left(\frac{dr}{d\theta}\right)^2 + r^2\right\}^{\frac{1}{2}} d\theta, \tag{1}$$

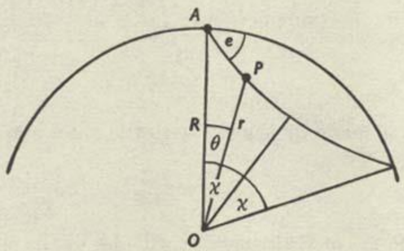


Fig. 1.

where ds is an element of length along the path. The actual path is such that this integral is stationary for small variations of the path. Putting for a moment V for the integrand in (1) and ρ for $dr/d\theta$, we know from the calculus of variations that this is true if r satisfies the differential equation

$$\frac{\partial V}{\partial r} - \frac{d}{d\theta}\left(\frac{\partial V}{\partial \rho}\right) = 0, \tag{2}$$

a first integral of which is known to be

$$V = \rho\frac{\partial V}{\partial \rho} + p, \tag{3}$$

where p is constant for a given ray. Substituting for V and simplifying we find

$$\frac{r^2}{c} = p\left\{\left(\frac{dr}{d\theta}\right)^2 + r^2\right\}^{\frac{1}{2}}, \tag{4}$$

whence

$$\frac{dr}{d\theta} = \pm r\left(\frac{r^2}{p^2c^2} - 1\right)^{\frac{1}{2}}, \tag{5}$$

$$\theta = \pm\int_R^r \frac{p\,dr}{r(r^2/c^2 - p^2)^{\frac{1}{2}}}. \tag{6}$$

If θ begins by increasing, then as r begins by decreasing we must take the negative sign. But $dr/d\theta$ vanishes when the ray reaches its deepest point, and if χ is the corresponding value of θ

$$\chi = \int_{r/c=p}^{r=R} \frac{p\,dr}{r(r^2/c^2 - p^2)^{\frac{1}{2}}}. \tag{7}$$

After passing this point the ray bends upwards, remaining symmetrical about the line joining the deepest point to the centre, and reaches the surface again at the point $(R, 2\chi)$. We also write $2\chi = \Delta$ and call Δ the epicentral distance of a point on the surface. Then if we know c as a function of r we can calculate the time of travel of the pulse to any distance Δ that it reaches; and conversely, if the time of travel is known from observation as a function of Δ one of our problems is to find what distribution of c is consistent with it.

If the ray makes an angle i with the radius we have for small displacements along the ray

$$r\,d\theta = \sin i\,ds = \sin i\left\{\left(\frac{dr}{d\theta}\right)^2 + r^2\right\}^{\frac{1}{2}} d\theta, \tag{8}$$

whence from (4)

$$p = \frac{r}{c}\sin i; \tag{9}$$

and if e is the angle made by the ray with the outer surface, and c_0 is the value of c there,

$$p = \frac{R}{c_0}\cos e. \tag{10}$$

Now if the ray emerges at P at time T, e will also be the angle of emergence at P. Let P' (Fig. 2) be a neighbouring point $(R, \Delta + d\Delta)$ in the same plane,

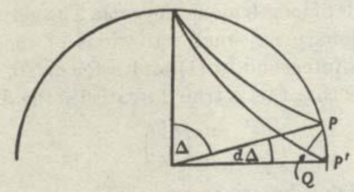


Fig. 2.

$T + dT$ the time needed by the wave to reach P'. Draw PQ perpendicular to the ray that reaches P'. Then PQ is nearly a part of a wave front and the times to P and Q differ by a quantity of order PQ^2. Hence, to the first order

$$QP' = c_0\,dT, \quad PP' = R\,d\Delta, \quad QP' = PP'\cos e, \tag{11}$$

whence

$$\frac{R}{c_0}\cos e = \frac{dT}{d\Delta}. \tag{12}$$

Thus the ray parameter p is identified as $dT/d\Delta$.

More than this is true; for we can apply the same argument to the rays to a pair of neighbouring points at the same distance r from the centre and use (9); then if t is given as a function of r, θ

$$\frac{\partial t}{\partial \theta} = p. \tag{13}$$

If we also compare a pair of neighbouring points on the same radius we get

$$\frac{\partial t}{\partial r} = \frac{\cos i}{c} = \pm\left(\frac{1}{c^2}-\frac{p^2}{r^2}\right)^{\frac{1}{2}}, \quad (14)$$

whence

$$r^2\left(\frac{\partial t}{\partial r}\right)^2 + \left(\frac{\partial t}{\partial \theta}\right)^2 = \frac{r^2}{c^2}, \quad (15)$$

which is a differential equation satisfied by the time to a general point (r, θ).

If $T = t(R, \Delta)$ is a known function of Δ, p is also found in terms of Δ by differentiation, by (10) and (12). Then we put

$$r/c = \eta, \quad (16)$$

and (7) becomes

$$\tfrac{1}{2}\Delta = \int_p^{R/c_0} \frac{p}{\sqrt{(\eta^2-p^2)}} \frac{d}{d\eta}(\log r)\, d\eta. \quad (17)$$

The left side is a known function of the parameter p; the right contains $\log r$, which is so far an unknown function of η. Hence (17) can be regarded as an integral equation to determine $\log r$ as a function of η. It has been solved by G. Herglotz (1907) and H. Bateman (1910). An elementary solution has been given by G. Rasch (during a joint work with I. Lehmann) and was communicated to me privately. Take a quantity μ such that (17) holds for $\mu \leqslant p \leqslant R/c_0$. Multiply (17) by $dp/\sqrt{(p^2-\mu^2)}$ and integrate from $p = \mu$ to R/c_0. Then

$$\frac{1}{2}\int_\mu^{R/c_0} \frac{\Delta\, dp}{\sqrt{(p^2-\mu^2)}} = \int_\mu^{R/c_0} \frac{p\, dp}{\sqrt{(p^2-\mu^2)}} \int_p^{R/c_0} \frac{d}{d\eta}(\log r) \frac{d\eta}{\sqrt{(\eta^2-p^2)}}. \quad (18)$$

Change the order of integration; the limits for p become μ to η, and those for η from μ to R/c_0. But if $\eta > \mu$

$$\int_\mu^\eta \frac{p\, dp}{\sqrt{(p^2-\mu^2)}\sqrt{(\eta^2-p^2)}} = \tfrac{1}{2}\pi. \quad (19)$$

Hence the integral is equal to

$$\tfrac{1}{2}\pi \int_\mu^{R/c_0} \frac{d}{d\eta}(\log r)\, d\eta = \tfrac{1}{2}\pi \log\frac{R}{r(\mu)}, \quad (20)$$

and

$$\log\frac{R}{r} = \frac{1}{\pi}\int_\mu^{R/c_0} \frac{\Delta\, dp}{\sqrt{(p^2-\mu^2)}}, \quad (21)$$

where r has the value that corresponds to $r/c = \mu$. This gives r as a function of r/c and hence c as a function of r.

The solution can be simplified by a transformation due to Wiechert and L. Geiger (1910). In (13) η and r vary along the ray considered, p remaining constant. But in (21) we are considering a set of rays, $p = \mu$ corresponding to that which makes $\eta = \mu$ at the deepest point, and $p = R/c_0$ to one that

grazes the surface. p itself is a function of the angle of emergence of a ray that descends at an intermediate angle. Then with a change of notation we write Δ for the value of Δ corresponding to $p = \mu$, and Δ_1 for a general p between μ and R/c_0. Put

$$p = \mu \cosh q. \quad (22)$$

Then

$$\cosh q = \frac{(dT/d\Delta)_1}{(dT/d\Delta)} = \frac{\cos e_1}{\cos e}, \quad (23)$$

and q ranges from 0 to the value q_0 found by taking the limiting value for short distances, R/c_0, for the numerator in the second expression. Then

$$\log\frac{R}{r} = \frac{1}{\pi}\int_0^{q_0} \Delta_1\, dq$$
$$= \left[\frac{1}{\pi}\Delta_1 q\right]_{\Delta_1=0}^{\Delta} - \frac{1}{\pi}\int_\Delta^0 q\, d\Delta_1. \quad (24)$$

But q vanishes at one limit and Δ_1 at the other; hence the integrated part is 0 and

$$\log\frac{R}{r} = \frac{1}{\pi}\int_0^\Delta q\, d\Delta_1. \quad (25)$$

The advantage of this form over (21) is that if T is given in terms of Δ by a table, it will usually be at equal intervals of Δ over large parts of the table; but it will not be at equal intervals of p. Numerical formulae of integration are much more manageable for equal intervals.

C. G. Knott (1919) used (21); but (25) had already been applied to an empirical table by S. Mohorovičić (1914, 1916).

As q behaves like $(\Delta-\Delta_1)^{\frac{1}{2}}$ at one terminus the usual Gregory and central-difference formulae of integration are unsatisfactory; but formulae suited to this case are known and can be applied to a few intervals at the end, while the usual ones are used for the rest of the range (Jeffreys, 1939*e*, p. 597; H. and B. S. Jeffreys, 1972, § 9·092). Other formulae of the same type are available for $\int_0^x f(x)\, dx$, where $f(x)$ behaves like $x^{-\frac{1}{2}}$ for x small.

If the velocity distribution is known we may still want to calculate times for parts of rays, especially for foci not on the surface. From (5), (6) and (1) we deduce

$$t = \int \frac{r\,|dr|}{c^2(r^2/c^2-p^2)^{\frac{1}{2}}}. \quad (26)$$

The procedure is to adopt a set of values of p and to work out θ, t for each by integration. In (6) and (26) the integrand behaves like $(\eta-p)^{-\frac{1}{2}}$ if the ray is nearly horizontal at some point, and therefore needs considerable accuracy in the computation. But

$$t - p\theta = \int \left(\frac{r^2}{c^2}-p^2\right)^{\frac{1}{2}} \frac{|dr|}{r}, \quad (27)$$

and its calculation is more manageable. In some cases, especially for rays that have not penetrated deeply, the right side of (27) varies slowly with p, and then even if there is a small error in the calculated θ we can use (27) to calculate t, which will be nearly correct for the calculated θ. This device often avoids the need for specially close intervals.

Hitherto, when the adopted travel times have been altered, the velocities have been completely recalculated. This can be avoided as follows. Suppose that a trial solution gives $dt/d\Delta = p$ and that the modification gives $dt/d\Delta = p + \delta p$; then to the first order in the changes

$$q + \delta q = \cosh^{-1} \frac{p_1 + \delta p_1}{p + \delta p}.$$

We find

$$\delta q = \frac{p\, \delta p_1 - p_1\, \delta p}{p(p_1^2 - p^2)^{\frac{1}{2}}}.$$

In most intervals this will vary smoothly, and as $\Delta_1 \to \Delta$, δq behaves like $(\Delta - \Delta_1)^{\frac{1}{2}}$ so long as δp is differentiable. Thus the integration presents no difficulty. It is of course necessary that the calculation for the trial solution shall have been done sufficiently accurately for accumulation of rounding off errors not to matter. This is not true for the J.B. solution of 1940; when the times of *PcP* were calculated, the time of *P* at grazing incidence on the core (where it coalesces with *PcP*) differed by $0{\cdot}6^{s}$ from the original *P* time. This exceeds the standard errors now attained in some studies. It was probably due to insufficient accuracy in integration formulae used in the early part of the work. No similar discrepancy was found for *S*.

Bullen (1960*a*) has suggested the use of his power law (2·082) as a first approximation, but as it makes d^2c/dr^2 positive and the actual values are mostly negative the above method is probably easier.

From: Jeffreys, H., 1976, *The Earth*, 6th ed., Cambridge University Press, Cambridge

APPENDIX IV: The Chandler Wobble

(modified from: http://en.wikipedia.org/wiki/Chandler_wobble)

The **Chandler wobble** is a small variation in Earth's axis of rotation, discovered by American astronomer **Seth Carlo Chandler** in 1891. It amounts to 0.7 arcseconds over a period of 435 days. In other words, Earth's poles move in an irregular circle of 3 to 15 metres in diameter, in an oscillation.

The wobble's diameter has varied since discovery, reaching its most extreme range recorded to date in 1910. The cause is unknown: barring any external force, the wobble should have eventually subsided. Originally it was believed that the wobble was caused by weather fluctuations from season to season causing shifts in atmospheric mass distribution, or possible geophysical movement beneath Earth's crust. On 18 July 2000, however, the Jet Propulsion Laboratory announced that "the principal cause of the Chandler wobble is fluctuating pressure on the bottom of the ocean, caused by temperature and salinity changes and wind-driven changes in the circulation of the oceans." This brings about a change in the shape of the earth.

The Chandler wobble is a factor considered by satellite navigation systems (especially military systems). It is also theorised as the cause of some tectonic activity, including earthquakes, which further shows its effects on the shape of the earth.