3.1 Viscous Resistance

Viscosity may be understood by assuming friction between hypothetic layers of fluid which are in contact with solid boundaries. That is, fluid layers are thought to be slid relative to each other with a shear stress in between them due to viscosity. This can be modelled simply by considering an element of fluid deformed under a flow effect:

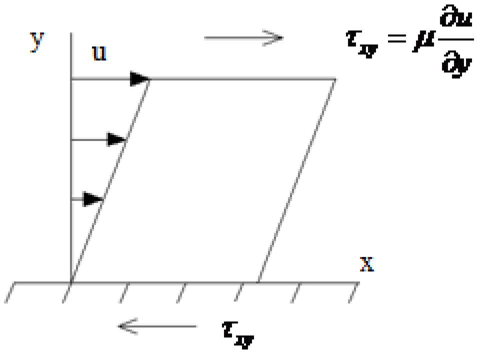

Fig. 3.3: Deformed fluid element under a flow effect.

Here τ_{xy} is the shear stress, $\partial u/\partial y$ is the rate of change of velocity (strain) as a function of y (distance from the solid boundary) and μ is the dynamic viscosity coefficient which is indeed shear stress per unit velocity gradient. But this is a very simple (2-D) way of defining shear stress. In a general velocity field of 3-dimensions, total stress tensor τ_{ij} is a linear function of 9 gradients of $\partial u_i/\partial x_j$ with an additional term for normal stress:

$$\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

where δ_{ij} is the *Kronecker's delta* and i,j=1,2,3 denote x,y,z components or directions. The first term of τ_{ij} gives the viscous pressure drag and the second term is related to the frictional resistance. Recall from *Fluid Mechanics* courses that we have two basic governing equations: one for the conservation of mass (continuity equation given by $\nabla \cdot V = 0$) and one for the conservation of momentum. Conservation of momentum requires the derivatives of the stress tensor, $\partial \tau_{ij} / \partial x_i$, and this yields *Navier-Stokes* equation in the vector form:

$$\frac{\partial V}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \upsilon \nabla^2 \vec{V}$$

where $\upsilon = \mu/\rho$ is the kinematic viscosity. This vector form of N-S equation gives a system of three nonlinear partial differential equations in terms of V=V(u,v,w) and, together with continuity equation, this system can only be solved analytically for some very simple restricted set of problems. The impossibility to solve analytically the N-S equations comes mainly from the nonlinear convective acceleration term $(\vec{V} \cdot \nabla)\vec{V}$ which is the main source of turbulence. Nowadays, N-S equations can be solved by computational (numerical) methods which requires powerful background on viscous flow theory and skills on numerical techniques and mesh generation.

Instead of going through the numerical solution of N-S equations, which is a vast area in fluid dynamics and in computational sciences, we will focus on the practical/engineering aspects of boundary layer, laminar and turbulent flows, separation and frictional resistance.

3.1.1. Boundary layer

The wetted hull surface is approximated as a flat-plate having equal surface area, and in this case boundary layer development can be investigated along a flat-plate in a simple way. The following figure represents laminar boundary layer along a flat plate. Here, $\delta(x)$ denotes perpendicular distance from the flat-plate where the flow velocity is theoretically equal to on-flow velocity U_∞. Practically the outer limit of the boundary layer is defined by the 99 percent of U_∞.

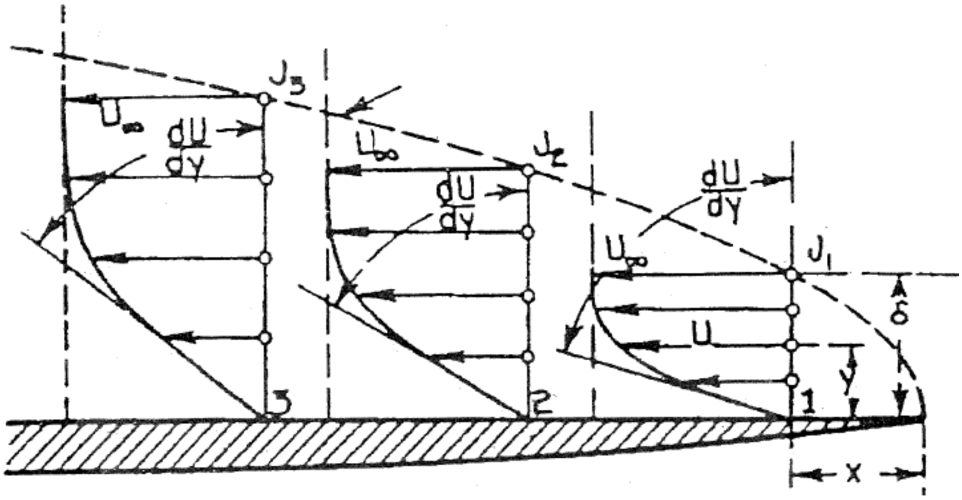

Fig. 3.4: Development of boundary layer on a flat-plate.

The viscous flow is laminar for $Re_x = \frac{Ux}{\upsilon} \leq 10^5$. The transition to turbulent flow is between $2x10^5 < Re_x < 3x10^6$. Laminar flow may be explained as well-organized flow in layers and/or along pathlines, whereas turbulent flow in characterized by a (high frequency) randomness and irregularity in the velocity and in the pressure field. This phenomenon is sketched in the following figure by a velocity distribution/profile in a turbulent boundary layer. The turbulence shows itself by relatively high vorticity as depicted in the right side of the figure. This explains the irregularity and high oscillations in the velocity profile. Note that velocity gradient $\partial u/\partial y$ at the flat-plate is significantly larger in the turbulent flow case as compared to that of laminar flow, since a larger exchange (or loss) of fluid momentum is experienced in the turbulent flow.

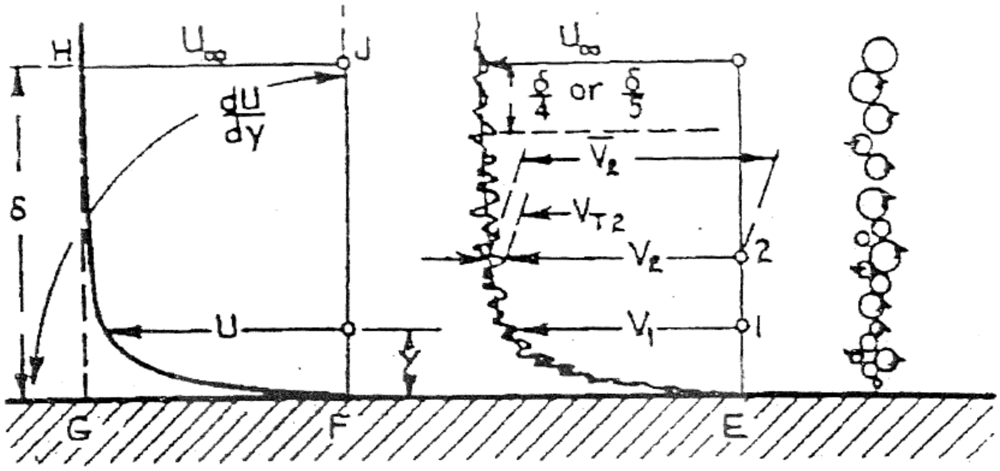

Fig. 3.5: Velocity profiles in a turbulent boundary layer (2-D).

It is customary to investigate flat-plate friction problem instead of a 3-D flow and boundary layer growth along a flat-plate. In low-Reynolds-number regime, the interaction between viscous and inviscid layers is strong whereas in high-Reynolds-number flow, either laminar or turbulent, the boundary layers are thinner. The (wide spread used) boundary layer thickness formulas are;

$$\frac{\delta}{x} = \begin{Bmatrix} \dfrac{5.0}{\mathrm{Re}_x^{1/2}}; \text{ laminar } (\textit{Blasius}) \\ \dfrac{0.16}{\mathrm{Re}_x^{1/7}}; \text{ turbulent } (\textit{von Karman}) \end{Bmatrix}$$

Note that, again, the no-slip wall condition retards the flow, resulting in a rounded velocity profile u(y) which reaches to the external (incoming) velocity $U=U_\infty$ at $y=\delta(x)$. Then the drag force or frictional resistance on the plate is given by taking the momentum loss at the end of the plate which is caused by the shear stress along the plate:

$$R_F = p\int_0^\delta u(U-u)dy = \int_0^x \tau_w dx \text{ (per unit width).}$$

It is convenient to express R_F in term of momentum thickness θ:

$$R_F = \rho U^2 \theta$$

where $\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$ is the momentum thickness.

The other important concept in a boundary layer flow is the displacement thickness δ^* which is given by

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

An approximate formula for δ^* is;

$$\frac{\delta^*}{x} \cong \frac{1.83}{\mathrm{Re}_x^{1/2}} \text{ (for laminar flow)}$$

and for turbulent flows it is approximated as:

$$\delta^* \cong \int_0^{\delta} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy = \frac{1}{8}\delta$$

The formulas for momentum thickness may be given as:

$$\frac{\theta}{x} = \frac{0.664}{\mathrm{Re}_x^{1/2}} \text{ for laminar plate and } \theta \cong \frac{7}{72}\delta \text{ for turbulent plate.}$$

The ratio of displacement thickness to momentum thickness gives the shape factor: $H = \frac{\delta^*}{\theta}$. Conventionally, H=2.59 (Blasius boundary layer) is a typical figure for laminar flows, while H=1.3-1.4 is typical for turbulent flows. A large shape factor may be taken as an indication for boundary layer separation.

3.1.2 Roughness Effect

There are differences in frictional drag coefficients when the effect of wall roughness is taken into account. An appropriate parameter for roughness is given by x/ε or L/ε where L is the characteristic length of the plate and ε denotes the roughness in m. The following figure shows the effect of roughness on C_D (drag coefficient=D/ρSV^2) as compared to smooth case.

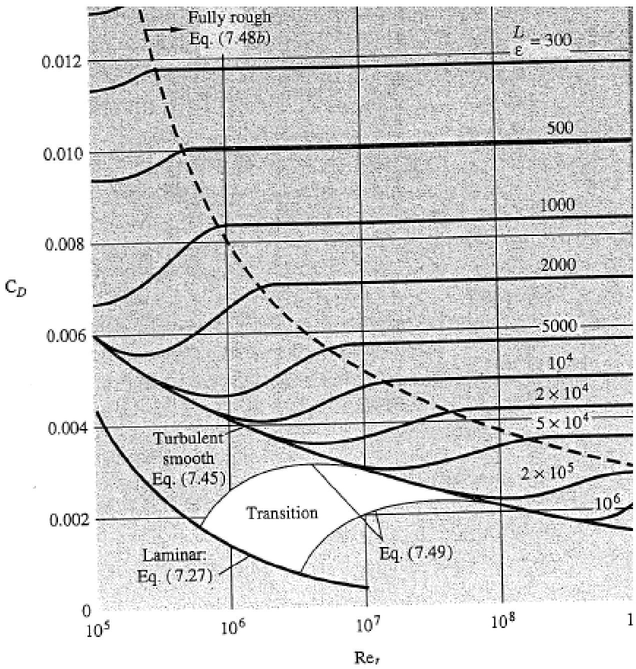

Fig. 3.6: Drag (frictional) coefficient of laminar and turbulent boundary layers on smooth and rough plates. (White, 1999)

Roughness effect can also be investigated in terms of Roughness Reynolds number, k^+. For this investigation we need to know the friction velocity $u_\tau = \sqrt{(\tau_w / \rho)}$ and viscous length scale $l_\upsilon=\upsilon/u_\tau$. Thus Roughness Reynolds number is $k^+=k/l_\upsilon$, where k is the wall roughness in meters. Then we may define the flow regimes by the help of k^+ as:

- hydraulically smooth, when $k^+<5$
- transitionally rough, when $5<k^+<70$
- fully rough, when $k^+>70$. In the 3rd case eddy shedding occurs which effects the overlap layer and form drag becomes dominant.

3.1.3 Separation and vortex shedding

Flow separation means detachment of the streamline flow from a solid boundary. In the separated region, there are eddies, vortices and even a reverse flow. Development of the separation can be seen in the following photograph. With the increase of the velocity the effects of viscosity increase and end up with a boundary layer growth. In the wake region, as the velocity increases, flow eventually separates which yields two symmetric vortices. With increasing speed the separation zone becomes unstable.

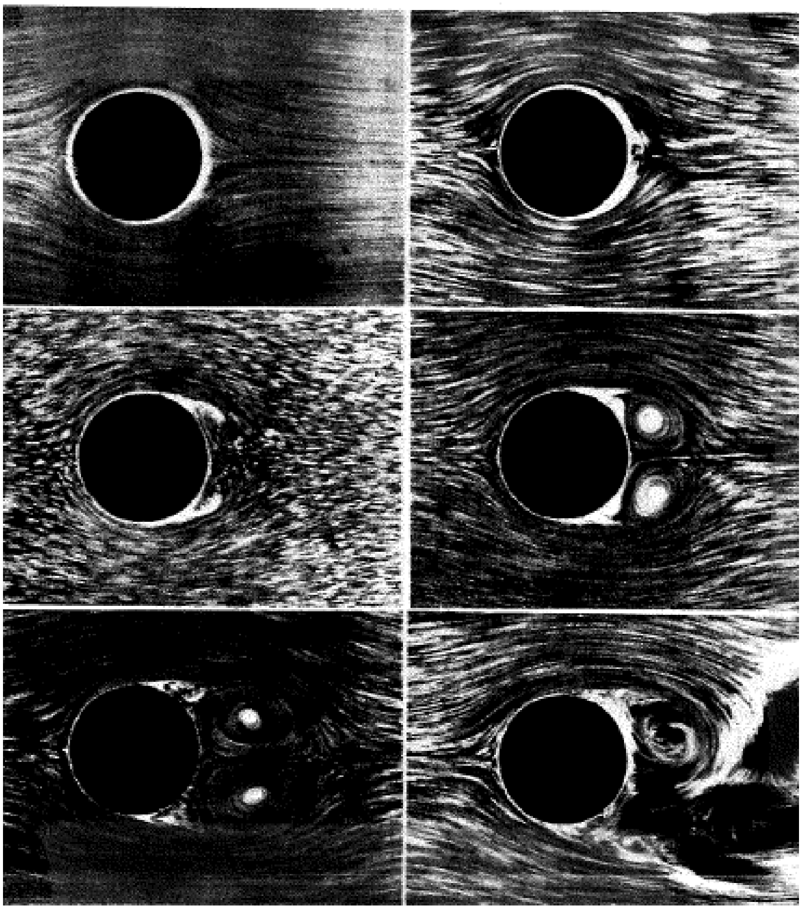

Fig. 3.7: Stages of the flow past a circular cylinder. (Prandtl, 1927)

We now know that the shear stress caused by viscosity has a retarding effect upon the flow. In a 3-D flow around a ship like body a negative pressure gradient may help to reduce this retarding effect to prevent separation of the flow. Thus negative pressure gradient ($\partial p/\partial x<0$) is termed as favourable pressure gradient. A positive pressure gradient, namely adverse pressure gradient, has the opposite effect which forces the fluid flow to climb the pressure hill.

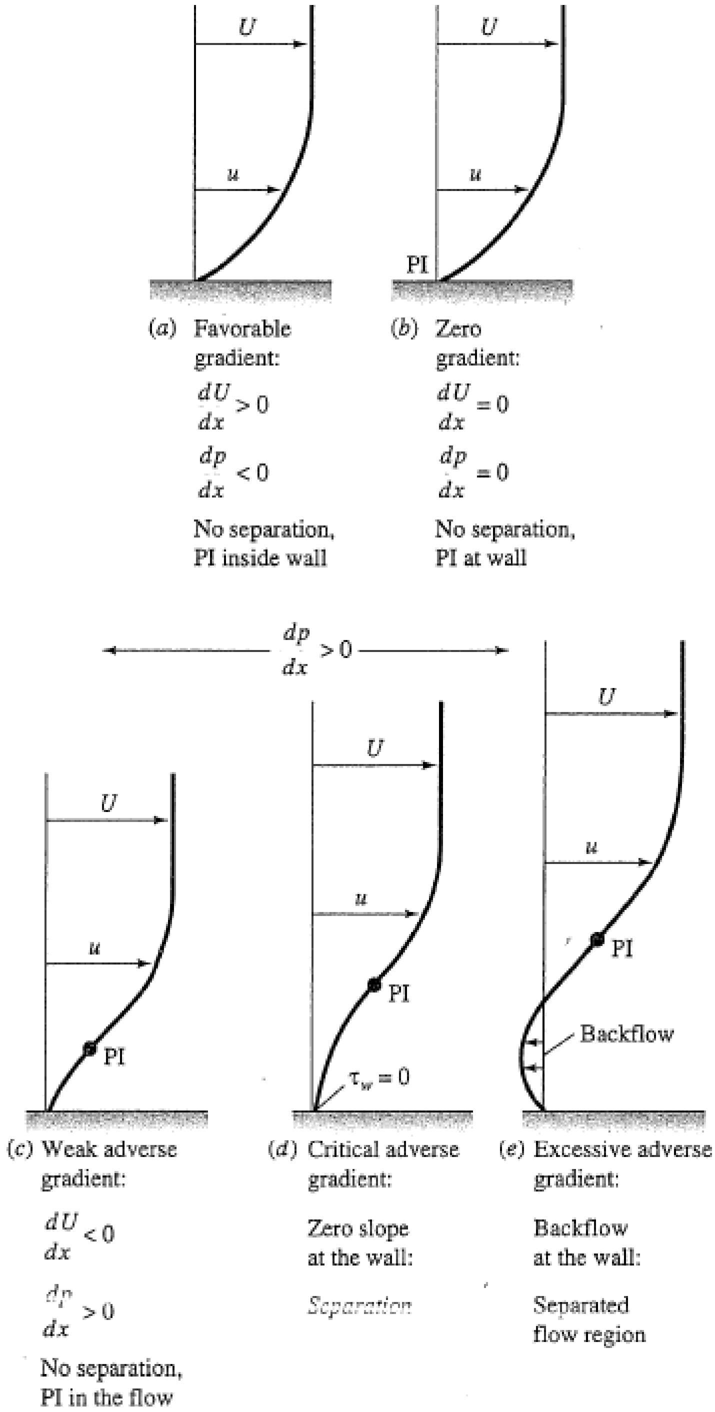

Fig. 3.8: Effect of pressure gradient. (White, 1999)

(PI: Point of inflection)

As understood from the above figure, adverse pressure gradient causes the flow separate from the rest of the flow which is streamlined. If we take the following figure into consideration, we can see that favourable (negative) pressure gradient at point A counteracts the retarding effect of viscosity up to the point B. Thereafter, adverse (positive) pressure gradient starts gradually and velocity near the wall reduces and boundary layer thickens. Critical adverse pressure gradient end up with stagnation point at C and separation starts beyond this point.

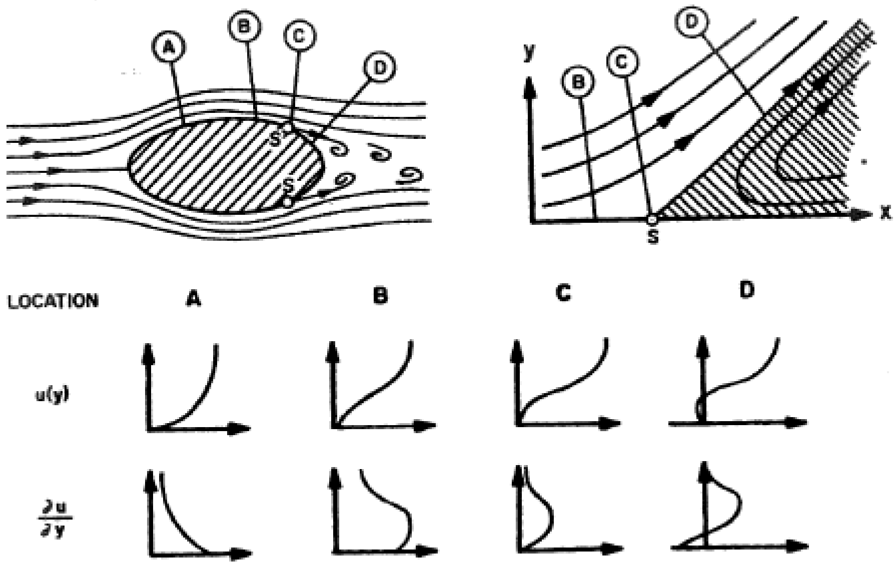

Fig. 3.9: Separation flow phenomena in a flow past a rigid body. (Newman, 1980)

Vortex shedding is another consequence of the viscous flow around bluff bodies. For example around Re~50 vortex shedding begin to take place in a viscous flow around the circular cylinder. The vortices create low-pressure zones which urge the vortex shedding body to move towards the vortices (low-pressure zones).

In summary, flow separation and vortex shedding are the main sources of the viscous pressure drag (or form drag)/viscous pressure resistance.

3.1.4 Frictional/viscous resistance in ships

Frictional resistance can be calculated by integrating the tangential shear stresses over the wetted surface of the ship. This is indeed not an easy process. Thus the researchers/engineers find it versatile to derive empirical formulas for flat-plate frictional resistance and then add the 3-D effects and viscous pressure effects.

Schoenherr’s flat-plate frictional drag coefficient formula (1932) is a pioneering example of this type of models:

$$\frac{1}{\sqrt{C_F}} = 4.13\log_{10}(\mathrm{Re}\cdot C_F)$$

(Note that first pioneering experimental research was carried out by Froude (1872-74)). Hughes (1953-54) then presented a formula to represent numerous experimental data which was then re-arranged as:

$$C_F = \frac{0.067}{(\log_{10} \mathrm{Re} - 2)^2}$$

The Hughes formula was then corrected in 1957 ITTC(International Towing Tank Conference) and agreed on the following formula:

$$C_F = \frac{0.075}{(\log_{10} \mathrm{Re} - 2)^2}$$

which is currently in use in today's calculations. One should bear in mind that ITTC-1957 formula does not take 3-D form effects into account as described before.

Note also that the above formulas are valid under the assumption that the rigid body surface is hydraulically smooth. But naturally there is always a roughness to some extent. ITTC introduced a correction factor (roughness allowance/ship-model correlation coefficient) to include roughness effects:

$$\Delta C_F = C_A = \left[105 \left(\frac{k}{L_{WL}} \right)^{1/3} - 0.64 \right] \cdot 10^{-3}$$

where k is the surface roughness in meters. The roughness of new ships having very well surface finishing may be less than 100 μm; for an average quality new ship it is about 120-150 μm and for old and very rough ships it may reach 1000 μm.

Currently, it is now possible to calculate 3-D frictional effects and viscous pressure effects, although this requires skilful work. Apart from computational possibilities; based on the past experimental studies, it is assumed/accepted to express the difference between total viscous pressure and frictional resistance by the following relationship:

$$C_V = C_F + C_{VP} = C_F \left(1 + \frac{C_{VP}}{C_F} \right) = (1 + k) C_F$$

where k is the form factor. (Some text books take (1+k) as the form factor). Note that in that in this case kC_F represents viscous pressure coefficient together with 3-D frictional effects.