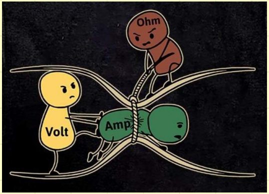
Current, Resistance, and Electromotive Force



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Contents

Learning outcomes

- The meaning of electric current, and how charges move in a conductor.
- What is meant by the resistivity and conductivity of a substance.
- How to calculate the resistance of a conductor from its dimensions and its resistivity.
- How an electromotive force (emf) makes it possible for current to flow in a circuit.
- How to do calculations involving energy and power in circuits.
- How to use a simple model to understand the flow of current in metals.

Introduction - 1

- Up to now we studied the interactions of electric charges at rest; now we're ready to study charges in motion.

- An *electric current* consists of charges in motion from one region to another.

- If the charges follow a conducting path that forms a closed loop, the path is called an *electric circuit*.

Introduction - 2

- Electric circuits convey energy from one place to another.
- As charged particles move within a circuit, electric potential energy is transferred from a source (such as a battery or generator) to a device in which that energy is either stored or converted to another form: e.g. into sound in a stereo system or into heat and light in a toaster or light bulb.
- Electric circuits are useful because they allow energy to be transported without any moving parts (other than the moving charged particles themselves).

Introduction - 3

Electric circuits are at the heart of

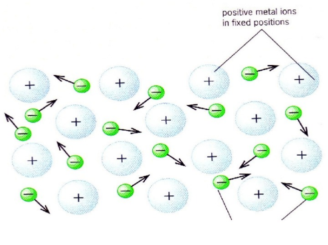
- computers,
- television transmitters and receivers, and
- household and industrial power distribution systems.
- Your nervous system is a specialized electric circuit that carries vital signals from one part of your body to another.

To prepare for the study of electric circuits in the next chapter, we'll examine the basic properties of electric currents here.

Current

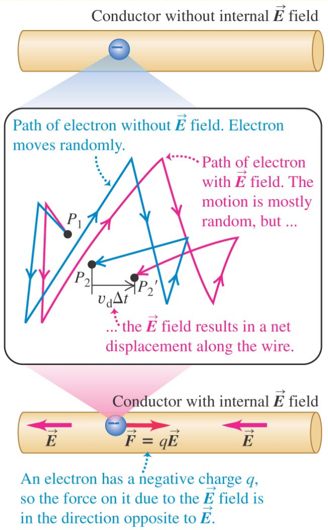
Electrons in a conductor in electrostatics

- In an ordinary metal such as copper or aluminum, some of the electrons, e^-, are free to move.
- The e^- do not escape from the conducting material, because they are attracted to the $+$ ions of the material.
- In electrostatic conditions $\vec{\mathbf{E}} = 0$ everywhere within the conductor.
- However, this does not mean that e^- are at rest; they move randomly in all directions with speeds, of the order of $10^6\,\mathrm{m/s}$.



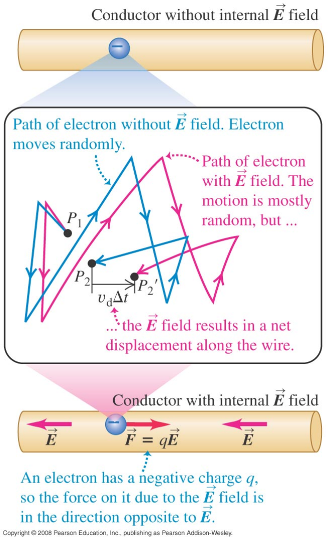
- The motion of the e^- is random, so there is no net flow of charge in any direction.

Current



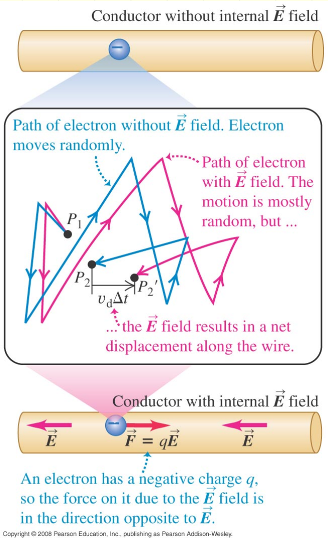
- A current is any motion of charge from one region to another:
- In electrostatic situations ($\vec{\mathbf{E}} = 0$), the random motion of the electrons produces no net flow of charge and so there is no current.
- If a constant, steady electric field $\vec{\mathbf{E}}$ is established inside a conductor, then there is a net flow of charge and a current is produced.

Drift velocity



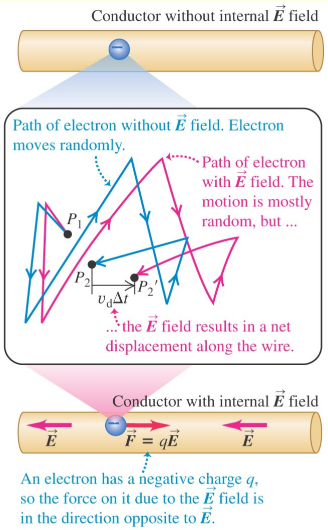
- if a constant, steady $\vec{\mathbf{E}}$ is established inside a conductor, a free e^- inside the conductor is then subjected to a steady force $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$.
- If the charged particle were moving in *vacuum*, this steady force would cause a steady acceleration $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m$.

Drift velocity



- But an e^- moving in a conductor undergoes frequent collisions with the massive, nearly stationary ions. In each such collision the particle's direction of motion undergoes a random change.
- The net effect of $\vec{\mathbf{E}}$ is that in addition to the random motion of e^-s within the conductor, there is also a very slow net motion of the e^-s opposite to the direction of $\vec{\mathbf{E}}$.
- This motion is described in terms of the *drift velocity* $\vec{\mathbf{v}}_d$ of e^-s. As a result, there is a net current in the conductor.

Drift velocity



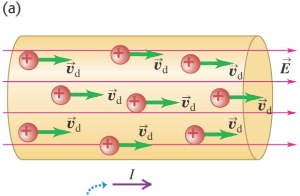
- While the random motion of the e^-s has a very fast average speed of about 10^6 m/s, the drift speed is very slow, often on the order of 10^{-4} m/s.
- Given that the e^- move so slowly, you may wonder why the light comes on immediately when you turn on the switch of a flashlight!

Drift velocity

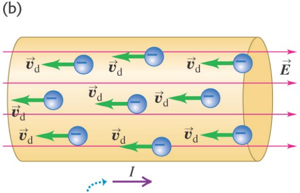
Analogy:
A group of soldiers standing at attention when the sergeant orders them to start marching; the order reaches the soldiers' ears at the speed of sound, which is much faster than their marching speed, so all the soldiers start to march essentially in unison.

Analogy:
Consider a garden hose connected to a tap. If the hose is filled with water in the beginning, water will flow from the end as soon as the tap is opened.

Direction of current



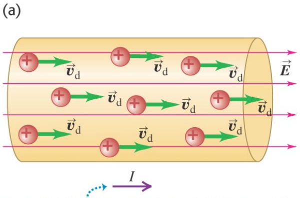
A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

- In metals charge-carrying particles are the e^-s.
- In other conducting materials, the charges of the moving particles may be $+$ or $-$.

Direction of current



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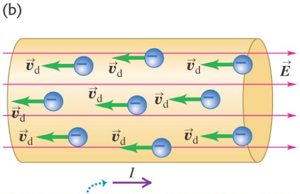

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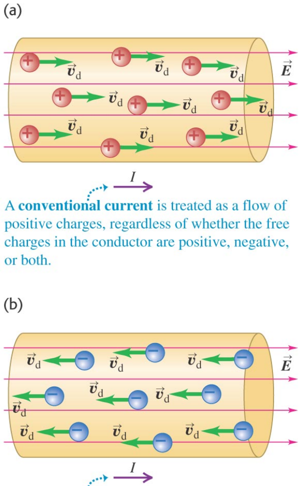
- In an ionized gas (plasma) or an ionic solution the moving charges may include both e^{-}s and $+$ charged ions.
- In a semiconductor material such as germanium or silicon, conduction is partly by e^{-}s and partly by motion of vacancies, also known as holes; these are sites of missing electrons and act like $+$ charges.

Direction of current



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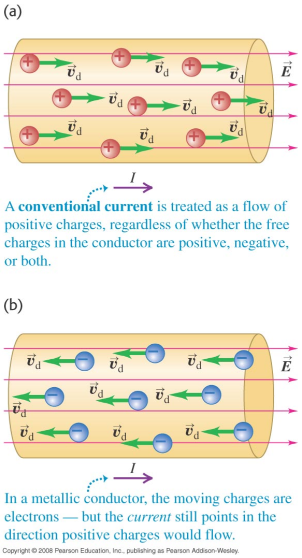
- If the charge carriers are $+$, $\vec{\mathbf{F}}$ is in the same direction with $\vec{\mathbf{E}}$ and the $\vec{\mathbf{v}}_\mathrm{d}$.
- If the charge carriers are negative $\vec{\mathbf{F}}$ is opposite to $\vec{\mathbf{E}}$, and the drift velocity $\vec{\mathbf{v}}_\mathrm{d}$ is from right to left.

Direction of current



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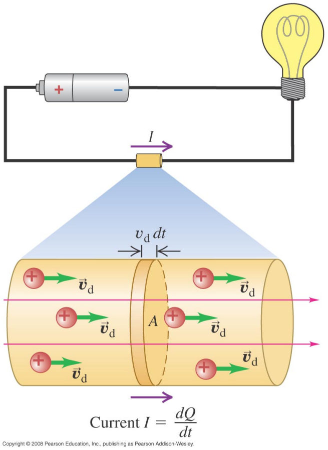
In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

- In both cases there is a net flow of + charge from left to right, and + charges end up to the right of − ones.
- We define the current, denoted by I, to be in the direction in which there is a flow of + charge.
- Thus we describe currents as though they consisted entirely of + charge flow, even in cases in which we know that the actual current is due to e^-s.

Current



- Here we consider the moving charges to be positive, so they are moving in the same direction as the current.
- We define the current through the cross-sectional area A to be the net charge flowing through the area per unit time:

electric current:

$$\boxed{I \equiv \frac{\mathrm{d}Q}{\mathrm{d}t}}$$

Current is not a vector!

- Although we refer to the direction of a current, current is not a vector quantity.
- In a current-carrying wire, the current is always along the length of the wire, regardless of whether the wire is straight or curved.
- No single vector could describe motion along a curved path.

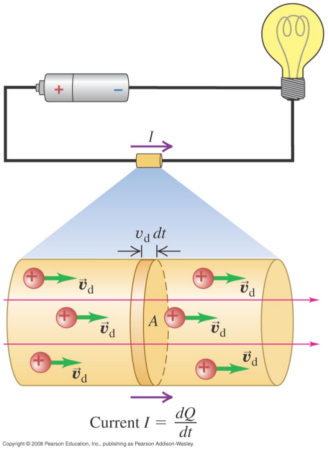
The unit of current

The SI unit of current is the *ampere*[1]; one ampere is defined to be one coulomb per second:

$$\boxed{1\,\mathrm{A} = 1\,\mathrm{C/s}}$$

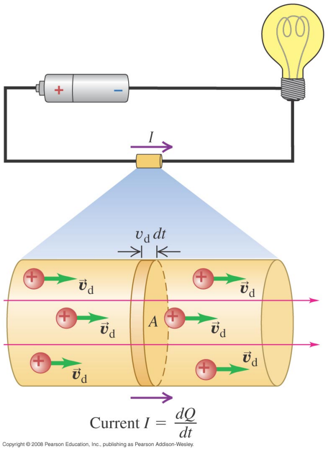
- Ordinary flashlight: $I = (0.5 - 1)\,\mathrm{A}$;
- A car engine's starter motor $I = 200\,\mathrm{A}$.
- Radio/television circuits: $1\,\mathrm{mA} = 10^{-3}\,\mathrm{A}$ or $1\,\mu\mathrm{A} = 10^{-6}\,\mathrm{A}$
- Computer circuits: $1\,\mathrm{nA} = 10^{-9}\,\mathrm{A}$ or $1\,\mathrm{pA} = 10^{-12}\,\mathrm{A}$.

[1] This unit is named in honor of the French scientist André Marie Ampère (1775-1836).



- Here we assume the charge carriers to be $+$, so they move in the same direction as I.
- n: charge carrier particles per unit volume (m^{-3}).
- Assume that all the particles move with the same drift velocity with magnitude v_d. In a time interval $\mathrm{d}t$, each particle moves a distance $v_\mathrm{d}\,\mathrm{d}t$.
- The particles that flow out of the right end of the shaded cylinder with length $v_\mathrm{d}\,\mathrm{d}t$ during $\mathrm{d}t$ are the particles that were within this cylinder at the beginning of the interval $\mathrm{d}t$.

Current & Drift velocity



- The volume of the cylinder is $\mathrm{d}V = Av_\mathrm{d}\,\mathrm{d}t$,
- the number of particles within it is $\mathrm{d}N = nAv_\mathrm{d}\,\mathrm{d}t$.
- If each particle has a charge q, the charge $\mathrm{d}Q$ that flows out of the end of the cylinder during time $\mathrm{d}t$ is

$$\mathrm{d}Q = q\mathrm{d}N = qnAv_\mathrm{d}\,\mathrm{d}t$$

Dividing both sides with $\mathrm{d}t$

$$\boxed{I = qnAv_\mathrm{d}}$$

Current density

Recall: $I = qnAv_\mathrm{d}$

- The current per unit cross-sectional area is called the current density J:

$$\boxed{J = I/A = qnv_\mathrm{d}}$$

- The SI unit of J is $\mathrm{A/m^2}$.
- We can also define a vector current density $\vec{\mathbf{J}}$ that includes the direction of the drift velocity:

$$\boxed{\vec{\mathbf{J}} = qn\vec{\mathbf{v}}_\mathrm{d}}$$

- Current density $\vec{\mathbf{J}}$ is a vector, but current I is not. $\vec{\mathbf{J}}$ describes how charges flow at a certain point (local), whereas I describes how charges flow through an extended object such as a wire.

Example: current density and drift velocity

Question

A copper wire of diameter $1\,\mathrm{mm}$ carries a constant current of $0.30\,\mathrm{A}$ to a $60\,\mathrm{W}$ bulb. The free-electron density in the wire is $n = 8.5 \times 10^{28}\,\mathrm{m}^{-3}$. Find (a) J and (b) v_{d}.

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Solution

- Crosssectinal area of the wire:
  $A = \pi r^2 = 3.14 \times \left(\frac{1\times10^{-3}\,\mathrm{m}}{2}\right)^2 = 7.85 \times 10^{-7}\,\mathrm{m}^2$.
- The magnitude of the current density:
  $J = \frac{I}{A} = \frac{0.30\,\mathrm{A}}{7.85\times10^{-7}\,\mathrm{m}^2} = 3.82 \times 10^7\,\mathrm{A/m}^2$.
- Drift velocity: $v_\mathrm{d} = \frac{J}{|q|n} = \frac{3.82\times10^7\,\mathrm{A/m}^2}{|-1.6\times10^{-19}\,\mathrm{C}|\times8.5\times10^{28}\,\mathrm{m}^{-3}} =$
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At this speed an e^- would require $\sim 6\,\mathrm{min}$ to travel $1\,\mathrm{m}$ along this wire! $v_{\mathrm{random}} \sim 10^{11} v_{\mathrm{d}}$. Electrons indeed *drift*!

Example: drift velocity

Question

How would the drift velocity change if we doubled the diameter of the copper wire in the previous problem while keeping the current the same.

a) None–v_{d} would be unchanged;

b) v_{d} would be twice as great;

c) v_{d} would be four times greater;

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Example: charge carrier density

Question

The mass density of silver at room temperature is $\rho_\mathrm{m} = 10.5 \times 10^3\,\mathrm{kg\,m^{-3}}$ and its atomic mass is $A = 108\,\mathrm{g/mol}$. If we assume there is $s = 1$ free electron per silver atom, what is the free-electron density for silver (n), in electrons/m^3?

Solution

- $A = 0.108\,\mathrm{kg/mol}$. $\frac{\rho_\mathrm{m}}{A}$ has unit $\mathrm{mol/m^3}$.
- $1\,\mathrm{mol} = N_0 = 6.02 \times 10^{23}$
- $\frac{N_0 \rho_\mathrm{m}}{A}$ Ag atoms per m^3
- $n = \frac{s N_0 \rho_\mathrm{m}}{A}$ electrons per m^3
- $n_\mathrm{Ag} = 1 \times 6.02 \times 10^{23} \times 10.5 \times 10^3 / 0.108 = 5.85 \times 10^{28}\,\mathrm{m^{-3}}$.
- Recall $n_\mathrm{Cu} = 8.5 \times 10^{28}\,\mathrm{m^{-3}}$.

Resistivity

Resistivity

The resistivity ρ of a material is defined as the ratio of the electric field applied to the current density:

$$\boxed{\rho \equiv \frac{E}{J}}$$

The greater the resistivity ρ, the greater the field E needed to cause a given current density, or the smaller the current density J caused by a given field.

Note that we have used ρ also for the volume charge density. What is meant by ρ should be evident from the context.

The unit of resistivity

Recall that

- the unit of E is $\mathrm{V/m}$
- the unit of J is $\mathrm{A/m^2}$

Thus $\rho = E/J$ implies that the unit of ρ is

$$\boxed{\frac{\mathrm{V\cdot m}}{\mathrm{A}}}$$

As we will soon see $1\,\mathrm{V/A} = 1\,\Omega$ (Ohm). We thus obtain the unit of ρ as

$$\boxed{\Omega\cdot\mathrm{m}}$$

Resistivities of materials

Note that resistivity is temperature dependent.

Resistivities at Room Temperature (20°C)

	Substance	ρ ($\Omega \cdot$ m)	Substance	ρ ($\Omega \cdot$ m)
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Ratio of the resistivity of quartz to silver is $\sim 10^{25}$!

Conductivity

The reciprocal of resistivity is conductivity:

$$\boxed{\sigma \equiv \frac{J}{E} = \frac{1}{\rho}}$$

Its unit is

$$\boxed{(\Omega \cdot \mathrm{m})^{-1} = \mathrm{S} \cdot \mathrm{m}^{-1} = \text{siemens per meter}}$$

Note that we have used σ also for the surface charge density. What is meant by σ should be evident from the context.

Ohm's "law"

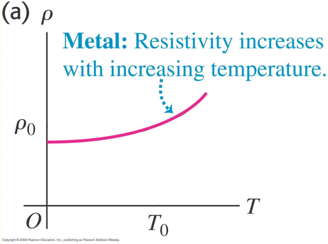
The relation between $\vec{\mathbf{E}}$ and $\vec{\mathbf{J}}$ can be very complex, but for some materials, especially metals it can be simply linear as shown by Georg Simon Ohm (1787-1854):

$$\boxed{\vec{\mathbf{E}} = \rho\vec{\mathbf{J}},} \qquad \boxed{\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}} \qquad \text{Ohm's “law”}$$

The materials that obey Ohm's law are called 'Ohmic'. For such materials, at a given temperature, ρ is a constant that does not depend on the value of E. Many materials show substantial departures from Ohm's-'law'; they are nonohmic, or nonlinear.

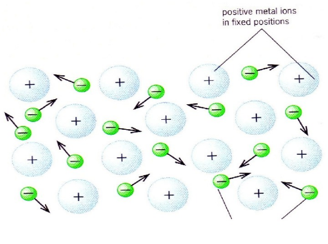
Note: We put the word "law" in quotation marks, since Ohm's "law", like the ideal-gas equation and Hooke's law, is an idealized model of an emprical relation that describes the behavior of *some* materials quite well but is not a *general* description of all matter.

Resistivity & Temperature: metals



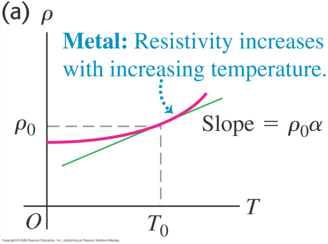
The resistivity of a metallic conductor increases with increasing temperature T.

Resistivity & Temperature: metals



As T increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving e^- will collide with an ion; this impedes the drift of e^- through the conductor and hence reduces the current.

Resistivity & Temperature: metals



Over a small temperature range (up to 100° C or so), the resistivity of a metal can be represented approximately by the equation

$$\boxed{\rho = \rho_0 \left[1 + \alpha(T - T_0)\right]}$$

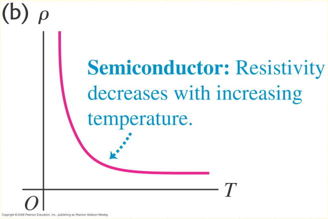
where the factor α is called the *temperature coefficient of resistivity*, and ρ_0 and T_0 are the reference values.

Temperature coefficients of resistivity

Table 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

Material	$\alpha\,[(^{\circ}\mathrm{C})^{-1}]$	Material	$\alpha\,[(^{\circ}\mathrm{C})^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

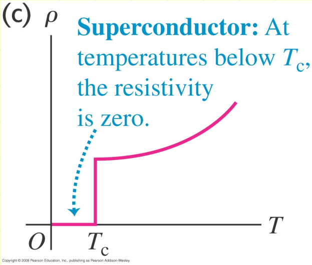
Resistivity & Temperature: semiconductors



- The resistivity ρ of semiconductors decreases with increasing temperature T.

- This is because at higher temperatures, more electrons become free from the atoms and become mobile; hence the temperature coefficient of resistivity (α) is negative.
- Measuring the resistivity of a small semiconductor crystal is therefore a sensitive measure of temperature; this is the principle of a type of thermometer called a *thermistor*.

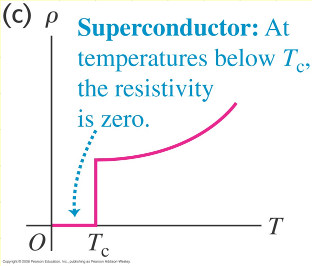
Resistivity& Temperature: superconductors



- Some materials, including several metallic alloys and oxides, show a phenomenon called *superconductivity* (SC) below a critical temperature, T_c.

- As the T decreases, ρ at first decreases smoothly, like that of any metal. But then at a certain T_c a phase transition occurs and ρ suddenly drops to zero.
- Once a current has been established in a SC ring, it continues indefinitely without the presence of any driving field.

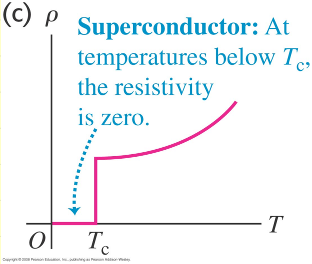
Resistivity& Temperature: superconductors



- Superconductivity was discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes (1853-1926).

- At very low temperatures, below 4.2 K, the resistivity of mercury suddenly drops to zero.
- For the next 75 years, the highest T_c attained was ~ 20 K.
- In 1986 Karl Müller and Johannes Bednorz discovered an oxide of barium, lanthanum, and copper with a T_c of nearly 40 K: "high-T_c" SC materials.

Resistivity& Temperature: superconductors



- By 1987 a complex oxide of yttrium, copper, and barium had been found that has a value of T_c well above the 77 K.
- By 2018 the record for T_c is 203.5 K, and it may soon be possible to fabricate materials that are SCs at room T.
- The implications for power-distribution systems, computer design, and transportation!
- Nowadays, SC electromagnets are used in particle accelerators and experimental magnetic-levitation railroads.
- SCs have other properties that require an understanding of magnetism to explore.

Resistance

Resistance

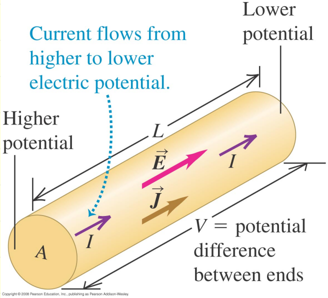
For a conductor with resistivity ρ, the current density $\vec{\mathbf{J}}$ and the electric field $\vec{\mathbf{E}}$ at a point are related by the Ohm's 'law'

$$\vec{\mathbf{E}} = \rho \vec{\mathbf{J}},$$

Often, however, we are more interested in the total current I in a conductor than in J and more interested in the potential difference V between the ends of the conductor than in E.

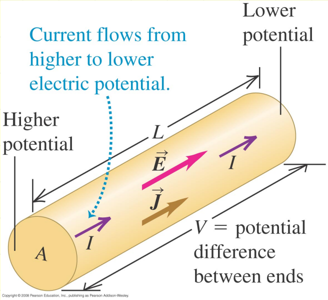
This is so, largely because I and V are much easier to measure than are J and E.

Consider a conducting wire with uniform cross-sectional area A and length L.



- Let V be the potential difference between the higher-potential and lower-potential ends of the conductor, so that $V > 0$.
- The direction of I is always from the higher-V end to the lower-V end. That's because I in a conductor flows in the direction of $\vec{\mathbf{E}}$, no matter what the sign of q, and because $\vec{\mathbf{E}}$ points in the direction of decreasing V.
- As the current flows through the potential difference, electric potential energy is transferred to the ions of the conducting material by collisions.

Resistance

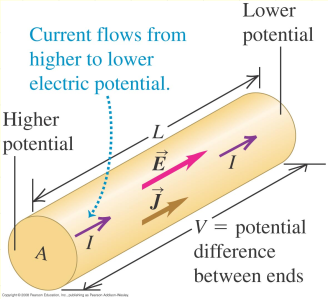


- $V = -\int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = EL$
- $J = \frac{I}{A}$
- $E = \rho J$

$$\Rightarrow \frac{V}{L} = \rho \frac{I}{A}$$

$$\Rightarrow V = \rho \frac{L}{A} I$$

Resistance



- We have found

$$V = \rho \frac{L}{A} I$$

When ρ is constant $I \propto V$

- Define the resistence R as

$$\boxed{R \equiv \frac{V}{I}}$$

- Comparison gives

$$\boxed{R = \rho \frac{L}{A}, \qquad \text{for uniform wire}}$$

Question

A ductile metal wire has resistance R. What will be the resistance of this wire in terms of R if it is stretched to three times its original length, assuming that the density and resistivity of the material do not change when the wire is stretched?

Exercise: dependence of resistivity on length and area

Question

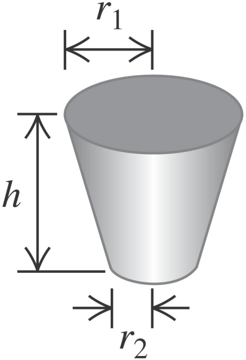
A ductile metal wire has resistance R. What will be the resistance of this wire in terms of R if it is stretched to three times its original length, assuming that the density and resistivity of the material do not change when the wire is stretched?

Answer

The volume will remain the same during stretching:
$\mathcal{V} = LA = L'A'$. Thus $A' = A/3$

$$R' = \rho \frac{L'}{A'} = \rho \frac{3L}{A/3} = 9\rho \frac{L}{A} = 9R$$

Exercise: resistivity of a conical wire

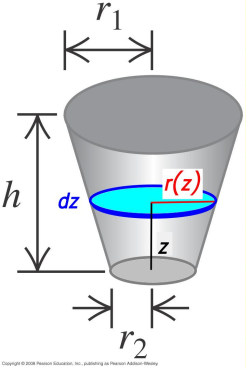


Question

A material of resistivity ρ is formed into a solid, truncated cone of height h and radii r_1 and r_2 at either end. Calculate the resistance of the cone between the two flat end faces.

Exercise: resistivity of a conical wire

Imagine slicing the cone into very many thin disks of thickness $\mathrm{d}z$.



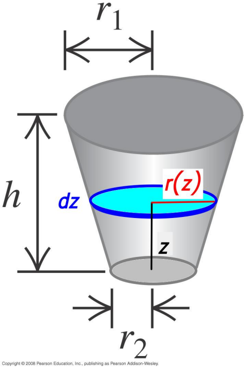
- Calculate the resistance of one such disk: $\mathrm{d}R = \rho \frac{\mathrm{d}z}{\pi r^2(z)}$.
- Note that $r(z)$ is a linear function: $r(z) = az + b$ where a and b are to be found by $r(z=0) = r_2$ and $r(z=h) = r_1$. These imply $b = r_2$ and $a = \frac{r_1 - r_2}{h}$:

$$r(z) = \frac{r_1 - r_2}{h} z + r_2, \qquad \mathrm{d}r = \frac{r_1 - r_2}{h} \mathrm{d}z$$

Thus $\mathrm{d}z = \frac{h}{r_1 - r_2} \mathrm{d}r$

Exercise: resistivity of a conical wire

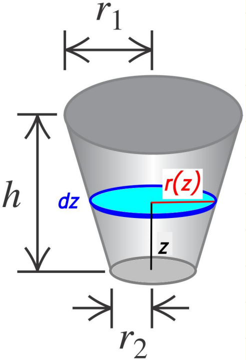
Imagine slicing the cone into very many thin disks of thickness $\mathrm{d}z$.



- $$R = \int \mathrm{d}R = \rho \int_0^h \frac{\mathrm{d}z}{\pi r^2(z)}$$
- $\mathrm{d}z = \frac{h}{r_1 - r_2}\mathrm{d}r$
  $$R = \frac{\rho h}{\pi(r_1 - r_2)} \left(-\frac{1}{r}\right)_{r_2}^{r_1}$$
- Finally,
  $$\boxed{R = \rho \frac{h}{\pi r_1 r_2}}$$

Exercise: resistivity of a conical wire

Imagine slicing the cone into very many thin disks of thickness dz.

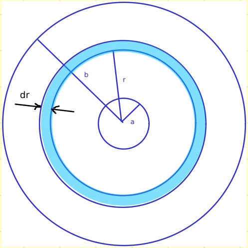


- Special case: Let us check that the result, $R = \rho\frac{h}{\pi r_1 r_2}$ agrees with $R = \rho L/A$ at the appropriate limit, i.e. for a uniform-cross section wire.
- For a uniform-cross section wire $r_1 = r_2$ and hence

$$R = \rho\frac{h}{\pi r_1 r_2} = \rho\frac{h}{\pi r_1^2} = \rho\frac{h}{A}$$

as expected.

Exercise: resistivity of a spherical shell

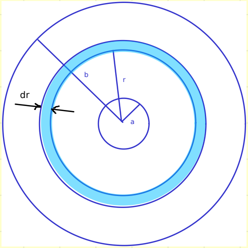


Question

The region between two concentric conducting spheres with radii a and b is filled with a conducting material with resistivity ρ.

(a) Show that the resistance between the spheres is given by

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$



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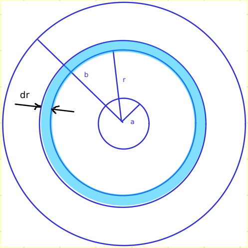
$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

(b) Derive an expression for the current density as a function of radius, in terms of the potential difference V_{ab} between the spheres.

Exercise: resistivity of a spherical shell



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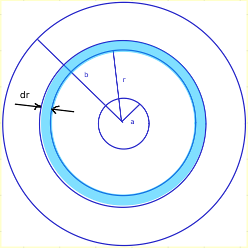
(b) Derive an expression for the current density as a function of radius, in terms of the potential difference V_{ab} between the spheres.

(c) Show that the result in part (a) reduces to $R = \rho L/A$ when the separation $L = b - a$ between the spheres is small.



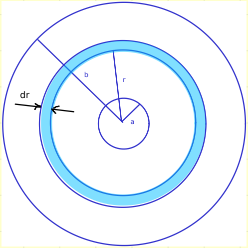
Question

Solution - (a)

Start by writing $\mathrm{d}R = \rho\frac{\mathrm{d}r}{4\pi r^2}$ Integrate over r from a to b

$$R = \rho \int_a^b \frac{\mathrm{d}r}{4\pi r^2} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Exercise: resistivity of a spherical shell

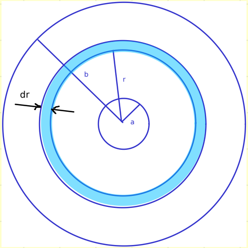


Question

Solution - (b)

Start by writing $J = I/A$ where $I = V/R$ and $A = 4\pi r^2$. Recall $R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$ as well.

Exercise: resistivity of a spherical shell



Question

Solution - (c)

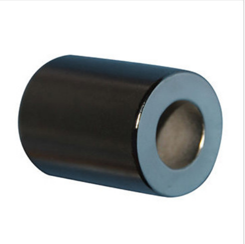
Start by rewriting R

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho}{4\pi}\frac{b-a}{ab}$$

For a small radius difference $a \simeq b$ you see that $4\pi ab \simeq 4\pi a^2 = A$ and let us call $L = b - a$. Thus the result is of the form

$$R = \rho\frac{L}{A}$$

Exercise: two resistances of a cylindrical shell



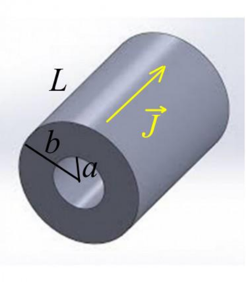
Question

Consider a hollow cylinder of length L, inner radius of a and outer radius b has resistivity ρ. Treat each surface (inner, outer, and the two end faces) as an equipotential surface. What is the resistence between

(a) the opposite faces and

(b) the inner and outer surfaces?

Exercise: two resistances of a cylindrical shell



Question - (a)

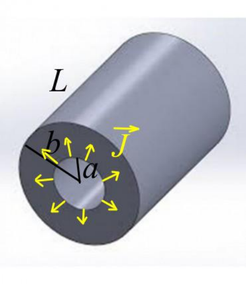
What is the resistence between the opposite faces?

Solution (a)

In this case $A = \pi(b^2 - a^2)$ and so

$$R = \rho \frac{L}{\pi(b^2 - a^2)}$$

Exercise: two resistances of a cylindrical shell



Question - (b)

What is the resistence between the inner and outer surfaces?

Solution (b)

In this case $A = 2\pi r L$ (adjacent area of a cylinder of radius r) and so

$$\mathrm{d}R = \rho \frac{\mathrm{d}r}{2\pi r L}$$

$$R = \frac{\rho}{2\pi L} \int_a^b \frac{\mathrm{d}r}{r} = \frac{\rho}{2\pi L} \ln \frac{b}{a}$$

Mechanical analogue of resistence

- In understanding $R = \rho L/A$ we can think of a narrow water hose.
- a narrow water hose offers more resistance to flow than a fat one
- a long hose has more resistance than a short one
- We can increase the resistance to flow by stuffing the hose with cotton or sand; this corresponds to increasing the resistivity.
- Flow rate is analogous to current.
- Potential difference is similar to the pressure difference between the tips.

Temperature dependence of resistence

- Because the resistivity of a material varies with T, the resistance of a specific conductor also varies with T.
- For temperature ranges that are not too great, this variation is approximately linear, analogous to

$$\rho = \rho_0 \left[1 + \alpha(T - T_0)\right]$$

- It is thus

$$R(T) = R_0 \left[1 + \alpha(T - T_0)\right]$$

- $R(T)$ is the resistance at T and R_0 is the resistance at T_0, often taken to be 0° C or 20° C.
- The temperature coefficient of resistance α is the same in both eqns if L and A do not change appreciably with T.

Exercise:

Question:

Suppose the resistance of a copper wire is $1.05\,\Omega$ at $20°C$. Find the resistance at $0°C$ and $100°C$.

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Suppose the resistance of a copper wire is $1.05\,\Omega$ at $20°\mathrm{C}$. Find the resistance at $0°\mathrm{C}$ and $100°\mathrm{C}$.

Solution: $T = 0°\mathrm{C}$

From Table, $\alpha = 0.00393\,(\mathrm{C}°)^{-1}$ for copper. Then Then by using

$$\begin{aligned} R(T) &= R_0\,[1 + \alpha(T - T_0)] \\ &= 1.05\,\Omega\,\left[1 + (0.00393\,(\mathrm{C}°)^{-1})\,(0°\mathrm{C} - 20°\mathrm{C})\right] \\ &= 0.97\,\Omega \end{aligned}$$

Exercise:

Question:

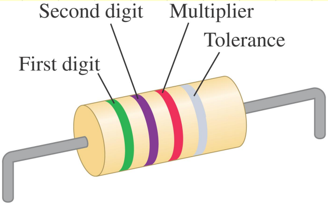
Suppose the resistance of a copper wire is $1.05\,\Omega$ at $20°\mathrm{C}$. Find the resistance at $0°\mathrm{C}$ and $100°\mathrm{C}$.

Solution: $T = 100°\mathrm{C}$

From Table, $\alpha = 0.00393\,(\mathrm{C}°)^{-1}$ for copper. Then Then by using

$$\begin{aligned} R(T) &= R_0\,[1 + \alpha(T - T_0)] \\ &= 1.05\,\Omega\,\left[1 + (0.00393\,(\mathrm{C}°)^{-1})\,(100°\mathrm{C} - 20°\mathrm{C})\right] \\ &= 1.38\,\Omega \end{aligned}$$

Color codes for resistors



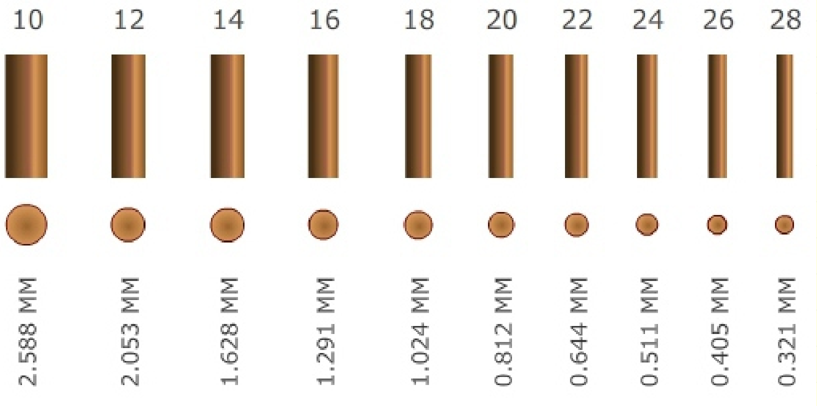
Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

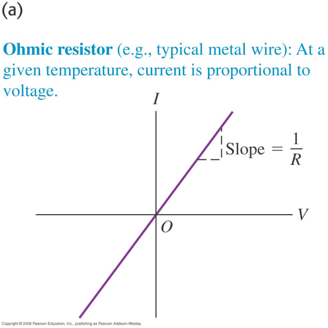
- A circuit device made to have a specific value of resistance between its ends is called a *resistor*.
- R may be marked with a standard code that uses three or four color bands near one end.
- The 4th band, if present, indicates the accuracy (tolerance) of the value;
  no band means 20%,
  a silver band 10%,
  a gold band 5%.

Wire gauges & resistivity

We rank wire diameter by "Gauge"



Ohm's law–again



- For a resistor that obeys Ohm's law, a graph of current as a function of potential difference (voltage) is a straight line.
- The slope of the line is $1/R$. (recall $V = RI$)
- If the sign of the potential difference changes, so does the sign of the current produced;
- This corresponds to interchanging the higher- and lower-potential ends of the conductor, so the electric field, current density, and current all reverse direction.

Exercise

Question:

An 18 gauge copper wire has a cross-sectional area of $8.17 \times 10^{-7}\,\mathrm{m}^2$. It carries a current of $1.67\,\mathrm{A}$. Find (a) the electricfield magnitude in the wire; (b) the potential difference between two points in the wire $50.0\,\mathrm{m}$ apart; (c) the resistance of a $50.0\,\mathrm{m}$ length of this wire.

Exercise

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Solution (a):

From Table $\rho = 1.72 \times 10^{-8}\,\Omega \cdot \mathrm{m}$. Hence
$E = \rho J = \rho I/A = 0.0352\mathrm{V/m}$

Exercise

Question:

An 18 gauge copper wire has a cross-sectional area of $8.17 \times 10^{-7}\,\mathrm{m}^2$. It carries a current of $1.67\,\mathrm{A}$. Find (a) the electricfield magnitude in the wire; (b) the potential difference between two points in the wire $50.0\,\mathrm{m}$ apart; (c) the resistance of a $50.0\,\mathrm{m}$ length of this wire.

Solution (b):

Potential difference is

$V = El = (0.0352\mathrm{V/m})(50\,\mathrm{m}) = 1.76\,\mathrm{V}$

Exercise

Question:

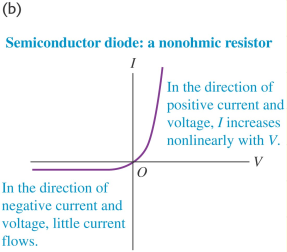
An 18 gauge copper wire has a cross-sectional area of $8.17 \times 10^{-7}\,\mathrm{m}^2$. It carries a current of $1.67\,\mathrm{A}$. Find (a) the electricfield magnitude in the wire; (b) the potential difference between two points in the wire $50.0\,\mathrm{m}$ apart; (c) the resistance of a $50.0\,\mathrm{m}$ length of this wire.

Solution (c):

$R = \rho \frac{l}{A} = 1.05\,\Omega$

Alternatively, we can find R from $R = V/I$.

Non-Ohmic devices



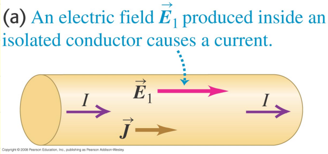
- In devices do not obey Ohm's law, I-V relation is not linear.
- *Semiconductor diode* is a device used to convert alternating current to direct current and to perform a wide variety of logic functions in computer circuitry.
- For $V > 0$, I increases exponentially with increasing V; I is extremely small for $V < 0$.
- Thus $V > 0$ causes a current to flow in the $+$ direction, but $V < 0$ causes little or no current. Hence a diode acts like a one-way valve in a circuit.

Electromotive force and circuits

Why circuits?

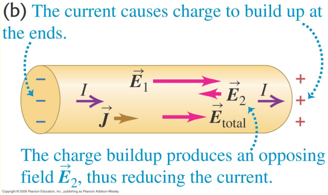
For a conductor to have a steady current, it must be part of a path that forms a closed loop or complete circuit. Why?

Why circuits?



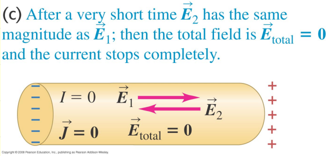
If you establish an electric field $\vec{\mathbf{E}}_1$ inside an isolated conductor with resistivity ρ that is *not* part of a complete circuit, a current begins to flow with current density $\vec{\mathbf{J}} = \vec{\mathbf{E}}_1/\rho$.

Why circuits?



As a result a net positive charge quickly accumulates at one end of the conductor and a net negative charge accumulates at the other end.

Why circuits?

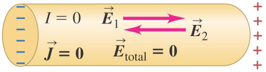


These charges themselves produce an electric field $\vec{\mathbf{E}}_2$ in the direction opposite to $\vec{\mathbf{E}}_1$, causing the total electric field and hence the current to decrease. Within a very small fraction of a second, enough charge builds up on the conductor ends that the total electric field

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \mathbf{0}$$

inside the conductor. Then $\vec{\mathbf{J}} = \vec{\mathbf{E}}/\rho = \mathbf{0}$ as well, and the current stops altogether.

(c) After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$; then the total field is $\vec{E}_{\text{total}} = \mathbf{0}$ and the current stops completely.



So there can be no steady motion of charge in such an incomplete circuit.

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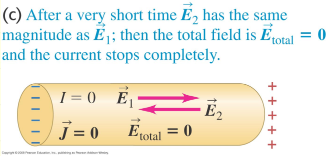
Why circuits?



So there can be no steady motion of charge in such an incomplete circuit.

We thus conclude that a complete circuit must be established in order to obtain a steady current.

These charges themselves produce an electric field $\vec{\mathbf{E}}_2$ in the direction opposite to $\vec{\mathbf{E}}_1$, causing the total electric field and hence the current to decrease. Within a very small fraction of a second, enough charge builds up on the conductor ends that the total electric field

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \mathbf{0}$$

inside the conductor. Then $\vec{\mathbf{J}} = \vec{\mathbf{E}}/\rho = \mathbf{0}$ as well, and the current stops altogether.

"Pumping" charges up!

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- But we know that if a charge q goes around a complete circuit and returns to its starting point, the potential energy U must be the same at the end of the round trip as at the beginning.
- We also know that there is always a *decrease* in U when charges move through an ordinary conducting material with resistance.
- So we conclude that in order to maintain a steady current in a *complete* circuit there must be an element of the circuit through which U increases!

"Pumping" charges up!

- The problem is analogous to an ornamental water fountain that recycles its water.
- Water moves in the direction of decreasing gravitational potential energy, and collects in a basin in the bottom. So how can it be recycled continuously?
- A *pump* keeps lifting the water up!

What drives a circuit?

- In an electric circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain.
- In this device a charge would travel "uphill," i.e. from lower to higher potential energy, even though the electrostatic force is trying to push it from higher to lower potential energy.
- The direction of current in such a device is from lower to higher potential, just the opposite of what happens in an ordinary conductor.

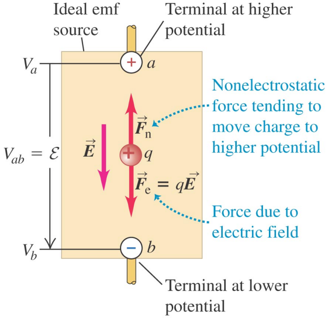
Electromotive force (emf) - 1

- The influence that makes current flow from lower to higher potential is called *electromotive force* (emf, $\mathcal{E}$)
- A circuit device that provides emf is called a *source of emf*.
- Note that “electromotive force” is a poor term because emf is not a force but an energy-per-unit-charge quantity, like potential.
- The SI unit of emf is the same as that for potential, the volt $1\,\mathrm{V} = 1\,\mathrm{J/C}$.
- A typical flashlight battery has an emf of $1.5\,\mathrm{V}$; this means that the battery does $1.5\,\mathrm{J}$ of work on every coulomb of charge that passes through it.

Electromotive force (emf) - 2

- Every complete circuit with a steady current must include a source of emf.
- Batteries, electric generators, solar cells, thermocouples, and fuel cells are all examples of sources of emf.
- All such devices convert energy of some form (mechanical, chemical, thermal, and so on) into electric potential energy and transfer it into the circuit to which the device is connected.
- An *ideal source of emf* maintains a constant potential difference between its terminals, independent of the current through it.
- Real-life sources of emf has internal resistence.

Ideal emf

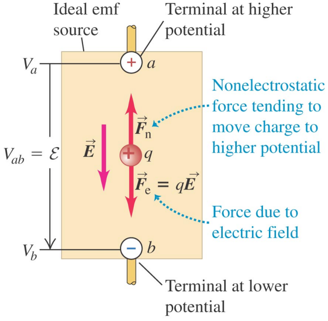


When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

- A charge q within the source experiences $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$.

- An ideal source of emf maintains a potential difference between conductors a and b, called the terminals of the device.
- Terminal a, marked $+$, is maintained at higher potential than terminal b, marked $-$.
- Associated with this potential difference is an electric field $\vec{\mathbf{E}}$ in the region around the terminals, both inside and outside the source.
- The electric field inside the device is directed from a to b.

Ideal emf

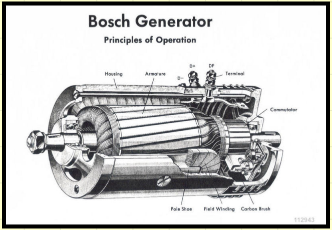


When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

- But the emf source also provides an additional influence, which we represent as a non-electrostatic force $\vec{\mathbf{F}}_n$.
- This force, operating inside the device, pushes charge from b to a in an "uphill" direction against $\vec{\mathbf{F}}_e$.
- Thus $\vec{\mathbf{F}}_n$ maintains the potential difference between terminals.
- If $\vec{\mathbf{F}}_n$ were not present, charge would flow between the terminals until potential difference was zero.

Origin of non-electrostatic force

The origin of the additional influence $\vec{\mathbf{F}}_{\mathrm{n}}$ depends on the kind of source:



- In a *generator* it results from magnetic-field forces on moving charges.
- In a battery or fuel cell it is associated with diffusion processes and varying electrolyte concentrations resulting from chemical reactions.
- In an electrostatic machine such as a *Van de Graaff generator*, an actual mechanical force is applied by a moving belt or wheel.

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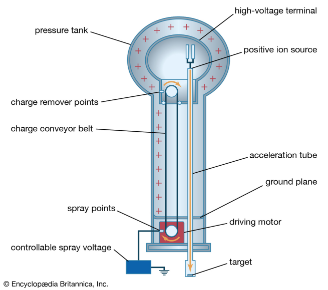

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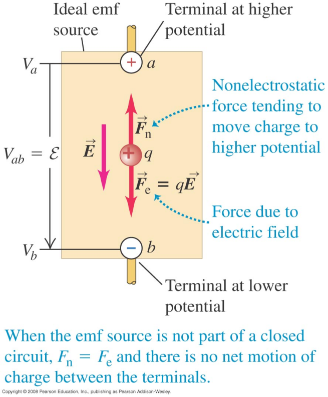
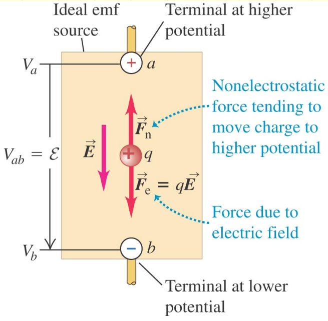
Ideal emf



- If a positive charge q is moved from b to a inside the source, $\vec{\mathbf{F}}_n$ does a positive amount of work $W_n = q\mathcal{E}$ on the charge.
- This displacement is opposite to $\vec{\mathbf{F}}_e$, so the U associated with q increases by an amount equal to qV_{ab}, where $V_{ab} = V_a - V_b$ is the (positive) potential of point a with respect to point b.

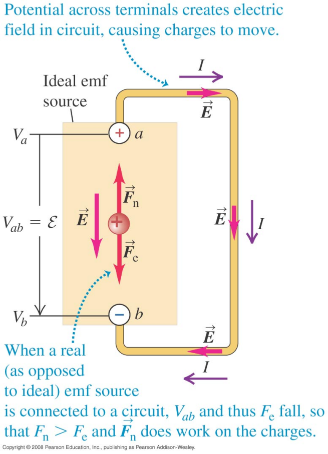
Ideal emf



When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

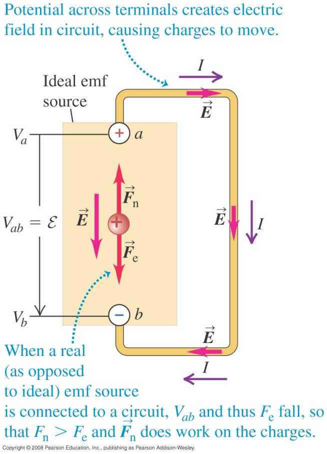
- For the ideal source of emf that we've described, $\vec{\mathbf{F}}_e$ and $\vec{\mathbf{F}}_n$ are equal in magnitude but opposite in direction, so the total work done on the charge q is zero; there is an increase in U but no change in the kinetic energy of the charge.
- It's like lifting a book from the floor to a high shelf at constant speed.
- The increase in U is just equal to the nonelectrostatic work W_n, so $q\mathcal{E} = qV_{ab}$, or

Ideal emf



- Now let's make a complete circuit by connecting a wire with R to the terminals of a source.
- The ΔV between terminals a and b sets up an E within the wire;
- This causes I to flow around the loop from a toward b, from higher to lower V.
- Where the wire bends, equal amounts of $+$ and $-$ charges persist on the "inside" and "outside" of the bend.

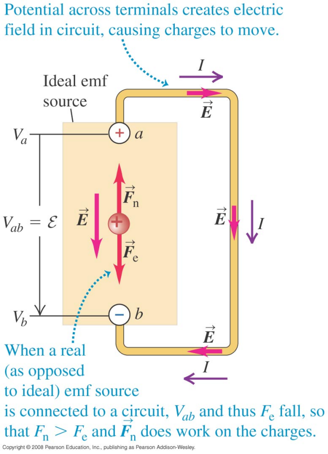
Ideal emf



- These charges exert the forces that cause I to follow the bends in the wire.
- From $V = RI$ the potential difference between the ends of the wire is given by $V_{ab} = IR$. Combining with $V_{ab} = \mathcal{E}$, we have

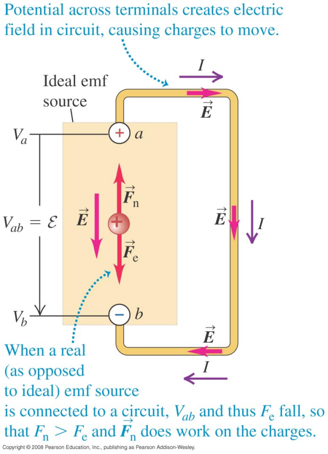
$$\mathcal{E} = V_{ab} = IR \;\; \text{(ideal source of emf)}$$

Ideal emf



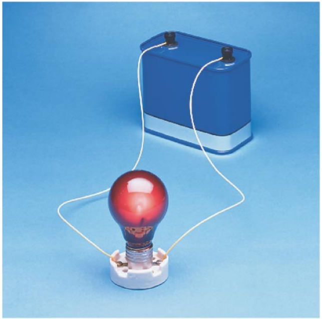
- That is, when a positive charge q flows around the circuit, the potential rise $\mathcal{E}$ as it passes through the ideal source is numerically equal to the potential drop $V_{ab} = IR$ as it passes through the remainder of the circuit.
- Once $\mathcal{E}$ and R are known, this relationship determines the current in the circuit.

Ideal emf



- A common misconception is to think that I is "used up" (consumed) in a circuit by the time it reaches the negative terminal. In fact I is the same at every point in a simple loop circuit. (Even if the wire thickness is not constant throughout the circuit)
- This is a result of Charge conservation!

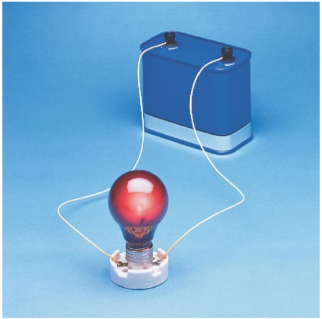
Internal resistence



- ΔV across a real source of emf in a circuit is not equal to the emf.
- q moving through the source of emf encounters resistance which we call the *internal resistance* of the source, and denote by r.
- Accordingly there is an associated drop in V equal to Ir
- Thus, when I is flowing through a source from the $-$ terminal b to the $+$ terminal a, V_{ab} between the terminals is

$$V_{ab} = \mathcal{E} - Ir$$

Internal resistence



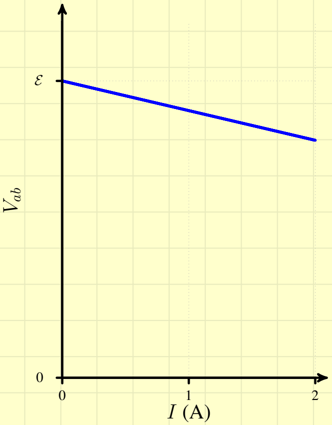
- The terminal voltage

$$V_{ab} = \mathcal{E} - Ir$$

is less than the emf $\mathcal{E}$ because of the term Ir representing V drop across the internal resistance r.

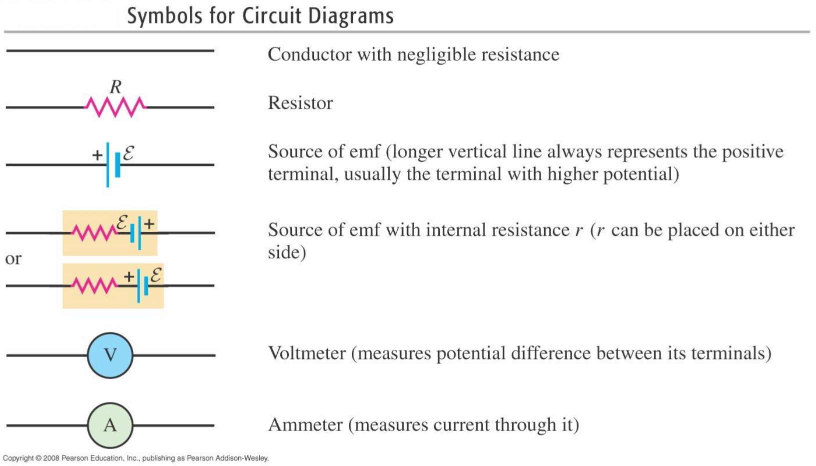
- Hence the increase in potential energy qV_{ab} as a charge q moves from b to a within the source is less than the work $q\mathcal{E}$ done by $\vec{\mathbf{F}}_{\mathrm{n}}$, since some U is lost in traversing r.

Internal resistence

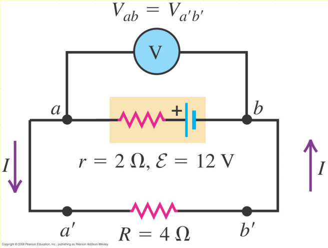


- A $1.5\,\mathrm{V}$ battery has an emf of $1.5\,\mathrm{V}$, but the terminal voltage V_{ab} of the battery is equal to $1.5\,\mathrm{V}$ only if no current is flowing through it so that $I = 0$
- If the battery is part of a complete circuit through which current is flowing, the terminal voltage will be less than $1.5\,\mathrm{V}$.
- For a real source of emf, the terminal voltage equals the emf only if no current is flowing through the source.

Symbols for Circuit Diagrams

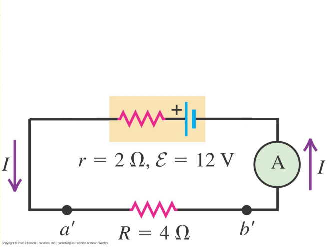


Internal resistance is an intrinsic part of a source of emf



- A voltmeter measures the potential difference between its terminals;
- An *idealized voltmeter* has infinitely large resistance and measures potential difference without having any current diverted through it.
- A voltmeter is connected in *parallel* to the circuit.

Ammeter



- A ammeter measures the current passing through it.
- An *idealized ammeter* has zero resistance and has no potential difference between its terminals.
- A ammeter is connected in *series* to the circuit.

Example: current and voltage across a resistor

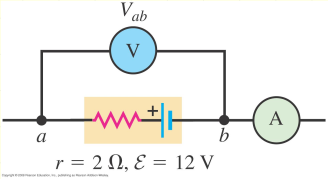
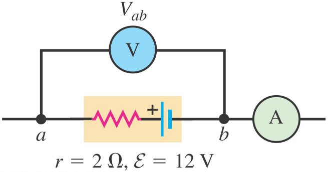


Figure shows a source (a battery) with emf $\mathcal{E} = 12\,\mathrm{V}$ and internal resistance $r = 2\,\Omega$. The wires to the left of a and to the right of the ammeter A are not connected to anything. Determine the respective readings V_{ab} and I of the idealized voltmeter V and the idealized ammeter A.

Example: current and voltage across a resistor



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There is zero current because there is no complete circuit. (Idealized voltmeter: $r \to \infty$, so no current flows through it.) Hence the ammeter reads $I = 0$. Because $I = 0$ through the battery, there is no potential difference across its internal resistance. V_{ab} across the battery terminals is equal to the emf. So the voltmeter reads $V_{ab} = \mathcal{E} = 12\,\text{V}$.

Example: current and voltage across a resistor

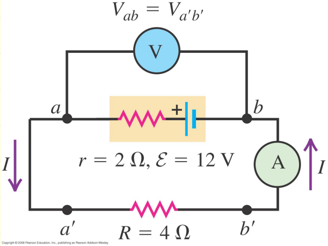
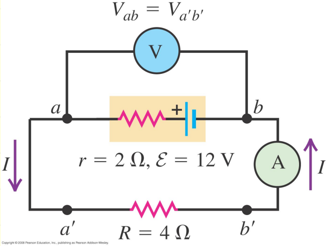


Figure shows a source (a battery) with emf $\mathcal{E} = 12\,\mathrm{V}$ and internal resistance $r = 2\,\Omega$. We add a $4\,\Omega$ resistor to the battery, forming a complete circuit. What are the voltmeter and ammeter readings V_{ab} and I now?



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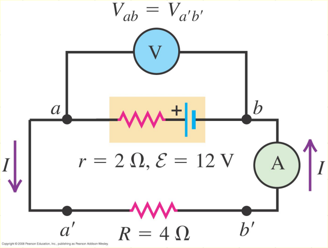
Total resistance $R + r$ and so $I = \frac{\mathcal{E}}{R+r} = \frac{12\,\mathrm{V}}{4\,\Omega + 2\,\Omega} = 2\,\mathrm{A}$

Example: current and voltage across a resistor



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Total resistance $R + r$ and so $I = \frac{\mathcal{E}}{R+r} = \frac{12\,\mathrm{V}}{4\,\Omega + 2\,\Omega} = 2\,\mathrm{A}$

$V_{a'b'} = V_{ab} = RI = (4\,\Omega)\,(2\,\mathrm{A}) = 8\,\mathrm{V}$

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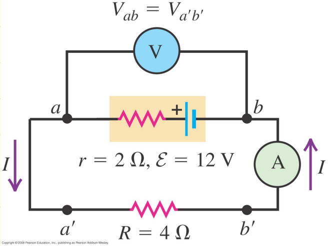

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Total resistance $R + r$ and so $I = \frac{\mathcal{E}}{R+r} = \frac{12\,\mathrm{V}}{4\,\Omega + 2\,\Omega} = 2\,\mathrm{A}$

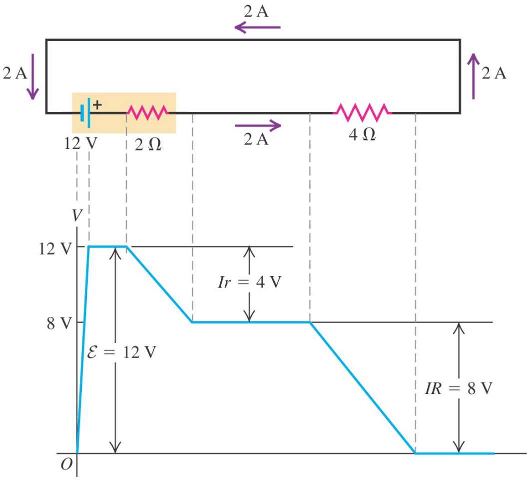
$V_{a'b'} = V_{ab} = RI = (4\,\Omega)\,(2\,\mathrm{A}) = 8\,\mathrm{V}$

Similarly, $V_{ab} = \mathcal{E} - rI = 12\,\mathrm{V} - (2\,\Omega)\,(2\,\mathrm{A}) = 8\,\mathrm{V}$

Charging a battery

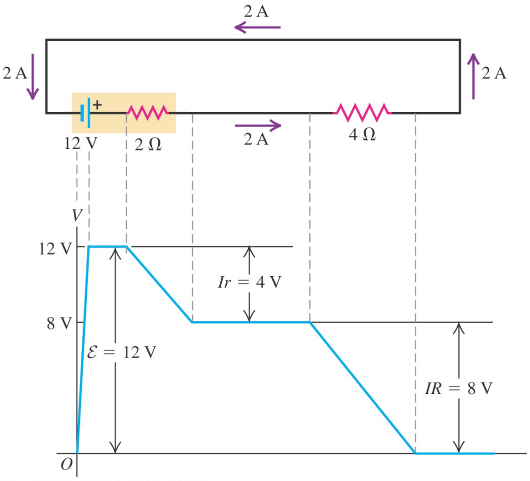
- If a battery is acting as a source that produces the current in the circuit, current flows through the battery from its $-$ terminal toward its $+$ terminal, and $V_{ab} < \mathcal{E}$.
- But if a battery is being recharged, current flows through it in the opposite direction, from its positive terminal toward its negative terminal. In such a case $I < 0$ in $V_{ab} = \mathcal{E} - rI$, so $V_{ab} > \mathcal{E}$.
- No matter which way current flows through the battery, the smaller the internal resistance r, the less the difference between V_{ab} and $\mathcal{E}$.

Potential Changes Around a Circuit



$$\mathcal{E} - rI - RI = 0$$

Potential Changes Around a Circuit

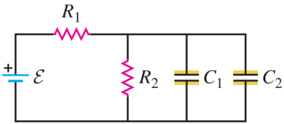


The net change in potential energy for a charge q making a round trip around a complete circuit must be zero.

Exercise: Capacitor circuit

25.70 •• **CP** Consider the circuit shown in **Fig. P25.70**. The battery has emf 52.0 V and negligible internal resistance. $R_2 = 3.00\,\Omega$, $C_1 = 4.00\,\mu\text{F}$, and $C_2 = 5.00\,\mu\text{F}$. After the capacitors have attained their final charges, the charge on C_1 is $Q_1 = 18.0\,\mu\text{C}$. What is (a) the final charge on C_2; (b) the resistance R_1?

Figure **P25.70**



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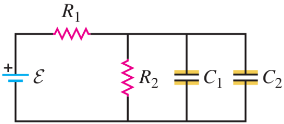


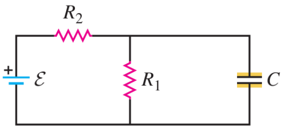
Solution

- No current on the capacitors. The current through the resistors is $I = \frac{\mathcal{E}}{R_1+R_2}$, but we don't know R_1!
- ΔV on R_2 is $\Delta V = R_2 I$ is the same as ΔV on capacitors.
- $Q_1 = C_1 \Delta V \Rightarrow \Delta V = \frac{Q_1}{C_1} = 4.5\,\text{V}$, $Q_2 = C_2 \Delta V = 22.5\,\mu\text{C}$.
- $I = \Delta V / R_2 = 1.5\,\text{A}$
- $R_1 + R_2 = \frac{\mathcal{E}}{I} = 34.7\,\Omega \Rightarrow R_1 = 31.7\,\Omega$.

Exercise

25.71 •• **CP** Consider the circuit shown in **Fig. P25.71**. The emf source has negligible internal resistance. The resistors have resistances $R_1 = 7.00\ \Omega$ and $R_2 = 4.00\ \Omega$. The capacitor has capacitance $C = 6.00\ \mu\text{F}$. When the capacitor is fully charged, the magnitude of the charge on its plates is $Q = 35.0\ \mu\text{C}$. Calculate the emf $\mathcal{E}$.

Figure **P25.71**



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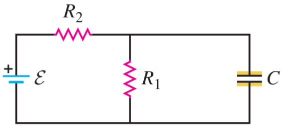
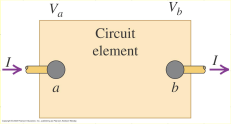


Solution

- No current on the capacitor. The current through the resistors is $I = \frac{\mathcal{E}}{R_1+R_2}$, but we don't know $\mathcal{E}$!
- ΔV on R_1 is $\Delta V = R_1 I$ is the same as ΔV on C.
- $\Delta V = \frac{Q}{C} = 5.83\,\mathrm{V}$.
- $I = \Delta V/R_1 = 0.83\,\mathrm{A}$
- $\mathcal{E} = (R_1 + R_2)I = 9.17\,\mathrm{V}$.

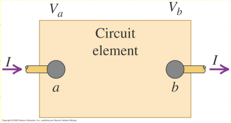
Energy and power in electric circuits

Energy and power delivered to an circuit element



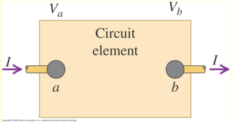
- a circuit element (a resistor, battery etc.) with potential difference $V_a - V_b = V_{ab}$ between its terminals and current I passing through it in the direction from a toward b.
- As charge passes through the circuit element, the electric field does work on the charge.
- As charge $\mathrm{d}Q$ passes through the circuit element, there is a change in potential energy $\mathrm{d}U = \mathrm{d}QV_{ab}$.
- Power is energy per unit time: $P = \frac{\mathrm{d}U}{\mathrm{d}t}$ (unit $\mathrm{J/s} = \mathrm{W}$
- Thus $\boxed{P = IV_{ab}}$ as $\mathrm{d}Q/\mathrm{d}t = I$.

Power Input to a Pure Resistance



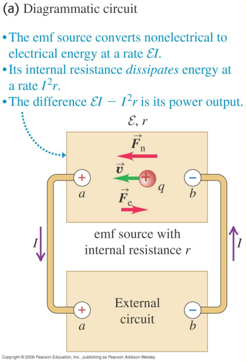
- For a resistor $V_{ab} = RI$
- $P = IV_{ab} = RI^2 = \frac{V_{ab}^2}{R}$
- In this case the potential at a (where I enters the resistor) is always higher than that at b (where the current exits). Current enters the higher-potential terminal of the device, and P represents the rate of transfer of electric potential energy *into* the circuit element.

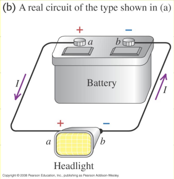
Power Input to a Pure Resistance



- What happens to this energy?
- The moving charges collide with atoms in the resistor and transfer some of their energy to these atoms, increasing the internal energy of the material.
- The temperature of the resistor increases.
- We say that energy is dissipated in the resistor at a rate RI^2.
- Power rating of a resisters gives the maximum power that may be delivered to the element without 'burning' it.

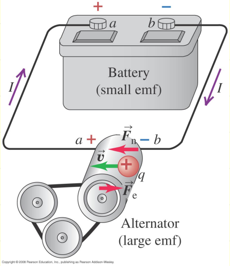
Power Output of a Source





- $P = V_{ab}I,\ V_{ab} = \mathcal{E} - rI$
- $P = \mathcal{E}I - rI^2$
- The term rI^2 is the rate at which electrical energy is dissipated in the internal resistance of the source. The difference $P = \mathcal{E}I - rI^2$ is the net electrical power output of the source

Power Input of a Source



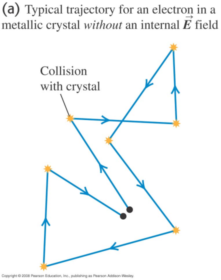
- $P = V_{ab} I$, $V_{ab} = \mathcal{E} + rI$
- $P = \mathcal{E} I + rI^2$
- Work is being done on, rather than by, the agent that causes the nonelectrostatic force in the upper source.
- There is a conversion of electrical energy into nonelectrical energy in the upper source at a rate $\mathcal{E} I$.

Theory of metallic conduction

Metallic conduction

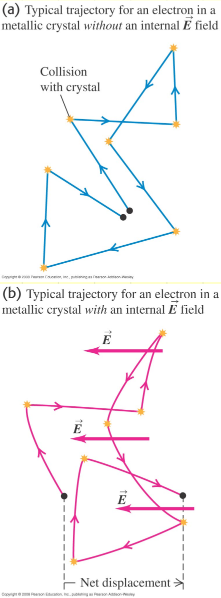
- What is the microscopic origin of Ohm's law, $\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$, in metals:
- We will consider a very simple model that treats the electrons as classical particles and ignores their QM behavior in solids.
- Using this model, we'll derive an expression for the resistivity, $\rho = E/J$ of a metal.
- $\vec{\mathbf{J}} = qn\vec{\mathbf{v}}_{\mathrm{d}}$ and $q = -e$
- We need to relate the drift velocity $\vec{\mathbf{v}}_d$ to the electric field $\vec{\mathbf{E}}$

Mean free time, drift velocity and resistivity



The average time between collisions is called the *mean free time*, denoted by τ.

Mean free time, drift velocity and resistivity



- The field exerts $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, and this causes

$$\vec{\mathbf{a}} = \frac{q}{m}\vec{\mathbf{E}}$$

where m is the electron mass. *Every* electron has this acceleration.

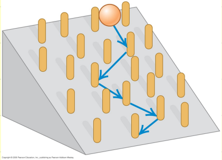
- The velocity of an electron at time τ is

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\tau$$

- For an average electron $\vec{\mathbf{v}}_0 = 0$. Thus

$$\vec{\mathbf{v}}_{\text{ave}} = \frac{q\tau}{m}\vec{\mathbf{E}}$$

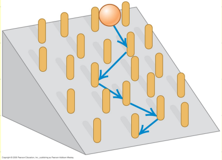
Mean free time, drift velocity and resistivity



- After time $t = \tau$, the tendency of the collisions to decrease the velocity of an average electron (by means of randomizing collisions) just balances the tendency of the $\vec{\mathbf{E}}$ field to increase this velocity

$$\vec{\mathbf{v}}_{\mathrm{d}} = \frac{q\tau}{m}\vec{\mathbf{E}}$$

Mean free time, drift velocity and resistivity



- Substituting this into $\vec{\mathbf{J}} = qn\vec{\mathbf{v}}_\mathrm{d}$ we get

$$\vec{\mathbf{J}} = \frac{q^2 n\tau}{m}\vec{\mathbf{E}}$$

- Thus, $\rho = E/J$ gives

$$\boxed{\rho = \frac{m}{q^2 n\tau}}$$

- If n and τ are independent of E, then ρ is independent of E and the conducting material obeys Ohm's law.