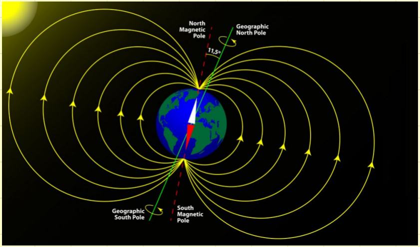
Magnetic field and magnetic forces

FIZ102E: Electricity & Magnetism



Yavuz Ekşi

İTÜ Fizik Mühendisliği Bölümü

Contents

Maxwell's Equations and the Lorentz Force

Gauss's law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0}$$

Gauss' law for magnetism

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

Faraday's law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

Generalized Ampere's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_{\mathrm{C}} + \mu_0 \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

Force on a particle with charge q and velocity $\vec{\mathbf{v}}$ moving in $\vec{\mathbf{E}}$ & $\vec{\mathbf{B}}$ fields

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Electrostatics: Charges are at rest.

Gauss's law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0}$$

charge Q is source of $\vec{\mathbf{E}}$

Faraday's law (static)

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = 0$$

we can define an electric potential.

Electric force on q in $\vec{\mathbf{E}}$

$$\boxed{\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}}$$

İSTANBUL TEKNİK ÜNİVERSİTESİ 1773

Gauss' law for magnetism

No magnetic charges.

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

Ampere's law (static)

Current i_C is source of $\vec{\mathbf{B}}$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_\mathrm{C}$$

Magnetic force on q moving with $\vec{\mathbf{v}}$ in $\vec{\mathbf{B}}$

$$\boxed{\vec{\mathbf{F}}_M = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}}$$

Learning goals

- The properties of magnets, and how magnets interact with each other.
- The nature of the force that a moving charged particle experiences in a magnetic field.
- How magnetic field lines are different from electric field lines.
- How to analyze the motion of a charged particle in a magnetic field.
- Some practical applications of magnetic fields in chemistry and physics.
- How to analyze magnetic forces on current-carrying conductors.
- How current loops behave when placed in a magnetic field.

Magnets vs electric charges

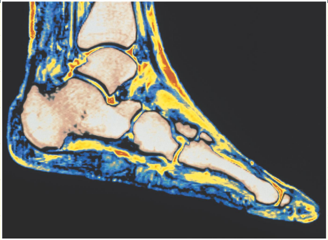
- permanent magnets attract unmagnetized iron objects and can also attract or repel other magnets.
- Electric forces act on electric charges whether they are moving or not.
- Magnetic forces act only on moving charges.

Fields

- the electric force arises in two stages: (1) a charge produces an electric field in the space around it, and (2) a second charge responds to this field.
- Magnetic forces also arise in two stages: (1) a moving charge or a collection of moving charges (that is, an electric current) produces a magnetic field, and (2) current or moving charge responds to this magnetic field, and so experiences a magnetic force.
- In this chapter we study the second stage in the magnetic interaction –that is, how moving charges and currents respond to magnetic fields.
- In the next Chapter we will examine how moving charges and currents produce magnetic fields.

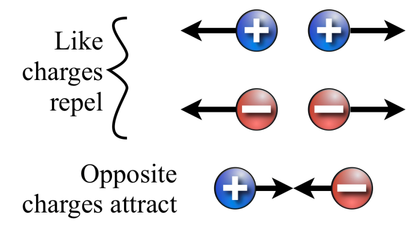
Magnetism

Introduction

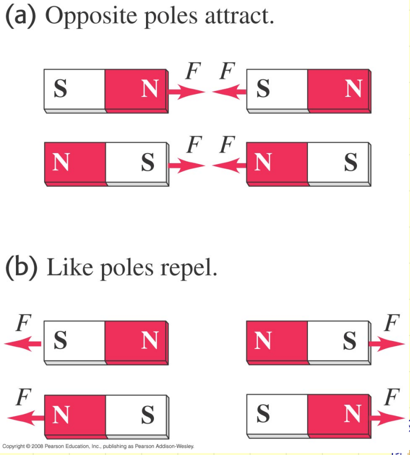


- How does magnetic resonance imaging (MRI) allow us to see details in soft nonmagnetic tissue?
- How can magnetic forces, which act only on moving charges, explain the behavior of a compass needle?

Magnetic poles

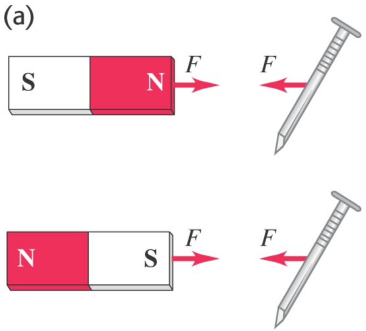


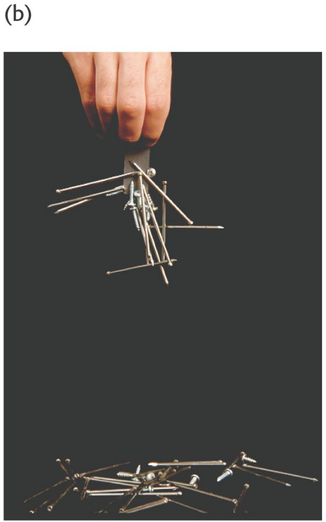
Forces between magnetic poles mimic forces between charges.



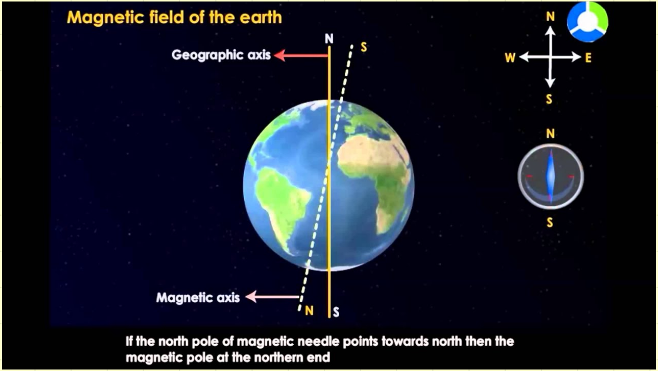
Magnetism and certain metals

BUT ... either pole of a permanent magnet will attract a metal like iron!





Magnetic field of the earth



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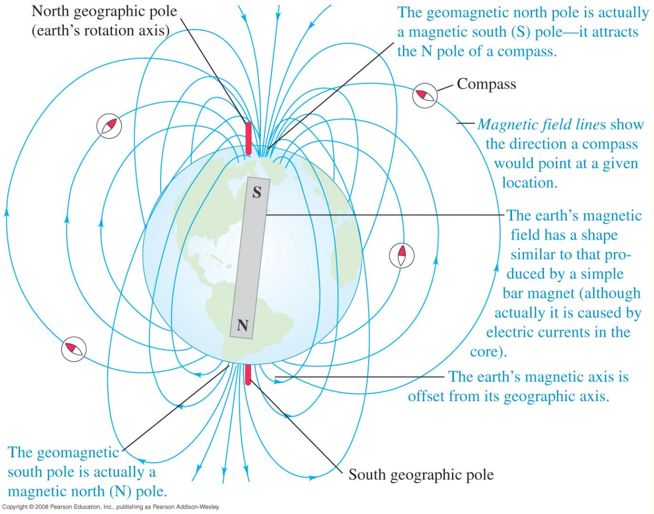
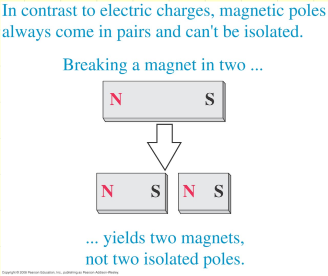


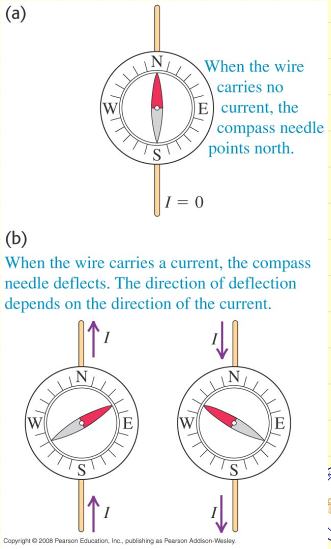
Magnetic monopoles

- Breaking a bar magnet does not separate its poles!
- There is no experimental evidence for *magnetic monopoles*.



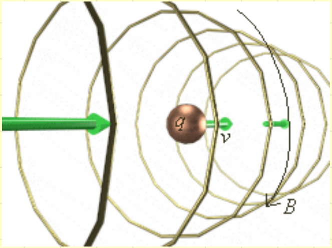
Electric current and magnets

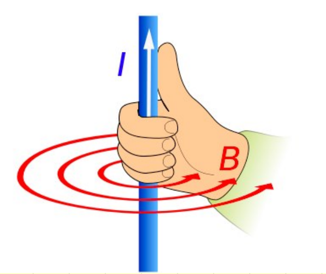
- (1820) Hans Oersted discovered wire carrying current causes compass to deflect.
- There is a connection between moving charges and magnetism.



Magnetic Field

The magnetic field



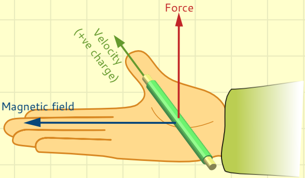


A *moving* charge (or current) creates a magnetic field in the surrounding space. (more in next chapter)

The magnetic force on moving charges

- A charge moving in a magnetic field is acted by a magnetic force

$$\vec{\mathbf{F}}_{\mathrm{M}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \qquad (7)$$



- Magnetic force on moving charge q is perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.
- Magnitude of the magnetic force is

$$F_{\mathrm{M}} = |q| vB \sin\phi \qquad (8)$$

The magnetic force on moving charges

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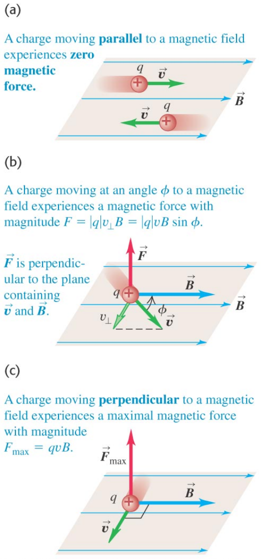


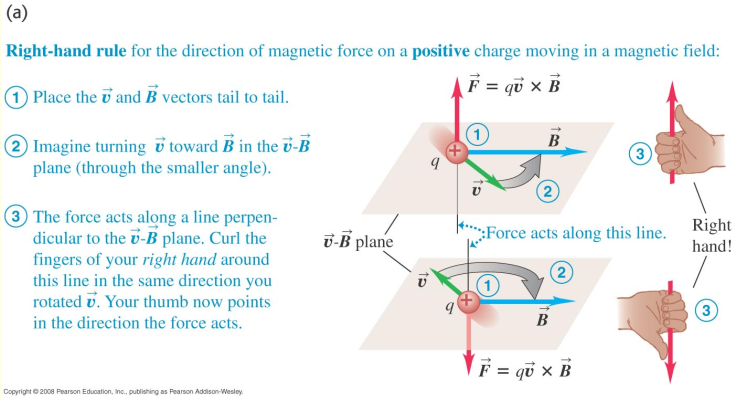
Unit of the magnetic field

- Magnetic field is denoted with letter "B"
- $F_{\mathrm{M}} = |q|vB\sin\phi$ suggests the unit of B as $\frac{\mathrm{N\,s}}{\mathrm{C\,m}}$
- measured in Tesla (in SI) or Gauss (in cgs)

$$1\,\mathrm{G} = 1 \times 10^{-4}\,\mathrm{T} \tag{9}$$

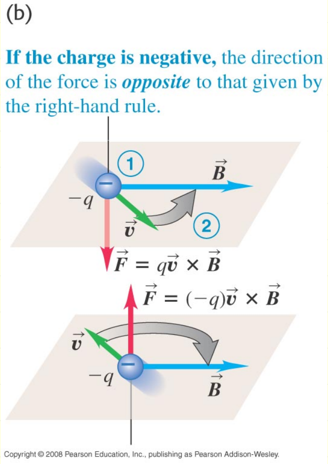
- Magnetic field is a vector field: $\vec{\mathbf{B}}$

The magnetic force on a moving charge



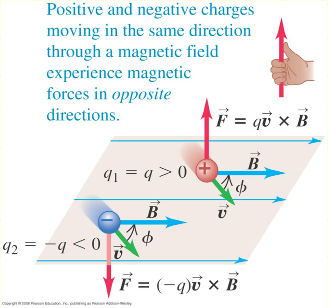
- Write magnetic force as vector cross product.
- Right-hand rule gives direction of force on positive charge.

The magnetic force on a moving charge



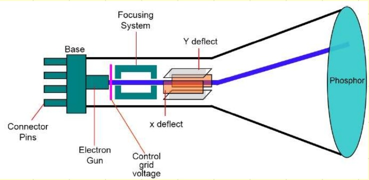
- Write magnetic force as vector cross product.
- Left-hand rule gives direction of force on positive charge.

Equal velocities but opposite signs

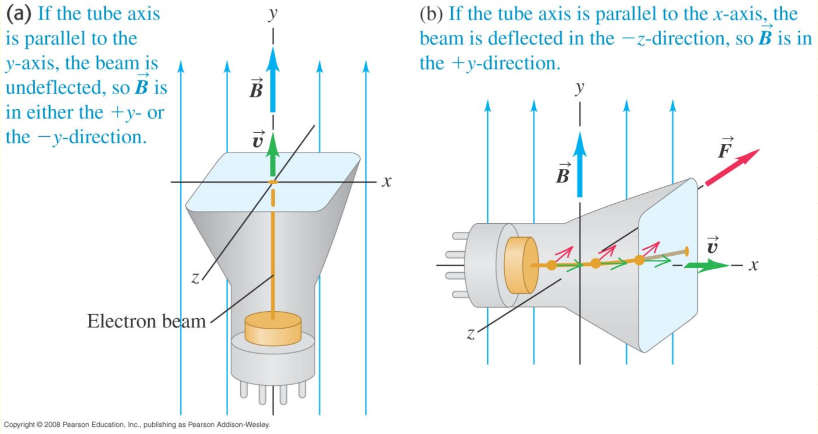


Two charges of equal magnitude but opposite signs moving in same direction in same field experience magnetic forces in opposite directions.

The cathode ray tube



Determining the direction of a magnetic field

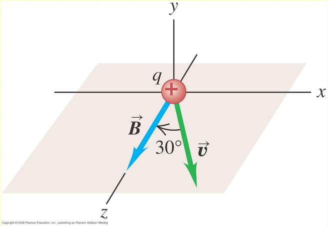


To explore an unknown magnetic field, we can measure the magnitude and direction of the force on a *moving* test charge and then use $\vec{\mathbf{F}}_{\mathrm{M}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ to determine $\vec{\mathbf{B}}$.

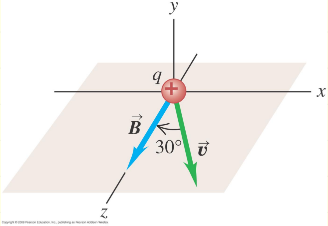
Ex: Magnetic force on a proton

Question

Beam of protons ($q = 1.6 \times 10^{-19}\,\mathrm{C}$) moves at $3.0 \times 10^{5}\,\mathrm{m/s}$ through $2.0\,\mathrm{T}$ field directed along z-axis. Velocity direction is $30°$ from the z-axis in the $x - y$ plane. Force on a proton?



Ex: Magnetic force on a proton



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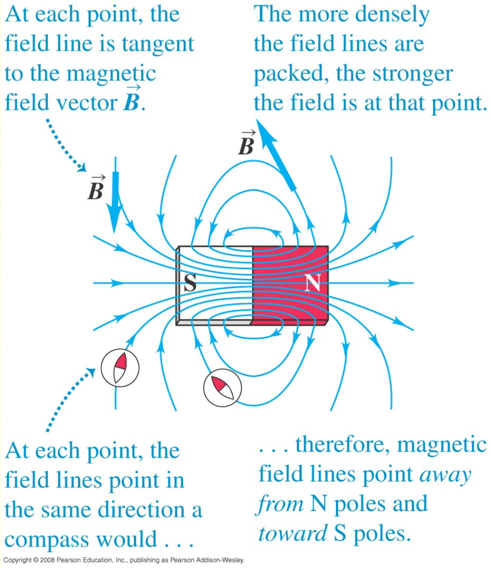
Solution

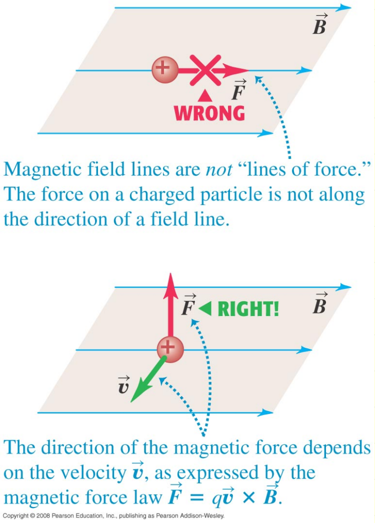
$$
\begin{aligned}
F_{\mathrm{M}} &= qvB\sin\phi \\
&= (1.6 \times 10^{-19}\,\mathrm{C})(3.0 \times 10^{5}\,\mathrm{m/s})(2.0\,\mathrm{T})\sin 30° \\
&= 4.8 \times 10^{-14}\,\mathrm{N}
\end{aligned}
$$

Magnetic Field Lines and Magnetic Flux

Magnetic field lines

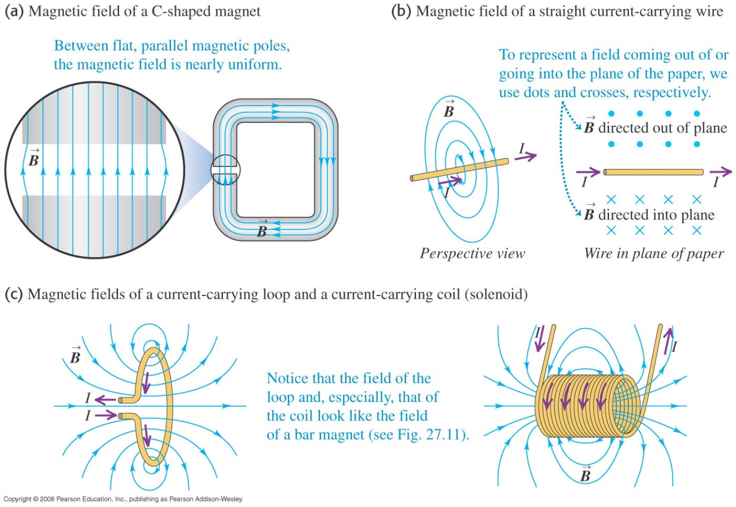


Magnetic field lines are *not* lines of force



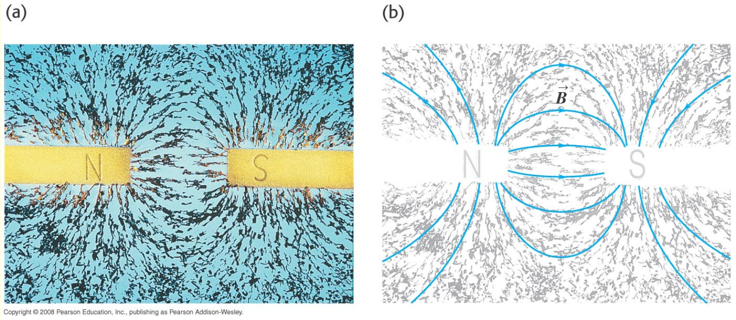
Important! Remember magnetic field lines are not lines of magnetic force.

Magnetic field lines



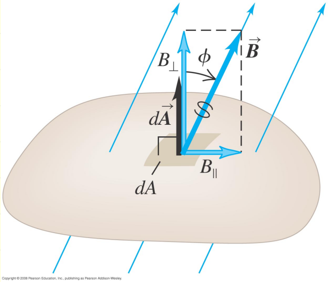
Magnetic field lines produced by some common sources of magnetic field.

Magnetic field lines



Iron filings, like compass needles, tend to align with magnetic field lines. Hence they provide an easy way to visualize field lines

Magnetic flux



- The magnetic flux through a surface is defined as

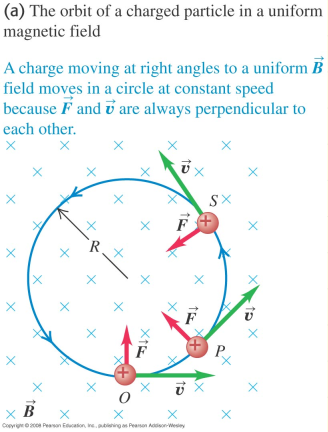
$$\Phi_B = \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} \qquad (10)$$

- The magnetic flux through a *closed* surface is zero.
- The SI unit of magnetic flux is Weber (Wb)

$$1\,\mathrm{Wb} = 1\,\mathrm{T}\cdot\mathrm{m}^2 = 1\,\mathrm{N}\cdot\mathrm{m}/\mathrm{A} \qquad (11)$$

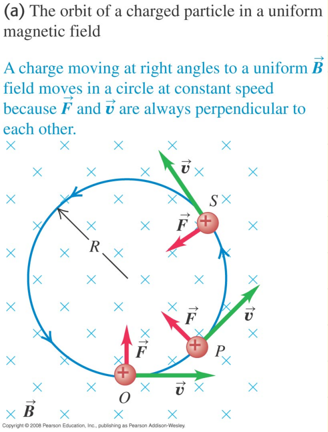
Motion of Charged Particles in a Magnetic Field

Motion of charged particles in a magnetic field



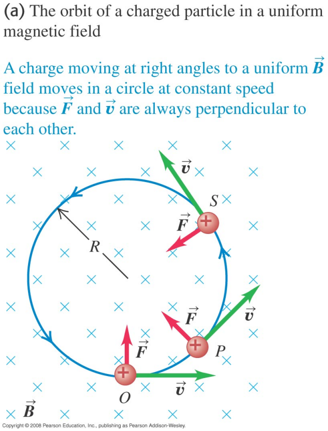
- Charged particle in magnetic field (only) moves with constant speed
- $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
  - $\vec{\mathbf{F}}$ is $\perp$ to $\vec{\mathbf{v}}$
  - So is acceleration!
  - No change in magnitude of velocity!

The magnetic force does no work on the charged particles



- $W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{l}}$
- $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ & $\mathrm{d}\vec{\mathbf{l}} = \vec{\mathbf{v}}\,\mathrm{d}t$
- $W = q\int (\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}}\,\mathrm{d}t$
- $W = 0$ as $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is $\perp$ to $\vec{\mathbf{v}}$.
- Work-Energy theorem $W = \Delta K$ $(K = \frac{1}{2}mv^2)$
- $W = 0 \Rightarrow \Delta K = 0$
- $\Delta K = 0 \Rightarrow v^2 = \text{constant}$

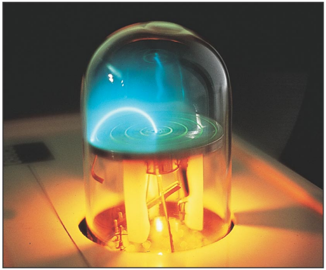
Motion of charged particles in a magnetic field



- If velocity of particle is ONLY perpendicular to B field, particle moves in a circle of radius $R = mv/|q|B$.
- Number of revolutions of particle per unit time is *cyclotron frequency*.

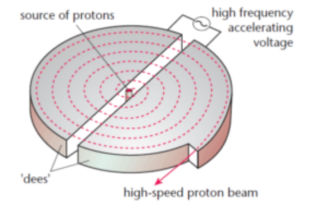
Motion of charged particles in a magnetic field

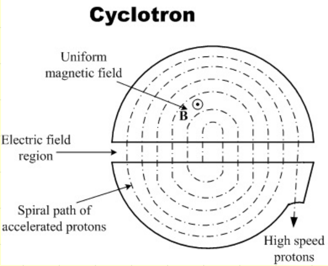
(b) An electron beam (seen as a blue arc) curving in a magnetic field



- $F = qvB = mv^2/R$
- $\omega = v/R = qB/m$
- $f = \omega/2\pi$

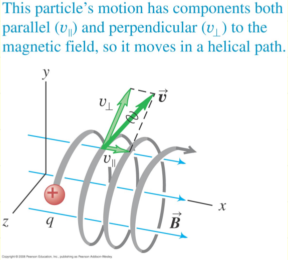
Motion of charged particles in a magnetic field





- If velocity of particle is perpendicular to B field, particle moves in a circle of radius $R = mv/|q|B$
- IF add a "kick" to speed, radius increases!
- A cyclotron!
- Number of revolutions of particle per unit time is *cyclotron frequency*.

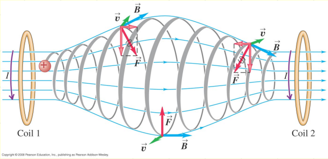
Helical motion

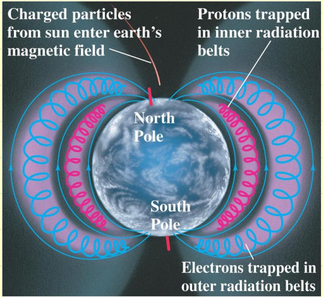


- If particle has velocity components parallel to and perpendicular to B field, its path is a helix.
- Speed & kinetic energy of the particle still remain constant.

A nonuniform magnetic field

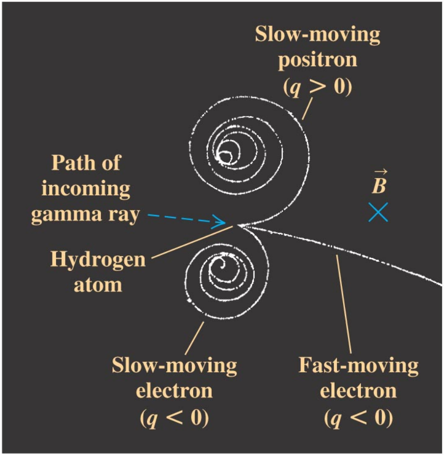
- Charges can be trapped in a magnetic bottle, which results from a non-uniform magnetic field.
- Van Allen radiation belts act like a magnetic bottle, and produce aurora. These belts are due to the earth's non-uniform field.





Applications of Motion of Charged Particles

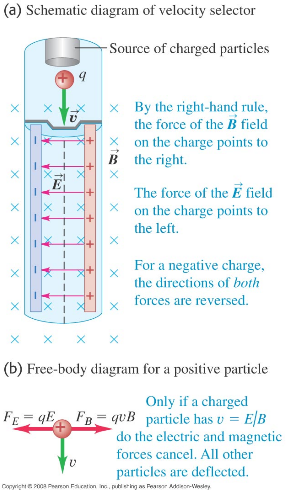
Bubble chamber



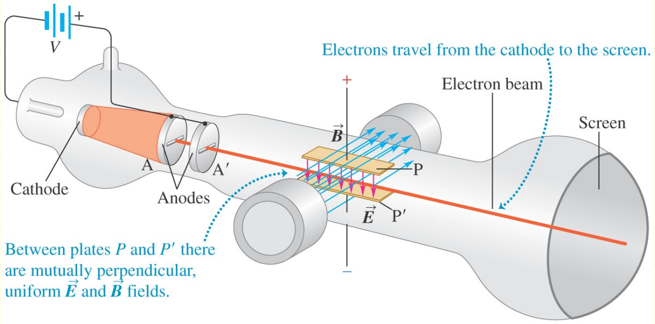
Track of charged particles in a bubble chamber experiment.

Velocity selector

- Velocity selectors use perpendicular electric & magnetic fields to select particles of specific speed from beam.
- Only particles having speed $v = E/B$ pass through undeflected.



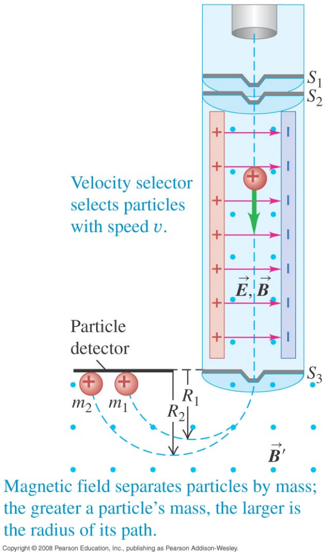
Thomson's e/m experiment



Measure ratio e/m for electron.

Velocity selector

- Mass spectrometer to measure masses of ions.
- Bainbridge mass spectrometer first uses velocity selector,
- then magnetic field separates particles by mass.

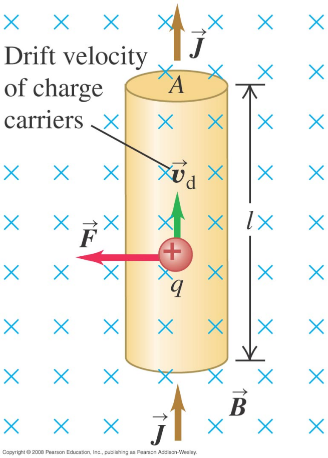


Magnetic Force on a Current-Carrying Conductor

The magnetic force on a current-carrying conductor

- Magnetic force on a moving positive charge in a conductor.

$$\boxed{\vec{\mathbf{F}} = I\,\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}} \qquad (12)$$



- For an infinitely small segment: $\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$
- Magnitude $F = I\ell B \sin\phi$
- I: scalar current (A)
  $\vec{\ell}$: direction of current (m)
  $\vec{\mathbf{B}}$: direction of mag. field (T).
- Magnetic force is perpendicular to the wire segment and the magnetic field.

The magnetic force on a current-carrying conductor

- Magnetic force on a moving positive charge in a conductor.

$$\boxed{\vec{\mathbf{F}} = I\,\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}} \qquad (12)$$

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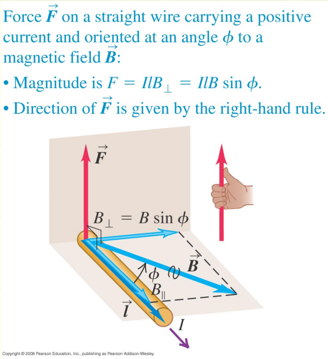


Derivation

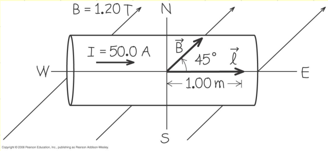
- The average force on each charge is $\vec{\mathbf{F}}_1 = q\vec{\mathbf{v}}_\mathrm{d} \times \vec{\mathbf{B}}$
- There are $nA\ell$ charges in motion (n is the number of charges per unit volume)
- The total force is then $\vec{\mathbf{F}} = qnA\ell\vec{\mathbf{v}}_\mathrm{d} \times \vec{\mathbf{B}}$
- Noting that $I = qnAv_\mathrm{d}$ (recall form Ch 25)
- $\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$

where in the last step we attributed the direction of $\vec{\mathbf{v}}_\mathrm{d}$ (and hence I) to $\vec{\boldsymbol{\ell}}$.

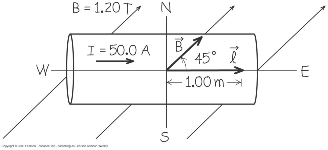
Example

Question

A straight horizontal copper rod carries a current of $50.0\,\mathrm{A}$ from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal $\vec{\mathbf{B}}$ toward the northeast (that is, $45°$ north of east) with magnitude $1.20\,\mathrm{T}$. (a) Find $\vec{\mathbf{F}}$ on a $1.00\,\mathrm{m}$ section of rod. (b) While keeping the rod horizontal, how should it be oriented to maximize $|\vec{\mathbf{F}}|$? What is $|\vec{\mathbf{F}}|$ in this case?



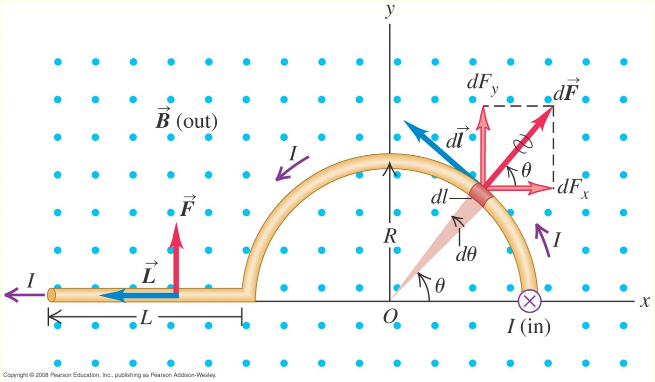
Example



Solution

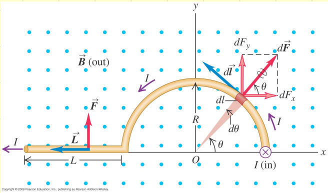
- $\vec{\boldsymbol{\ell}} = 1.00\,\mathrm{m}\hat{\mathbf{i}}$
- $\vec{\mathbf{B}} = 1.20\,\mathrm{T}[\cos 45°\hat{\mathbf{i}} + \sin 45°\hat{\mathbf{j}}]$
- $$\begin{aligned}\vec{\mathbf{F}} &= I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} \\ &= (50\,\mathrm{A})\,(1.00\,\mathrm{m})\,(1.20\,\mathrm{T})\hat{\mathbf{i}} \times [\cos 45°\hat{\mathbf{i}} + \sin 45°\hat{\mathbf{j}}] \\ &= 42.4\,\mathrm{N}\hat{\mathbf{k}}.\end{aligned}$$
- $F_{\max} = F/\sin 45° = 60\,\mathrm{N}$
  achieved when $\vec{\mathbf{B}}$ is $\perp$ to $\vec{\boldsymbol{\ell}}$.

Ex: Magnetic force on a curved conductor



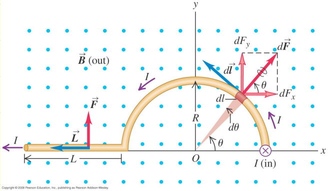
What is the magnetic force on the curved part of the wire?

Solution



- $\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$
- $\mathrm{d}\vec{\boldsymbol{\ell}} \perp \vec{\mathbf{B}} \Rightarrow \mathrm{d}F = I\,\mathrm{d}\ell\, B$
- $\mathrm{d}\ell = R\,\mathrm{d}\theta$, $\mathrm{d}F_y = \mathrm{d}F\sin\theta$
- $\mathrm{d}F_y = IRB\sin\theta\,\mathrm{d}\theta$
- $F_y = IRB\int_0^{\pi}\sin\theta\ \mathrm{d}\theta = 2IRB$
- Show that
  $F_x = IRB\int_0^{\pi}\cos\theta\ \mathrm{d}\theta = 0$

Solution



Note that $F_y = 2IRB$ is exactly the force that would be exerted if we replaced the semicircle with a straight segment of length $2R$ along the x-axis.

- $\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$
- $\mathrm{d}\vec{\boldsymbol{\ell}} \perp \vec{\mathbf{B}} \Rightarrow \mathrm{d}F = I\,\mathrm{d}\ell\,B$
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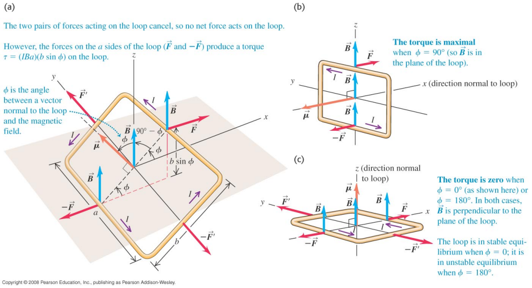
  $F_x = IRB\int_0^\pi \cos\theta\ \mathrm{d}\theta = 0$

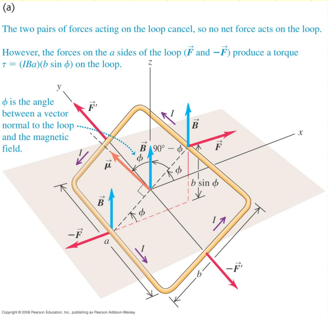
Force and Torque on a Current Loop

Force and torque on a current loop in B field



- *Net force* on a current loop in a *uniform* magnetic field is zero.
- But the net torque is not, in general, equal to zero.

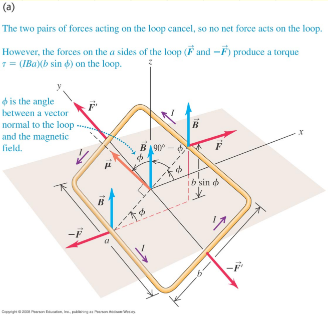
Force and torque on a current loop in B field



- $F = IaB$, $F' = IbB \sin(90^\circ - \phi) = IbB \cos\phi$
- Thus $F_{\text{net}} = 0$
- The moment arm for F is ($\perp$ distance from the rotation axis to the line of action of the force) $(b/2) \sin\phi$
- $\tau = 2F \frac{b}{2} \sin\phi$ i.e.

$$\tau = (IaB) b \sin\phi \tag{13}$$

Magnetic dipole moment



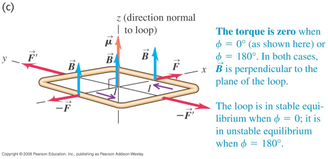
- $A = ab$ (area of the rectangle)
- The product IA is called the *magnetic dipole moment* of the loop

$$\mu = IA \tag{14}$$

- In terms of μ we can write the torque as

$$\tau = \mu B \sin\phi \tag{15}$$

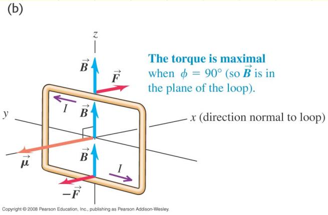
Magnetic torque on a loop



As $\tau = \mu B \sin\phi$

- $\tau = 0$ for the aligned ($\phi = 0$) and anti-aligned ($\phi = 180°$) case.
- $\tau = \tau_{\max} = \mu B$ for the $\perp$ case.

Magnetic torque on a loop



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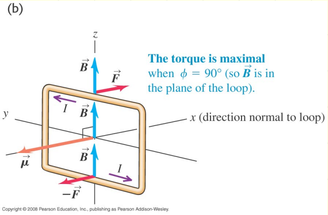

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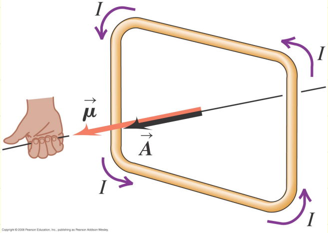
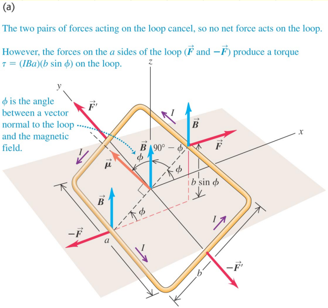


- Magnetic dipole moment is a vector quantity: $\vec{\mu}$
- The torque vector is thus

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}} \tag{16}$$

Magnetic torque on a loop



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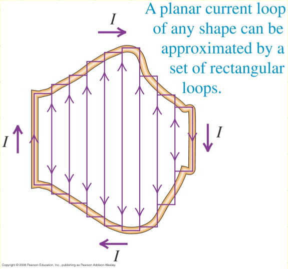
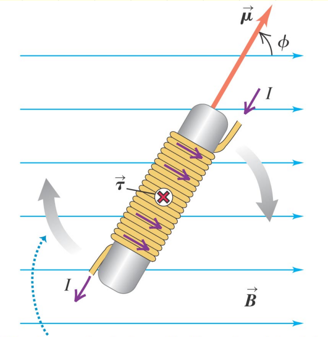
Magnetic moment for arbitrary shapes



$$\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}} \tag{17}$$

- Although we have derived $\tau = \mu B \sin\phi$ and other relations for a rectangular current loop, all these relationships are valid for a plane loop of any shape at all.
- Any planar loop may be approximated as closely as we wish by a very large number of rectangular loops
- The forces and torques on the sides of two loops adjacent to each other cancel, and the only forces and torques that do not cancel are due to currents around the boundary.

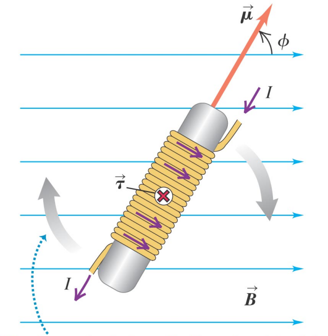
Magnetic moment for coils



The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$.

- a coil approximates N planar loops close together.
- simply multiply each force, the magnetic moment and the torque.
- $\vec{\boldsymbol{\mu}} \equiv N I \vec{\mathbf{A}}$.
- $\tau = NIAB \sin\phi = \mu B \sin\phi$
- $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

Potential Energy for a Magnetic Dipole



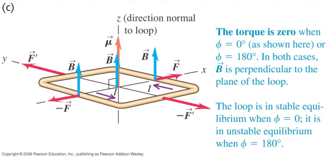
The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$.

- When a $\vec{\boldsymbol{\mu}}$ changes orientation in a $\vec{\mathbf{B}}$, the field does work on it.
- In an infinitesimal angular displacement $\mathrm{d}\phi$, the work $\mathrm{d}W$ is given by $\tau\,\mathrm{d}\phi$, and there is a corresponding change in potential energy.
- The potential energy for a magnetic dipole is

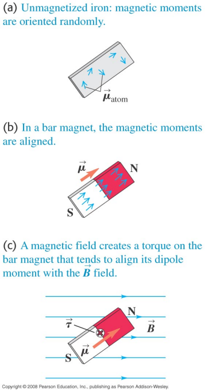
$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}} = -\mu B \cos\phi \qquad (18)$$

- U is least $(-\mu B)$ when $\vec{\boldsymbol{\mu}}$ and $\vec{\mathbf{B}}$ are parallel.

Stability

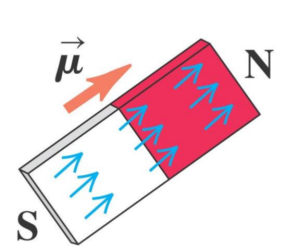


- Current loops tend to orient themselves with their axes parallel to $\vec{\mathbf{B}}$ so that U is minimum.
- $\tau = 0$ for this configuration.
- $\tau = 0$ also for anti-parallel ($\phi = 180°$), but this is unstable equilibrium as U is maximum (μB).



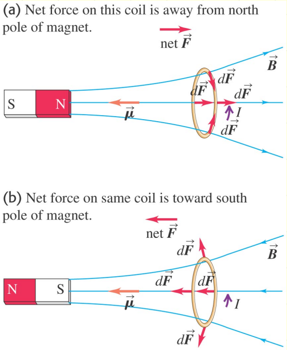
- An e^- has a net magnetic moment, μ_e.
- In an iron atom a substantial fraction of the μ_es align with each other, $\mu \neq 0$ for the atom.
- In an unmagnetized piece of iron there is no overall alignment of the $\vec{\mu}$'s of the atoms; their vector sum is zero, and the net $\vec{\mu} = \vec{0}$ (a).
- In an iron bar magnet $\vec{\mu}$'s of many of the atoms are $\parallel$, and there is a net $\vec{\mu}$ (b).
- If the magnet is placed in a $\vec{\mathbf{B}}$, the field exerts a $\vec{\tau}$ that tends to align $\vec{\mu}$ with $\vec{\mathbf{B}}$ (c).

How magnets work



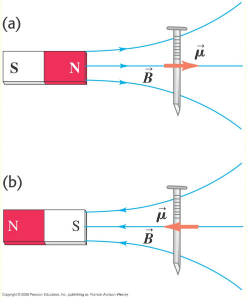
- A bar magnet tends to align with a $\vec{\mathbf{B}}$ field so that a line from the south pole to the north pole of the magnet is in the direction of $\vec{\mathbf{B}}$
- hence the real significance of a magnet's north and south poles is that they represent the head and tail, respectively, of the magnet's dipole moment $\vec{\boldsymbol{\mu}}$

Magnetic Dipole in a Nonuniform Magnetic Field



- In a nonuniform B field, the net force on the loop is not, in general, zero.
- The force is toward the region where the field lines are farther apart and the field magnitude B is less.
- If the polarity of the bar magnet is reversed so that $\vec{\mu}$ and $\vec{\mathbf{B}}$ are parallel, the net force on the loop is to the left, i.e. toward the region of greater field magnitude near the magnet.
- Case (a) repelling (N-N), case be attractive (N-S)

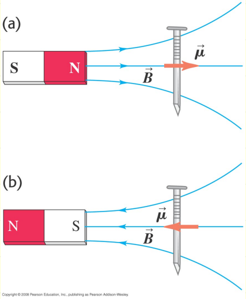
How a magnet can attract an unmagnetized iron object



A previously unmagnetized object containing iron is attracted to either pole of a magnet. How?

- First, the atomic magnetic moments of the iron tend to align with the $\vec{\mathbf{B}}$ field of the magnet, so the iron acquires a net $\vec{\mu}$ parallel to the field.
- The north pole of the magnet is closer to the nail (which contains iron), and $\vec{\mu}$ produced in the nail is to the right (N is on the rightside)
- Changing the polarity of the magnet, as in reverses the directions of both $\vec{\mathbf{B}}$ and $\vec{\mu}$. And the nail is again attracted toward the magnet.

How a magnet can attract an unmagnetized iron object



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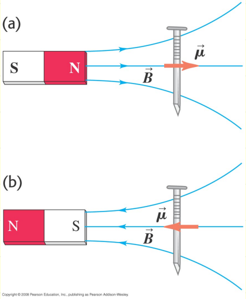

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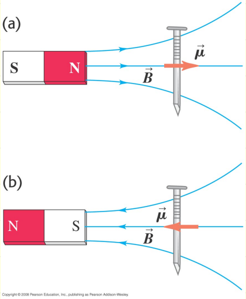

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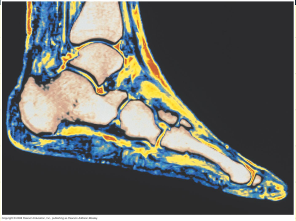
How MRI work



In magnetic resonance imaging (MRI), a patient is placed in a strong magnetic field.

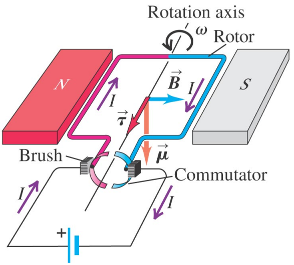
- Each hydrogen nucleus in the patient acts like a miniature current loop with a $\vec{\mu}$ that tends to align with the applied $\vec{\mathbf{B}}$.
- Radio waves of just the right frequency then flip these $\vec{\mu}$ out of alignment.
- The extent to which the radio waves are absorbed is proportional to the amount of hydrogen present.
- This makes it possible to image details in hydrogen-rich soft tissue.

The Direct-Current Motor

The direct-current motor

- In a motor a magnetic torque acts on a current-carrying conductor, and electric energy is converted to mechanical energy.
- The moving part of the motor is the rotor, a length of wire formed into an open-ended loop and free to rotate about an axis.
- The ends of the rotor wires are attached to circular conducting segments that form a commutator.

(a) Brushes are aligned with commutator segments.

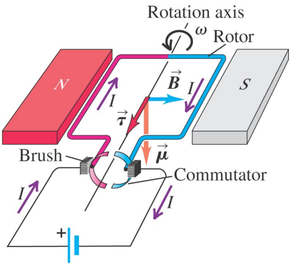


- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

The direct-current motor

- The rotor is a current loop with $\vec{\mu}$.
- The rotor lies between opposing poles of a permanent magnet $\vec{\mathbf{B}}$ so that $\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$ acts on the rotor.
- For the rotor orientation shown μ causes the rotor to turn ccw to align $\vec{\mu}$ with $\vec{\mathbf{B}}$.

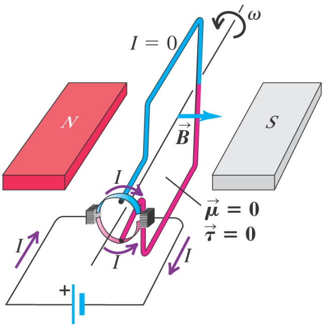
(a) Brushes are aligned with commutator segments.



- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

- Now, consider the rotor has rotated by $90°$.
- It would simply oscillate around this orientation.
- But here's where the *commutator* comes into play; each brush is now in contact with both segments of the commutator.
- There is no potential difference between the commutators, so at this instant $I = 0$, and $\mu = 0$ in the rotor.
- The rotor continues to rotate ccw because of its inertia.

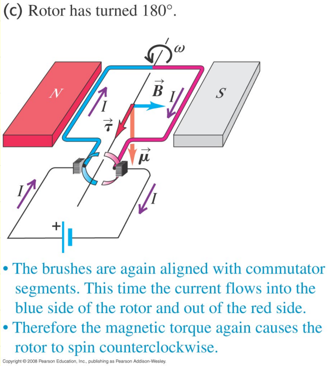
(b) Rotor has turned 90°.



- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

The direct-current motor

- When the rotor reaches (c) I enters on the blue side of the rotor and exits on the red side,
- just the opposite of the situation in (a)
- While the direction of I has reversed wrt the rotor, the rotor itself has rotated $180°$ and $\vec{\boldsymbol{\mu}}$ is in the same direction with respect to $\vec{\mathbf{B}}$.
- Thanks to the commutator, I reverses after every $180°$ of rotation, so $\vec{\boldsymbol{\mu}}$ is always in the direction to rotate the rotor ccw.

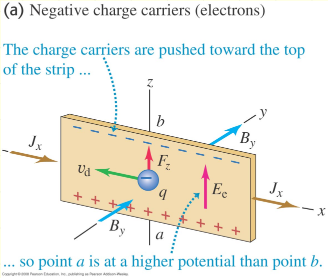


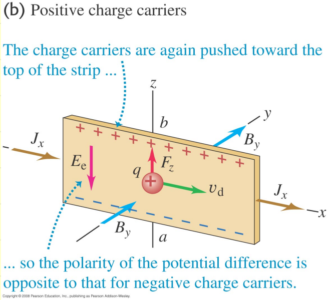
The Hall Effect

The Hall Effect

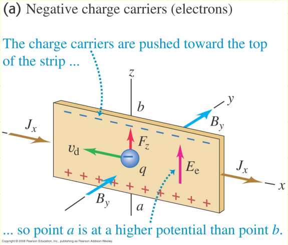
- Discovered by the American physicist Edwin Hall in 1879
- Demonstrates the reality of the forces acting on the moving charges in a conductor in a magnetic field.
- An ingenious set up to determine the sign of charge carriers in metals.

The Hall Effect



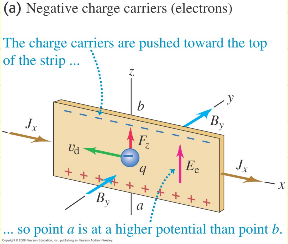


In both cases the magnetic force is upward, just as the magnetic force on a conductor is the same whether the moving charges are positive or negative. In either case a moving charge is driven toward the upper edge of the strip by the magnetic force $F_z = |q|v_d B$. The polarity E depends on whether the moving charges are $+$ or $-$.



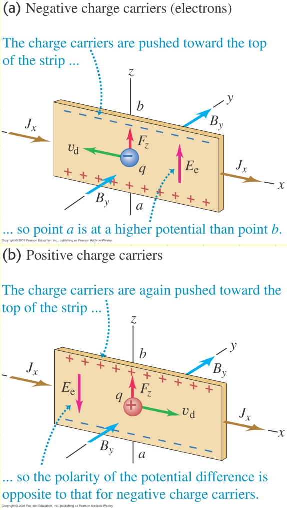
- If the charge carriers are electrons, an excess negative charge accumulates at the upper edge of the strip, leaving an excess positive charge at its lower edge.
- This accumulation continues until the resulting transverse electrostatic field E_e becomes large enough to cause a force (magnitude $|q|E$) that is equal and opposite to the magnetic force (magnitude $|q|v_d B$).

The Hall Effect



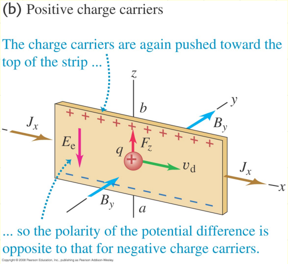
- After that, there is no longer any net transverse force to deflect the moving charges.
- This electric field causes a transverse potential difference between opposite edges of the strip, called the *Hall voltage* or the *Hall emf*.

The Hall Effect



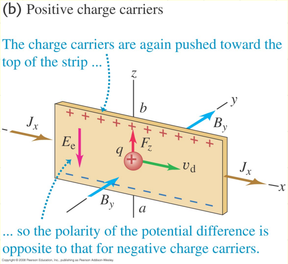
- The polarity depends on whether the moving charges are positive or negative.
- *Experiment* shows that for metals the upper edge of the strip becomes negatively charged, showing that the charge carriers in a metal are indeed negative electrons.

The Hall Effect



- In terms of the coordinate axes in Fig, the electrostatic field E_e for the positive-q case is in the $-z$-direction.
- The magnetic field is in the $+y$-direction, and we write it as B_y.
- The magnetic force (in the $+z$-direction) is qv_dB_y.
- The current density J_x is in the $+x$-direction.

The Hall Effect

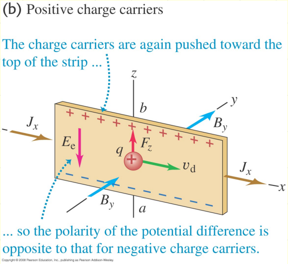


- In the steady state, when the forces qE_z and qv_dB_y sum to zero: $qE_z + qv_dB_y = 0 \Rightarrow E_z = -v_dB_y$
- This confirms that when q is positive, E_z is negative.
- Recall: $J_x = nqv_d$ and eliminate v_d to find

$$nq = -\frac{J_xB_y}{E_z} \tag{19}$$

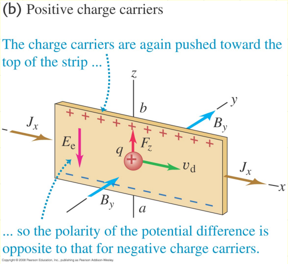
- Note that this result is valid for both $+$ and $-$ q. When q is $-$, E_z is $+$, and conversely.

The Hall Effect



- We can measure J_x , B_y, and E_z, so we can compute the product nq.
- In both metals and semiconductors, q is equal in magnitude to the electron charge, so the Hall effect permits a direct measurement of n, the concentration of current-carrying charges in the material.
- The *sign* of the charges is determined by the polarity of the *Hall emf*.

The Hall Effect



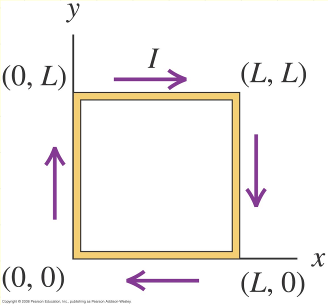
- The Hall effect can also be used for a direct measurement of electron drift speed v_d in metals.
- These speeds are very small, often of the order of $1\,\mathrm{mm/s}$ or less.
- If we move the entire conductor in the opposite direction to the current with a speed equal to the drift speed, then the electrons are at rest with respect to B, and the Hall emf disappears.
- Thus the conductor speed needed to make the Hall emf vanish is equal to the drift speed.

Problem

It was shown that the net force on a current loop in a uniform magnetic field is zero. But what if $\vec{\mathbf{B}}$ is not uniform? Here the magnetic field has no x-component but has both y- and z-components:

$$\vec{\mathbf{B}} = \left(\frac{B_0 y}{2L}\right)\hat{\mathbf{i}} + \left(\frac{B_0 x}{L}\right)\hat{\mathbf{j}}$$

where B_0 is a positive constant.



Force on a loop

Solution-1

Low segment: $y = 0$, thus

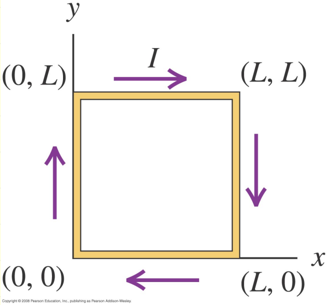
$\vec{\mathbf{B}} = \left(\cancel{\frac{B_0 y}{2L}}\right)\hat{\mathbf{i}} + \left(\frac{B_0 x}{L}\right)\hat{\mathbf{j}}$, $\mathrm{d}\vec{\boldsymbol{\ell}} = -\,\mathrm{d}x\,\hat{\mathbf{i}}$

thus

$\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = -I\left(\frac{B_0 x}{L}\right)\mathrm{d}x\,\hat{\mathbf{i}} \times \hat{\mathbf{j}}$,

$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ thus

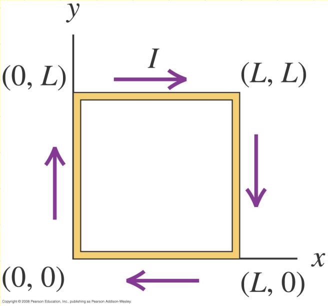
$$\begin{aligned}\vec{\mathbf{F}}_{\text{low}} &= -I\left(\frac{B_0}{L}\right)\int_0^L x\,\mathrm{d}x\,\hat{\mathbf{k}} \\ &= -I\left(\frac{B_0}{L}\right)\frac{L^2}{2}\hat{\mathbf{k}} \\ &= -\frac{1}{2} I B_0 L\hat{\mathbf{k}}\end{aligned}$$



Solution-2

Up segment: $y = L$, thus
$\vec{\mathbf{B}} = \left(\frac{B_0}{2}\right)\hat{\mathbf{i}} + \left(\frac{B_0 x}{L}\right)\hat{\mathbf{j}}$, $\mathrm{d}\vec{\ell} = \mathrm{d}x\,\hat{\mathbf{i}}$ thus
$\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\ell} \times \vec{\mathbf{B}} = I\left(\frac{B_0 x}{L}\right)\mathrm{d}x\,\hat{\mathbf{i}} \times \hat{\mathbf{j}}$,
$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ thus

$$\begin{aligned}\vec{\mathbf{F}}_{\text{up}} &= I\left(\frac{B_0}{L}\right)\int_0^L x\,\mathrm{d}x\,\hat{\mathbf{k}} \\ &= I\left(\frac{B_0}{L}\right)\frac{L^2}{2}\hat{\mathbf{k}} \\ &= \frac{1}{2}IB_0L\hat{\mathbf{k}}\end{aligned}$$



Solution-3

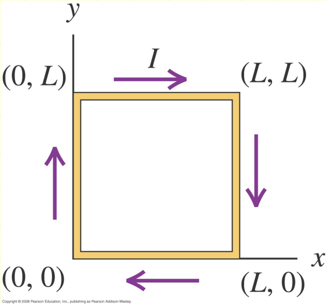
Left segment: $x = 0$, thus

$\vec{\mathbf{B}} = \left(\frac{B_0 y}{2L}\right)\hat{\mathbf{i}} + \left(\cancel{\frac{B_0 x}{L}}\right)\hat{\mathbf{j}}$, $\mathrm{d}\vec{\boldsymbol{\ell}} = \mathrm{d}y\,\hat{\mathbf{j}}$ thus

$\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = I\left(\frac{B_0 y}{2L}\right)\mathrm{d}y\,\hat{\mathbf{j}} \times \hat{\mathbf{i}}$,

$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$ thus

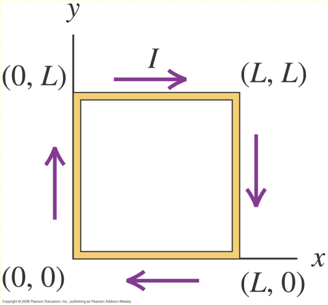
$$\begin{aligned}\vec{\mathbf{F}}_{\text{left}} &= -I\left(\frac{B_0}{2L}\right)\int_0^L y\,\mathrm{d}y\,\hat{\mathbf{k}} \\ &= -I\left(\frac{B_0}{2L}\right)\frac{L^2}{2}\hat{\mathbf{k}} \\ &= -\frac{1}{4}IB_0L\hat{\mathbf{k}}\end{aligned}$$



Solution-4

Right segment: $x = L$, thus
$\vec{\mathbf{B}} = \left(\frac{B_0 y}{2L}\right)\hat{\mathbf{i}} + B_0\hat{\mathbf{j}}$, $\mathrm{d}\vec{\ell} = -\,\mathrm{d}y\,\hat{\mathbf{j}}$ thus
$\mathrm{d}\vec{\mathbf{F}} = I\,\mathrm{d}\vec{\ell} \times \vec{\mathbf{B}} = -I\left(\frac{B_0 y}{2L}\right)\mathrm{d}y\,\hat{\mathbf{j}} \times \hat{\mathbf{i}}$,
$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$ thus

$$\begin{aligned}\vec{\mathbf{F}}_{\text{left}} &= I\left(\frac{B_0}{2L}\right)\int_0^L y\,\mathrm{d}y\,\hat{\mathbf{k}} \\ &= I\left(\frac{B_0}{2L}\right)\frac{L^2}{2}\hat{\mathbf{k}} \\ &= \frac{1}{4}IB_0L\hat{\mathbf{k}}\end{aligned}$$



Problem

Determine the magnetic moment of a disk of radius R, uniform charge Q rotating with angular speed $\vec{\boldsymbol{\Omega}} = \omega\hat{\mathbf{k}}$.

Solution

The answer is $\mu = \frac{1}{4}QR^2\omega$ as seen in
`http://www.phys.uri.edu/gerhard/PHY204/tsl199.pdf`

Problem

Determine the magnetic moment of a spherical shell of radius R, uniform charge Q rotating with angular speed $\vec{\mathbf{\Omega}} = \omega\hat{\mathbf{k}}$.

Solution

The answer is $\mu = \frac{1}{3}QR^2\omega$ as seen in
https://socratic.org/questions/
a-spherical-shell-of-radius-r-and-uniformly-charged-with-c
You can see it explained in a video
https://www.youtube.com/watch?v=4pFlrDIh4Yo

Magnetic moment of a sphere

Problem

Determine the magnetic moment of a sphere of radius R, uniform charge Q rotating with angular speed $\vec{\mathbf{\Omega}} = \omega\hat{\mathbf{k}}$.

Solution

The answer is $\mu = \frac{1}{5}QR^2\omega$ as seen in
https://www.toppr.com/content/story/amp/magnetic-dipole-moment-due-to-rotation-sphere-65427/
You can see it explained in a video
https://www.youtube.com/watch?v=VvoXhQbFBUI