Sources of magnetic field

FIZ102E: Electricity & Magnetism

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Contents

Maxwell's Equations and the Lorentz Force

Gauss' law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Gauss' law for magnetism

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

Faraday's law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

Generalized Ampere's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_{\mathrm{C}} + \mu_0 \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

Force on a particle with charge q and velocity $\vec{\mathbf{v}}$ moving in $\vec{\mathbf{E}}$ & $\vec{\mathbf{B}}$ fields

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Electrostatics: Charges are at rest.

Gauss' law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0}$$

charge Q is source of $\vec{\mathbf{E}}$

Faraday's law (static)

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = 0$$

we can define an electric potential.

Electric force on q in $\vec{\mathbf{E}}$

$$\boxed{\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}}$$

Magnetostatics: Steady currents

Gauss' law for magnetism

No magnetic charges.

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

Ampere's law (static)

Current i_C is source of $\vec{\mathbf{B}}$

$$\boxed{\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_{\mathrm{C}}}$$

Magnetic force on q moving with $\vec{\mathbf{v}}$ in $\vec{\mathbf{B}}$

$$\vec{\mathbf{F}}_M = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Learning goals

- To determine the magnetic field produced by a moving charge
- To study the magnetic field of an element of a current-carrying conductor
- To calculate the magnetic field of a long, straight, current-carrying conductor and a wire bent into a circle.
- To study the magnetic force between current- carrying wires.
- To use Ampere's Law to calculate magnetic fields of symmetric current distributions.
- How microscopic currents within materials give them their magnetic properties.

Summary of last week

Reminder: Lorentz Force

- In the previous chapter we studied the forces exerted on moving charges ($\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$) and on current-carrying conductors ($\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$) in a magnetic field.
- We didn't worry about how the magnetic field got there; we simply took its existence as a given fact.
- We have also seen that there is no magnetic charge (monopole) as implied by Gauss' law for magnetic fields $\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$.
- But how, then, are magnetic fields created?
- Magnetic fields are created by moving electric charges.

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Fields: Particle–Field–Particle interactions

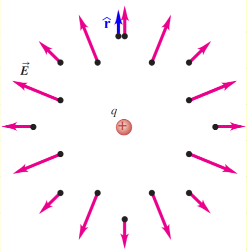
- The electric force arises in two stages:
  (1) a charge produces an electric field in the space around it
  (2) a second charge responds to this field.
- Magnetic forces also arise in two stages:
  (1) a moving charge or a collection of moving charges (that is, an electric current) produces a magnetic field
  (2) current or moving charge responds to this magnetic field, and so experiences a magnetic force.
- In this chapter we study the first stage in the magnetic interaction –that is, how magnetic fields are produced by moving charges and currents.

Magnetic field of a moving charge

The magnetic field of a moving charge

- In *electrostatics* the electric field of a point charge was

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$



- This is not a separate law, but is derived from Gauss' law

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0}$$

$(EA = 4\pi r^2 = q/\epsilon_0)$

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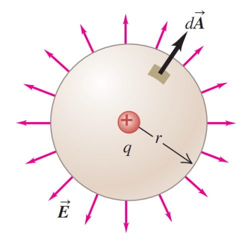
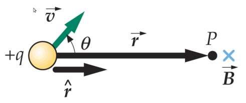


The magnetic field of a moving charge

- Is there a similar expression for the magnetic field produced by a point charge?
- A moving charge generates a magnetic field that depends on the velocity of the charge.

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \tag{1}$$



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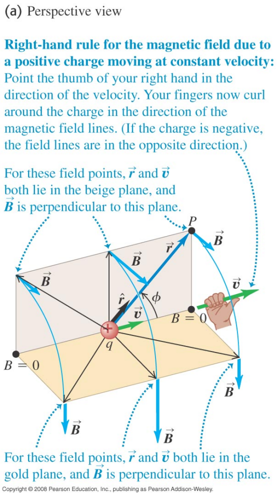
$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \tag{1}$$

- A moving charge q generates a magnetic field that depends on the velocity, $\vec{\mathbf{v}}$ of the charge.

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \qquad (2)$$

- $\hat{\mathbf{r}}$ is a unit vector from the source point to the field point.
- The magnitude of the field

$$|\vec{\mathbf{B}}| = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} \qquad (3)$$



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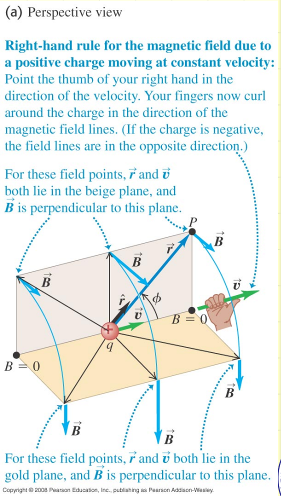

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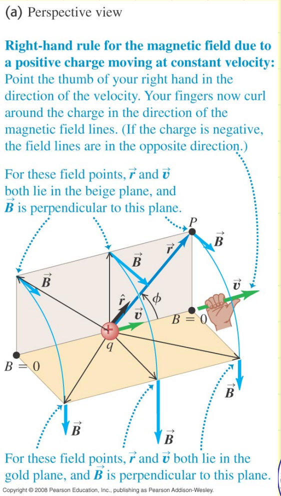


The magnetic field of a moving charge

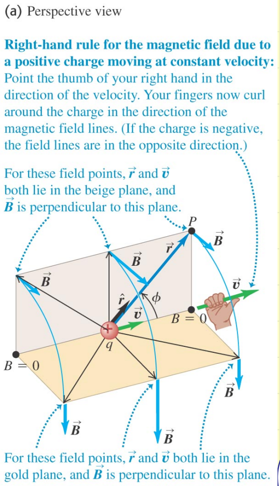
- Magnetic permeability constant

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A} \qquad (2)$$

- Electric and magnetic constants are related:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.998 \times 10^{8}\,\mathrm{m/s} \quad (3)$$

where c is *the speed of light.*



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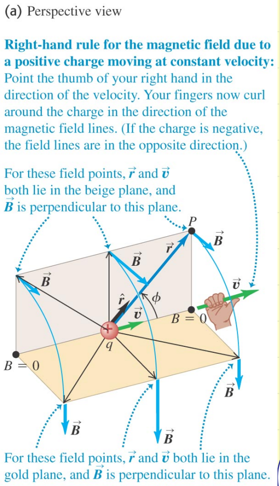
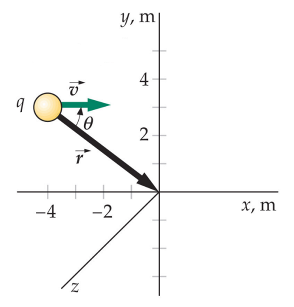


Question

A charge of $q = 4.5\,\mu\text{C}$ moves with velocity $v = 3 \times 10^3\,\text{m/s}$. What is the magnetic field at the origin of the coordinate system when the charge is at position $x = -4\,\text{m}$ and $y = 3\,\text{m}$.



Ex: Magnetic field of a moving charge

Solution

- $\vec{\mathbf{r}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$, $r = \sqrt{4^2 + (-3)^2} = 5\,\mathrm{m}$.

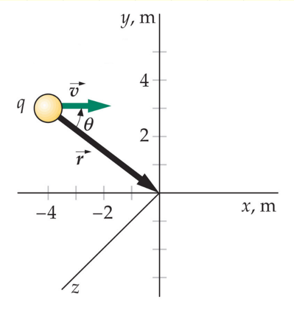
  $\hat{\mathbf{r}} = 4/5\hat{\mathbf{i}} - 3/5\hat{\mathbf{j}}, \quad \vec{\mathbf{v}} = 3 \times 10^3\,\mathrm{m/s}\hat{\mathbf{i}}$

  $$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

  $$= 10^{-7}\frac{(4.5\,\mu\mathrm{C})(3 \times 10^3\,\mathrm{m/s}\hat{\mathbf{i}}) \times (4/5\hat{\mathbf{i}}}{(5\,\mathrm{m})^2}$$

  $$= -3.24 \times 10^{-11}\,\mathrm{T}\,\hat{\mathbf{k}}$$

  where we used $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0$.

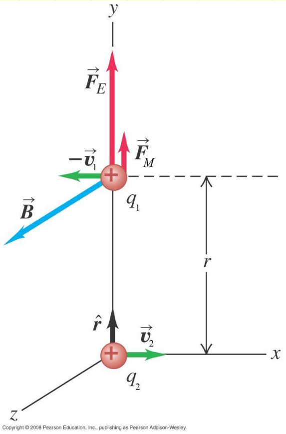


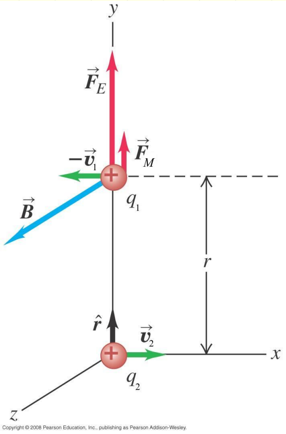
Ex: Magnetic force between two positive charges

Question

Consider two positive charges moving in opposite directions in space a distance r apart at some moment in time.

$F_M/F_E = ?$





Solution

E field exists (moving or not!)

$$\vec{\mathbf{F}}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{j}}$$

Ex: Magnetic force between two positive charges

Solution

- Motion ($\vec{\mathbf{v}}_2 = v_2\hat{\mathbf{i}}$) of q_2 produces

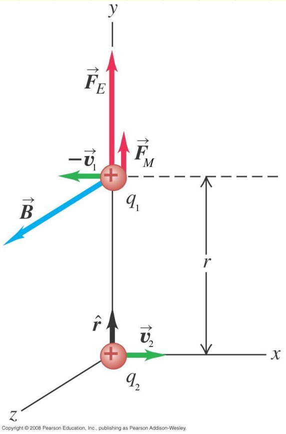
$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q_2 v_2 \hat{\mathbf{i}} \times \hat{\mathbf{j}}}{r^2}, \quad \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$$

- Charge q_1 is moving with $\vec{\mathbf{v}}_1 = -v_1\hat{\mathbf{i}}$ in this magnetic field and so is acted on by the force

$$\vec{\mathbf{F}}_M = q_1\vec{\mathbf{v}}_1 \times \vec{\mathbf{B}} = -q_1 v_1 \frac{\mu_0}{4\pi}\frac{q_2 v_2}{r^2}\hat{\mathbf{i}} \times \hat{\mathbf{k}}$$

$$= \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{r^2}\hat{\mathbf{j}}$$

where we used $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$.



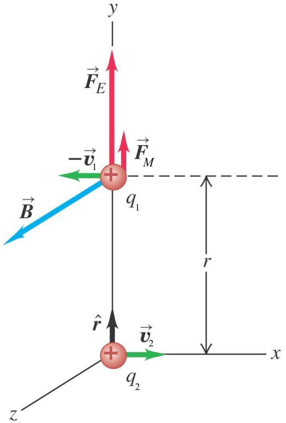
Solution

Bringing the results together

$$\vec{\mathbf{F}}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_M = \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{r^2}\hat{\mathbf{j}}$$

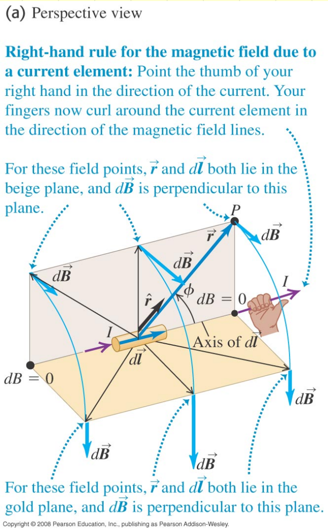
$$\begin{aligned}\frac{F_M}{F_E} &= \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{r^2}\frac{4\pi\epsilon_0 r^2}{q_1 q_2}\\ &= \mu_0\epsilon_0 v_1 v_2\\ &= v_1 v_2/c^2\end{aligned}$$



Magnetic force is much smaller than the electrical force.

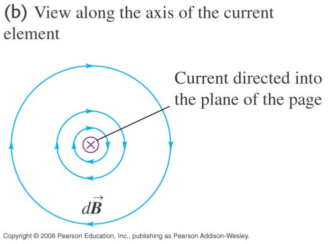
Magnetic field of a current element

Magnetic field of a current element



- The total magnetic field of several moving charges is the vector sum of each field.
- So a current of moving charges creates a B field!

Magnetic field of a current element



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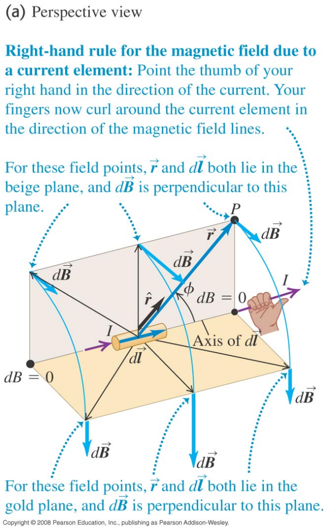
Magnetic field of a current element

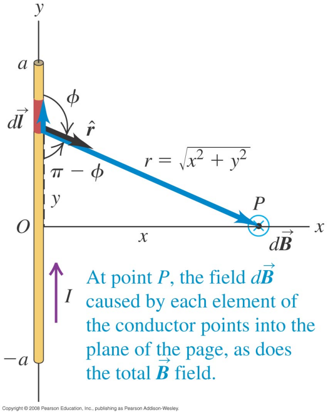


- $$\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{\mathrm{d}Q\,\vec{\mathbf{v}}_d \times \hat{\mathbf{r}}}{r^2} \tag{4}$$
- $$\mathrm{d}Q = nqA\,\mathrm{d}\ell \tag{5}$$

law of Biot and Savart

$$\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2} \tag{6}$$

Magnetic field of a current carrying wire



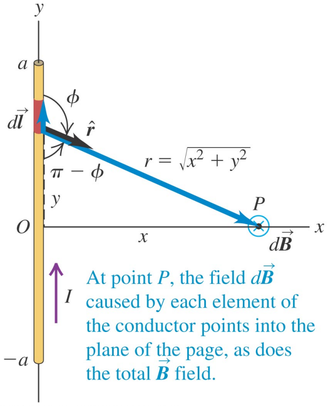
$$\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2}$$

As $\vec{\mathbf{B}} = \int \mathrm{d}\vec{\mathbf{B}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\int \frac{I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2}$$

Magnetic field of a straight current carrying conductor

Magnetic field of a straight wire



$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int \frac{I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2}$$

$$\mathrm{d}\vec{\boldsymbol{\ell}} = \mathrm{d}y\,\hat{\mathbf{j}},\ \vec{\mathbf{r}} = x\hat{\mathbf{i}} - y\hat{\mathbf{j}},\ r = \sqrt{x^2 + y^2}$$

$$\hat{\mathbf{r}} \equiv \frac{\vec{\mathbf{r}}}{r} = \frac{x}{r}\hat{\mathbf{i}} - \frac{y}{r}\hat{\mathbf{j}}$$

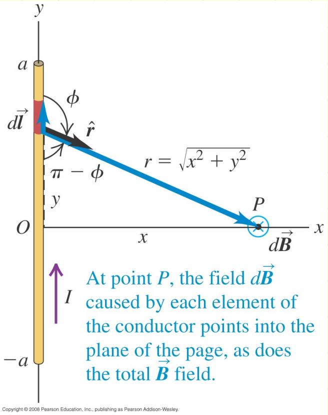
$$\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}} = \mathrm{d}y\,\hat{\mathbf{j}} \times \left(\frac{x}{r}\hat{\mathbf{i}} - \frac{x}{r}\hat{\mathbf{j}}\right)$$

$$= \mathrm{d}y\,\hat{\mathbf{j}} \times \frac{x}{r}\hat{\mathbf{i}} = -\frac{x\,\mathrm{d}y}{r}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = -\frac{\mu_0 I}{4\pi} \int \frac{x\,\mathrm{d}y}{r^3}\hat{\mathbf{k}} = -\frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{\mathrm{d}y}{(x^2 + y^2)^{3/2}}\hat{\mathbf{k}}$$

$$= -\frac{\mu_0 I x}{4\pi} \left.\frac{y}{x^2\sqrt{x^2 + y^2}}\right|_{-a}^{a} \hat{\mathbf{k}} = -\frac{\mu_0 I}{2\pi x} \frac{a}{\sqrt{a^2 + x^2}}\hat{\mathbf{k}}$$

Magnetic field of an infinitely long straight wire



So we found

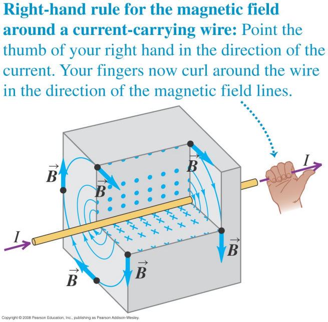
$$\vec{\mathbf{B}} = -\frac{\mu_0 I}{2\pi x}\frac{a}{\sqrt{a^2+x^2}}\hat{\mathbf{k}}$$

for the wire lying between $-a$ and a. If the wire is infinitely long ($a \to \infty$) we obtain

$$\vec{\mathbf{B}} = -\frac{\mu_0 I}{2\pi x}\hat{\mathbf{k}} \tag{7}$$

This is a good approximation for the field of a wire with $a \gg x$.

Direction of the magnetic field of a straight wire

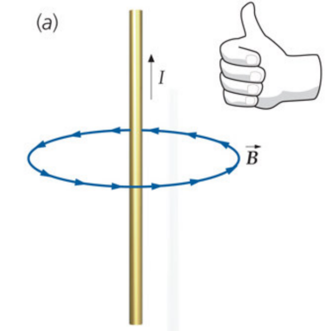


At a distance r from the wire the magnitude of the field is

$$B = \frac{\mu_0 I}{2\pi r} \tag{8}$$

and the direction is determined by the RHR.

Direction of the magnetic field of a straight wire



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Ex: Field of a long wire

Question

A long wire carries a current of 10 A.

(a) What is the magnetic field at a distance 1 cm?

(b) Is it hazardeous for living organisms?

Ex: Field of a long wire

Question

A long wire carries a current of $10\,\mathrm{A}$.

(a) What is the magnetic field at a distance $1\,\mathrm{cm}$?

(b) Is it hazardeous for living organisms?

Answer (a)

$I = 10\,\mathrm{A}$ and $r = 1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}) \times 10\,\mathrm{A}}{2\pi \times 10^{-2}\,\mathrm{m}}$$

$$= 2 \times 10^{-4}\,\mathrm{T}$$

Ex: Field of a long wire

Question

A long wire carries a current of $10\,\mathrm{A}$.
(a) What is the magnetic field at a distance $1\,\mathrm{cm}$?
(b) Is it hazardeous for living organisms?

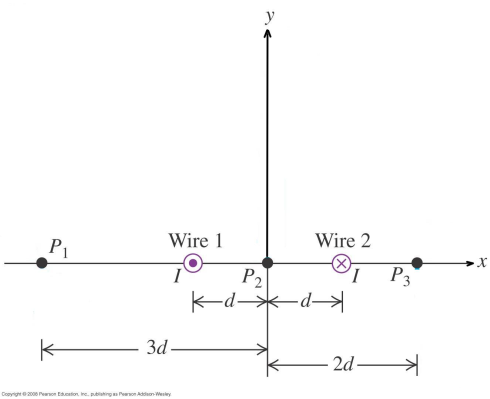
Answer (b)

In order to decide whether a magnetic field of $2 \times 10^{-4}\,\mathrm{T}$ is hazardeous for living organisms we should compare it with some other fields humans are exposed to.
The Earth has a magnetic field of $B_{\oplus} \simeq 0.5 \times 10^{-4}\,\mathrm{T}$ to which all living organisms on Earth a subjected to since at least a billion years! We are usually more than $1\,\mathrm{cm}$ away from such wires. Note also that a patient in an MRI device is exposed to about $1\,\mathrm{T}$ which also is not found to be hazardous.

Ex: Magnetic field of two wires

Question

Two long, straight, parallel wires perpendicular to the xy-plane are seperated by a distance $2d$. Each wire carries a current I but in opposite directions. Find $\vec{\mathbf{B}}$ at points P_1, P_2 and P_3.

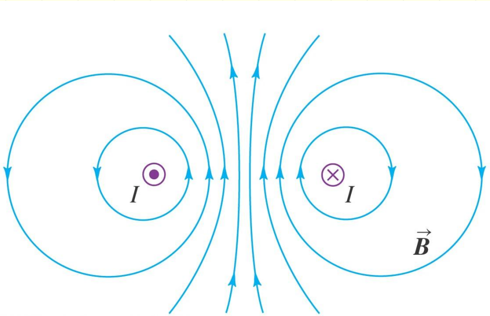


End-on view of the two-wire system described in the question.

Ex: Magnetic field of two wires

Solution

The fields support each other in the region between the wires and partially cancel each other on the left of the first wire and right of the second wire.



Map of the magnetic field produced by the two conductors. The field lines are closest together between the conductors, where the field is strongest.

Ex: Magnetic field of two wires

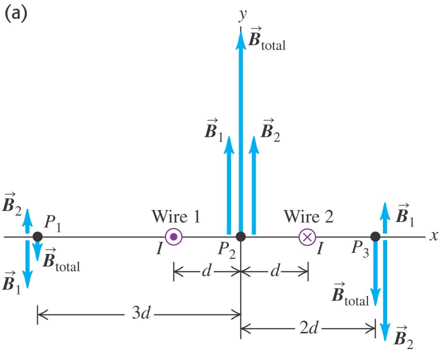
Solution

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

$$\vec{\mathbf{B}} = -\frac{\mu_0 I}{2\pi(2d)}\hat{\mathbf{j}} + \frac{\mu_0 I}{2\pi(4d)}\hat{\mathbf{j}} \quad \text{At } P_1$$

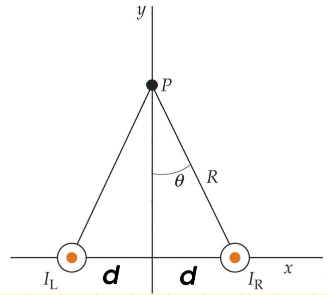
$$\vec{\mathbf{B}} = +\frac{\mu_0 I}{2\pi(d)}\hat{\mathbf{j}} + \frac{\mu_0 I}{2\pi(d)}\hat{\mathbf{j}} \quad \text{At } P_2$$

$$\vec{\mathbf{B}} = +\frac{\mu_0 I}{2\pi(3d)}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(d)}\hat{\mathbf{j}} \quad \text{At } P_3$$



Question

Two long, straight, parallel wires perpendicular to the xy-plane are seperated by a distance $2d$. Each wire carries a current I both in the $+z$-direction. Find $\vec{\mathbf{B}}$ at point P on the y-axis.



Ex: Magnetic field of two wires

Solution

The magnitudes are the same.

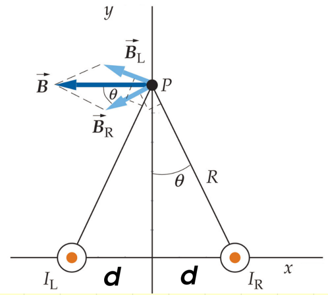
$$B_R = B_L = \frac{\mu_0 I}{2\pi R}$$

The y-components will cancel each other. Only the x-components will prevail and add-up:

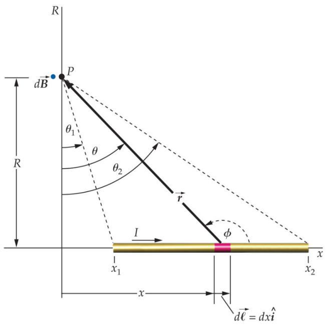
$$B = 2B_R \cos\theta, \quad \vec{\mathbf{B}} = -2B_R \cos\theta \hat{\mathbf{i}}$$

where

$\cos\theta = \sqrt{R^2 - d^2}/R = \sqrt{1 - d^2/R^2}$

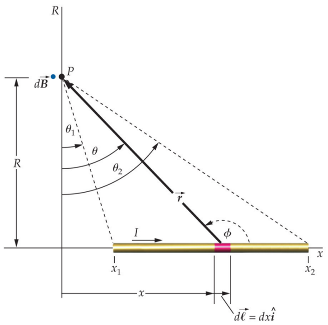


Ex: Magnetic field of current segment



This is a calculation for a general position relative to the current segment but we will be using it for highly symmetric situatiuons.

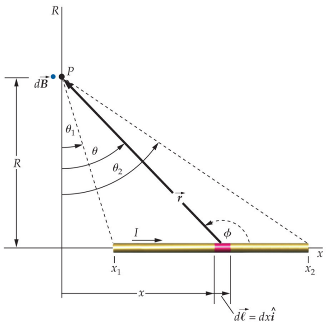
Ex: Magnetic field of current segment



Show that the $\vec{\mathbf{B}}$ at point P is

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1)\hat{\mathbf{k}}$$

Ex: Magnetic field of current segment



Show that the $\vec{\mathbf{B}}$ at point P is

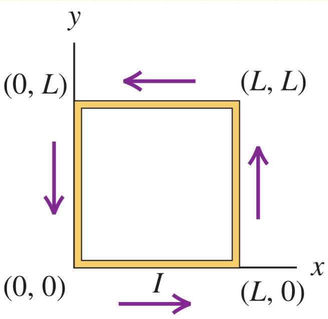
$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1)\hat{\mathbf{k}}$$

For an infinitely long wire
$\theta_1 = -\pi/2$ & $\theta_2 = +\pi/2$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi R}[1 - (-1)]\hat{\mathbf{k}} = \frac{\mu_0 I}{2\pi R}\hat{\mathbf{k}}$$

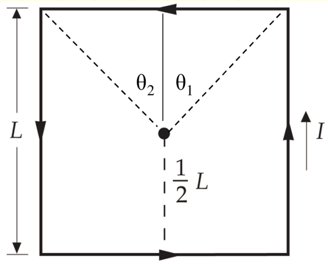
which is what we obtained before.

Ex: Magnetic field at the center of a square loop



What is the magnetic field at the center of the loop?

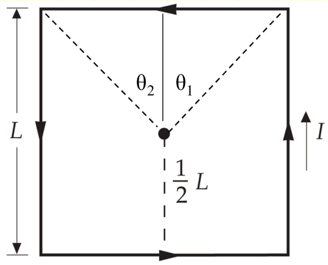
Ex: Magnetic field at the center of a square loop



Note that $\theta_1 = -\pi/4$ and $\theta_2 = +\pi/4$ and $R = L/2$. For one segment

$$\begin{aligned}
\vec{\mathbf{B}}_1 &= \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1)\hat{\mathbf{k}} \\
&= \frac{\mu_0 I}{2\pi L}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right]\hat{\mathbf{k}} \\
&= \frac{\sqrt{2}\mu_0 I}{2\pi L}\hat{\mathbf{k}}
\end{aligned}$$

Ex: Magnetic field at the center of a square loop



Note that $\theta_1 = -\pi/4$ and $\theta_2 = +\pi/4$ and $R = L/2$. For one segment

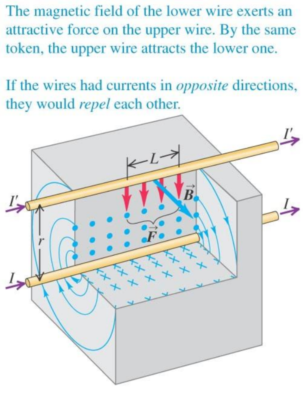
$$\begin{aligned}\vec{\mathbf{B}}_1 &= \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1)\hat{\mathbf{k}} \\ &= \frac{\mu_0 I}{2\pi L}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right]\hat{\mathbf{k}} \\ &= \frac{\sqrt{2}\mu_0 I}{2\pi L}\hat{\mathbf{k}}\end{aligned}$$

Each segment makes an identical contribution

$$\vec{\mathbf{B}} = 4\vec{\mathbf{B}}_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}\hat{\mathbf{k}}$$

Force between parallel conductors

Force between parallel conductors



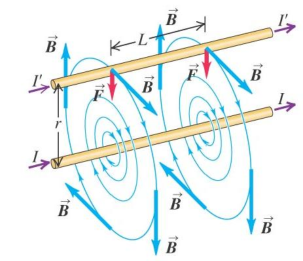
- The lower conductor produces a $\vec{\mathbf{B}}$ field that, at the position of the upper conductor has magnitude $B = \mu_0 I/2\pi r$.
- The force that this field exerts on a length L of the upper conductor is $\vec{\mathbf{F}} = I'\vec{\ell} \times \vec{\mathbf{B}}$. Thus

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r} \tag{9}$$

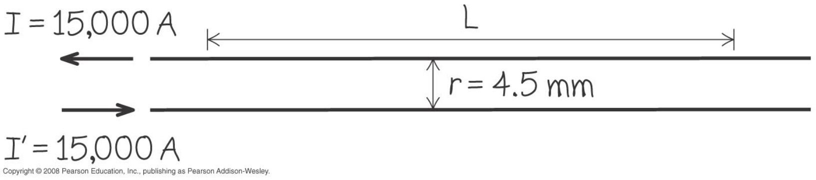
- The force per unit length on each conductor is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

Force between parallel conductors



- The conductors attract each other if the currents are in the same direction and repel if they are in opposite directions.

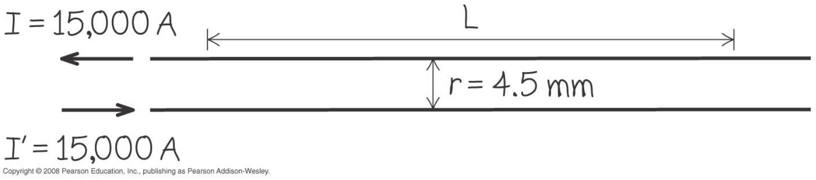


Question

What force does each wire exert on the other?

Solution

$$\begin{aligned}\frac{F}{L} &= \frac{\mu_0 I I'}{2\pi r} \\ &= \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1.5 \times 10^{4}\,\mathrm{A})^2}{2\pi \times 4.5 \times 10^{-3}\,\mathrm{m}} \\ &= 1 \times 10^{4}\,\mathrm{N/m}\end{aligned}$$



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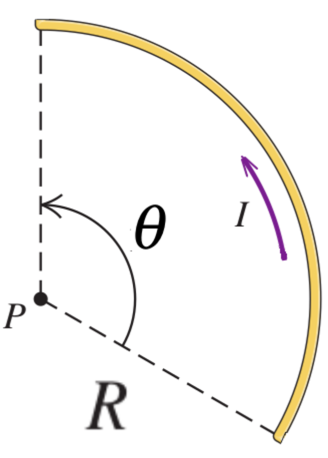
Magnetic field of a circular current loop

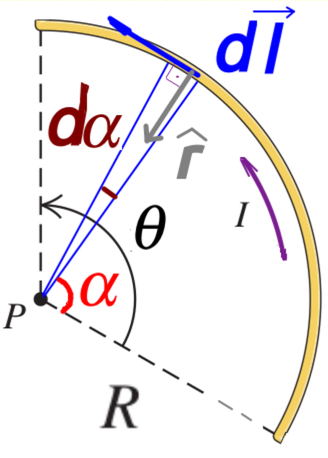
Magnetic field of curved segment



Queston

What is the magnetic field at the center of the curved segment?

Magnetic field of curved segment



Soluton

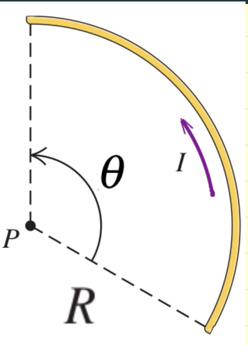
- $\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}}{r^2}$
- $\mathrm{d}\vec{\boldsymbol{\ell}} \perp \hat{\mathbf{r}}$

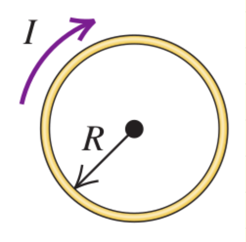
$$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\ell\,\cancelto{1}{|\hat{\mathbf{r}}|}\cancelto{1}{\sin 90^\circ}}{r^2} \quad \odot$$

- $\mathrm{d}\ell = R\,\mathrm{d}\alpha$, $r = R$
- $\mathrm{d}B = \frac{\mu_0 I}{4\pi}\frac{R\mathrm{d}\alpha}{R^2}$
- $B = \frac{\mu_0 I}{4\pi R}\int_0^\theta \mathrm{d}\alpha$

$$B = \frac{\mu_0 I}{4\pi R}\theta \quad \odot$$

where θ is in *radians*.





Curved segment

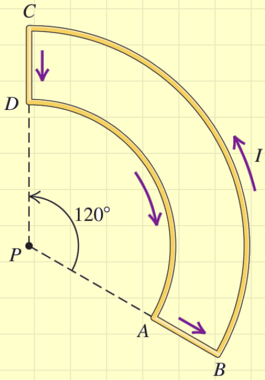
$$B = \frac{\mu_0 I}{4\pi R}\theta$$

Full circle

$$B = \frac{\mu_0 I(2\pi)}{4\pi R} = \frac{\mu_0 I}{2R}$$

Question

Calculate the magnetic field (magnitude and direction) at a point P due to a current $I = 12.0\,\mathrm{A}$ in the wire shown in Figure. Segment BC is an arc of a circle with radius $R_2 = 30.0\,\mathrm{cm}$, and point P is at the center of curvature of the arc. Segment DA is an arc of a circle with radius $R_1 = 20.0\,\mathrm{cm}$, and point P is at its center of curvature. Segments CD and AB are straight lines of length $10.0\,\mathrm{cm}$ each.

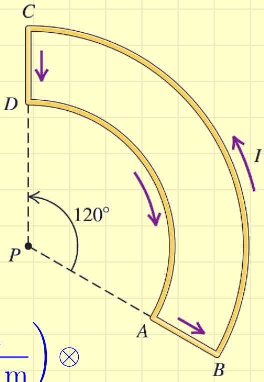


Ex: A loop

Solution

$\theta = 120^\circ = 2\pi/3\,\text{radians}$

$$B = \frac{\mu_0 I\theta}{4\pi R} = \frac{\mu_0 I}{6R} \quad \text{Single segment of radius } R$$



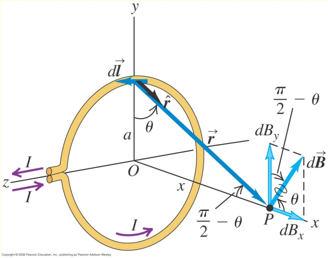
$$\vec{\mathbf{B}} = \frac{\mu_0 I}{6R_1}\otimes + \frac{\mu_0 I}{6R_2}\odot$$

$$= \frac{\mu_0 I}{6}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\otimes$$

$$= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(12.0\,\mathrm{A})}{6}\left(\frac{1}{0.2\,\mathrm{m}} - \frac{1}{0.3\,\mathrm{m}}\right)\otimes$$

$$\simeq 4\times 10^{-6}\,\mathrm{T}\otimes$$

Magnetic field of a circular current loop

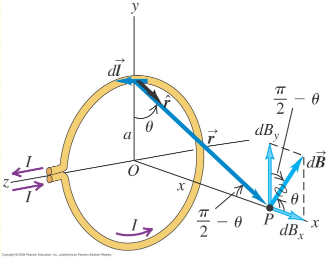


$r = \sqrt{a^2 + x^2}$,

$\cos\theta = a/r$

Question

- Use Biot-Savart Law to find field a distance x away on axis from a coil with current I.
- The result should give $B = \mu_0 I/2a$ for $x = 0$.



$r = \sqrt{a^2 + x^2}$,

$\cos\theta = a/r$

Solution

- $\hat{\mathbf{r}} = \vec{\mathbf{r}}/r$

$$\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{r^2}$$

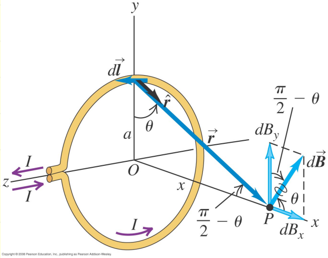
- $\mathrm{d}\vec{\boldsymbol{\ell}} \perp \hat{\mathbf{r}}$,

$$\mathrm{d}B = \frac{\mu_0}{4\pi} \frac{I\,\mathrm{d}\ell\,\cancelto{1}{|\hat{\mathbf{r}}|}\cancelto{1}{\sin 90^\circ}}{r^2}$$

- $\mathrm{d}B_x = \mathrm{d}B\cos\theta$

$$\mathrm{d}B_x = \frac{\mu_0}{4\pi} \frac{I\,\mathrm{d}\ell\cos\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{I\,\mathrm{d}\ell\,a}{r^3}$$

Magnetic field of a circular current loop



$$r = \sqrt{a^2 + x^2},$$

$$\cos\theta = a/r$$

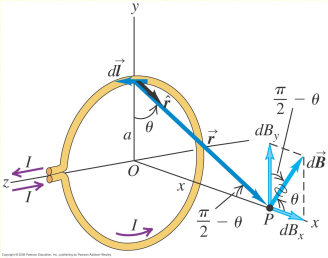
Solution

- We found

$$\mathrm{d}B_x = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\ell\,a}{r^3}$$

- Integrate

$$\begin{aligned} B_x &= \frac{\mu_0 I a}{4\pi r^3}\cancelto{2\pi a}{\int \mathrm{d}\ell} \\ &= \frac{\mu_0 I a^2}{2r^3} \\ &= \frac{\mu_0}{2}\frac{Ia^2}{(a^2+x^2)^{3/2}} \end{aligned}$$



$r = \sqrt{a^2 + x^2}$,
$\cos\theta = a/r$

Solution

- We found

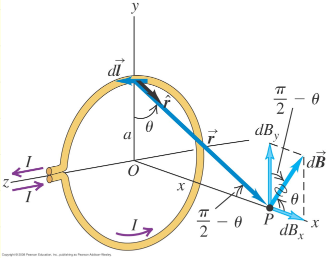
$$B_x = \frac{\mu_0}{2} \frac{Ia^2}{(a^2 + x^2)^{3/2}}$$

- For $x = 0$ this simplifies to

$$B_x = \frac{\mu_0}{2} \frac{Ia^2}{a^3} = \frac{\mu_0 I}{2a}$$

matching the field we found at the center of a circular loop.

Magnetic field of a circular current loop



$r = \sqrt{a^2 + x^2}$,
$\cos\theta = a/r$

Result in terms of μ

- We found

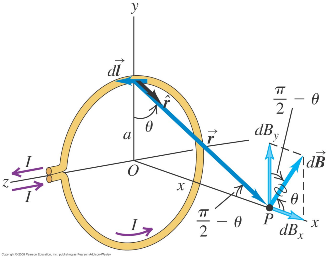
$$B_x = \frac{\mu_0}{2} \frac{Ia^2}{(a^2 + x^2)^{3/2}}$$

- Recall $\mu = IA = I\pi a^2$ is the *magnetic moment*

$$B_x = \frac{\mu_0}{2\pi} \frac{\mu}{(a^2 + x^2)^{3/2}}$$

- If $x \gg a$

$$B_x = \frac{\mu_0}{2\pi} \frac{\mu}{|x|^3}$$



$r = \sqrt{a^2 + x^2}$,
$\cos\theta = a/r$

Field of a coil

- If there are N turns in the loop

$$B_x = \frac{\mu_0}{2} \frac{NIa^2}{(a^2 + x^2)^{3/2}}$$

- Recall $\mu = NIA = NI\pi a^2$ is the magnetic moment

$$B_x = \frac{\mu_0}{2\pi} \frac{\mu}{(a^2 + x^2)^{3/2}}$$

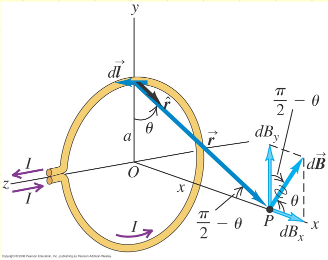
remains unchanged

- If $x \gg a$

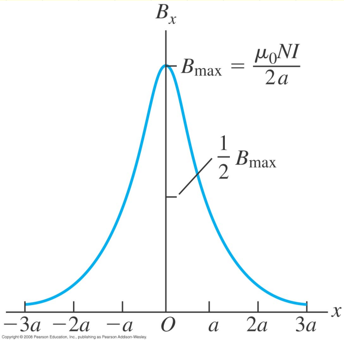
$$B_x = \frac{\mu_0}{2\pi} \frac{\mu}{|x|^3}$$

Magnetic field of a circular current loop

$$B_x = \frac{\mu_0}{2}\frac{NIa^2}{(a^2+x^2)^{3/2}}$$

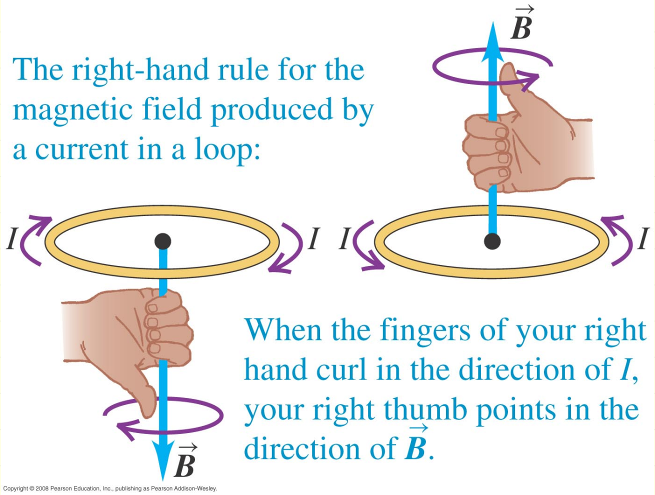


$r = \sqrt{a^2+x^2}$,
$\cos\theta = a/r$



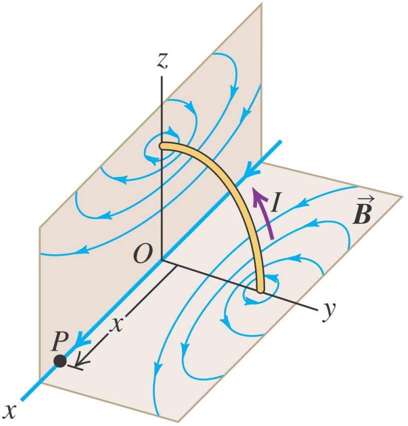
Direction of the field

Direction of field using right-hand rule.



Direction of the field

Magnetic field lines produced by the current in a circular loop. At points on the axis, the $\vec{\mathbf{B}}$ field has the same direction as $\vec{\mu}$ of the loop.



Direction of the field

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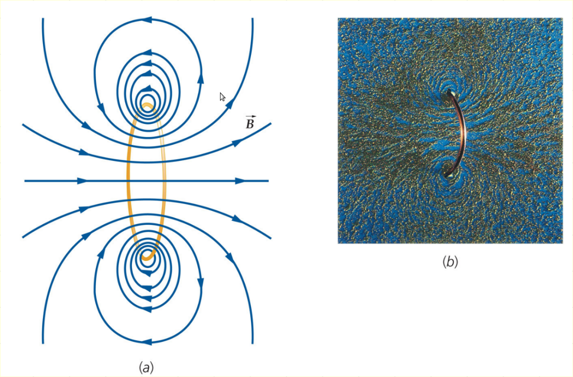
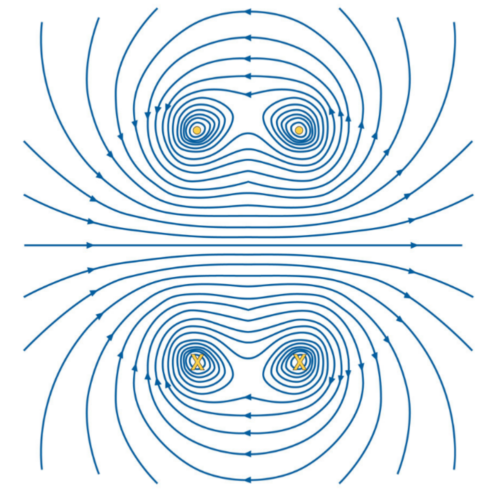


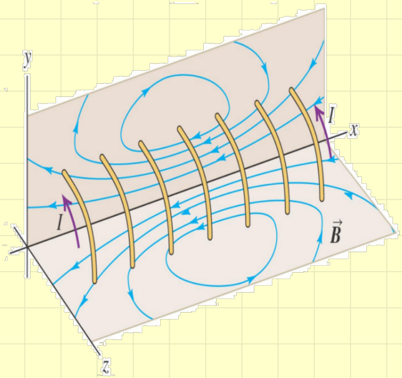
Magnetic field of two closely spaced rings

With the addition of a second current loop the magnetic field becomes more uniform in the near the center of the loops.

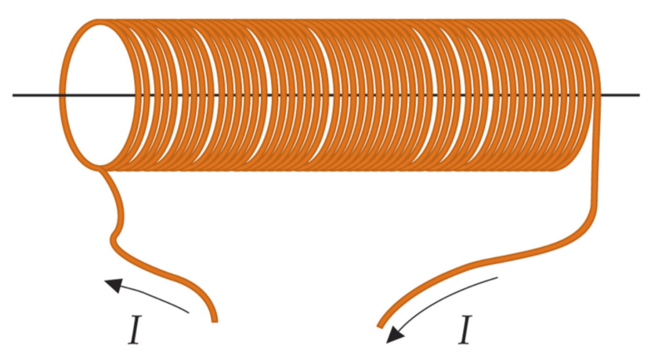


Magnetic field of many closely spaced rings

With the addition of further current loops the magnetic field becomes more uniform near the center of the loops.

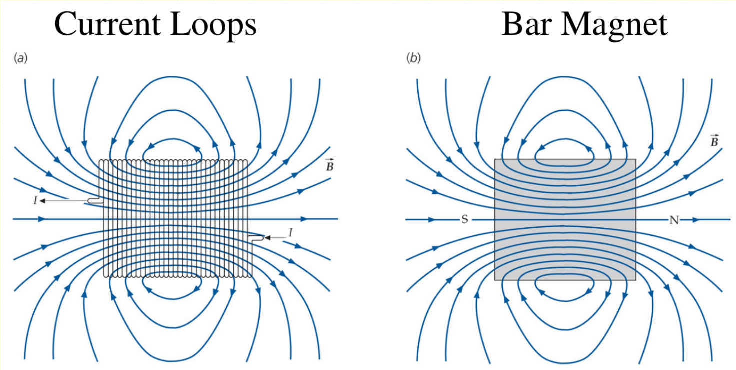


Solenoid

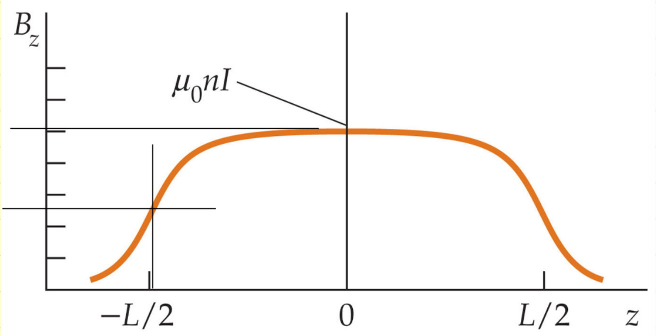


A circuit element with $N =$ thousands of closely spaced turns (windings), each of which can be regarded as a circular loop carrying te same current.

Solenoid



The fields are similar but you can't get access to the internal region of the bar magnet.



Exact calculations show that for a long, closely wound solenoid, half of these field lines emerge from the ends and half "leak out" through the windings between the center and the end, as the figure suggests. Here $n = N/L$

Solenoid

$z_2 - z_1 = L$, $n = N/L$. In length $\mathrm{d}z$ there are $n\,\mathrm{d}z$ turns each with current I:

$$\mathrm{d}i = In\,\mathrm{d}z$$

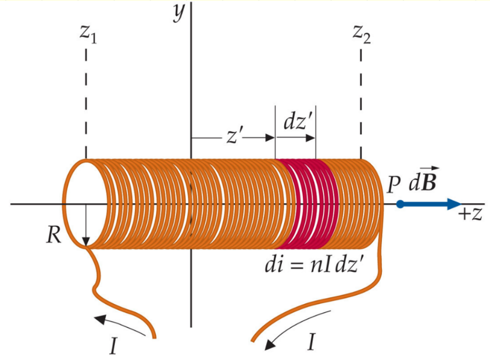
Recall that for a circular loop of radius a, current I we habe obtained the field as

$$B_x = \frac{\mu_0}{2}\frac{Ia^2}{(a^2+x^2)^{3/2}}$$

Replace $I \to \mathrm{d}i$, $a \to R$, $x \to z$

$$\mathrm{d}B_z = \frac{\mu_0}{2}\frac{\mathrm{d}i\,R^2}{(R^2+z^2)^{3/2}}$$

Integrating this from z_1 to z_2

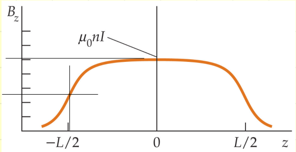


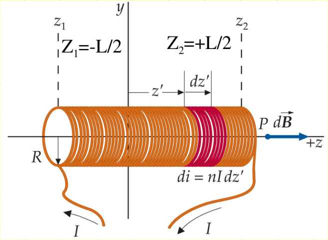
$$B_z = \frac{\mu_0}{2}nI\left(\frac{z_2}{\sqrt{R^2+z_2^2}} - \frac{z_1}{\sqrt{R^2+z_1^2}}\right)$$

(Field at the center $z = 0$)

Let us add some symmetry by choosing $z_1 = -L/2$ and $z_2 = L/2$

$$
\begin{aligned}
B_z &= \frac{\mu_0}{2} nI \left(\frac{z_2}{\sqrt{R^2 + z_2^2}} - \frac{z_1}{\sqrt{R^2 + z_1^2}} \right) \\
&= \frac{\mu_0}{2} nI \left(\frac{L/2}{\sqrt{R^2 + (L/2)^2}} - \frac{-L/2}{\sqrt{R^2 + (-L/2)^2}} \right) \\
&= \frac{\mu_0}{2} nI \left(\frac{L}{\sqrt{R^2 + (L/2)^2}} \right)
\end{aligned}
$$





Ideal solenoid limit

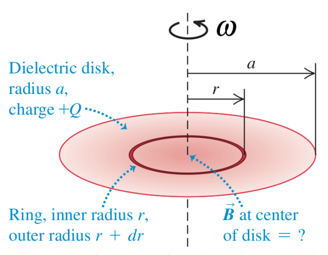
Ideal solenoid ($L \gg R$) assumption simplifies further

$$
\begin{aligned}
B_z &= \frac{\mu_0}{2} n I \left(\frac{L}{\sqrt{\cancelto{0}{R^2} + (L/2)^2}} \right) \\
&= \frac{\mu_0}{2} n I \left(\frac{L}{L/2} \right) \\
&= \mu_0 n I
\end{aligned}
$$

We later will derive this from Ampere's law.

Question

A thin dielectric disk with radius a has a total charge $+Q$ distributed uniformly over its surface. It rotates with angular velocity ω about an axis perpendicular to the surface of the disk and passing through its center.
(a) Find the magnetic field at the center of the disk.
(b) Find the magnetic field at a distance x along the rotation axis.

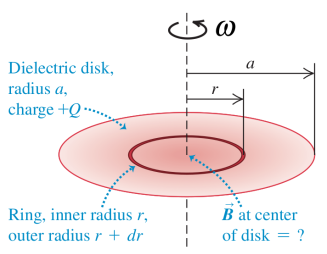


Ex: Field produced by a rotating disc of charge

Solution (a)

- We have found the magnetic field at the center of a ring of radius r carrying current I as $B = \mu_0 I/2r$
- If the total charge is $+Q$ the surface charge density is $\sigma = Q/\pi a^2$
- A ring with radius r and thickness $\mathrm{d}r$ has area $\mathrm{d}A = 2\pi r\,\mathrm{d}r$ and hence its charge is

$$\mathrm{d}q = \sigma\,\mathrm{d}A = \frac{Q}{\cancel{\pi} a^2} 2\cancel{\pi} r\,\mathrm{d}r$$



- The period of rotation is $T = 2\pi/\omega$ and the current corresponding of the rotation of the ring is then $\mathrm{d}I = \mathrm{d}q\,/T$

$$\mathrm{d}I = \frac{Q\omega}{\pi a^2} r\,\mathrm{d}r$$

Ex: Field produced by a rotating disc of charge

- The field due to current $\mathrm{d}I$ is then $\mathrm{d}B = \mu_0 \, \mathrm{d}I \,/2r$

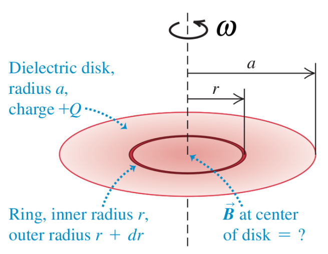
$$\mathrm{d}B = \frac{\mu_0 Q \omega}{2\pi a^2} \, \mathrm{d}r$$

- Integration gives

$$B = \frac{\mu_0 Q \omega}{2\pi a^2} \int_0^a \mathrm{d}r = \frac{\mu_0 Q \omega}{2\pi a}$$

- Recall the magnetic moment of the disc is $\mu = \frac{1}{4} Q a^2 \omega$. Then

$$B = \frac{2\mu_0 \mu}{\pi a^3}$$



Ex: Field produced by a rotating disc of charge

Solution (b)

- For a ring of radius a with current I we have found

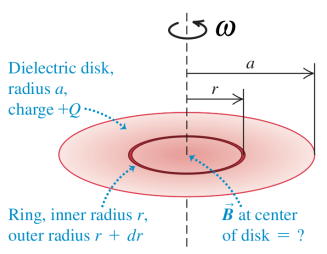
$$B = \frac{\mu_0}{2} \frac{I a^2}{(a^2 + x^2)^{3/2}}$$

- For radius r and current $\mathrm{d}I = \frac{Q\omega}{\pi a^2} r \, \mathrm{d}r$

$$\mathrm{d}B = \frac{\mu_0}{2} \frac{\mathrm{d}I \, r^2}{(r^2 + x^2)^{3/2}}$$

- Thus integration gives

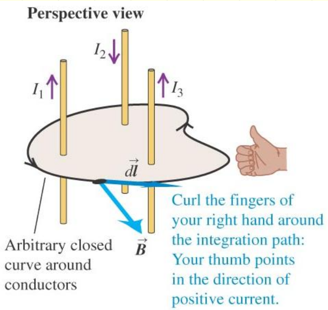
$$B = \frac{\mu_0 Q\omega}{2\pi a^2} \int_0^a \frac{r^3 \, \mathrm{d}r}{(r^2 + x^2)^{3/2}}$$
$$= \frac{\mu_0 Q\omega}{2\pi a^2} \left(\frac{a^2 + 2x^2}{\sqrt{a^2 + x^2}} - 2x \right)$$



$$\int \frac{r^3 \, \mathrm{d}r}{(r^2 + x^2)^{3/2}} = \frac{r^2 + 2x^2}{\sqrt{r^2 + x^2}} + C$$

see `https://www.integral-calculator.com/`

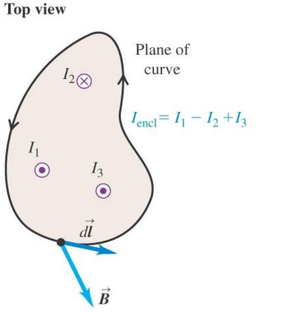
Ampere's law



$$\overbrace{\oint \underbrace{\vec{B}\cdot d\vec{l}}_{\text{Scalar product of magnetic field and vector segment of path}}}^{\text{Line integral around a closed path}} = \underset{\text{Magnetic constant}}{\mu_0}\,\underset{\text{Net current enclosed by path}}{I_{\text{encl}}}$$

- Suppose several long, straight conductors pass through surface bounded by closed loop path.
- The line integral of total magnetic field is proportional to *algebraic* sum of currents.

Ampere’s law



Ampere’s law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

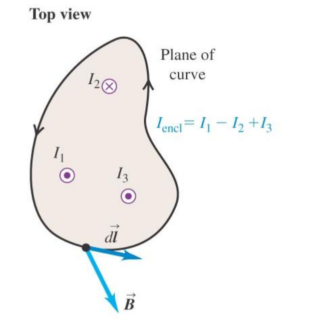
$$\overbrace{\oint \underbrace{\vec{B} \cdot d\vec{l}}_{\text{Scalar product of magnetic field and vector segment of path}}}^{\text{Line integral around a closed path}} = \mu_0 I_{encl}$$

Magnetic constant (μ_0)

Net current enclosed by path (I_{encl})

- I_{enc} = algebraic sum of currents enclosed or linked by integration path
- Evaluate by using right- hand sign rule.

Ampere's law



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

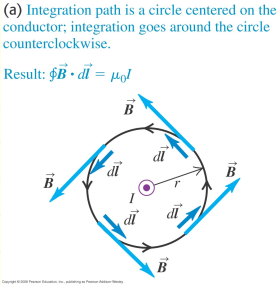
$$\overbrace{\oint}^{\text{Line integral around a closed path}} \underbrace{\vec{B} \cdot d\vec{l}}_{\text{Scalar product of magnetic field and vector segment of path}} = \mu_0 I_{encl}$$

Magnetic constant (μ_0)

Net current enclosed by path (I_{encl})

- Valid for conductors & paths of any shape.
- If integral around closed path is zero...
  - does not necessarily mean that B field is zero everywhere
  - only that total current through an area bounded by path is zero.

Field of a infinitely long straight wire



- Ampere's law relates electric current to line integral *around a closed path.*

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \oint B\,\mathrm{d}l = B \oint \mathrm{d}l = B\,2\pi r$$

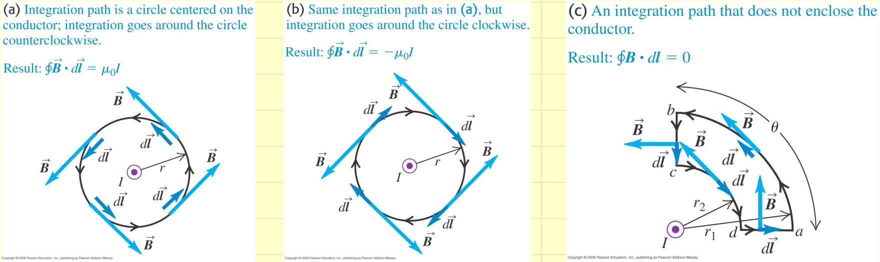
- $I_{\text{enc}} = I$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} \Rightarrow B\,2\pi r = \mu_0 I$$

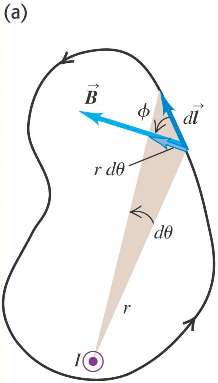
$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

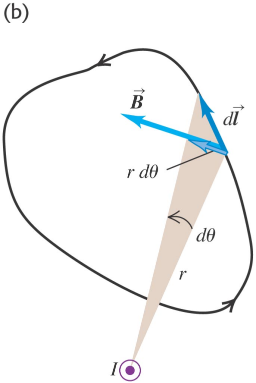
(same with the result we obtained with Biot-Savart's law.)

Ampere’s law for a *circular* path around a long straight conductor



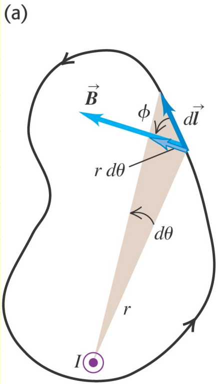
Ampere's law for a *general* path around a long straight conductor

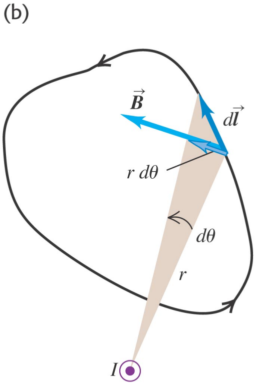




$$\vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = B\,\mathrm{d}l\cos\phi \;\&\; \mathrm{d}l\cos\phi = r\,\mathrm{d}\theta$$

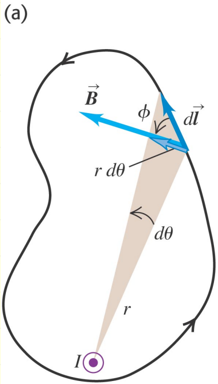
Ampere's law for a *general* path around a long straight conductor

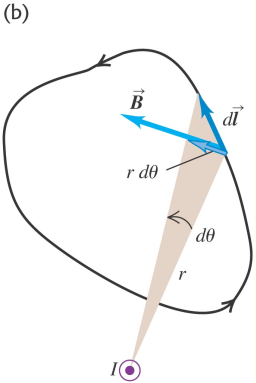




$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \oint \frac{\mu_0 I}{2\pi r} r \,\mathrm{d}\theta = \frac{\mu_0 I}{2\pi} \oint \mathrm{d}\theta$$

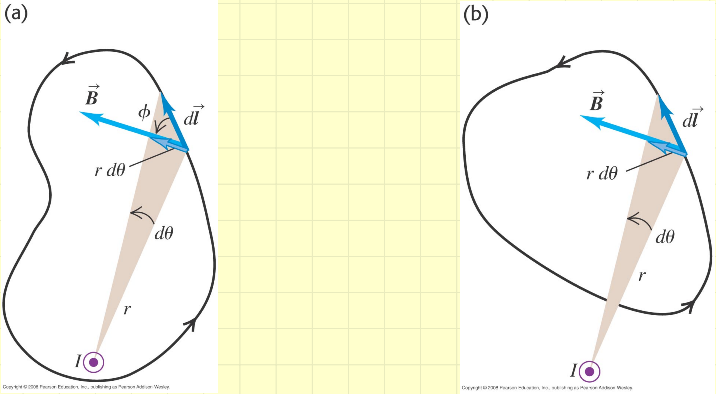
Ampere's law for a *general* path around a long straight conductor





$\oint d\theta$ is the net change in θ during the trip around the integration path

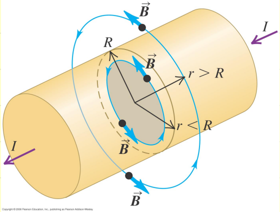
Ampere's law for a *general* path around a long straight conductor



$\oint \mathrm{d}\theta = 2\pi$ if I is inside the loop (Fig a) and $\oint \mathrm{d}\theta = 0$ if I is outside the loop (Fig b)

Applications of Ampere's law

Field of a long cylindrical conductor

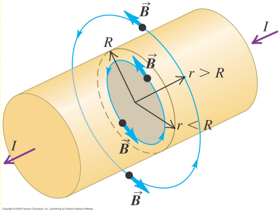


Question

- Cylindrical conductor with radius R carries current I.
- Current is uniformly distributed over cross-sectional area of conductor.
- $B = ?$ everywhere.

$$B = \begin{cases} ? & \text{for } r < R \\ ? & \text{for } r > R \end{cases}$$

Field of a long cylindrical conductor



Answer: For $r > R$

- $$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \oint B\,\mathrm{d}l = B \oint \mathrm{d}l = B\,2\pi r$$

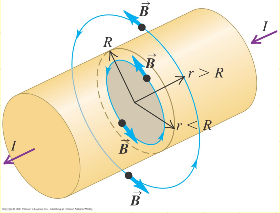
- $I_{\text{enc}} = I$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} \Rightarrow B\,2\pi r = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \text{ for } r > R}$$

(same with the result we obtained for the straight wire)

Field of a long cylindrical conductor



Answer: For $r < R$

- $$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \oint B \,\mathrm{d}l = B \oint \mathrm{d}l = B\, 2\pi r$$

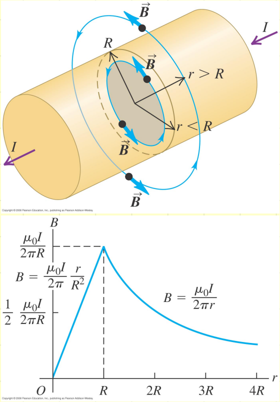
- $I_{\text{enc}} = I(\pi r^2/\pi R^2)$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} \Rightarrow B\, 2\pi r = \mu_0 I \frac{r^2}{R^2}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R^2} r \text{ for } r < R}$$

(increases linearly)

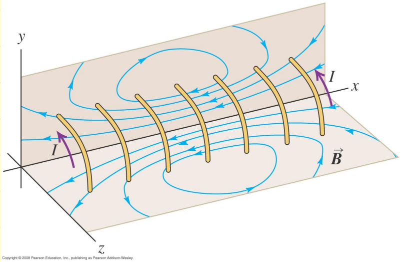
Field of a long cylindrical conductor

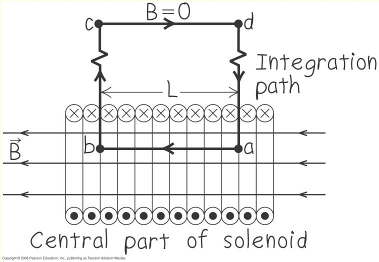


Answer: both

$$B = \begin{cases} \frac{\mu_0 I}{2\pi R^2} r & \text{for } r < R \\ \frac{\mu_0 I}{2\pi r} & \text{for } r > R \end{cases}$$

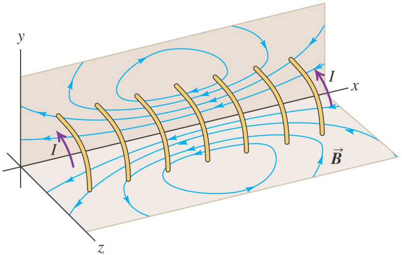
Field of an ideal solenoid

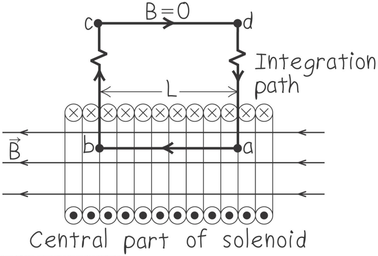




A solenoid consists of a helical winding of wire on a cylinder.

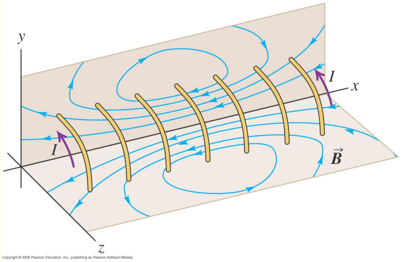
Field of an ideal solenoid

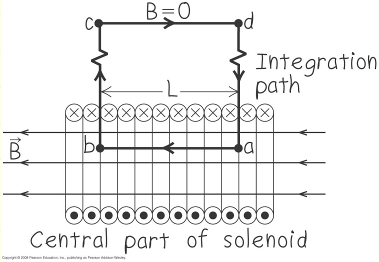




Field depends on # of turns per meter (n) and current I

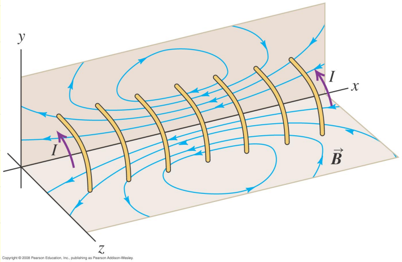
Field of an ideal solenoid

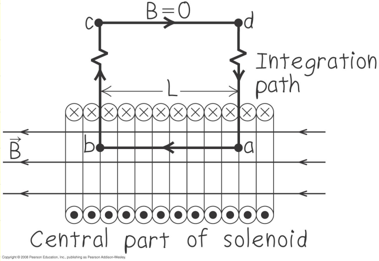




$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = BL$$

Field of an ideal solenoid

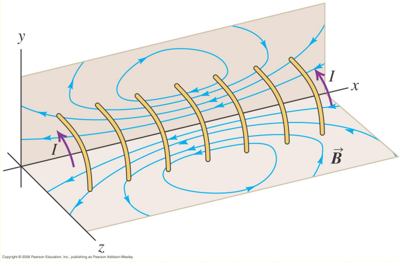


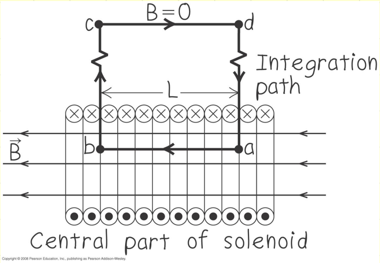


$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = BL$$

$$I_{\mathrm{enc}} = nLI$$

Field of an ideal solenoid



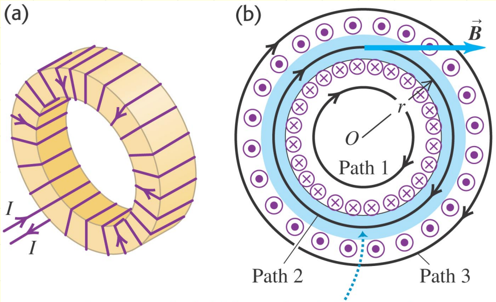


$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = BL$$

$$I_{\mathrm{enc}} = nLI$$

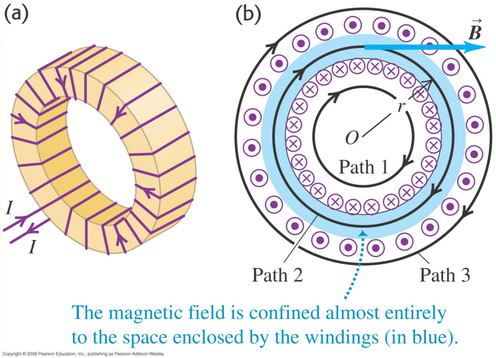
$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\mathrm{enc}} \Rightarrow BL = \mu_0 nLI \Rightarrow \boxed{B = \mu_0 nI}$$

Field of an toroidal solenoid



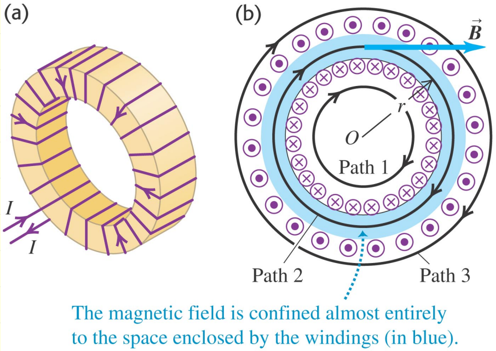
A doughnut-shaped toroidal solenoid, tightly wound with N turns of wire carrying a current I.

Field of an toroidal solenoid



$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B2\pi r$$

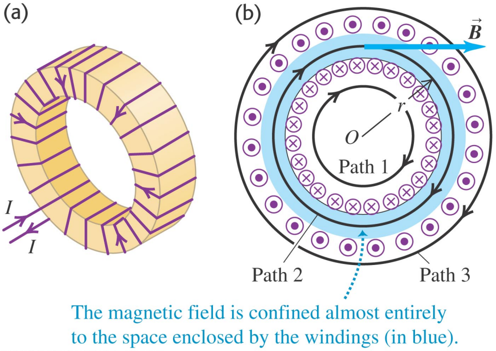
Field of an toroidal solenoid



$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = B2\pi r$$

$$I_{\mathrm{enc}} = NI$$

Field of an toroidal solenoid



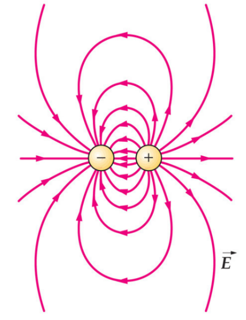
$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = B 2\pi r$$

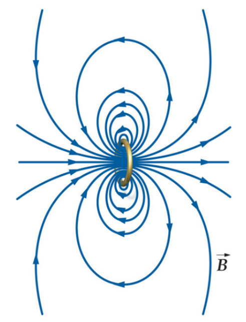
$$I_{\text{enc}} = NI$$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} \Rightarrow \boxed{B = \frac{\mu_0 NI}{2\pi r}}$$

Gauss' law for magnetism

Gauss' law for magnetism

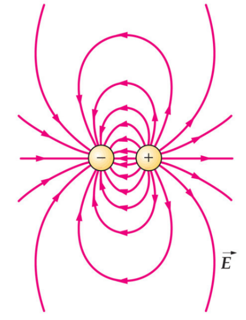


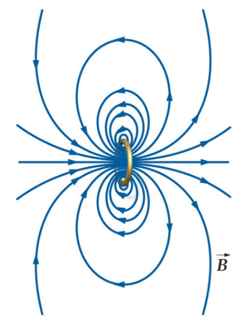


$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = Q_{\mathrm{enc}}/\epsilon_0$$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

The geometry in this case is similar (dipolar) except between the charges.



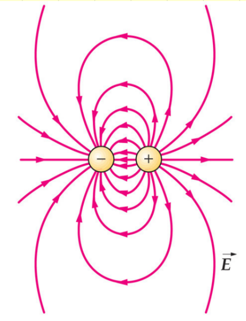


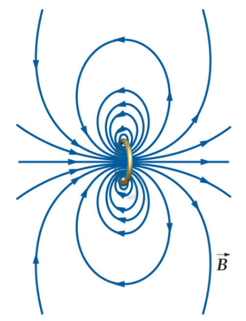
$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = Q_{\mathrm{enc}}/\epsilon_0$$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

Electric field lines start from the positive charge and terminate on the negative one.

Gauss' law for magnetism





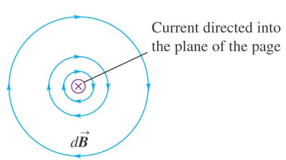
$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = Q_{\mathrm{enc}}/\epsilon_0 \qquad \oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0$$

Magnetic field lines are continuous. No magnetic monopoles!

Magnetic materials

Magnetic materials

- We said that all magnetic fields arise from moving charges.
- Are there really moving charges in permanent magnets?
- Why do we use iron cores in coils of transformers?



(b) View along the axis of the current element

Magnetic materials

- We said that all magnetic fields arise from moving charges.
- Are there really moving charges in permanent magnets?
- Why do we use iron cores in coils of transformers?

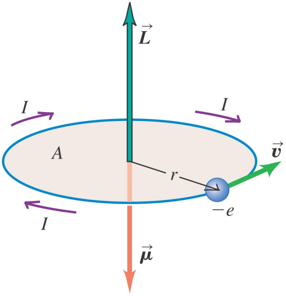
Magnetic materials

- We said that all magnetic fields arise from moving charges.
- Are there really moving charges in permanent magnets?
- Why do we use iron cores in coils of transformers?

The Bohr Magneton

- The atoms that make up all matter contain moving e^-s, and these e^-s form microscopic current loops that produce B fields of their own.
- $\mu = IA$ where $A = \pi r^2$
- The orbital period $T = 2\pi r/v$
- The equivalent current I is the total charge passing any point on the orbit per unit time: $I = \frac{e}{T} = \frac{ev}{2\pi r}$
- Thus the magnetic moment is

$$\mu = \frac{ev}{2\pi r}\pi r^2 = \frac{1}{2}evr$$



An e^- moving with $\vec{\mathbf{v}}$ in a circular orbit of radius r has an angular momentum $\vec{\mathbf{L}}$ and an oppositely directed *orbital* magnetic dipole moment $\vec{\boldsymbol{\mu}}$.

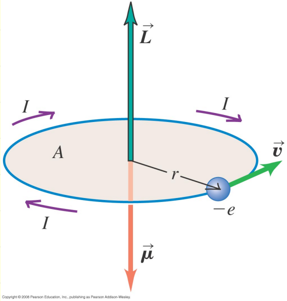
The Bohr Magneton

- Using $L = mvr$ in $\mu = \frac{1}{2}evr$

$$\mu = \frac{e}{2m}L$$

- According to QM angular momentum is quantized in multiples of $h/2\pi$ where $h = 6.62607015 \times 10^{-34}\ \mathrm{J \cdot s}$ is the Planck's constant.
- The unit of magnetic moment thus becomes (by $L \to h/2\pi$)

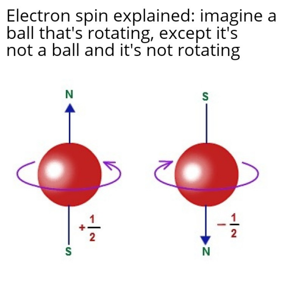
$$\mu_\mathrm{B} = \frac{e}{2m}\frac{h}{2\pi} = 9.274 \times 10^{-24}\ \mathrm{A \cdot m^2}$$



An e^- moving with $\vec{\mathbf{v}}$ in a circular orbit of radius r has an angular momentum $\vec{\mathbf{L}}$ and an oppositely directed *orbital* magnetic dipole moment $\vec{\boldsymbol{\mu}}$.

Spin angular momentum and magnetic moment

- Electrons also have an intrinsic angular momentum, called *spin*
- The spin angular momentum also has an associated magnetic moment, and its magnitude turns out to be almost exactly one Bohr magneton.

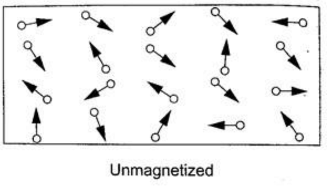


It also has a spin angular momentum and an oppositely directed *spin* magnetic dipole moment.

Magnetization

- In many materials these magnetic moments are randomly oriented and cause no net magnetic field.
- But in some materials an external field can cause the magnetic moments to become oriented preferentially with the field ($\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$), so their magnetic fields add to the external field.
- classes of magnetic behavior
  - paramagnetism
  - diamagnetism
  - ferromagnetism

to be read from the book.



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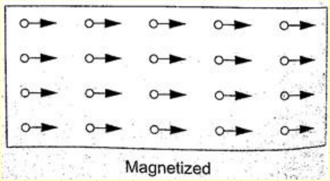

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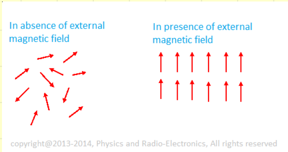
Magnetization

- the $\vec{\mathbf{B}}$ field produced by a current loop is proportional to the loop's $\vec{\boldsymbol{\mu}}$.
- The additional B field produced by microscopic electron current loops is proportional to $\vec{\boldsymbol{\mu}}_{\text{tot}}$ total per unit volume V in the material.
- We call this vector quantity the magnetization of the material $\vec{\mathbf{M}} = \vec{\boldsymbol{\mu}}_{\text{tot}}/V$



Magnetic permeability and susceptibility

- The additional magnetic field due to magnetization of the material turns out to be equal simply to $\mu_0\vec{\mathbf{M}}$:

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \mu_0\vec{\mathbf{M}}$$

- The result is that the magnetic field at any point in such a material is greater by a dimensionless factor K_m, called the *relative permeability* of the material, than it would be if the material were replaced by vacuum:

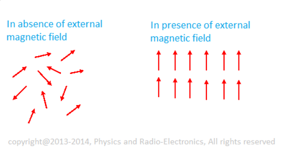
$$\mu = K_m\mu_0$$

- The amount by which the relative permeability differs from unity is called the *magnetic susceptibility*

$$\chi_m = K_m - 1$$

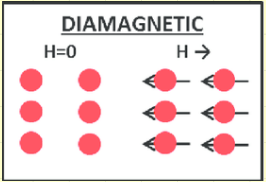
Paramagnetism

- Materials having the behavior just described is said to be *paramagnetic*.
- Paramagnetic materials having K_m values slightly greater than 1.
- Ex: Aluminum has $K_m = 1.000022$ and so $\chi_m = 2.2 \times 10^{-5}$.
- You can hardly notice paramagnetism in your daily life since it is a weak effect. See Youtube link



Diamagnetism

- In some materials $\vec{\boldsymbol{\mu}}_{\text{tot}}$ of all the atomic current loops is zero when no B is present.
- But even these materials have magnetic effects because an external B alters e^{-} motions within the atoms, causing additional current loops and induced magnetic dipoles.
- In this case the additional B caused by these current loops is always *opposite* in direction to that of the external B.
- Such *diamagnetic* materials always have $\chi_m < 0$ and K_m slightly less than unity.



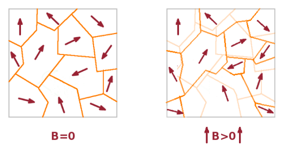
Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials

Table 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20°C$

Material	$\chi_m = K_m - 1\ (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

Ferromagnetism

- Includes iron, nickel, cobalt, and many alloys containing these elements.
- Strong interactions between atomic magnetic moments cause them to line up parallel to each other in regions called *magnetic domains*, even when no external B is present.
- Within each domain, nearly all of the atomic magnetic moments are parallel.
- $K_m \gg 1$, typically of the order of 1000 to 100,000.



Summary