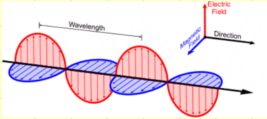
Electromagnetic waves

FIZ102E: Electricity & Magnetism



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Contents

Learning Goals

- How EM waves are generated.
- How and why the speed of light is related to the fundamental constants of electricity and magnetism.
- Why there are both electric and magnetic fields in a light wave.
- How to describe the propagation of a sinusoidal electromagnetic wave.
- What determines the amount of energy and momentum carried by an EM wave.
- How to describe standing electromagnetic waves.

Displacement current

Displacement current

- We have seen that a varying magnetic field gives rise to an induced E field through Faraday's law:

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

- Maxwell (1865) proposed that a varying E field gives rise to a B field:

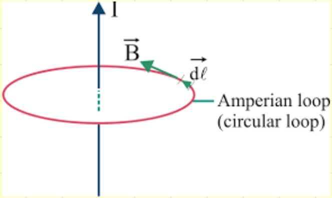
$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} \propto \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}}$$

- There was no experimental evidence, he just hoped there is such a symmetry in Nature.
- This effect is very important, for it leads to the prediction of the existence of EM waves.

Generalizing Ampere's Law

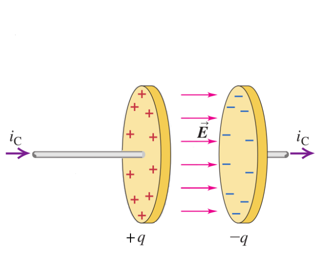
- Let's recall to Ampere's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\mathrm{enc}}$$



- The problem with Ampere's law in this form is that it is *incomplete.*
- Why?

- Consider the process of charging a capacitor
- Conducting wires lead conduction current i_{C} into one plate and out of the other;
- the charge q increases,
- and the electric field $\vec{\mathbf{E}}$ between the plates increases.

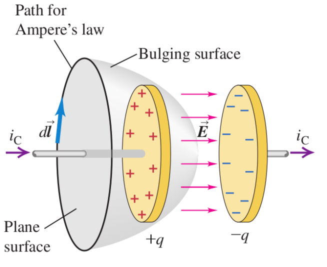


Note: We use lowercase i's and v's to denote instantaneous values of currents and potential differences, respectively, that may vary with time.

Generalizing Ampere's Law

- Let's apply Ampere's law to the circular path shown.

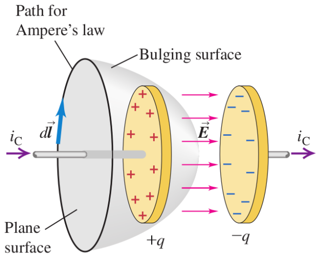
$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 I_{\mathrm{enc}}$$



- For the plane circular area bounded by the circle, I_{enc} is just the current i_{C} in the left conductor.
- But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero.

Generalizing Ampere's Law

- However, something else is happening on the bulged-out surface.
- As the capacitor charges, the electric field $E = \sigma/\epsilon_0 = q/A\epsilon_0$ and the electric flux $\Phi_E = \int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = EA = q/\epsilon_0$ through the surface are increasing. This suggests $q = \epsilon_0 \Phi_E$.

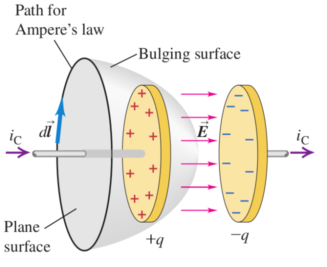


q: instantaneous charge
C: the capacitance
v: instantaneous potential

Generalizing Ampere's Law

- We obtained $q = \epsilon_0 \Phi_E$
- As $i_C = \frac{dq}{dt}$

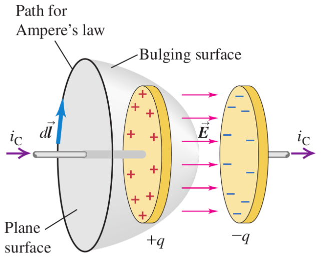
$$i_C = \epsilon_0 \frac{d\Phi_E}{dt} \qquad (1)$$



- In order that the magnetic field is continuous we invent a fictitious *displacement current* i_D in the region between the plates

$$i_D \equiv \epsilon_0 \frac{d\Phi_E}{dt} \qquad (2)$$

Generalizing Ampere's Law



- We imagine that the changing flux

$$i_D \equiv \epsilon_0 \frac{d\Phi_E}{dt}$$

  through the curved (bulged-out) surface in the figure is equivalent, in Ampere's law, to a conduction current, i_C through that surface.
- We include this fictitious current, along with the real conduction current i_C , in Ampere's law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \left(i_C + i_D\right)_{\text{enc}} \qquad \text{(Gen. Ampere's law)}$$

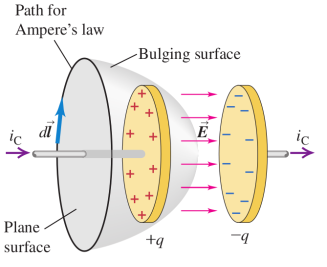
Generalizing Ampere's Law

- Generalized Ampere's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 \left(i_{\mathrm{C}} + i_{\mathrm{D}}\right)_{\mathrm{enc}}$$

  is obeyed no matter which surface we use.

- For the flat surface, i_{D} is zero;
- for the curved surface, i_{C} is zero
- and i_{C} for the flat surface equals i_{D} for the curved surface.

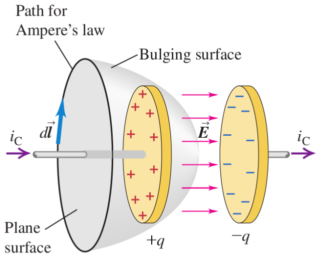


How can a current go through a capacitor

- Another benefit of generalized Ampere's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 \left(i_\mathrm{C} + i_\mathrm{D}\right)_\mathrm{enc}$$

  is that it lets us generalize Kirchhoff's junction rule.
- Considering the left plate of the capacitor, we have i_C into it but none out of it.
- But when we include the i_D, we have i_C coming in one side and an equal i_D coming out the other side.



- With this generalized meaning of the term "current," we can speak of current going through the capacitor.

The reality of displacement current

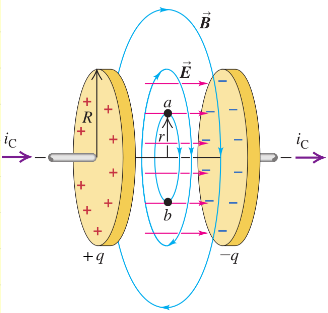
- Does the displacement current have any real physical significance or is it just a ruse to satisfy Ampere's law and Kirchhoff's junction rule?
- Test: If i_{D} really plays the role in Ampere's law, then there ought to be a B in the region between the plates while the capacitor is charging.

- For $r < R$ we obtain $i_{\mathrm{D}}(r) = \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \epsilon_0 \pi r^2 \, \mathrm{d}E/\mathrm{d}t$ and $E = \sigma/\epsilon_0 = q/\epsilon_0 \pi R^2$ and hence

$$i_{\mathrm{D}}(r) = \frac{r^2}{R^2} \frac{\mathrm{d}q}{\mathrm{d}t}, \qquad r < R$$

- Hence $\oint B \, \mathrm{d}l = B 2\pi r = \mu_0 i_{\mathrm{D}}(r)$ implies

$$B = \mu_0 \frac{r}{2\pi R^2} \frac{\mathrm{d}q}{\mathrm{d}t}, \qquad r < R$$



- When we measure the B in this region, we find that it really is there and that it behaves just as the equation predicts.

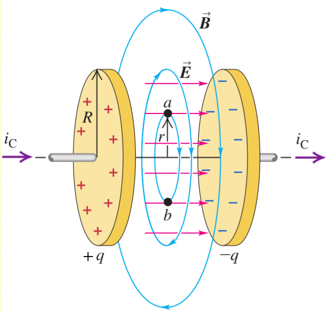
The Reality of Displacement Current

- Outside the region between the plates it becomes

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = B2\pi r = \mu_0 \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$B = \frac{\mu_0}{2\pi r}\frac{\mathrm{d}q}{\mathrm{d}t}, \qquad r > R$$

- Thus outside the region between the plates, B is the same as though the wire were continuous and the plates not present at all.



Maxwell's equations of electromagnetism

- We, finally, are now in a position to wrap up in a single package all of the relationships between E and B fields and their sources.
- This package consists of 4 equations, called Maxwell's equations.
- Maxwell did not discover all of these equations alone, but he was the one to put them together, added the displacement term to make them consistent and predicting the existence of EM waves.

Maxwell's equations of electromagnetism

- The 1st of Maxwell's equations is Gauss's law for E fields:

Gauss's law for $\vec{E}$:

$$\underbrace{\oint \vec{E} \cdot d\vec{A}}_{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Charge enclosed by surface: Q_{encl}

Electric constant: ϵ_0

- It involves an integral of E over a closed surface.
- This states that electric charges (Q_{enc}) are the sources of E fields.

Maxwell's equations of electromagnetism

- The 2nd of Maxwell's equations is Gauss's law for B fields:

Gauss's law for $\vec{B}$:

$$\overbrace{\oint \vec{B} \cdot d\vec{A}}^{\text{Flux of magnetic field through any closed surface ...}} = 0 \quad \text{... equals zero.}$$

- Involves an integral of B over a closed surface.
- States that there are no *magnetic monopoles* (single magnetic charges) to act as sources of B fields.

Maxwell's equations of electromagnetism

- The 3rd of Maxwell's equations is Faraday's law:

Faraday's law for a stationary integration path:

$$\underbrace{\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}}_{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

- Involves a line integral of E over a closed path.
- Faraday's law states that a changing magnetic flux acts as a source of E field.
- If there is a changing B, the line integral—which must be carried out over a stationary closed path—is not zero.
- Thus the E produced by a changing B is not *conservative*.

Maxwell's equations of electromagnetism

- The 4th of Maxwell's eqns is Generalized Ampere's law:

Ampere's law for a stationary integration path:

$$\overbrace{\oint \vec{B} \cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underset{\text{Magnetic constant}}{\mu_0}\Big(\underset{\text{Conduction current through path}}{i_C} + \underbrace{\overset{\text{Electric constant}}{\epsilon_0} \overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\Big)_{\text{encl}}$$

- Involves a line integral of B over a closed path.
- States that both a conduction current and a changing electric flux act as sources of B field

The role of E in Maxwell's equations

- In general, the total $\vec{\mathbf{E}}$ field at a point in space can be the superposition of an electrostatic field $\vec{\mathbf{E}}_\mathrm{c}$ caused by a distribution of charges at rest and a magnetically induced, nonelectrostatic field $\vec{\mathbf{E}}_\mathrm{n}$. That is,

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_\mathrm{c} + \vec{\mathbf{E}}_\mathrm{n} \tag{3}$$

- We mentioned that the E produced by a changing B is not *conservative*. But still we do not write Fraday's law as $\oint \vec{\mathbf{E}}_\mathrm{n} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$
- because the electrostatic part $\vec{\mathbf{E}}_\mathrm{c}$ is always conservative:

$$\oint \vec{\mathbf{E}}_\mathrm{c} \cdot \mathrm{d}\vec{\mathbf{l}} = 0$$

and does not contribute to the integral in Faraday's law.

The role of E in Maxwell's equations

- Similarly, the nonconservative part $\vec{\mathbf{E}}_{\mathrm{n}}$ of the $\vec{\mathbf{E}}$ field does not contribute to the integral in Gauss's law, because this part of the field can not contribute to the *net* flux since the corresponding field lines do not start from a $+$ charge and terminate on a $-$ one, but are continuous.
- Hence $\oint \vec{\mathbf{E}}_{\mathrm{n}} \cdot \mathrm{d}\vec{\mathbf{A}}$ is always zero.
- We conclude that in all the Maxwell equations, $\vec{\mathbf{E}}$ is the total electric field; these equations don't distinguish between conservative and nonconservative fields.

Symmetry in Maxwell's Equations

In empty space there is no charge $Q_{\text{encl}} = 0$ and no conduction current, $i_{\text{C}} = 0$.

In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

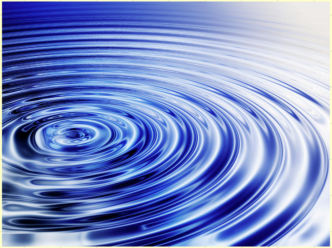
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

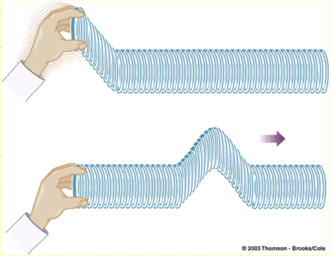
Review: Waves

What is a wave?

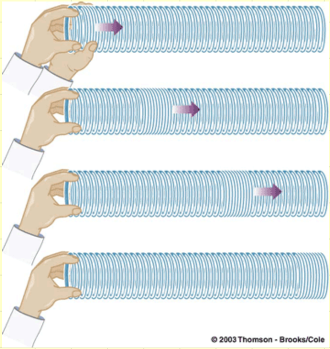


- A wave is a disturbance that travels through a *medium* from one location to another location.
- There is always a force acting upon the particles that restores them to their original position.
- When a wave travels in a medium, the individual particles of the medium do not travel but only are only displaced temporarily from their rest position transfering their energy to the adjacent particles.

Transverse and longitudinal waves

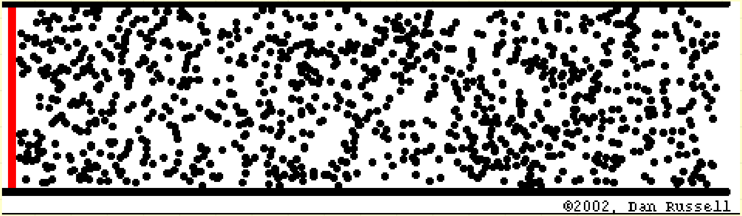


Transverse waves oscillate in a direction perpendicular to the direction of propagation.



Longitudinal waves oscillate in the same direction as the direction of propagation.

Sound waves



- Vibrating material medium produces sound waves.
- Sound waves are longitudinal.
- Only energy is transfered, not the material of the medium (air in this case).
- Relatively dilute and dense regions of molecules.

Wave equation in physics

- The wave equation

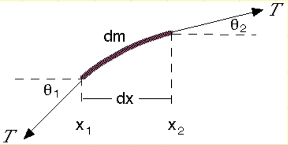
$$\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2}$$

  relates space and time derivatives of the wave function Ψ.

- v is the speed of the wave

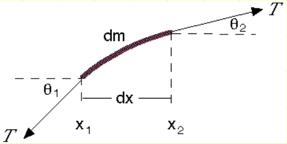
$$v = \sqrt{\frac{\text{restoring property}}{\text{inertial property}}}$$

Waves on a string



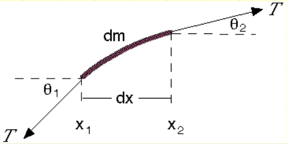
- the stretched string as a simple case
- mass per unit length μ.
- tension T and equilibrium position is along the x-axis.
- Restriction to small deformations: $\theta \ll 1 \Rightarrow \sin\theta \simeq \theta, \cos\theta \simeq 1 \Rightarrow \tan\theta \simeq \sin\theta$.

Waves on a string



- Newton's 2nd law in the vertical y-direction: $F_y = (\mathrm{d}m)a_y$
- The net force in the y direction is $F_y = T\sin\theta_2 - T\sin\theta_1$
- $\mathrm{d}m = \mu\,\mathrm{d}x$
- $a_y = \partial^2 y/\partial t^2$

Waves on a string



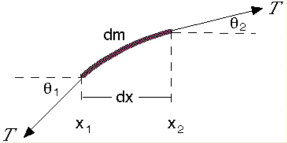
- For small angles $\sin\theta \simeq \tan\theta \simeq \frac{\partial y}{\partial x}$ and so

$$F_y = T\left(\frac{\partial y}{\partial x}\right)_{x_2} - T\left(\frac{\partial y}{\partial x}\right)_{x_1}$$

- $F_y = (\mathrm{d}m)a_y$ becomes

$$T\left(\frac{\partial y}{\partial x}\right)_{x_2} - T\left(\frac{\partial y}{\partial x}\right)_{x_1} = \mu\,\mathrm{d}x\,\frac{\partial^2 y}{\partial t^2}$$

Waves on a string

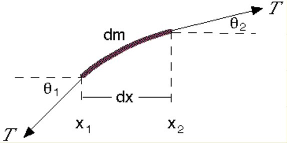


- $$T\left(\frac{\partial y}{\partial x}\right)_{x_2} - T\left(\frac{\partial y}{\partial x}\right)_{x_1} = \mu \, \mathrm{d}x \, \frac{\partial^2 y}{\partial t^2}$$

- Noting that $\frac{\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}}{\mathrm{d}x} \simeq \frac{\partial^2 y}{\partial x^2}$ we obtain

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}}$$

Waves on a string



Comparing

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}$$

with the generic wave equation

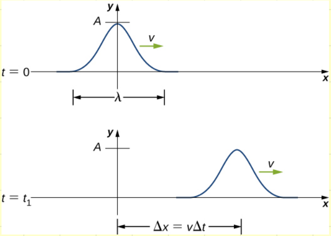
$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

we obtain

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\text{restoring property}}{\text{inertial property}}}$$

The speed of the wave is determined by the medium.

Solution of the wave equation



The solution of the generic wave equation

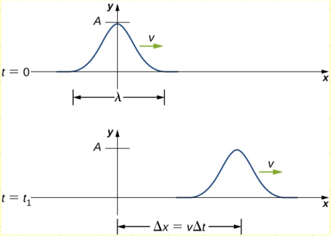
$$\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2}$$

is

$$\Psi(x,t) = f_+(x - vt) + f_-(x + vt)$$

$f_+(x - vt)$ describes a wave moving in the $+x$ direction and vice versa.

Solution of the wave equation



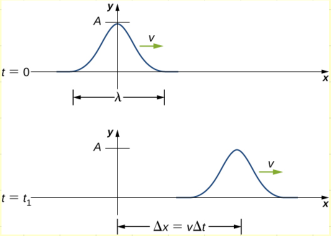
Ex: Show that

$$\Psi(x,t) = f_+(x - vt) + f_-(x + vt)$$

satisfies the wave equation

$$\frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} = \frac{\partial^2 \Psi}{\partial x^2}$$

Solution of the wave equation



Ex: Show that

$$\Psi(x,t) = A\sin[\alpha(x - vt)]$$

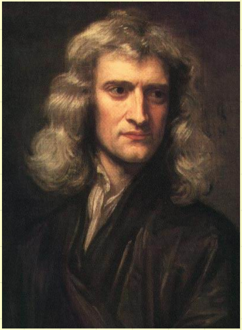
where α is a constant (with dimension $1/L$ and A is a constant to be determined by the initial conditions) satisfies the wave equation

$$\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2}$$

Nature of light

What is light?

The question of the nature of light is very old, but we can start it from the 17th century:



Newton (1643 1727)
Light is a stream of particles.

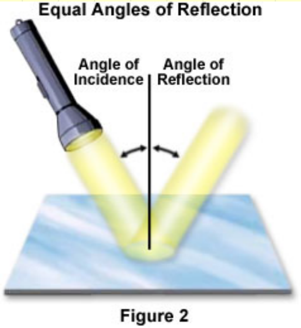
Huygens (1629-1695)
Light is a wave phenomena.

What is light?

We can not answer this question by analysing light under some kind of microscope.

We should make hypothesis about the nature of light and check if we can explain phenomena like reflection, refraction etc.

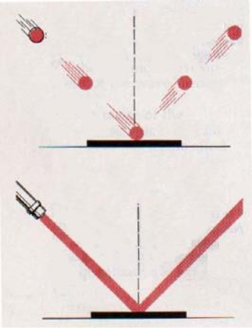
Two phenomena

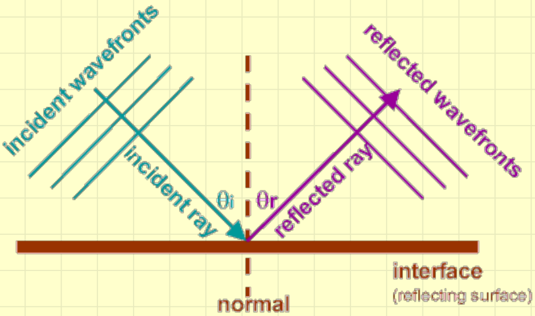


Reflection

Refraction

Reflection



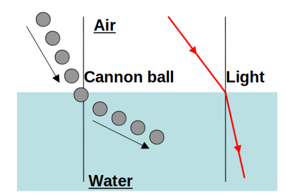


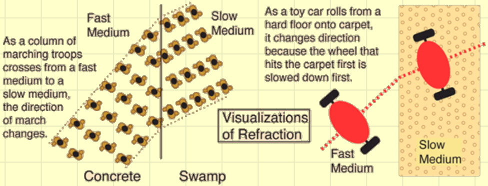
Particle hypothesis

Wave hypothesis

Both particle and wave hypothesis can explain the reflection phenomena. We can not decide the nature of light by analyzing the reflection of light.

Refraction





Particle hypothesis

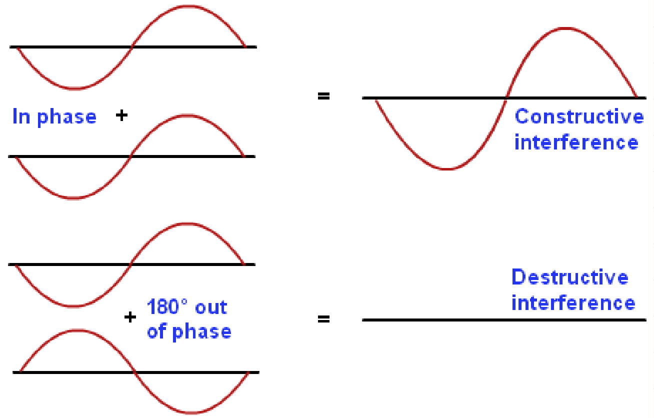
Wave hypothesis

Particle hypothesis can explain refraction by assuming light propagates *faster* in a dense medium. Wave hypothesis can explain refraction by assuming light propagates *slower* in a dense medium. As the speed of light was not measured then it is not possible to decide on the nature of light studying this phenomena as well.

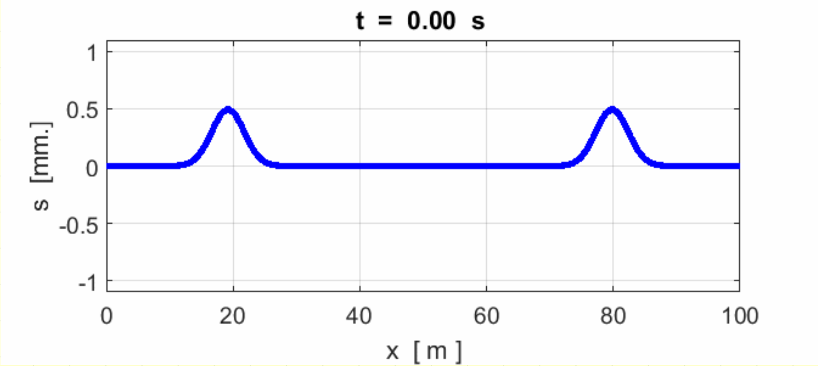
18th century: Newton won

- Mechanistic world view. Everything explained in terms of particles.
- If the space is empty, how can light travel from stars to us without a medium for waves.

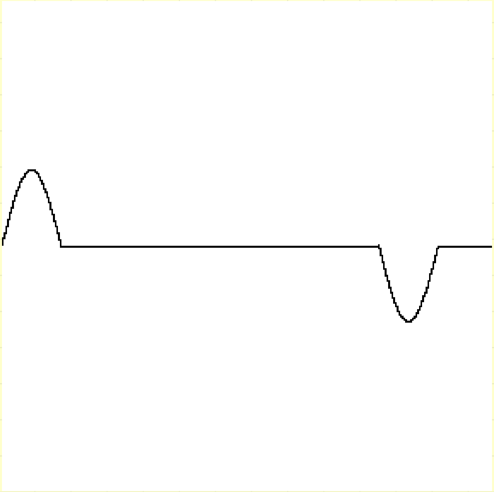
Waves can get superposed.



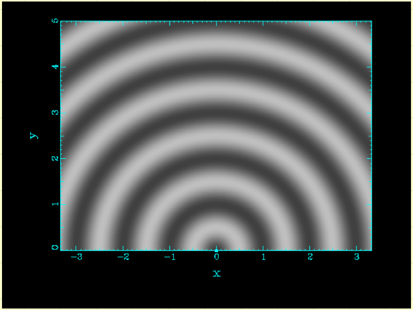
Constructive superposition

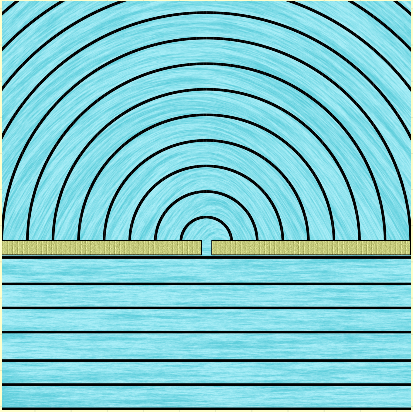


Destructive superposition



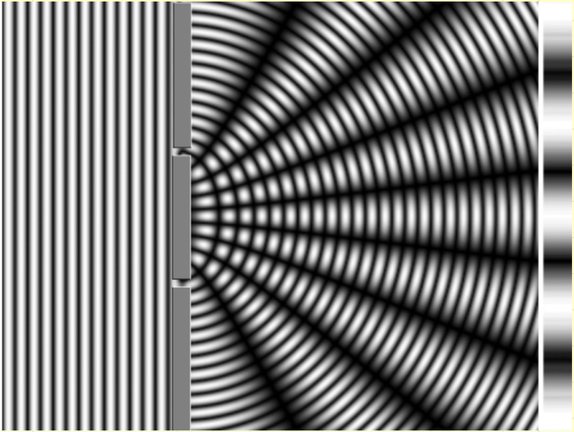
Diffraction





Waves can diffract.

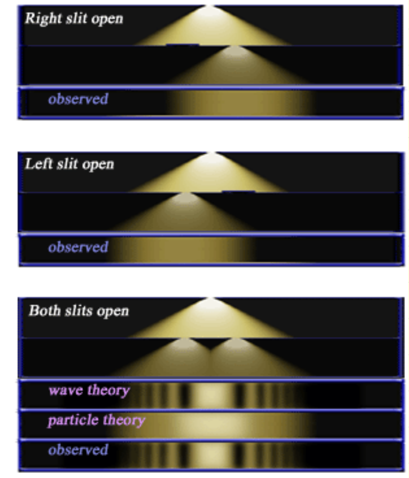
Interference in double slit

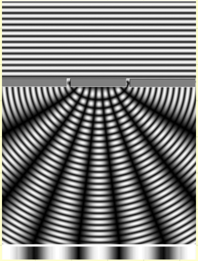


Waves can interfere.

Light and double slit experiment

Thomas Young
(1773-1829)

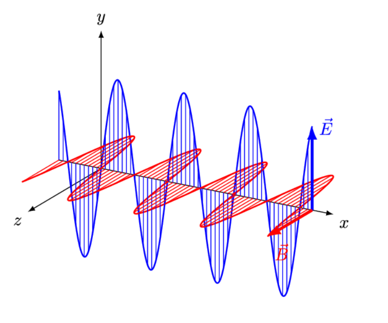




Light produces double-slit pattern. Light is a wave phenomenon!

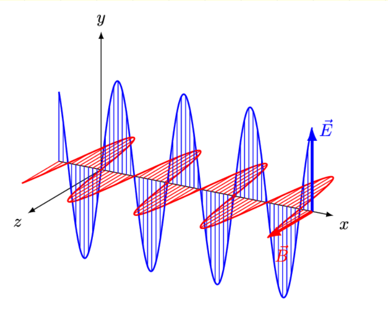
EM waves of Maxwell

- Maxwell's equations show that a time-varying $\vec{\mathbf{B}}$ field acts as a source of $\vec{\mathbf{E}}$ field and that a time-varying $\vec{\mathbf{E}}$ field acts as a source of $\vec{\mathbf{B}}$ field.
- These $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields can sustain each other, forming an EM wave that propagates through space.



EM waves of Maxwell

- Visible light emitted by the glowing filament of a light bulb is one example of an EM.
- Other kinds of EM waves are produced by TV and radio stations, x-ray machines, and radioactive nuclei.



What we do today

- Maxwell's equations are the theoretical basis for understanding EM waves.
- EM waves carry both energy and momentum.
- Visible light, radio, x-rays, and other types of EM waves differ only in their frequency and wavelength.
- Unlike waves on a string or sound waves in a fluid, EM waves do not require a material medium; the light that you see coming from the stars at night has traveled without difficulty across tens or hundreds of light-years of (nearly) empty space.
- Nonetheless, EM waves and mechanical waves have much in common and are described in much the same language.

Maxwell’s Equations and EM Waves

Electrodynamics

- In the bulk of the course (i.e. except for Faraday's law we considered steady state fields.)
- In the static case $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ decoupled from each other.
- According to Faraday's law a changing magnetic field induces an electric field.
- Maxwell (1864) proposed that a changing electric field induces a magnetic field.
- Thus, when either an $\vec{\mathbf{E}}$ or a $\vec{\mathbf{B}}$ field is changing with time, a field of the other kind is induced in adjacent regions of space.

Electromagnetism

Maxwell's Equations

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0} \qquad \text{Gauss' Law for E fields}$$

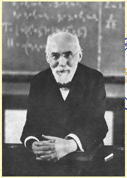
$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}}{\mathrm{d}t}\int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} \qquad \text{Faraday's law}$$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0 \qquad \text{Gauss' Law for B fields}$$

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_{\mathrm{enc}} + \mu_0 \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t}\int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} \qquad \text{Generalized Ampèré's Law}$$

Lorentz force

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$



Electrostatics

Interaction of charges at rest.

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}} = \frac{Q_{\mathrm{enc}}}{\epsilon_0}$$
$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = 0 \tag{4}$$

Electric force

$$\vec{\mathbf{F}}_{\mathrm{E}} = q\vec{\mathbf{E}} \tag{5}$$

Interaction of charges in motion.

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{A}} = 0 \qquad \oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 i_{\mathrm{C}} \tag{6}$$

Magnetic force

$$\vec{\mathbf{F}}_{\mathrm{M}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \tag{7}$$

The idea of fields

- The electric force arises in two stages: (1) a charge produces an electric field in the space around it, and (2) a second charge responds to this field.
- Magnetic forces also arise in two stages: (1) a moving charge or a collection of moving charges (that is, an electric current) produces a magnetic field, and (2) current or moving charge responds to this magnetic field, and so experiences a magnetic force.

EM waves

- An electromagnetic disturbance, consisting of time-varying E and B fields, can propagate through space from one region to another, even when there is no matter in the intervening region.
- Such a disturbance have the properties of a wave, and an appropriate term is *EM wave*.
- Maxwell showed that EM waves would travel with the speed

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \simeq 3 \times 10^8 \, \mathrm{m/s} \tag{8}$$

which is numerically equal to the measured speed of light.
- Maxwell concluded that *light is an EM wave.*
- Other forms of EM waves are radio and television transmission, x-rays, microwaves, γ-rays etc.

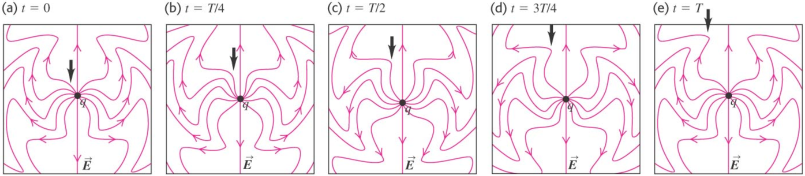
Generating EM waves

According to Maxwell's equations,

- a point charge at rest produces a static $\vec{\mathbf{E}}$ field but no $\vec{\mathbf{B}}$ field.
- a point charge moving with a constant velocity produces both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields.
- an accelerating point charge produces EM waves.

In every situation where EM energy is radiated, the source is accelerated charges

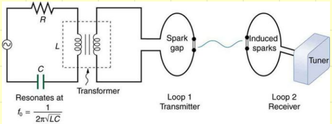
EM waves by an oscillating point charge



- One way in which a point charge can be made to emit EM waves is by making it oscillate in SHM.
- Oscillating the charge up and down makes waves that propagate outward from the charge along field lines.
- Emission is not equal in all directions; the EM waves are strongest at $90°$ to the axis of motion of the charge, while there are no waves along this axis.
- The magnetic disturbance that spreads outward from the charge is not shown.

Discovery of EM waves

- Electromagnetic waves with macroscopic wavelengths were first produced in the laboratory in 1887 by *Heinrich Hertz*.
- As a source of waves, he used charges oscillating in L-C circuits.
- Hertz detected the resulting EM waves with other circuits tuned to the same frequency.
- Marconi and others made radio communication a familiar household experience.



Youtube link

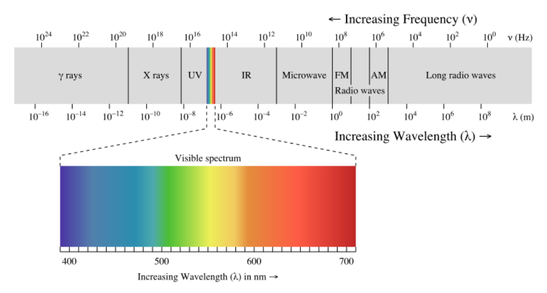
https://www.youtube.com/watch?v=FWCN_uI5ygY

EM spectrum

The EM spectrum encompasses EM waves of all frequencies ν and wavelengths λ related by

$$c = \nu\lambda \tag{9}$$

where $c = 299\,792\,458\,\mathrm{m/s}$ is the speed of light (in vacuum).



Despite vast differences in their uses and means of production, these are all EM waves with the same propagation speed, c.

Visible range

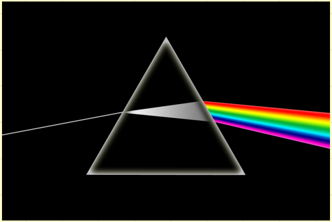
We can detect only a very small segment ($\lambda = 380 - 750\text{nm}$) of this spectrum directly through our sense of sight. We call this range visible light.

Table 32.1 Wavelengths of Visible Light

400 to 440 nm	Violet
440 to 480 nm	Blue
480 to 560 nm	Green
560 to 590 nm	Yellow
590 to 630 nm	Orange
630 to 700 nm	Red

Monochromatic light

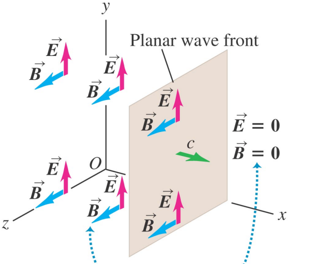
- Ordinary white light includes all visible wavelengths.
- Approximately monochromatic (single-color) light can be obtained by using special sources or filters.
- Absolutely mono-chromatic light with only a single wavelength is an unattainable idealization.
- Light from a laser is much more nearly monochromatic than is light obtainable in any other way.
- Street lights are also monochromatic to a good extend.



Plane EM waves

Plane EM waves

- Assume an electric field $\vec{\mathbf{E}}$ that has only a y-component and a magnetic field $\vec{\mathbf{B}}$ with only a z-component,
- And assume that both fields move together in the $+x$-direction with a speed c that is initially unknown.

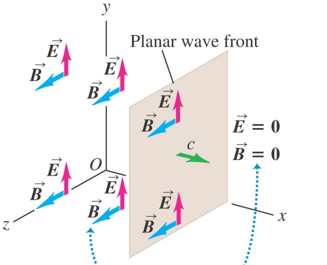


The electric and magnetic fields are uniform behind the advancing wave front and zero in front of it.

- Are these consistent with Maxwell's equations?

Plane EM waves

- We suppose that the boundary plane, which we call the *wave front*, moves in the $+x$-direction with a constant speed c.
- Such a wave, in which at any instant the fields are uniform over any plane $\perp$ to the direction of propagation, is called a *plane wave.*

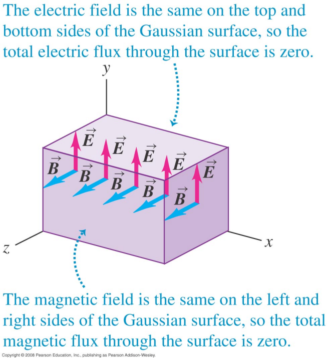


The electric and magnetic fields are uniform behind the advancing wave front and zero in front of it.

- Are these consistent with Maxwell's equations?

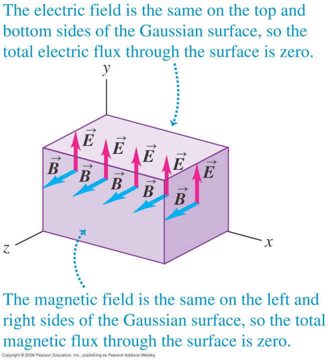
Gauss' laws

- Let us first verify that this wave satisfies Gauss' laws for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields.
- we take as our Gaussian surface a rectangular box with sides parallel to the $xy-$, $xz-$, and $yz-$coordinate planes.
- The box encloses no q: Φ_E and Φ_B through the box are both zero.



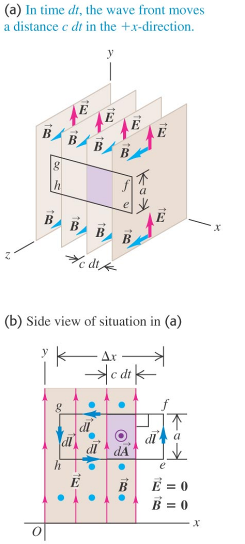
Gauss' laws

- This would not be the case if $\vec{\mathbf{E}}$ or $\vec{\mathbf{B}}$ had an x-component, $\parallel$ to the direction of propagation;
- Thus to satisfy Gauss' laws, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ must be $\perp$ to the direction of propagation: the wave must be *transverse*.



Faraday's law

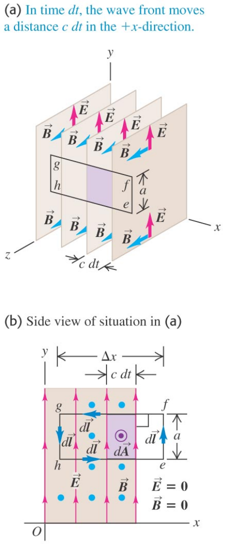
- Apply Faraday's law to a rectangle $efgh$ that is parallel to the $xy-$plane
- As shown in Fig. b, a cross section in the $xy-$plane, this rectangle has height a and width Δx.
- At the time shown, the wave front has progressed partway through the rectangle, and $\vec{\mathbf{E}} = 0$ along the side ef.



Faraday's law

- In a time $\mathrm{d}t$, the magnetic flux through the rectangle in the $xy-$plane increases by an amount $\mathrm{d}\Phi_B$.
- This increase equals the flux through the shaded rectangle with area $ac\,\mathrm{d}t$:

$$\mathrm{d}\Phi_B = Bac\,\mathrm{d}t \Rightarrow \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = Bac$$



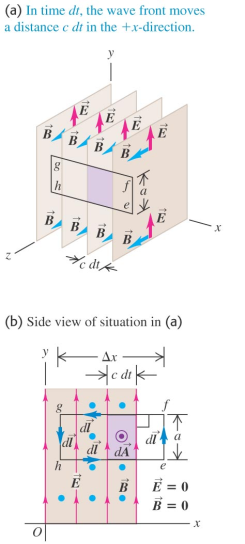
Faraday’s law

- Only side gh contributes to the integral on the LHS of Faraday’s law:

$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -Ea$$

- Thus $\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$ becomes

$$-Ea = Bac \Rightarrow \boxed{E = cB}$$

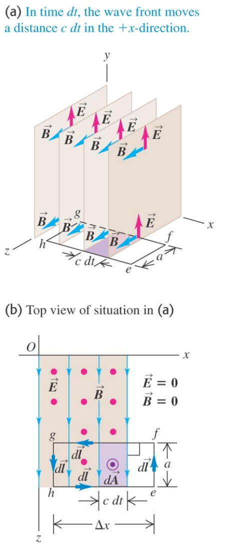


Ampere's law

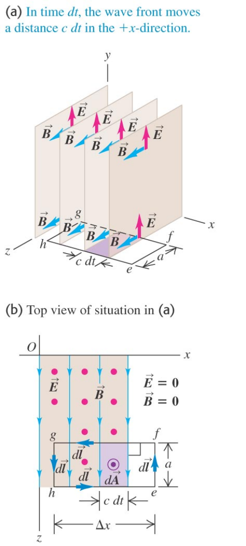
- There is no conduction current $i_C = 0$, so Ampere's law is

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = \mu_0 \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

- We move our rectangle so that it lies in the $xz-$plane, and we again look at the situation at a time when the wave front has traveled partway through the rectangle.



- The $\vec{\mathbf{B}}$ field is zero at every point along side ef, and at each point on sides fg and he it is either 0 or $\perp$ to $\mathrm{d}\vec{\mathbf{l}}$. Only side gh contributes: $\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = Ba$.
- In order that the RHS of Ampere's law is also non-zero, $\vec{\mathbf{E}}$ must have a $y-$component ($\perp$ to $\vec{\mathbf{B}}$) so that Φ_E and hence its derivative are non-zero.
- We thus conclude that in an EM wave, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ must be mutually perpendicular.



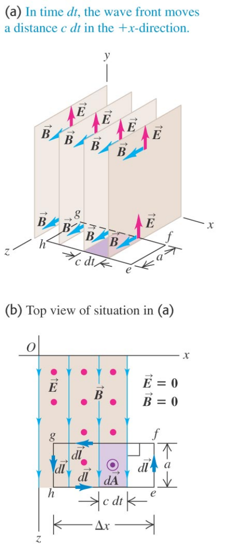
Ampere's law

- In a time $\mathrm{d}t$, the electric flux through the rectangle in the $xz-$plane increases by an amount $\mathrm{d}\Phi_E$.
- This increase equals the flux through the shaded rectangle with area $ac\,\mathrm{d}t$:

$$\mathrm{d}\Phi_E = Eac\,\mathrm{d}t \Rightarrow \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = Eac$$

- Substituting these into Ampere's law we find

$$B = \epsilon_0 \mu_0 c E \tag{10}$$



Speed of EM waves

We obtained

- $E = cB$ from Faraday's law
- $B = \epsilon_0 \mu_0 c E$ from Ampere's law

These imply

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{11}$$

Inserting the numerical values of these quantities, we find

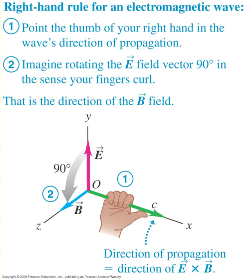
$$\begin{aligned} c &= \frac{1}{\sqrt{(8.854 \times 10^{-12}\,\mathrm{C^2/N \cdot m})(1.257 \times 10^{-6}\,\mathrm{N/A^2})}} \\ &= 2.998 \times 10^{8}\,\mathrm{m/s} \end{aligned}$$

Our assumed wave is consistent with all of Maxwell's equations, provided that the wave front moves with the speed given above, which is the speed of light!

Key Properties of Electromagnetic Waves

We chose a simple wave for our study in order to avoid mathematical complications, but this special case illustrates several important features of all EM waves:

- The wave is transverse; both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are $\perp$ to the direction of propagation of the wave.
- The $\vec{\mathbf{E}}$ and $\mathbf{B}$ fields are also $\perp$ to each other.
- The direction of propagation is the direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.
- There is a definite ratio between the magnitudes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$: $E = cB$.
- The wave travels in vacuum with a definite and unchanging speed, c.
- Unlike mechanical waves, which need the particles of a medium such as air to transmit a wave, EM waves require no medium.



Polarization of EM waves

We obtained

- Electromagnetic waves have the property of polarization.
- The choice of the $y-$direction for $\vec{\mathbf{E}}$ was arbitrary.
- If we had chosen z for $\vec{\mathbf{E}}$ then $\vec{\mathbf{B}}$ would be in the $-y$-direction.
- A wave in which $\vec{\mathbf{E}}$ is always parallel to a certain axis is said to be *linearly polarized* along that axis.
- More generally, any wave traveling in the $x-$direction can be represented as a superposition of waves linearly polarized in the $y-$ and $z-$directions.

Derivation of the EM Wave Equation

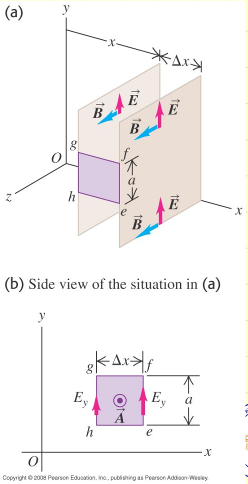
- Recall the wave equation for the displacement of a string:

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

  describing a mechanical wave traveling along the $x-$axis.

- To derive the corresponding equation for an EM wave, we again consider a plane wave: E_y and B_z are uniform over any plane $\perp$ to the $x-$axis, the direction of propagation.

- But now we let E_y and B_z vary continuously as we go along the $x-$axis: $E_y = E_y(x,t)$ and $B_z = B_z(x,t)$



- For the rectangle

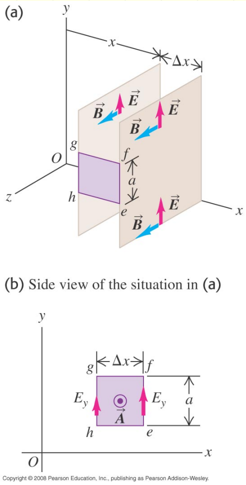
$$\oint \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{l}} = -E_y(x,t)a + E_y(x+\Delta x,t)a$$
$$= a\left[E_y(x+\Delta x,t) - E_y(x,t)\right]$$

- The magnetic flux Φ_B through this rectangle is $B_z(x,t)a\Delta x$

$$\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = \frac{\partial B_z}{\partial t} a\Delta x$$

- Applying Faraday's law and $\Delta x \to 0$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \tag{12}$$



Derivation of the EM Wave Equation

- For the rectangle

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{l}} = -B_z(x + \Delta x, t)a + B_z(x, t)a$$

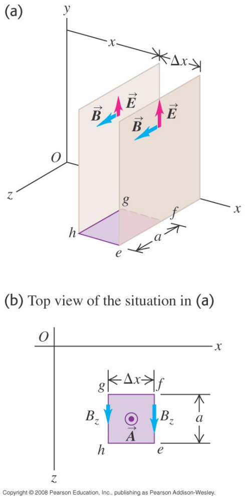
$$= -a\left[B_z(x + \Delta x, t) - B_z(x, t)\right]$$

- The magnetic flux Φ_E through this rectangle is $E_y(x, t)a\Delta x$

$$\frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = \frac{\partial E_y}{\partial t} a\Delta x$$

- Applying Ampere's law and $\Delta x \to 0$

$$\frac{\partial B_z}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} \qquad (13)$$



Derivation of the EM Wave Equation

- We obtained

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$
$$\frac{\partial B_z}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$$

- Take the partial derivative of the upper equation wrt x and lower eqution wrt t

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial x \partial t}$$
$$\frac{\partial^2 B_z}{\partial t \partial x} = -\mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

- Combining these two equations we get

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \tag{14}$$

Derivation of the EM Wave Equation

Compairing this equation

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \tag{15}$$

with the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

we find

$$\frac{1}{v^2} = \mu_0 \epsilon_0 \Longrightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \tag{16}$$

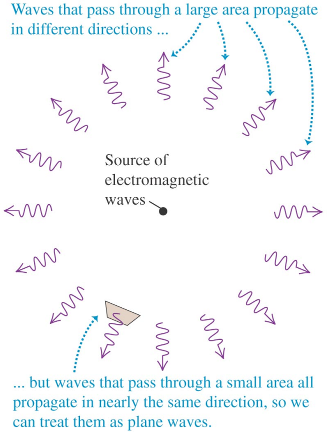
Similarly, we can show that B_z also must satisfy the same wave equation

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2} \tag{17}$$

Sinusoidal EM waves

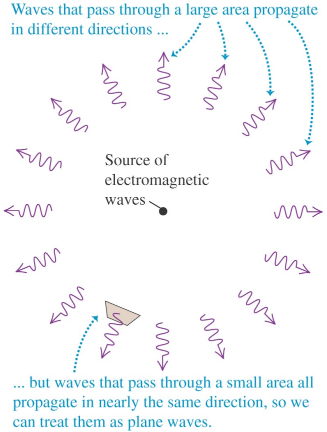
Sinusoidal EM waves

- In a sinusoidal EM wave, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ at any point in space are sinusoidal functions of time,
- and at any instant of time the *spatial* variation of the fields is also sinusoidal.
- Some sinusoidal EM waves are plane waves; at any instant the fields are uniform over any plane perpendicular to the direction of propagation.



Sinusoidal EM waves

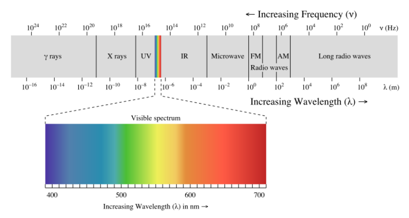
- But some EM waves, such as those in the figure, are *not* sinusoidal.
- but if we restrict our observations to a relatively small region of space at a sufficiently great distance from the source, even these waves are well approximated by plane waves



Frequency and wavelength

- The frequency f, the wavelength λ, and the speed of propagation c of any periodic wave are related by

$$c = \lambda f \qquad (18)$$



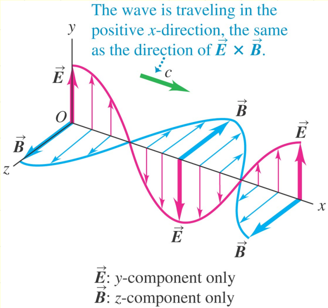
- If f is $10^8\,\mathrm{Hz}$ (100 MHz), typical of commercial FM radio broadcasts,

$$\lambda = \frac{3 \times 10^8\,\mathrm{m/s}}{10^8\,\mathrm{s}^{-1}} = 3\,\mathrm{m}$$

Note that $f \propto 1/\lambda$

Fields of a sinusoidal Wave

- Linearly polarized sinusoidal EM wave traveling in the $+x$-direction.
- Fields oscillate in phase:
  - $\vec{\mathbf{E}}$ is max where $\vec{\mathbf{B}}$ is max
  - $\vec{\mathbf{E}}$ is zero where $\vec{\mathbf{B}}$ is zero
  - where $\vec{\mathbf{E}}$ is in $+y$-direction $\vec{\mathbf{B}}$ is in $+z$-direction.
  - where $\vec{\mathbf{E}}$ is in $-y$-direction $\vec{\mathbf{B}}$ is in $-z$-direction.
- At all points $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ gives the direction of propagation (the $+x$-direction).



One wavelength of the wave is shown at $t = 0$. Although the fields only at points on the x-axis are shown there are electric and magnetic fields at all points in space.

Wave number and angular frequency

- The relation $c = \lambda f$ can also be written as

$$c = \frac{\omega}{k}$$

  where
- $\omega = 2\pi f$ is the *angular frequency*
- and $k = 2\pi/\lambda$ is the *wave number*.
- Accordingly,

$$y(x,t) = y_{\max} \cos(kx - \omega t)$$

  describes a wave moving to the $+x$-directionon the string.
- $y_{\max}$ is the *amplitude*.

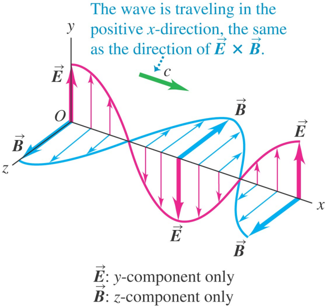
Fields of a sinusoidal wave

The EM fields in the figure then can be described as

$$\vec{\mathbf{E}} = \hat{\mathbf{j}}E_y(x,t) = \hat{\mathbf{j}}E_{\text{max}}\cos(kx - \omega t)$$
$$\vec{\mathbf{B}} = \hat{\mathbf{k}}B_z(x,t) = \hat{\mathbf{k}}B_{\text{max}}\cos(kx - \omega t)$$

The sine curves in the figure represent the fields as functions of x at time $t = 0$.

Note the two different k's in the above equation: the unit vector $\hat{\mathbf{k}}$ in the z-direction and the wave number k. Don't get these confused!

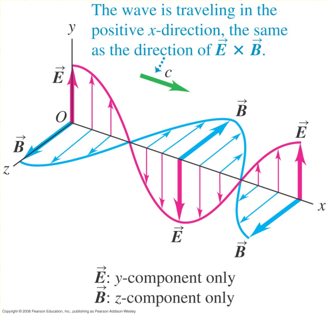


Amplitudes are related

- We have seen $E = cB$
- For sinusoidal waves

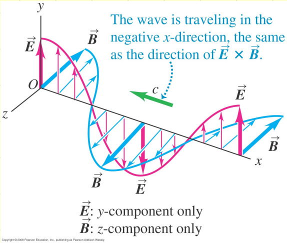
$$E_y(x,t) = E_{\max}\cos(kx - \omega t)$$
$$B_z(x,t) = B_{\max}\cos(kx - \omega t)$$

- This implies $E_{\max} = cB_{\max}$
- And at any point the oscillations of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are in phase.



EM wave traveling in the $-x$-direction

- The figure shows the $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields of a wave traveling in the $-x$-direction.
- At points where $\vec{\mathbf{E}}$ is in the $+y$-direction $\vec{\mathbf{B}}$ is in the $-z$-direction.
- where $\vec{\mathbf{E}}$ is in the $-y$-direction $\vec{\mathbf{B}}$ is in the $+z$-direction.

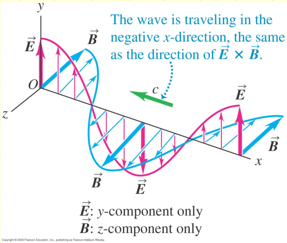


EM wave traveling in the $-x$-direction

- At any point the oscillations of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields are in phase.
- and $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ points in the propagation direction.
- The wave functions for this wave are

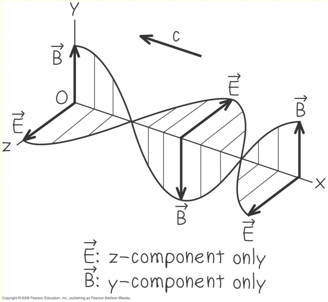
$$\vec{\mathbf{E}} = +\hat{\mathbf{j}} E_{\max} \cos(kx + \omega t)$$

$$\vec{\mathbf{B}} = -\hat{\mathbf{k}} B_{\max} \cos(kx + \omega t)$$



Question

A carbon dioxide laser emits a sinusoidal EM wave that travels in vacuum in the $-x$-direction. The wavelength is $10.6\,\mu\text{m}$ (in the infrared). and the $\vec{\mathbf{E}}$ field is parallel to the z-axis, with $E_{\max} = 1.5\,\text{MV/m}$. Write vector equations for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ as functions of time and position.

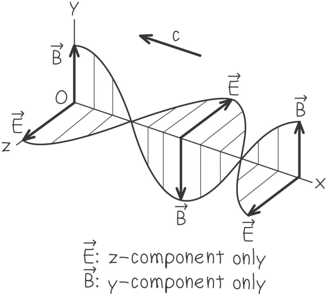


Ex: Electric and magnetic fields of a laser beam

Solution

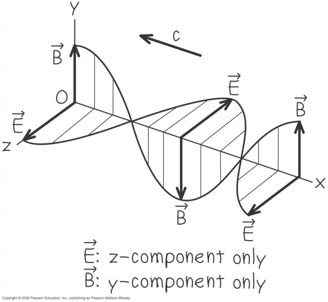
- the wave in this example is given to be linearly polarized along the z-axis.
- Given that the direction is $-x$, $\vec{\mathbf{B}}$ must be in the $+y$-direction where $\vec{\mathbf{E}}$ is in the $+z$-direction so that $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ points in the $-x$-direction.
- A possible pair of wave functions

$$\vec{\mathbf{E}} = \hat{\mathbf{k}} E_{\max} \cos(kx + \omega t)$$
$$\vec{\mathbf{B}} = \hat{\mathbf{j}} B_{\max} \cos(kx + \omega t)$$



Ex: Electric and magnetic fields of a laser beam

Solution

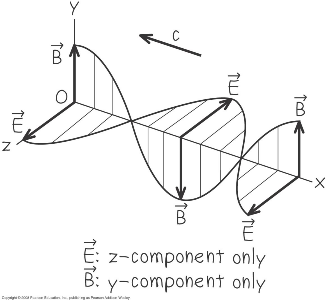


$$B_{\max} = \frac{E_{\max}}{c} = \frac{1.5 \times 10^6\,\mathrm{V/m}}{3 \times 10^8\,\mathrm{m/s}}$$
$$= 5 \times 10^{-3}\,\mathrm{T}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{10.6 \times 10^{-6}\,\mathrm{m}}$$
$$= 5.93 \times 10^5\,\mathrm{rad/m}$$

$$\omega = ck = (3 \times 10^8\,\mathrm{m/s})\,(5.93 \times 10^5\,\mathrm{rad/m})$$
$$= 1.78 \times 10^{14}\,\mathrm{rad/s}$$

Ex: Electric and magnetic fields of a laser beam



Solution

Hence

$$\vec{\mathbf{E}} = \hat{\mathbf{k}} E_{\max} \cos(kx + \omega t)$$
$$\vec{\mathbf{B}} = \hat{\mathbf{j}} B_{\max} \cos(kx + \omega t)$$

with

$B_{\max} = 5 \times 10^{-3}\,\mathrm{T}$,
$k = 5.93 \times 10^{5}\,\mathrm{rad/m}$,
$\omega = 1.78 \times 10^{14}\,\mathrm{rad/s}$ become

$$\vec{\mathbf{E}} = \hat{\mathbf{k}}(1.5 \times 10^{6}\,\mathrm{V/m}) \cos\left[(5.93 \times 10^{5}\,\mathrm{rad/m})x + (1.78 \times 10^{14}\,\mathrm{rad/s})t\right]$$
$$\vec{\mathbf{B}} = \hat{\mathbf{j}}(5 \times 10^{-3}\,\mathrm{T}) \cos\left[(5.93 \times 10^{5}\,\mathrm{rad/m})x + (1.78 \times 10^{14}\,\mathrm{rad/s})t\right]$$

Electromagnetic Waves in Matter

- So far, our discussion of EM waves has been restricted to waves in vacuum.
- But EM waves can also travel in matter.

Electromagnetic Waves in Matter

- Here we extend our analysis to EM waves in non-conducting materials—that is, *dielectrics*.
- $E = vB$ and $v = \frac{1}{\sqrt{\epsilon\mu}}$ where $\epsilon = K\epsilon_0$ and $\mu = K_m\mu_0$.

$$v = \frac{c}{\sqrt{KK_m}}$$

  and for most dielectric materials $K_m \simeq 1$.
- Index of refraction $n \equiv c/v = \sqrt{KK_m} \simeq \sqrt{K}$.

The dielectric constant of water is $K \simeq 1.8$ for visible light, so the speed of visible light in water is slower than in vacuum by a factor of $1/\sqrt{K} = 0.75$.

Energy and momentum in EM waves

Energy in EM waves

- EM waves carry energy; the energy in sunlight is a familiar example.
- Recall the electric and magnetic energy densities

$$u_E = \frac{1}{2}\epsilon_0 E^2, \qquad u_B = \frac{1}{2\mu_0} B^2$$

- a region of empty space where E and B fields are present, the total energy density u is

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

Energy in EM waves

- Using $E = cB$

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 c^2 B^2$$

- Using $c^2 = \frac{1}{\epsilon_0 \mu_0}$

$$u_E = \frac{1}{2\mu_0} B^2 = u_B$$

- The energy density in $\vec{\mathbf{E}}$ field is the same as the energy density in $\vec{\mathbf{B}}$ field: $u_E = u_B$

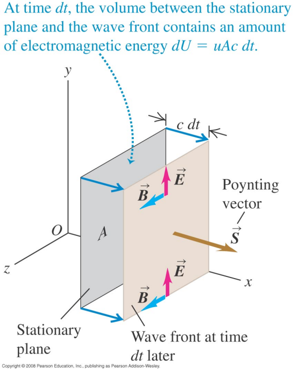
$$u = u_E + u_B = 2u_E = 2u_B = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

EM Energy Flow

- EM waves such as those we have described are *traveling waves* that transport energy from one region to another.
- We can describe this energy transfer in terms of energy transferred per unit time per unit cross-sectional area, or power per unit area, for an area perpendicular to the direction of wave travel.
- How is the energy flow related to the fields?

EM Energy Flow

- Consider a stationary plane, $\perp$ to the x-axis, that coincides with the wave front at a certain time.
- In a time $\mathrm{d}t$ after this, the wave front moves a distance $\mathrm{d}x = c\,\mathrm{d}t$ to the right of the plane.
- The energy in the space to the right of area A had to pass through the area to reach the new location.



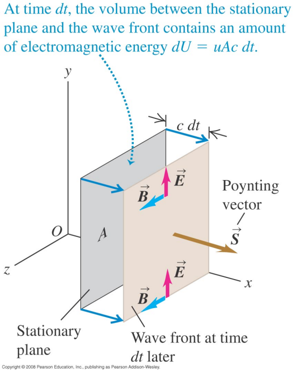
EM Energy Flow

- The volume $\mathrm{d}V = Ac\,\mathrm{d}t$ and the energy $\mathrm{d}U$ are related as:

$$\mathrm{d}U = u\,\mathrm{d}V$$

- The energy passing through A in time $\mathrm{d}t$ is called the *instantaneous intensity*

$$S \equiv \frac{1}{A}\frac{\mathrm{d}U}{\mathrm{d}t} = uc$$



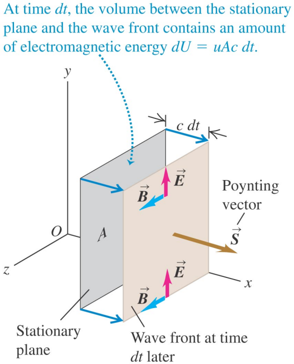
EM Energy Flow

- Since $u = \epsilon_0 E^2$ and $S = uc$ we obtain

$$S = \epsilon_0 E^2 c$$

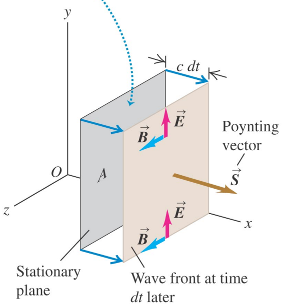
- Using $E = cB$ & $c^2 = \frac{1}{\epsilon_0 \mu_0}$ we can also write this as

$$S = \frac{EB}{\mu_0}$$



EM Energy Flow

- The units of S are energy per unit time per unit area, or power per unit area.
- The SI unit of S is $1\,\mathrm{J/s \cdot m^2}$
- or $1\,\mathrm{W/m^2}$.



Poynting vector

- We can define a vector quantity that describes both the magnitude and direction of the energy flow rate.
- Poynting vector[1] is defined as

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

- $\vec{\mathbf{S}}$ points in the direction of propagation of the wave.
- Since $\vec{\mathbf{E}} \perp \vec{\mathbf{B}}$ the magnitude of $\vec{\mathbf{S}}$ is EB/μ_0.
- Recall that this is the energy flow per unit area and per unit time through a cross-sectional area $\perp$ to the propagation direction.

[1] Introduced by the British physicist John Poynting (1852–1914)

Poynting vector

- The total energy flow per unit time (power, P) out of any closed surface is the integral of $\vec{\mathbf{S}}$ over the surface:

$$P = \oint \vec{\mathbf{S}} \cdot \mathrm{d}\vec{\mathbf{A}}\,.$$

Example: Joule heating: $P = i^2 R$

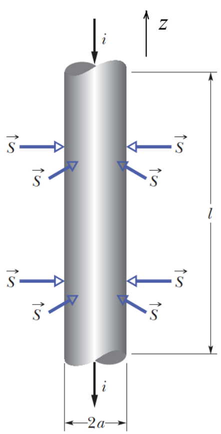
As you know the conversion of electrical energy into heat in a resistor is referred to a "Joule-heating" of the resistor. Sometimes the process is described by saying that electrical energy is "dissipated" in the resistor as heat. If energy is dissipated one might wonder about the source of that energy, for example did it come from the voltage source through the wires? To answer these questions consider the following problem.

Question:

The figure shows a cylindrical resistor of length l, radius a, and resistivity ρ, carrying current i.

(a) Show that the Poynting vector S at the surface of the resistor is everywhere directed normal to the surface, as shown.

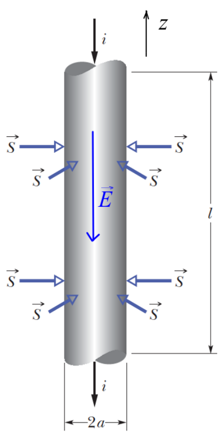
(b) Show that the rate P at which energy flows into the resistor through its cylindrical surface, calculated by integrating the Poynting vector over this surface, is equal to the rate at which thermal energy is produced: integral $\oint \vec{\mathbf{S}} \cdot \mathrm{d}\vec{\mathbf{A}} = i^2 R$, where $\mathrm{d}A$ is an element of area on the cylindrical surface and R is the resistance.



Solution:

As $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$ and $\vec{\mathbf{J}} = \frac{I}{\pi a^2}(-\hat{\mathbf{k}})$ we obtain

$$\vec{\mathbf{E}} = -\frac{\rho I}{\pi a^2}\hat{\mathbf{k}} \tag{19}$$



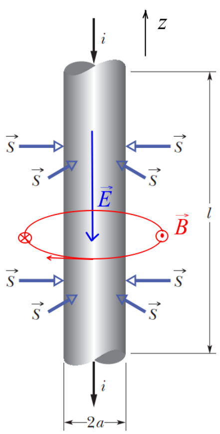
Solution:

As $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$ and $\vec{\mathbf{J}} = \frac{I}{\pi a^2}(-\hat{\mathbf{k}})$ we obtain

$$\vec{\mathbf{E}} = -\frac{\rho I}{\pi a^2}\hat{\mathbf{k}}$$

The magnetic field (for $r < a$) on the other hand is

$$\vec{\mathbf{B}} = -\frac{\mu_0 I}{2\pi}\frac{r}{a^2}\hat{\phi}$$



Solution:

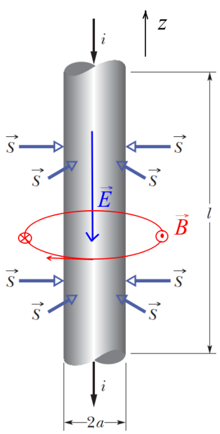
With $\vec{\mathbf{E}} = -\frac{\rho I}{\pi a^2}\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = -\frac{\mu_0 I}{2\pi}\frac{r}{a^2}\hat{\phi}$ we obtain

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0} = -\frac{\rho I}{\pi a^2}\frac{I}{2\pi}\frac{r}{a^2}\hat{\mathbf{r}}$$

Thus the energy flowing into a cylinder of radius r is $\oint \vec{\mathbf{S}} \cdot \mathrm{d}\vec{\mathbf{A}} = S\,2\pi r l$ giving

$$P = \frac{I^2 l \rho}{\pi a^4} r^2$$

For $r = a$ this becomes $P = \frac{I^2 l \rho}{\pi a^2} = I^2 R$!!



Poyting vector for sinusoidal waves

- For the sinusoidal waves that we considered before

$$\begin{aligned}\vec{\mathbf{S}}(x,t) &= \frac{1}{\mu_0}\vec{\mathbf{E}}(x,t) \times \vec{\mathbf{B}}(x,t) \\ &= \frac{1}{\mu_0}\hat{\mathbf{j}}E_{\max}\cos(kx-\omega t) \times \hat{\mathbf{k}}B_{\max}\cos(kx-\omega t) \\ &= \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx-\omega t)\hat{\mathbf{i}} \\ &= S_{\max}\cos^2(kx-\omega t)\hat{\mathbf{i}}\end{aligned}$$

- $\cos^2(kx-\omega t) > 0$ and so $\vec{\mathbf{S}}$ points in the $+x$-direction.
- $S_{\max} = \frac{E_{\max}B_{\max}}{\mu_0}$

Intensity

- The Poynting vector at any point is a function of time.
- Because the frequencies of typical electromagnetic waves are very high, the time variation of the Poynting vector is so rapid that it's most appropriate to look at its average value.
- The magnitude of the average value of $\vec{\mathbf{S}}$ at a point is called the *intensity* of the radiation at that point.

Intensity

- The SI unit of intensity is the same as for S, $1\,\mathrm{W/m^2}$.
- The average of $S = S_{\max}\cos^2(kx - \omega t)$ is $\langle S\rangle = S_{\max}/2$ since

$$\cos^2(kx-\omega t) \equiv \frac{1}{2}(1 + 2\cos[2(kx-\omega t)])$$

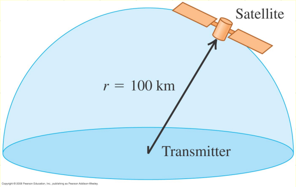
and $\langle \cos[2(kx-\omega t)]\rangle = 0$ (at any point, it is $+$ during one half-cycle and $-$ during the other half). Thus

$$\langle S\rangle = I = \frac{1}{2}S_{\max} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c}$$

Ex:

Question

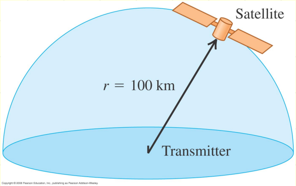
A radio station on the earth's surface emits a sinusoidal wave with average total power $50\,\mathrm{kW}$. Assuming that the transmitter radiates equally in all directions above the ground (which is unlikely in real situations), find the electric-field and magnetic-field amplitudes $E_{\max}$ and $B_{\max}$ detected by a satellite $100\,\mathrm{km}$ from the antenna.



Ex:

Answer

We are given the transmitter's average total power P. The intensity I is the average power per unit area; to find I at $100\,\mathrm{km}$ from the transmitter we divide P by the surface area of the hemisphere $A = 2\pi r^2$. For a sinusoidal wave, I is also equal to the magnitude of the average value $\langle S \rangle$ of the Poynting vector, so we can use $\langle S \rangle = I = \frac{E_{\text{max}}^2}{2\mu_0 c}$ to find E_{max}; then $B_{\text{max}} = E_{\text{max}}/c$.

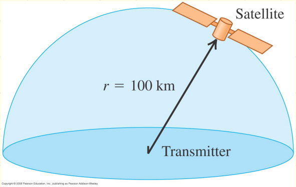


Ex:

Answer

- The area of the hemisphere of radius $r = 100\,\text{km} = 10^5\,\text{m}$ is $A = 2\pi r^2 = 6.28 \times 10^{10}\,\text{m}^2$.
- All the radiated power passes through this surface, so the average power per unit area (that is, the intensity) is

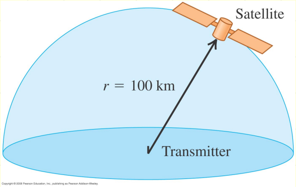
$$I = P/A = 50 \times 10^3\,\text{W}/6.28 \times 10^{10}\,\text{m}^2$$
$$= 7.96 \times 10^{-7}\,\text{W}/\text{m}^2$$



Ex:

Answer

- Using $I = E^2_{\max}/2\mu_0 c$ we obtain
  $E_{\max} = \sqrt{2\mu_0 c I} = 2.45 \times 10^{-2}\,\mathrm{V/m}$.
- Finally,
  $B_{\max} = E_{\max}/c = 8.17 \times 10^{-11}\,\mathrm{T}$.



EM Momentum Flow and Radiation Pressure

- We've shown that EM waves transport energy.
- It can also be shown that EM waves carry momentum p, with a corresponding momentum density of magnitude

$$\frac{\mathrm{d}p}{\mathrm{d}V} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2}$$

- This momentum is a property of the field; it is not associated with the mass of a moving particle in the usual sense.
- There is also a corresponding momentum flow rate. Using $\mathrm{d}V = Ac\,\mathrm{d}t$

$$\frac{1}{A}\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$

- Average rate of momentum transfer would then be I/c.

EM Momentum Flow and Radiation Pressure

- This momentum is responsible for radiation pressure.
- When an EM wave is completely absorbed by a surface, the wave's momentum is also transferred to the surface.
- For simplicity we'll consider a surface perpendicular to the propagation direction.
- Recall that that the rate $\mathrm{d}p/\mathrm{d}t$ at which momentum is transferred to the absorbing surface equals the force on the surface.
- The average force per unit area due to the wave, or *radiation pressure* p_{rad}, is the average value of $\mathrm{d}p/\mathrm{d}t$ divided by the absorbing area A.

Radiation pressure

The radiation pressure is then

$$p_{\text{rad}} = \begin{cases} I/c, & \text{wave totally absorbed} \\ 2I/c, & \text{wave totally reflected} \end{cases}$$

Radiation pressure

The radiation pressure is then

$$p_{\mathrm{rad}} = \begin{cases} I/c, & \text{wave totally absorbed} \\ 2I/c, & \text{wave totally reflected} \end{cases}$$

Why is the pressure doubled when radiation is reflected?

Radiation pressure

The radiation pressure is then

$$p_{\text{rad}} = \begin{cases} I/c, & \text{wave totally absorbed} \\ 2I/c, & \text{wave totally reflected} \end{cases}$$

Why is the pressure doubled when radiation is reflected?

Consider momentum transfer to a wall by particles. The change in the momentum of the particles would be doubled compared to the particles sticking to the surface.

Radiation pressure

The radiation pressure is then

$$p_{\text{rad}} = \begin{cases} I/c, & \text{wave totally absorbed} \\ 2I/c, & \text{wave totally reflected} \end{cases}$$

Ex: What is the radiation pressure for a surface which reflects a fraction r and absorbs the rest.

Radiation pressure

The radiation pressure is then

$$p_{\text{rad}} = \begin{cases} I/c, & \text{wave totally absorbed} \\ 2I/c, & \text{wave totally reflected} \end{cases}$$

Ex: What is the radiation pressure for a surface which reflects a fraction r and absorbs the rest.

In this case

$$\begin{aligned} p_{\text{rad}} &= r\frac{2I}{c} + (1-r)\frac{I}{c} \\ &= (1+r)\frac{I}{c} \end{aligned}$$

Radiation pressure

The radiation pressure is then

$$p_{\text{rad}} = \begin{cases} I/c, & \text{wave totally absorbed} \\ 2I/c, & \text{wave totally reflected} \end{cases}$$

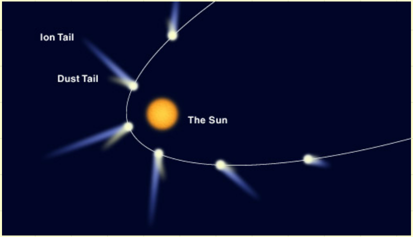
Intensity for direct sunlight, before it passes through the earth's atmosphere, is approximately $I = 1.4\,\mathrm{kW/m^2}$. The corresponding average pressure on a completely absorbing surface is $p_{\text{rad}} = I/c = 4.4 \times 10^{-6}\,\mathrm{Pa}$ which is $\sim 10^{-10} p_{\text{atm}}$. Radiation pressure can not be *felt*!

Radiation pressure in astronomy

Comet Hale-Bopp

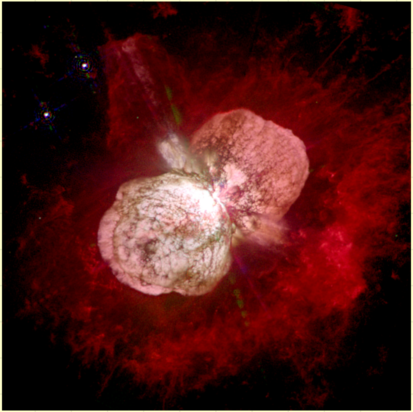
Radiation pressure in astronomy

Comet tails are “combed” away from the Sun.



Radiation pressure in astronomy

Radiation pressure sets the maximum mass of stars



Eta Carinae: A star trying to be 'big'!

Example:

Question:

A great amount of dust exists in interplanetary space. Although in theory these dust particles can vary in size from molecular size to a much larger size, very little of the dust in our solar system is smaller than about $a = 0.2\,\mu\text{m}$. Why?

Example:

Answer

- The dust particles are subject to two significant forces: the gravitational force that draws them toward the Sun and the radiation-pressure force that pushes them away from the Sun.
- The gravitational force is proportional to the cube of the radius of a spherical dust particle because it is proportional to the mass and therefore to the volume $4\pi a^3/3$ of the particle.
- The radiation pressure is proportional to the planar cross-section of the particle, πa^2.

Example:

Answer

- For large particles, the gravitational force is greater than the force from radiation pressure.
- For particles having radii less than about $0.2\,\mu\text{m}$, the radiation-pressure force is greater than the gravitational force and they are swept out of our solar system by sunlight.

Example:

Question:
Consider a small, spherical particle of radius a located in space a distance r from the Sun, of mass $M_\odot$. Assume the particle has a perfectly absorbing surface and a mass density ρ. The value of the solar luminosity is $L_\odot$. Calculate the value of a, in terms of $L_\odot$, a and ρ for which the particle is in equilibrium between the gravitational force and the force exerted by solar radiation.

Example:

Answer:

- The gravitational force on the particle is

$$F_{\mathrm{G}} = \frac{GM_\odot m}{r^2} = \frac{GM_\odot \frac{4}{3}\pi a^3 \rho}{r^2}$$

- The value of the solar intensity at the particle's location is

$$I = \frac{L_\odot}{4\pi r^2}$$

- This causes radiation pressure $p_{\mathrm{rad}} = I/c$ and this applies a radiative force $F_{\mathrm{rad}} = p_{\mathrm{rad}}\pi a^2$

$$F_{\mathrm{rad}} = \frac{L_\odot}{4\pi r^2 c}\pi a^2$$

Example:

Answer:

- Now, $F_\mathrm{G} = F_\mathrm{rad}$ implies

$$a = \frac{3L_\odot}{16\pi c G M_\odot \rho}$$

- Plugging in the constants

$$a = \frac{3 \times (4 \times 10^{26}\,\mathrm{W})}{16\pi(3 \times 10^{8}\,\mathrm{m/s})(6.67 \times 10^{-11}\,\mathrm{m^3/kg\,s^2})(2 \times 10^{30}\,\mathrm{kg})\rho}$$

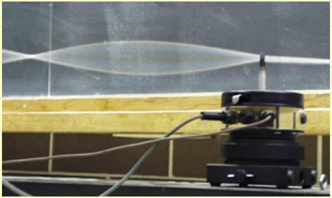
- Assuming $\rho = 3\,\mathrm{g/cm^3} = 3 \times 10^{3}\,\mathrm{kg/m^3}$ we obtain
  $a = 0.2\,\mu\mathrm{m}$!

Standing EM waves

Standing waves

- Waves on a string can be reflected from the edges.
- The superposition of an incident wave and a reflected wave forms a *standing wave*.
- Condition for a standing wave to form is

$$L = n\frac{\lambda}{2}, \qquad n = 1, 2, \cdots$$



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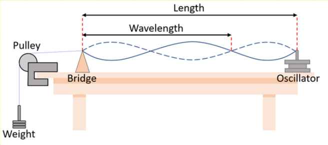

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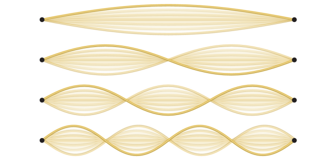
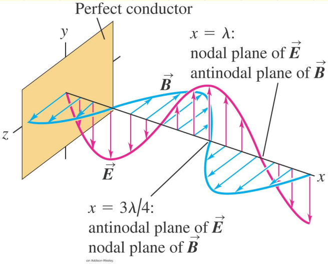


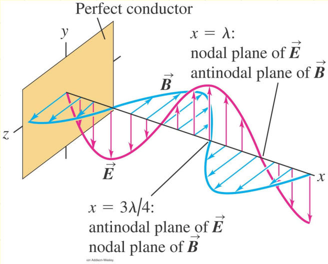
Standing EM waves

- EM waves can be reflected by the surface of a conductor (like a polished sheet of metal) or of a dielectric (such as a sheet of glass).
- The superposition of an incident EM wave and a reflected wave forms a *standing EM wave*.
- The situation is analogous to standing waves on a stretched string.



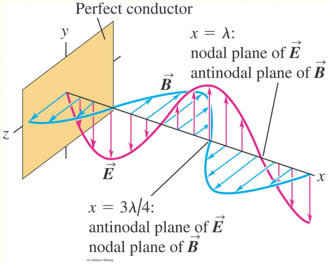
Standing EM waves

- Suppose a sheet of a perfect conductor (zero resistivity) is placed in the yz-plane
- A linearly polarized EM wave, traveling in the $-x$-direction, strikes it.



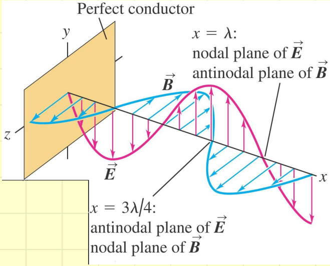
Standing EM waves

- $\vec{\mathbf{E}}$ cannot have a component parallel to the surface of a perfect conductor.
- $\vec{\mathbf{E}} = 0$ everywhere on the yz-plane as a result of induced currents cancelling the field of the incident wave.
- The currents induced on the surface of the conductor also produce a reflected wave that travels out from the plane in the $+x$-direction.



Standing EM waves

- The superposition principle states that the total $\vec{\mathbf{E}}$ field at any point is the vector sum of the $\vec{\mathbf{E}}$ fields of the incident and reflected waves, and similarly for the $\vec{\mathbf{B}}$ field.



- Therefore

$$E_y(x,t) = E_{\max}[\cos(kx+\omega t) - \cos(kx-\omega t)]$$
$$B_z(x,t) = B_{\max}[-\cos(kx+\omega t) - \cos(kx-\omega t)]$$

- We can simplify these by

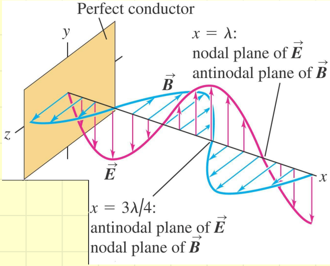
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Standing EM waves

- The results are

$$E_y(x,t) = -2E_{\max} \sin kx \sin \omega t$$
$$B_z(x,t) = -2B_{\max} \cos kx \cos \omega t$$

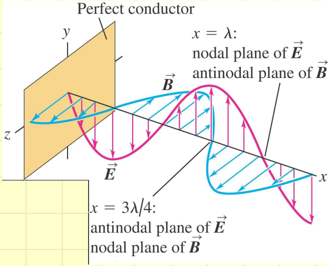
- Check that at $x = 0$ the electric field $E_y(x=0,t)$ is always zero; this is required by the nature of the ideal conductor, which plays the same role as a fixed point at the end of a string.



Standing EM waves

- Furthermore, $E_y(x,t)$ is zero at all t at points in those planes $\perp$ to the x-axis for which $\sin kx = 0$
- that is, $kx = 0, \pi, 2\pi, \ldots$.
- since $k = 2\pi/\lambda$, the positions of these planes are

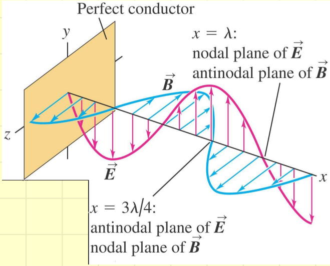
$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \quad \text{(nodal planes of } \vec{\mathbf{E}}\text{)}$$



- Midway between any two adjacent nodal planes is the anti-nodal plane on which $\sin kx = \pm 1$

Standing EM waves

- The total $\vec{\mathbf{B}}$ is zero at all times at points in planes on which $\cos kx = 0$.



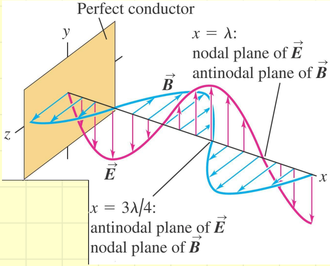
- These are the nodal planes of $\vec{\mathbf{B}}$, and they occur where

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \quad \text{(nodal planes of } \vec{\mathbf{B}}\text{)}$$

- There is an antinodal plane of $\vec{\mathbf{B}}$ midway between any two adjacent nodal planes.

Standing EM waves

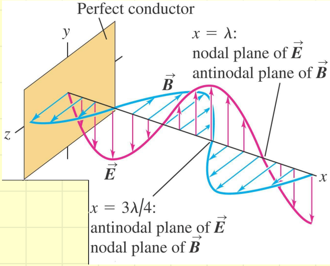
- The magnetic field is not zero at the conducting surface $x = 0$.
- The surface currents that must be present to make $\vec{\mathbf{E}}$ exactly zero at the surface cause B at the surface.
- The nodal planes of each field are separated by one half-wavelength.



- This is in contrast to a wave traveling where E and B at any particular point are *in phase*.

Standing EM waves

- Hence the nodes of $\vec{\mathbf{E}}$ coincide with the antinodes of$\vec{\mathbf{B}}$.
- The total E (B) is a sine (cosine) function of t.
- the two fields are therefore $90°$ out of phase at each point.



Standing Waves in a cavity

- Let's now insert a second conducting plane, parallel to the first and a distance L from it, along the $+x$-axis.
- The cavity between the two planes is analogous to a stretched string held at the points $x = 0$ and $x = L$.

A typical microwave oven sets up a standing electromagnetic wave with $\lambda = 12.2\,\text{cm}$, a wavelength that is strongly absorbed by the water in food. The wave has nodes spaced $\lambda/2 = 6.1\,\text{cm}$ apart. The food must be rotated while cooking; otherwise the portion that lies at a node will remain cold.

Standing Waves in a cavity

- Both conducting planes must be nodal planes for $\vec{\mathbf{E}}$.
- A standing wave can exist only when the second plane is placed at one of the positions where $E(x,t) = 0$, so L must be an integer multiple of $\lambda/2$:

$$\lambda_n = 2L/n, \qquad (n = 1, 2, 3, \ldots)$$

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The corresponding frequencies are

$$f_n = c/\lambda_n = nc/2L, \quad (n = 1, 2, 3, \ldots)$$

Thus there is a set of normal modes, each with a characteristic frequency, wave shape, and node pattern.

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Ex: Intensity in a standing wave

Question

Calculate the intensity of the standing wave represented by

$$E_y(x,t) = -2E_{\max} \sin kx \sin \omega t$$
$$B_z(x,t) = -2B_{\max} \cos kx \cos \omega t$$

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Solution

Let us first find the instantaneous value of $\vec{\mathbf{S}}$ and then average it over a whole number of cycles of the wave.

Ex: Intensity in a standing wave

Solution

- $$\begin{aligned}\vec{\mathbf{S}} &= \vec{\mathbf{E}} \times \vec{\mathbf{B}}/\mu_0 \\ &= [-2\hat{\mathbf{j}} E_{\max} \sin kx \sin \omega t] \times [-2\hat{\mathbf{k}} B_{\max} \cos kx \cos \omega t]/\mu_0 \\ &= \hat{\mathbf{i}}(E_{\max} B_{\max}/\mu_0) 2 \sin kx \cos kx \, 2 \sin \omega t \cos \omega t \\ &= \hat{\mathbf{i}}(E_{\max} B_{\max}/\mu_0) \sin 2kx \, \sin 2\omega t\end{aligned}$$

- The average value of a sine function over any whole number of cycles is zero: $I = \langle S_x \rangle = 0$.
- All the energy transferred by one wave is cancelled by an equal amount transferred in the opposite direction by the other wave.

Ex: Standing waves in a cavity

Question

Electromagnetic standing waves are set up in a cavity with two parallel, highly conducting walls $1.50\,\mathrm{cm}$ apart. (a) Calculate the longest wavelength λ and lowest frequency of these standing waves. (b) For a standing wave of this wavelength, where in the cavity does $\vec{\mathbf{E}}$ have maximum magnitude? Where is $\vec{\mathbf{E}}$ zero? Where does $\vec{\mathbf{B}}$ have maximum magnitude? Where is $\vec{\mathbf{B}}$ zero?

Ex: Standing waves in a cavity

Identify

Only certain normal modes are possible for EM waves in a cavity The longest possible wavelength and lowest possible frequency correspond to the $n = 1$ mode in

$$\lambda_n = 2L/n, \quad f_n = c/\lambda_n \quad (n = 1, 2, 3, \ldots)$$

After finding λ and f, the locations of the nodal planes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \quad (\text{nodal planes of } \vec{\mathbf{E}})$$

and

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \quad (\text{nodal planes of } \vec{\mathbf{B}})$$

The antinodal planes of each field are midway between adjacent nodal planes.

Ex: Standing waves in a cavity

Solution

- For $n = 1$ we have $\lambda_1 = 2L/1 = 2 \times 1.5\,\mathrm{cm} = 3\,\mathrm{cm}$.
- $f_1 = c/2L = 1.00 \times 10^{10}\,\mathrm{Hz}$.
- With $n = 1$ there is a single half-wavelength between the walls.
- The electric field has nodal planes ($\vec{\mathbf{E}} = \vec{\mathbf{0}}$) at the walls and an anti- nodal plane (where $\vec{\mathbf{E}}$ has its maximum magnitude) midway between them. $\vec{\mathbf{B}}$ has *antinodal* planes at the walls and a nodal plane midway between them.