ITU PHYSICS ENGINEERING. STUDENT, **nov28**, **16**
IDENTITY NUMBER :
NAME LASTNAME :
FIZ 411E QUIZ – 6 POINT :

[**Question.1**] (*20/100 Pnts*) Please write in the SI units, the Gauss' Law both forms, differential and integral in the presence of the dielectric material.

Hint-1: Think about the displacement field, $\boldsymbol{D}$ and free charge and its density.

[**Answer.1**] (*20/100 Pnts*)
With the help of Hint-1,

$$\begin{aligned} &\text{Differential form}: \ \boldsymbol{\nabla}\cdot\boldsymbol{D} \ = \ \rho_{\text{free}} \ , \\ &\text{Integral form}: \ \oint \boldsymbol{D}\cdot d\boldsymbol{a} \ = \ Q_{\text{free}} \ . \end{aligned} \tag{1}$$

[**Question.2**] (*80/100 Pnts*) About the capasitance per unit length calculation of cylindrical capasitor.

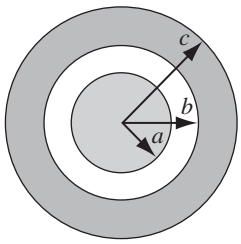

FIG. 1: coaxial cable

A certain coaxial cable consists of a copper wire, radius a, surrounded by a concentric copper tube of inner radius c and this cable length l, in above figure. The space between is partially filled (from b out to c) with material of dielectric constant, ϵ_r as shown.
Find the capacitance per unit length of this cable.

Hint-2: Think about the Gauss' Law in the presence of the dielectric material.

Hint-3: Take the free charge on a length l of the inner conductor as Q.

[**Answer.2**] (*80/100 Pnts*)
By using Hint-2 and -3,
in order to find the the capasitance per unit length $\frac{C}{l}$, we should obtain $\frac{Q}{V\,l}$.
Therefore we should find the displacement electric field, $\mathbf{D}$ and then electric field $\mathbf{E} = \epsilon\,\mathbf{D}$,
and finally potential. Let us start,

$$\begin{aligned} \oint \boldsymbol{D}\cdot d\boldsymbol{a} &= Q_{\text{free}}\,, \\ D\,2\pi\,r\,l &= Q\,, \\ \therefore D &= \frac{Q}{2\pi r l}\,. \end{aligned} \tag{2}$$

⋆ for $a < r < b$,

$$\therefore E = \frac{Q}{2\pi\,r\,l\,\epsilon_o}\,. \tag{3}$$

⋆ for $b < r < c$,

$$\begin{aligned} \mathbf{E} &= \frac{\boldsymbol{D}}{\epsilon}\,, \\ \therefore E &= \frac{Q}{2\pi\,r\,l\,\epsilon}\,. \end{aligned} \tag{4}$$

⋆ by using the equations (3) and (4), we can obtain the potential as:

$$\begin{aligned} V &= -\int_a^c \boldsymbol{E}\cdot d\boldsymbol{l}\,, \\ V &= \Big(\int_a^b \frac{Q}{2\epsilon_o\,\pi\,l}\frac{dr}{r} + \int_b^c \frac{Q}{2\epsilon\,\pi\,l}\frac{dr}{r}\Big)\,, \\ \therefore V &= \frac{Q}{2\,\pi\,\epsilon_o\,l}\,\Big[\,\ln\big(\frac{b}{a}\big) + \frac{\epsilon_o}{\epsilon}\,\ln\big(\frac{c}{b}\big)\,\Big]\,. \end{aligned} \tag{5}$$

Finally,

$$\therefore \frac{C}{l} = \frac{Q}{V\,l} = 2\,\pi\,\epsilon_o\,\Big[\,\ln\big(\frac{b}{a}\big) + \frac{\epsilon_o}{\epsilon}\,\ln\big(\frac{c}{b}\big)\,\Big]^{-1}\,. \tag{6}$$