Fundamentals of Power Electronics

Second edition

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Chapter 1: Introduction

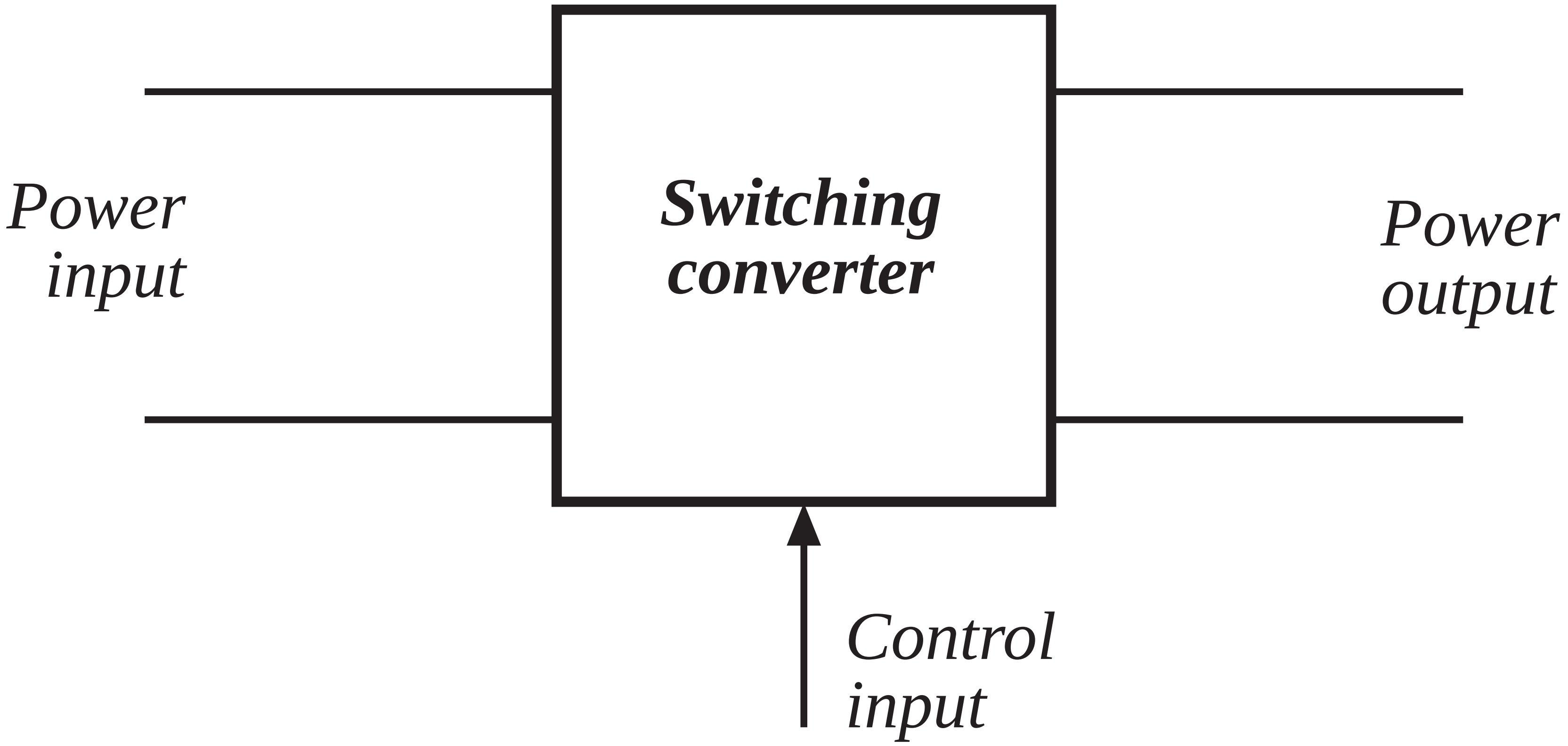
1.1. Introduction to power processing

1.2. Some applications of power electronics

1.3. Elements of power electronics

Summary of the course

1.1 Introduction to Power Processing



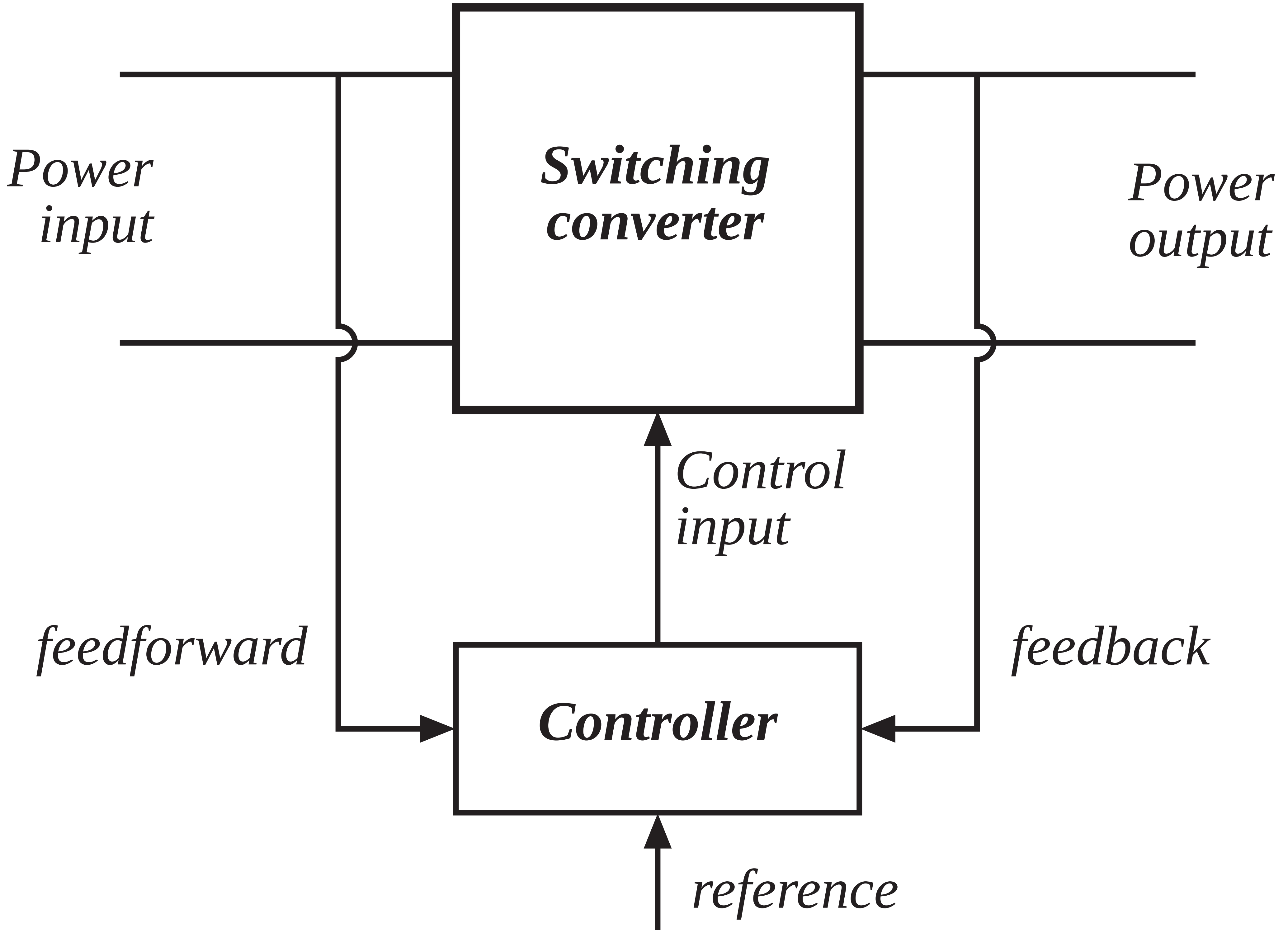
Dc-dc conversion: Change and control voltage magnitude

Ac-dc rectification: Possibly control dc voltage, ac current

Dc-ac inversion: Produce sinusoid of controllable magnitude and frequency

Ac-ac cycloconversion: Change and control voltage magnitude and frequency

Control is invariably required



High efficiency is essential

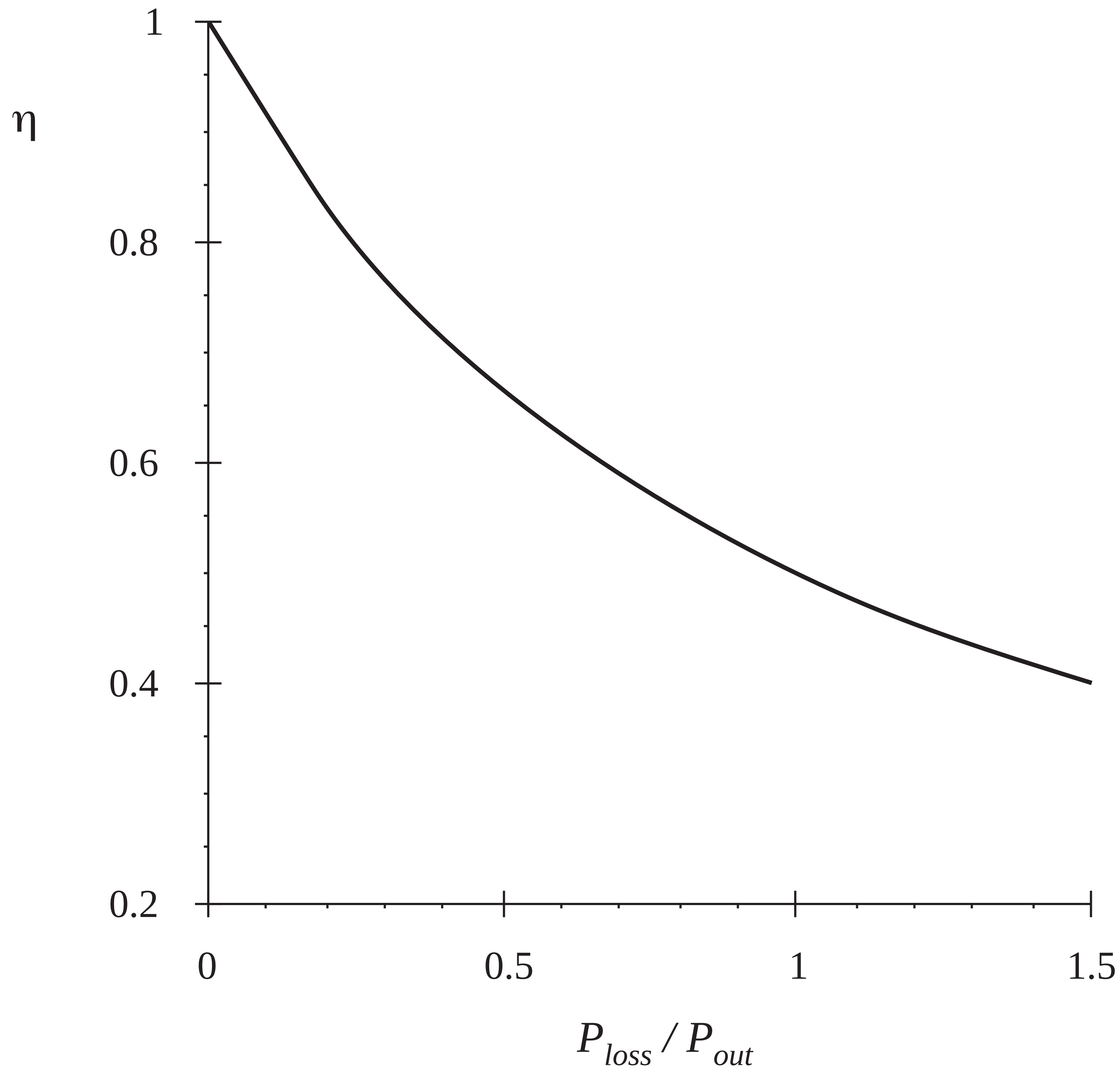
$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{loss} = P_{in} - P_{out} = P_{out}\left(\frac{1}{\eta} - 1\right)$$

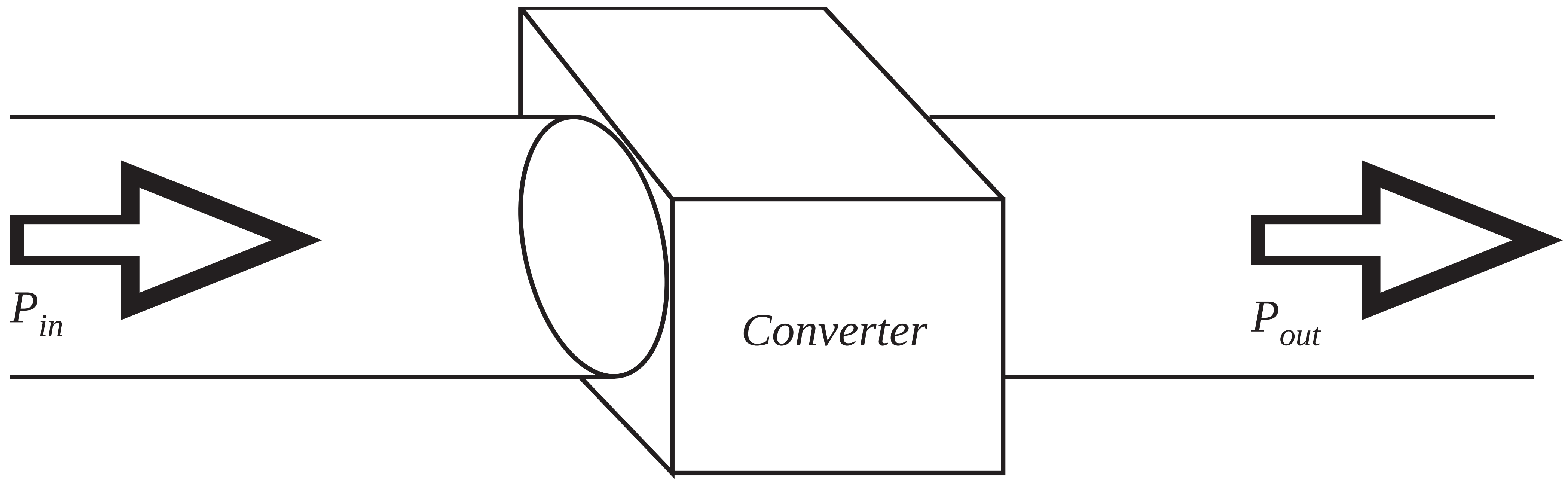
High efficiency leads to low power loss within converter

Small size and reliable operation is then feasible

Efficiency is a good measure of converter performance

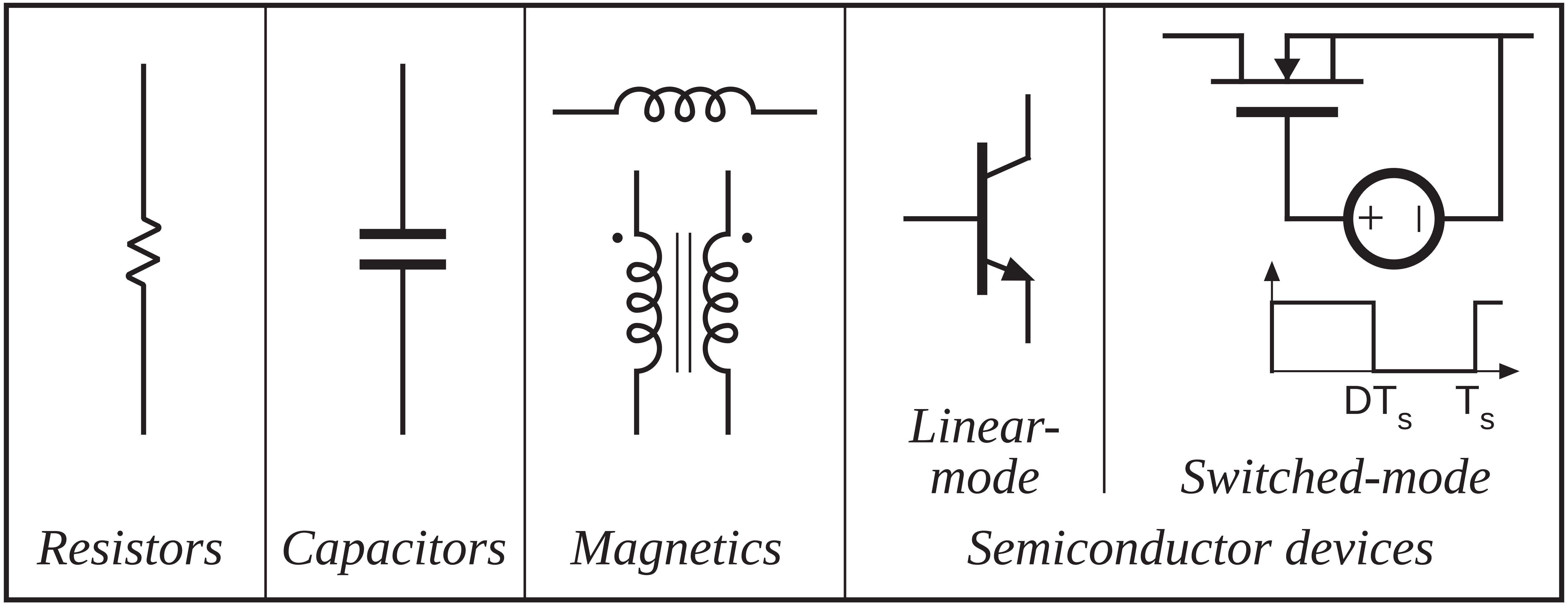


A high-efficiency converter

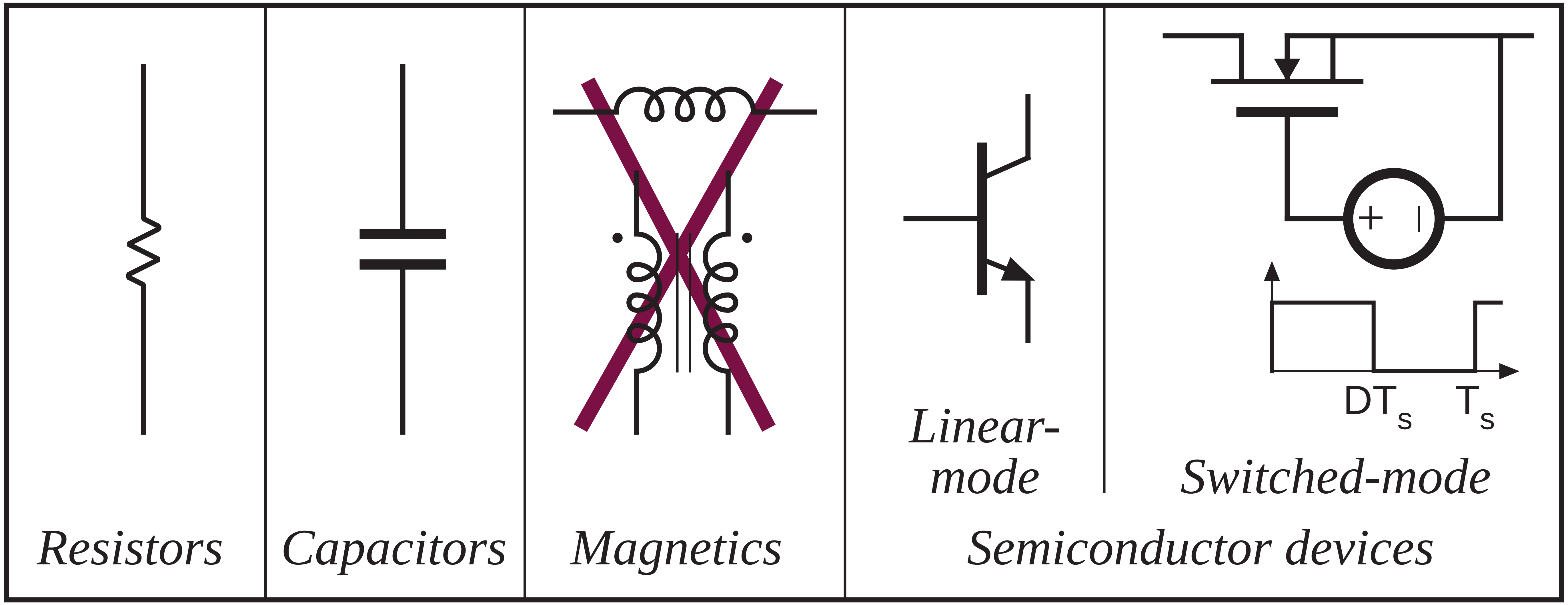


A goal of current converter technology is to construct converters of small size and weight, which process substantial power at high efficiency

Devices available to the circuit designer

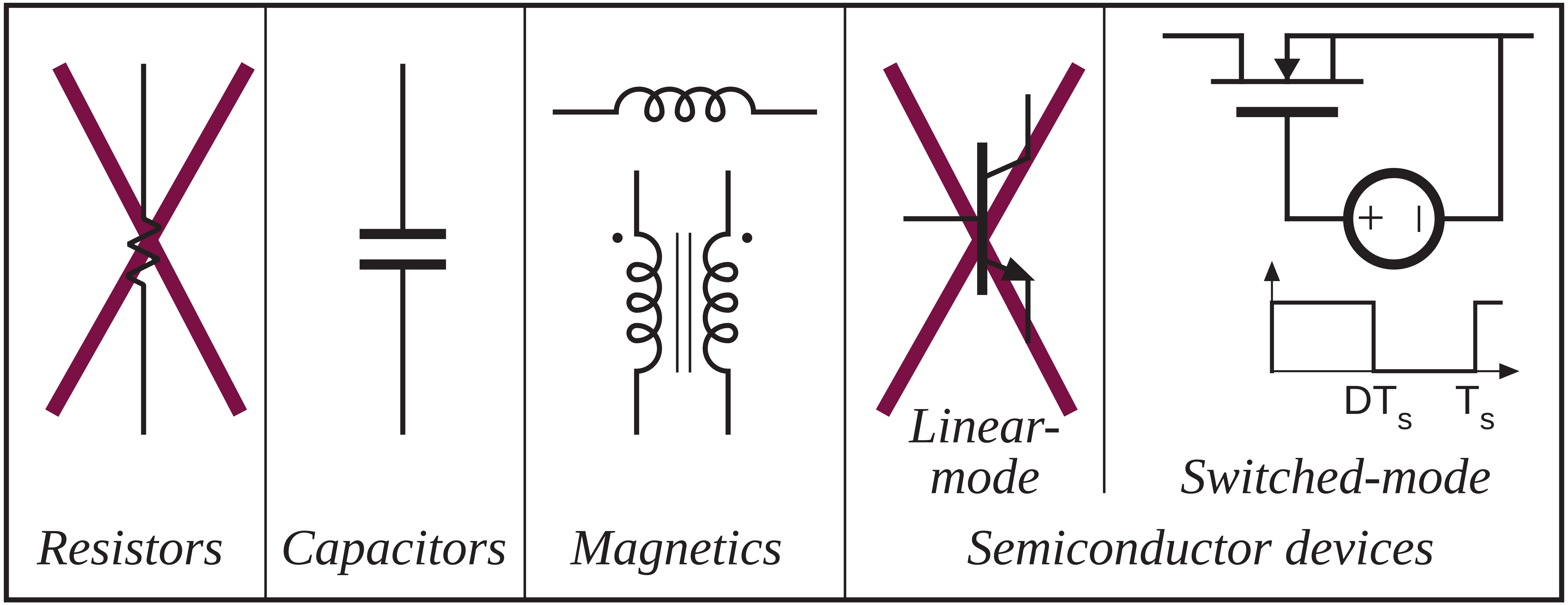


Devices available to the circuit designer



Signal processing: avoid magnetics

Devices available to the circuit designer



Power processing: avoid lossy elements

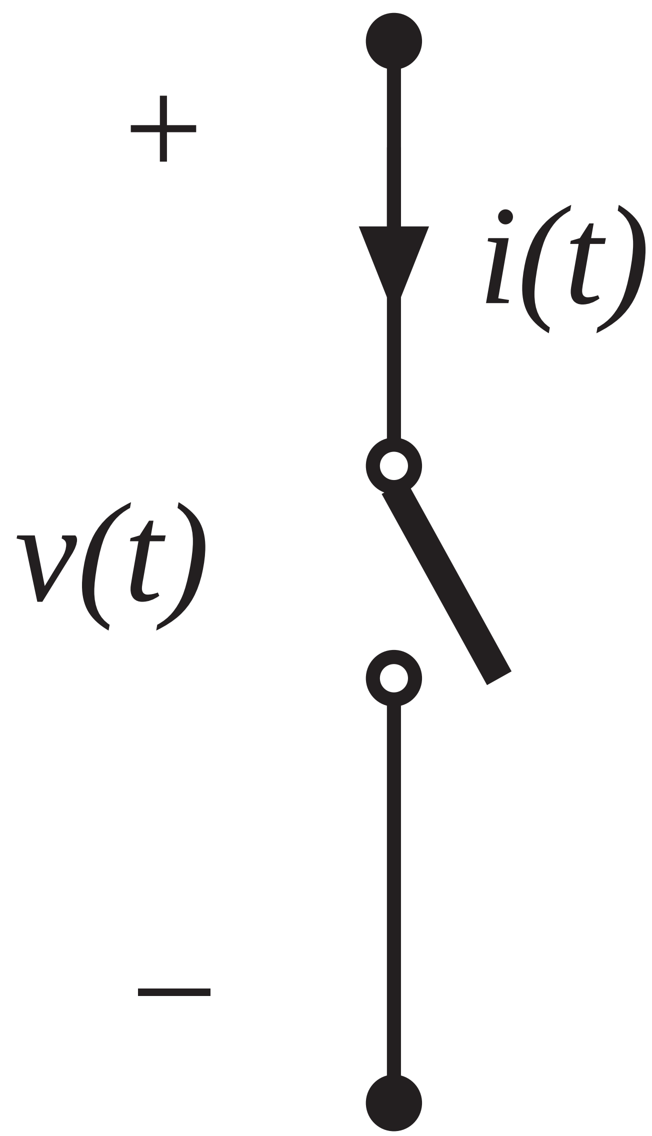
Power loss in an ideal switch

Switch closed: $v(t) = 0$

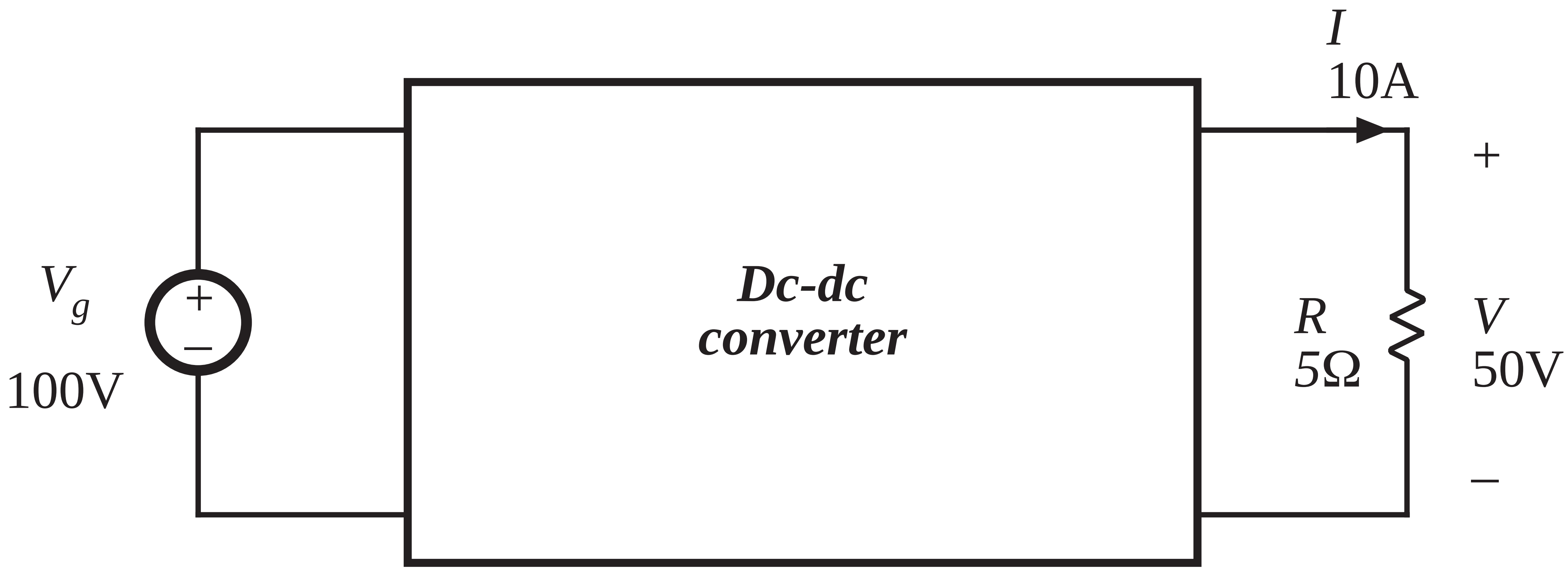
Switch open: $i(t) = 0$

In either event: $p(t) = v(t)\, i(t) = 0$

Ideal switch consumes zero power



A simple dc-dc converter example



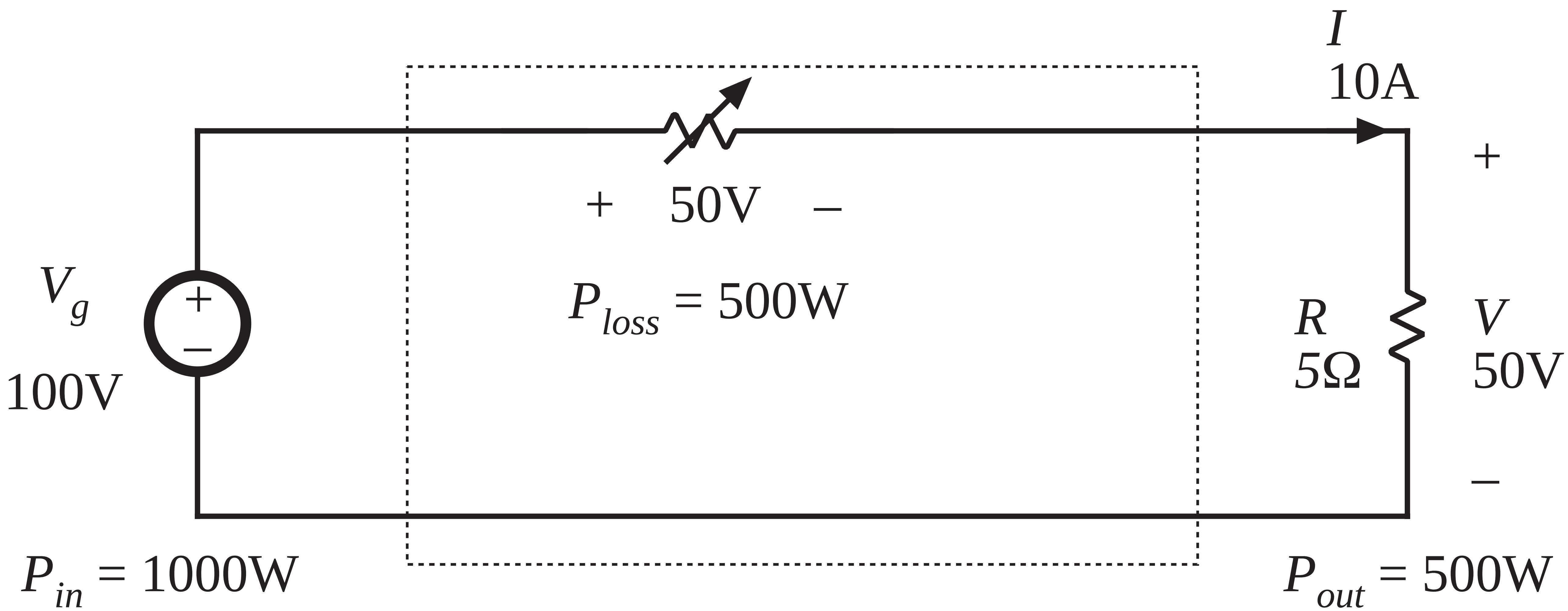
Input source: 100V

Output load: 50V, 10A, 500W

How can this converter be realized?

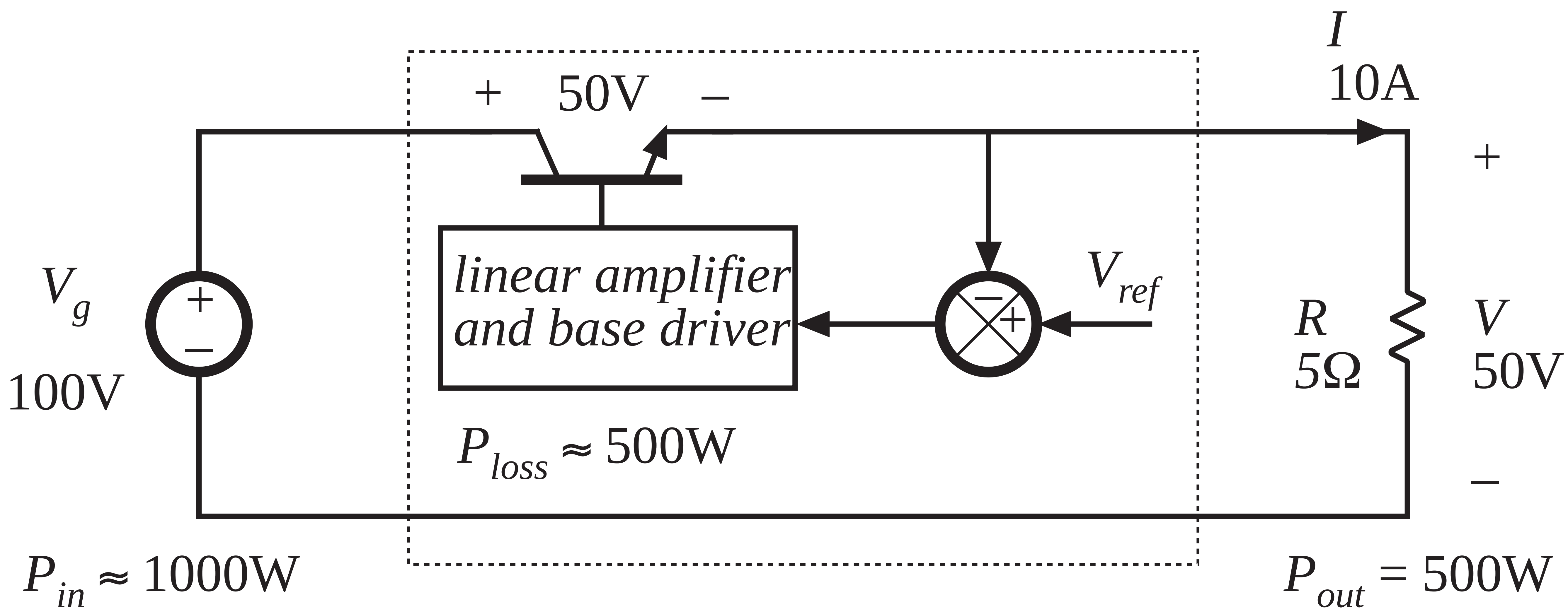
Dissipative realization

Resistive voltage divider

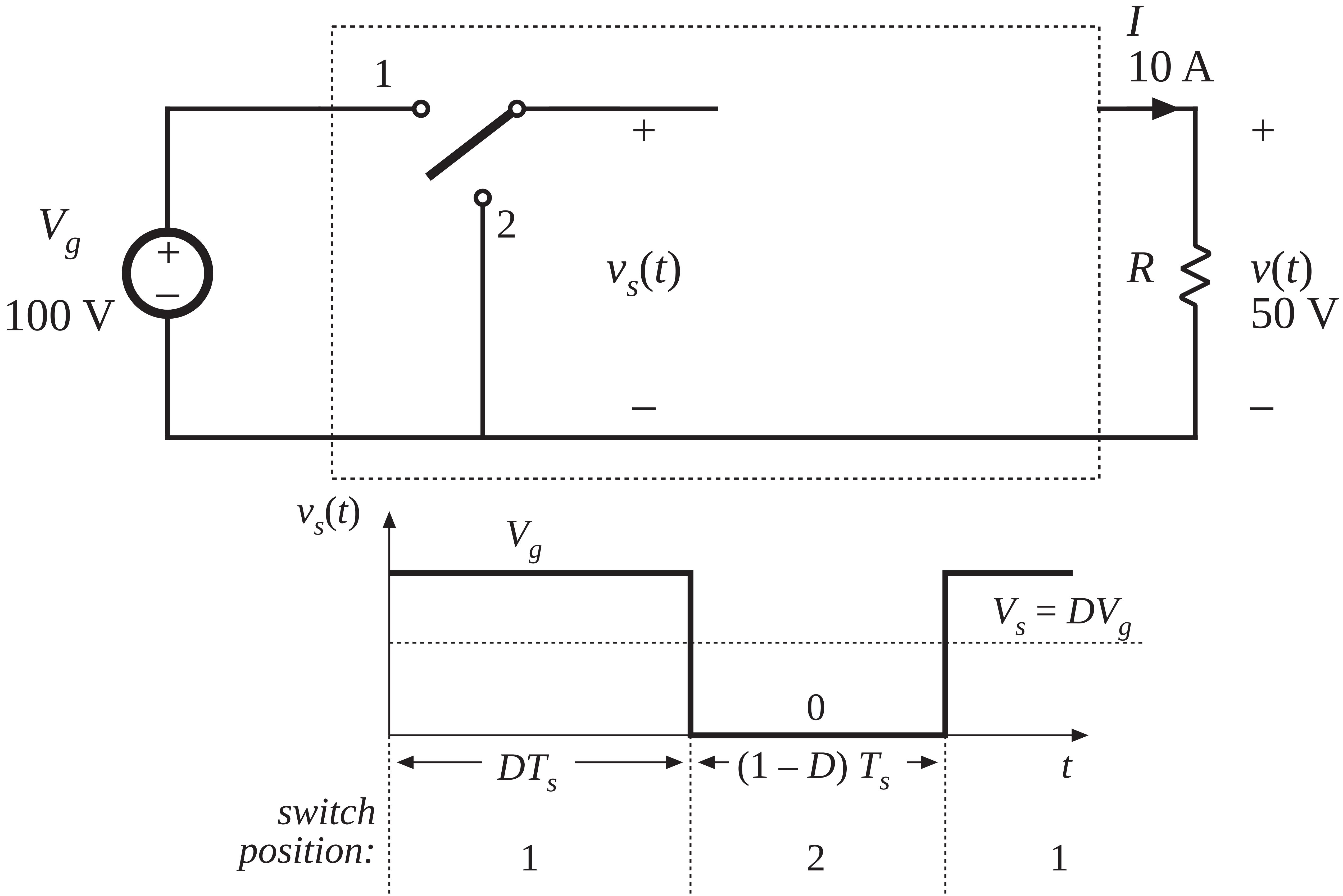


Dissipative realization

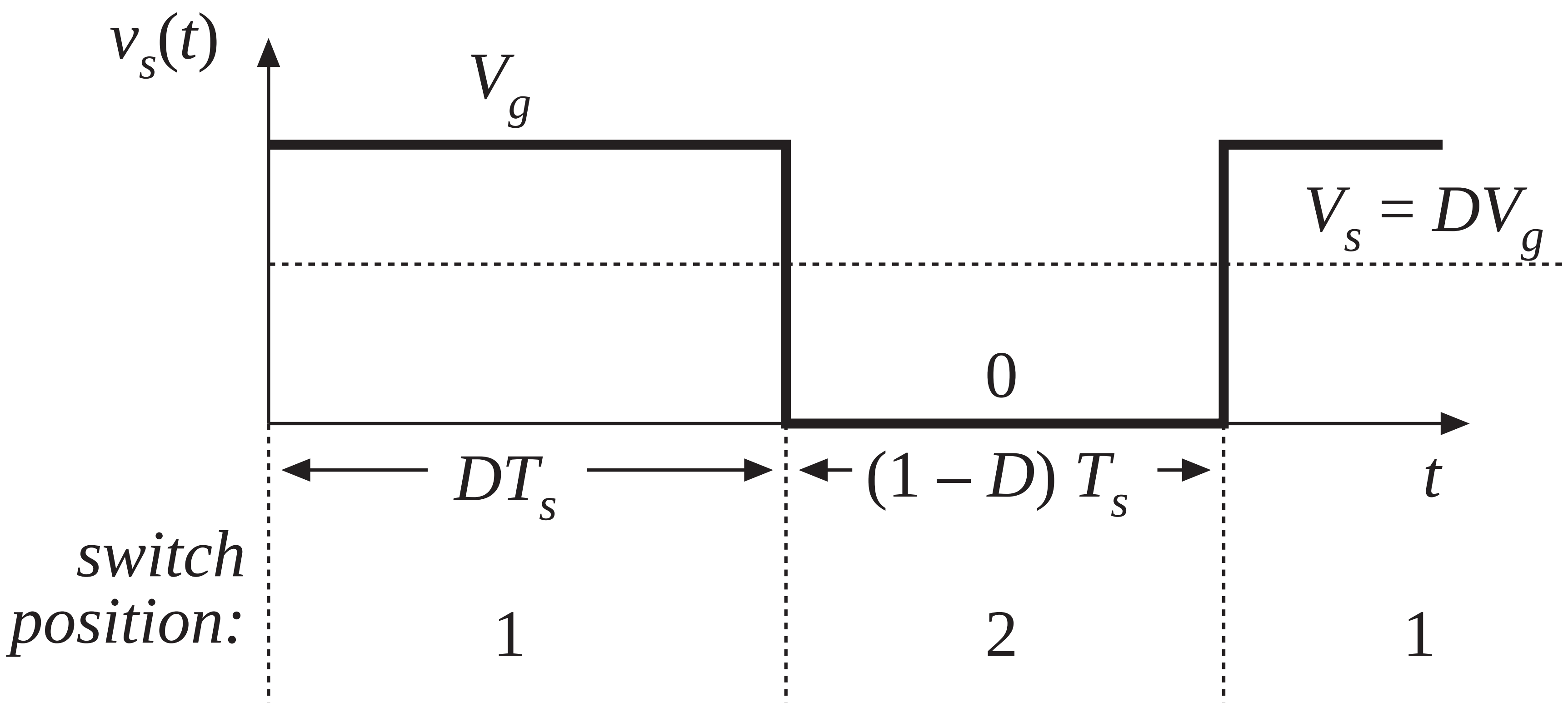
Series pass regulator: transistor operates in active region



Use of a SPDT switch



The switch changes the dc voltage level



D = switch duty cycle

$0 \le D \le 1$

T_s = switching period

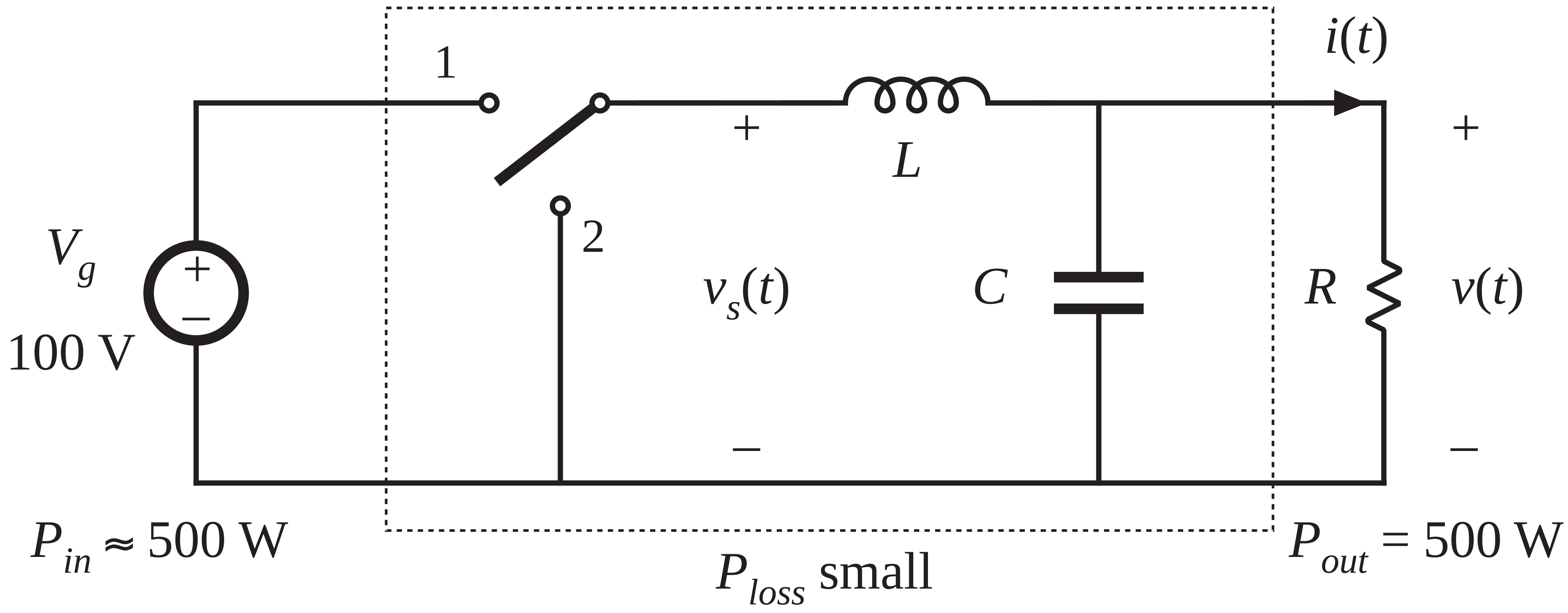
f_s = switching frequency
$= 1 / T_s$

DC component of $v_s(t)$ = average value:

$$V_s = \frac{1}{T_s} \int_0^{T_s} v_s(t)\, dt = DV_g$$

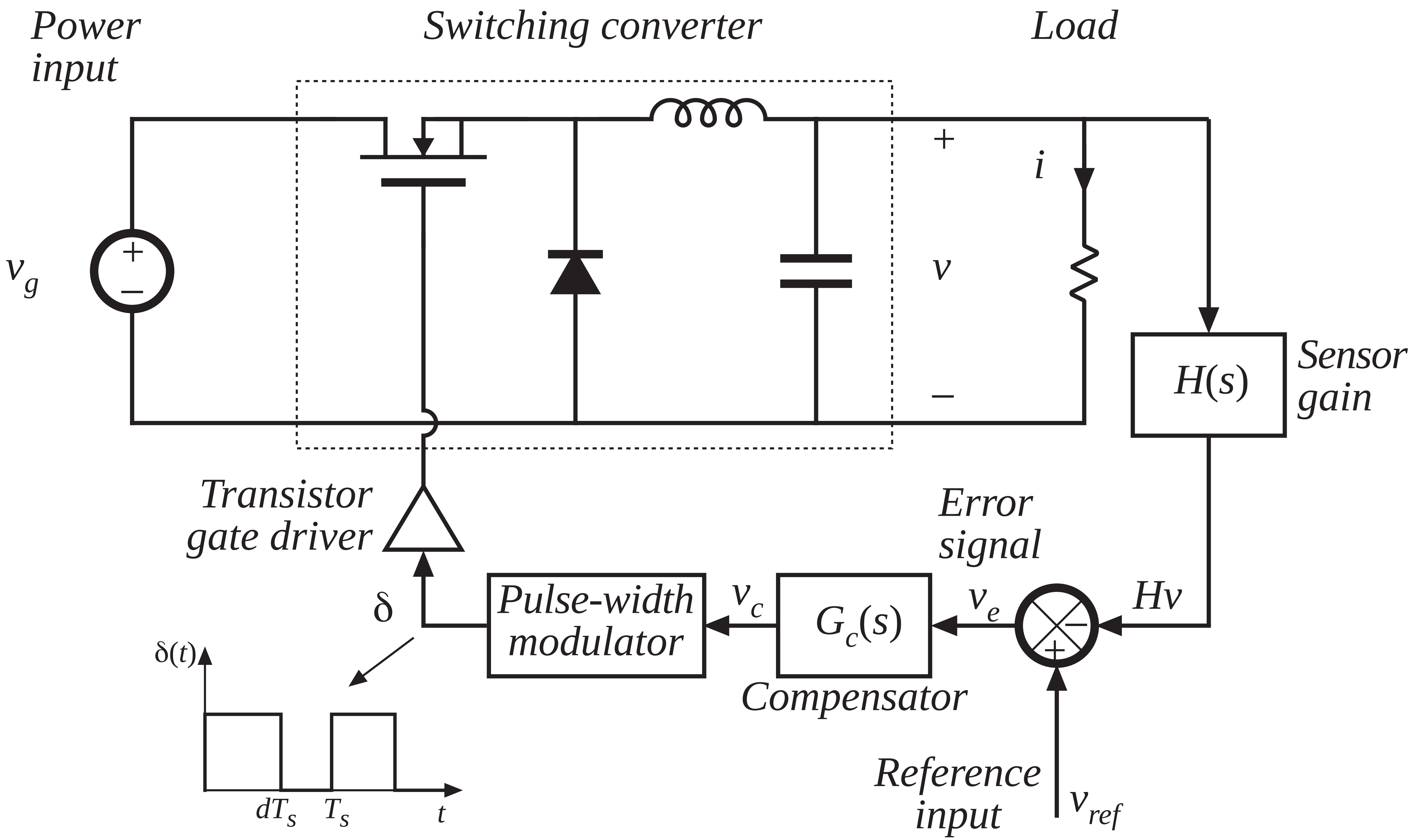
Addition of low pass filter

Addition of (ideally lossless) L-C low-pass filter, for removal of switching harmonics:

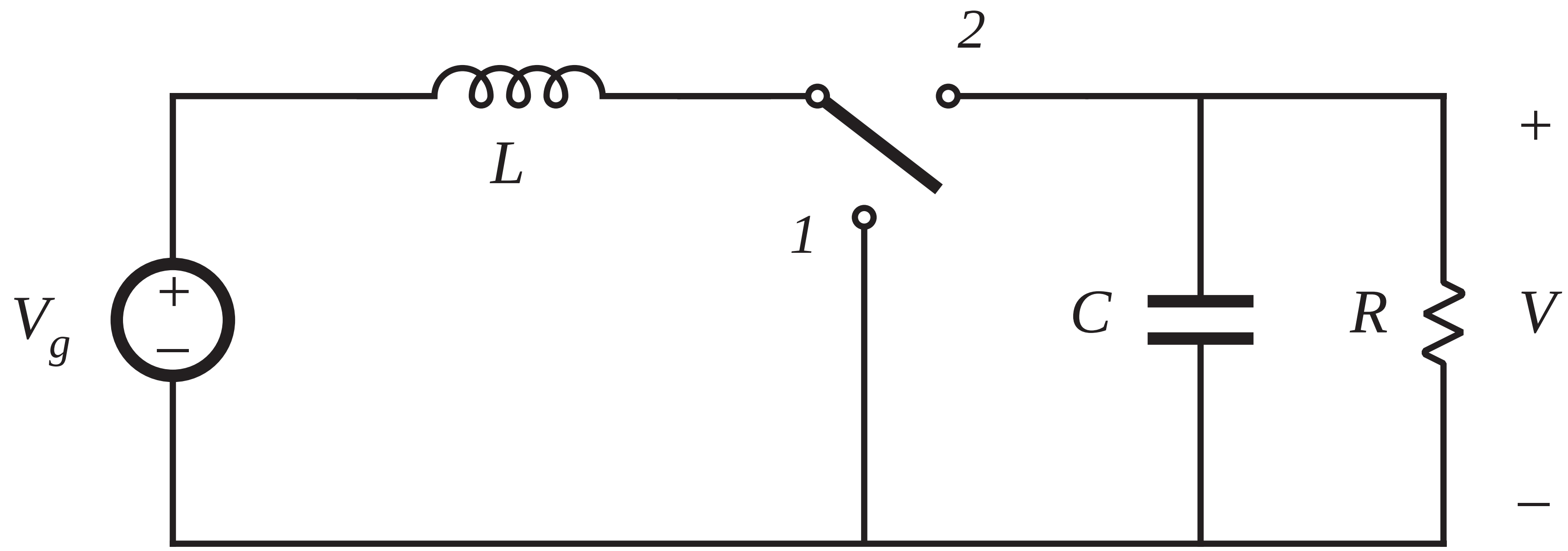


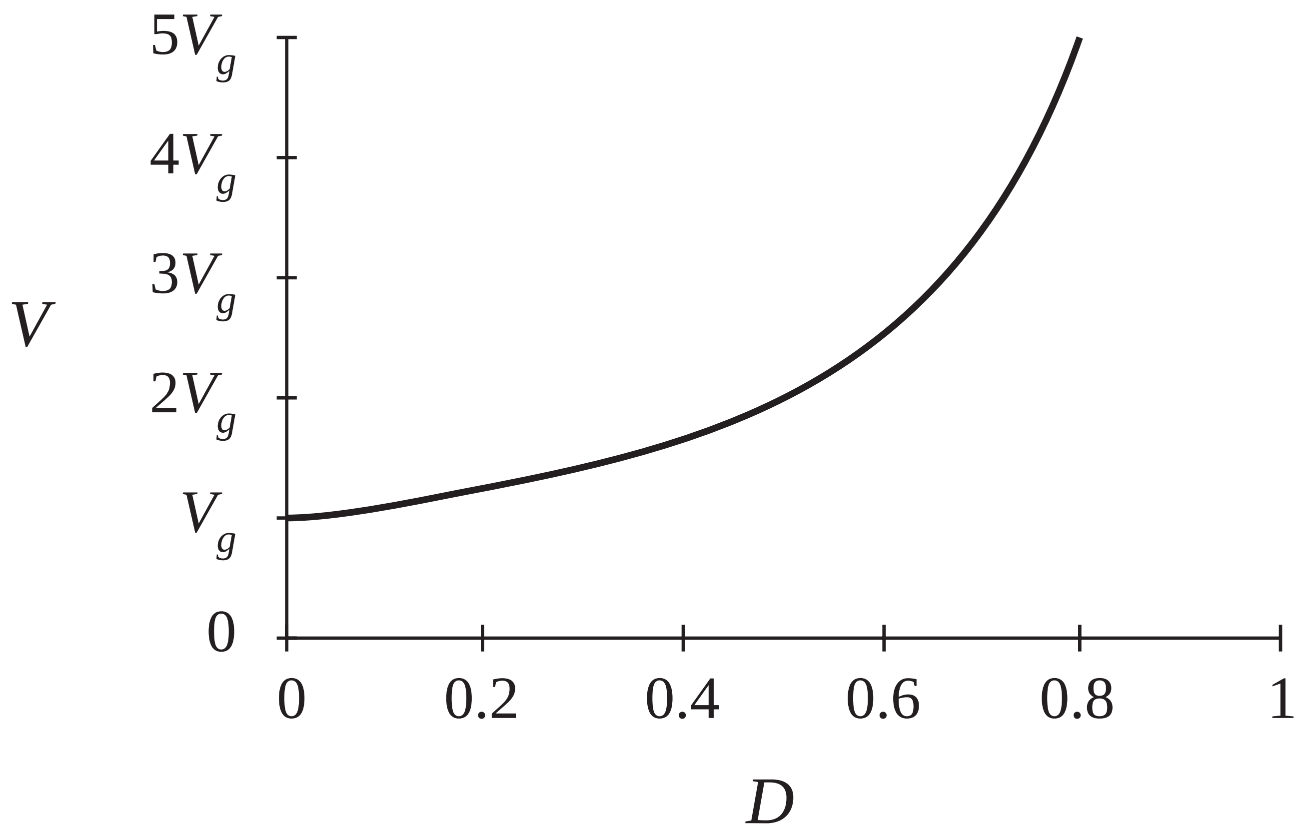
- Choose filter cutoff frequency f_0 much smaller than switching frequency f_s
- This circuit is known as the “buck converter”

Addition of control system for regulation of output voltage

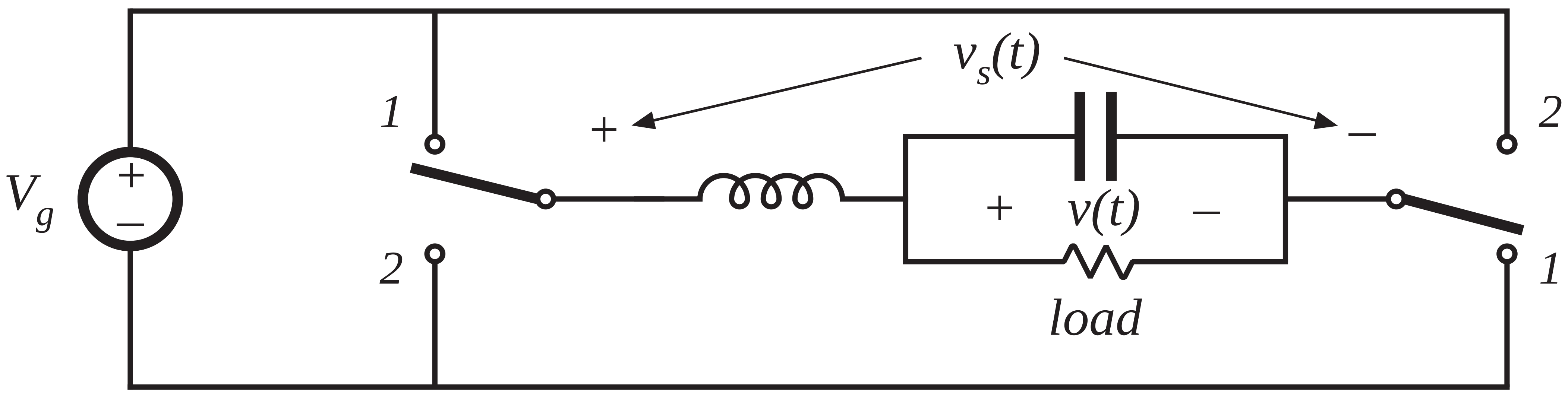


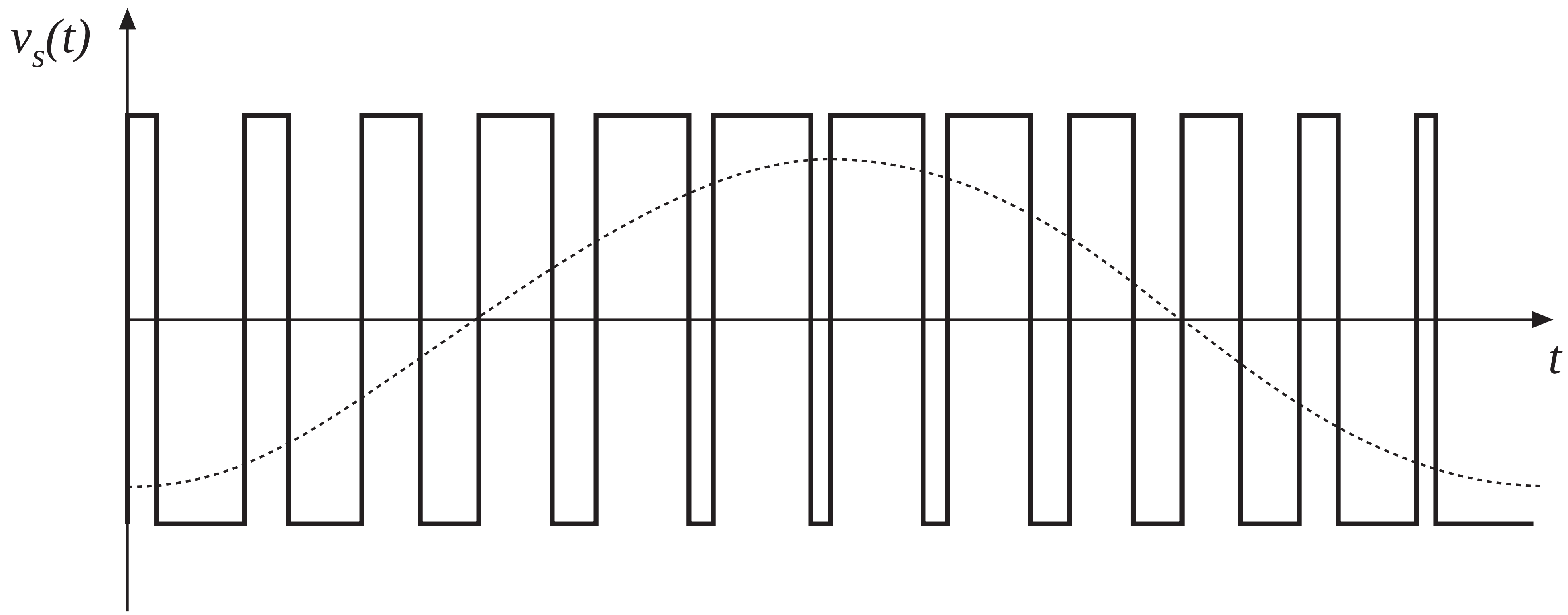
The boost converter





A single-phase inverter





"H-bridge"

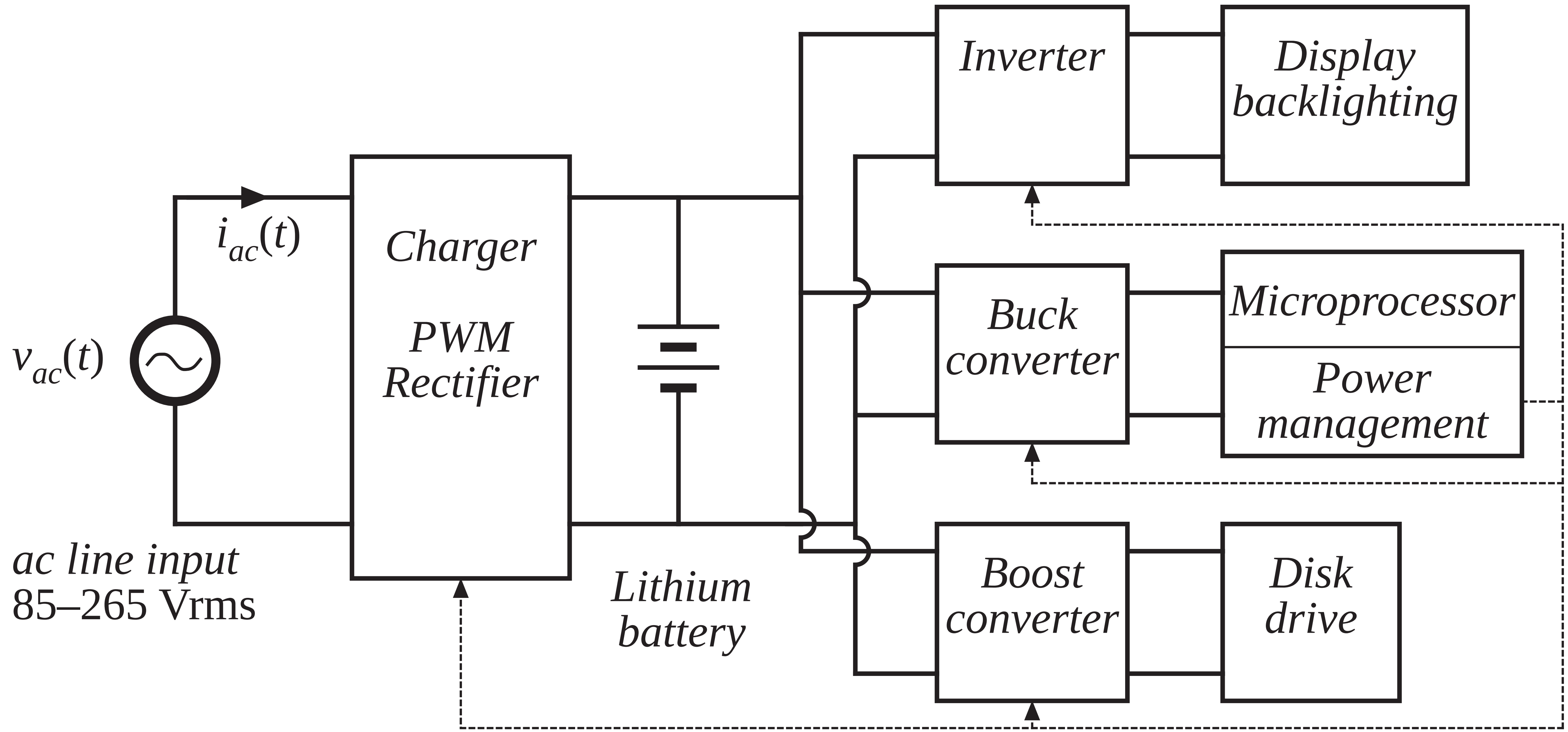
Modulate switch duty cycles to obtain sinusoidal low-frequency component

1.2 Several applications of power electronics

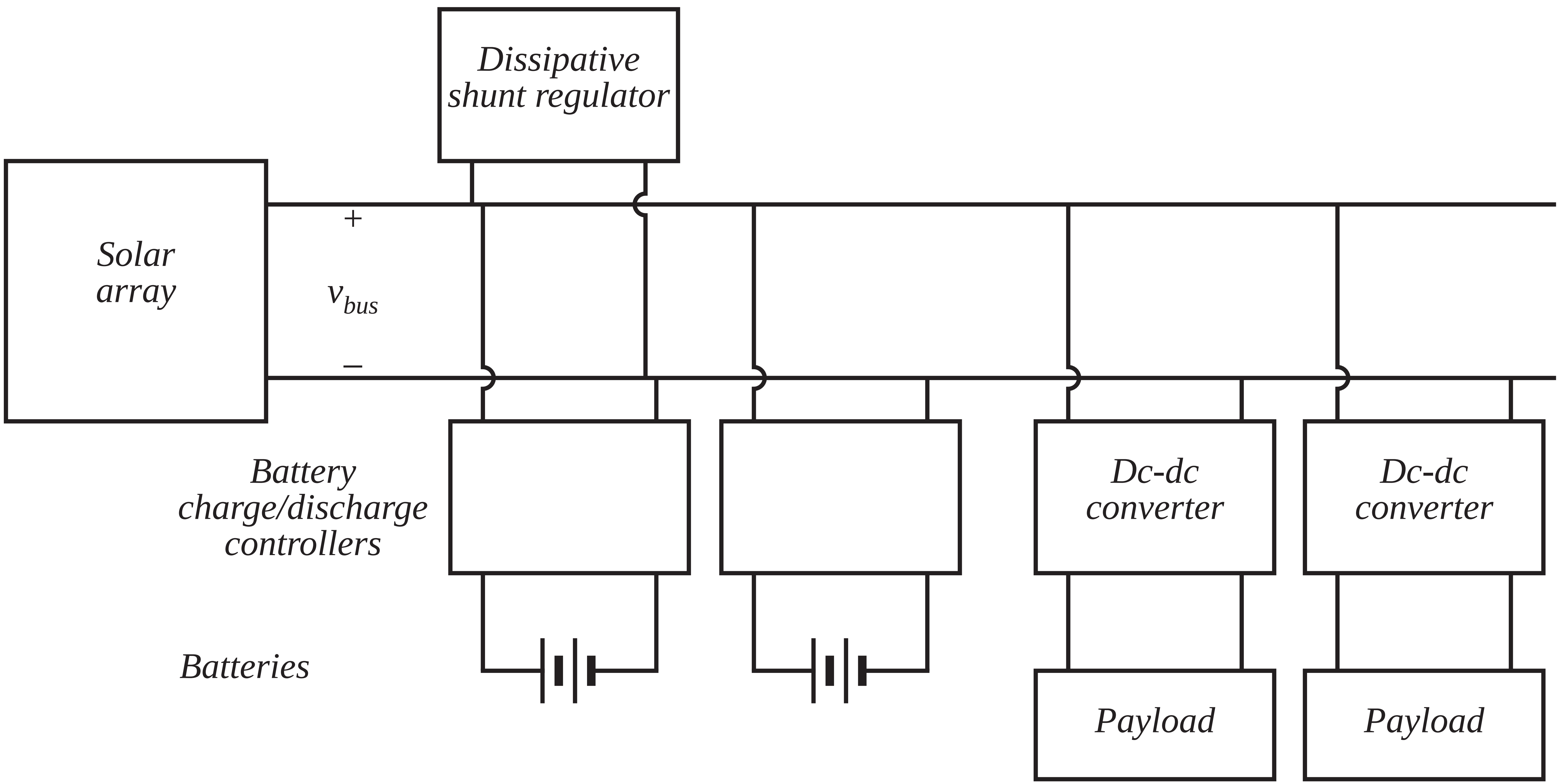
Power levels encountered in high-efficiency converters

- less than 1 W in battery-operated portable equipment
- tens, hundreds, or thousands of watts in power supplies for computers or office equipment
- kW to MW in variable-speed motor drives
- 1000 MW in rectifiers and inverters for utility dc transmission lines

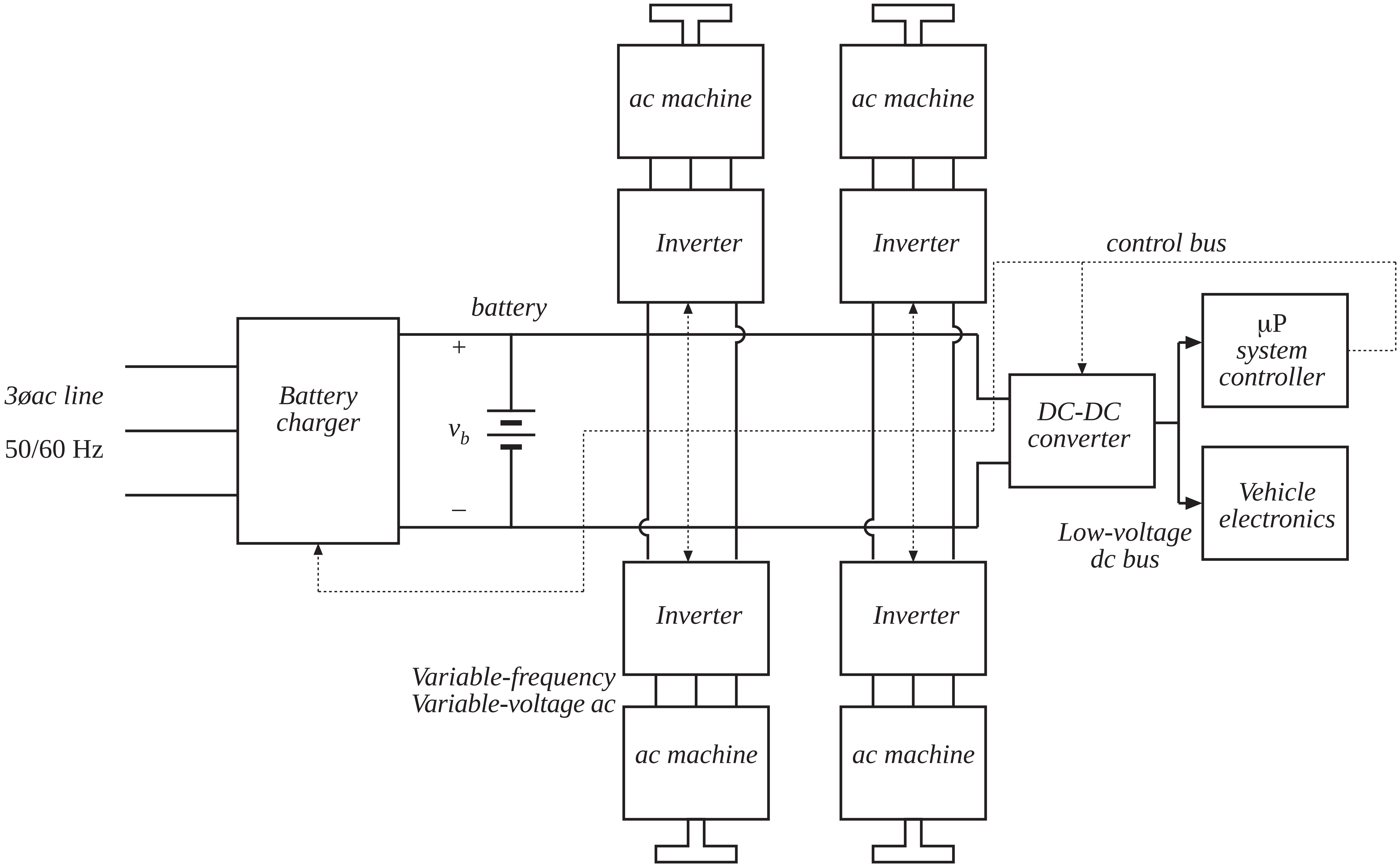
A laptop computer power supply system



Power system of an earth-orbiting spacecraft



An electric vehicle power and drive system



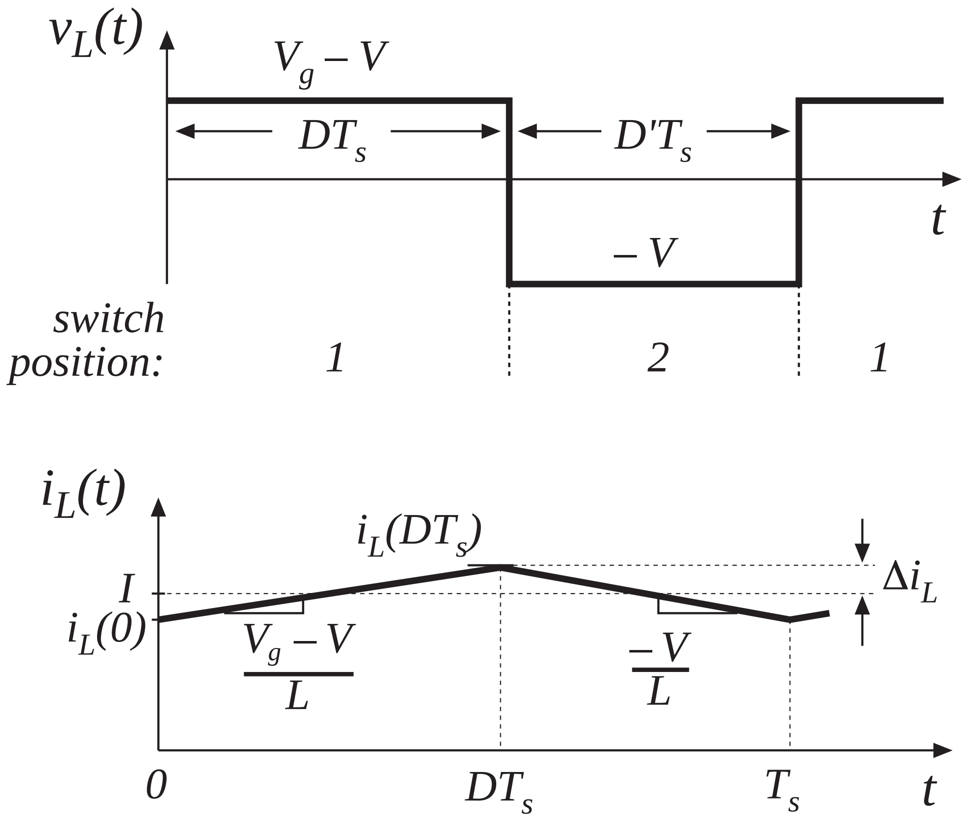
1.3 Elements of power electronics

Power electronics incorporates concepts from the fields of

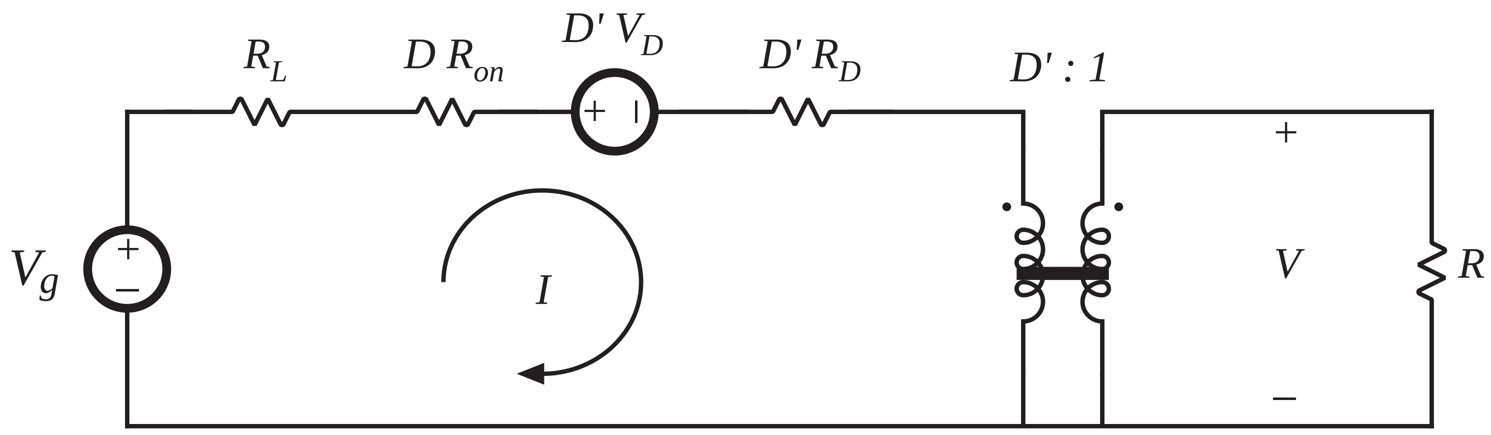
- analog circuits
- electronic devices
- control systems
- power systems
- magnetics
- electric machines
- numerical simulation

Part I. Converters in equilibrium

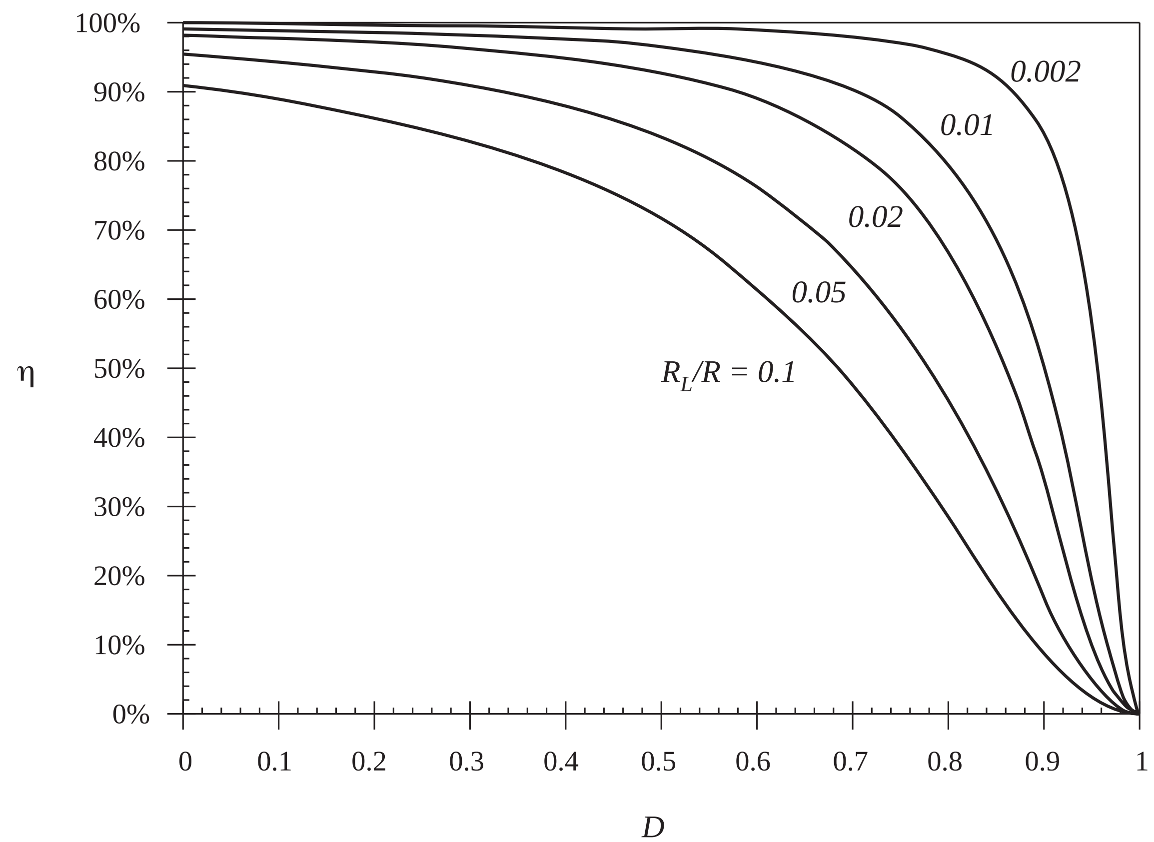
Inductor waveforms



Averaged equivalent circuit



Predicted efficiency

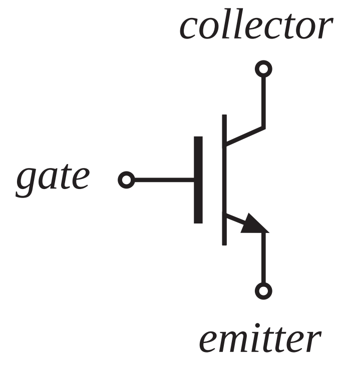


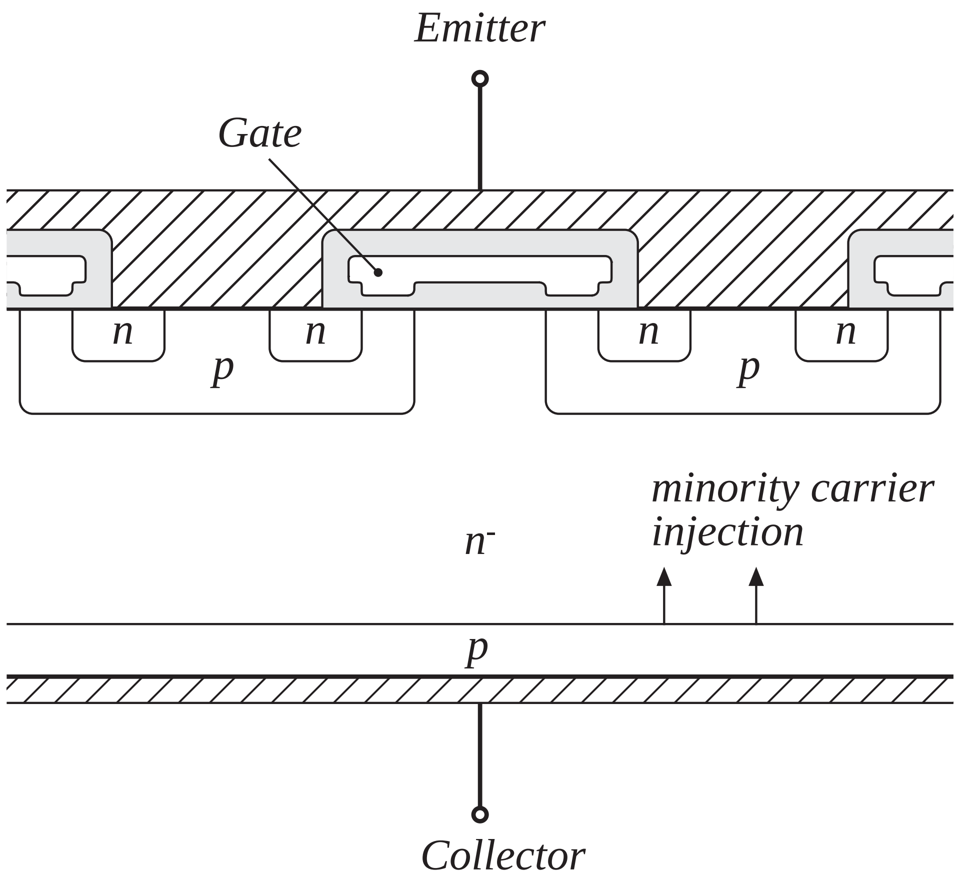
Discontinuous conduction mode

Transformer isolation

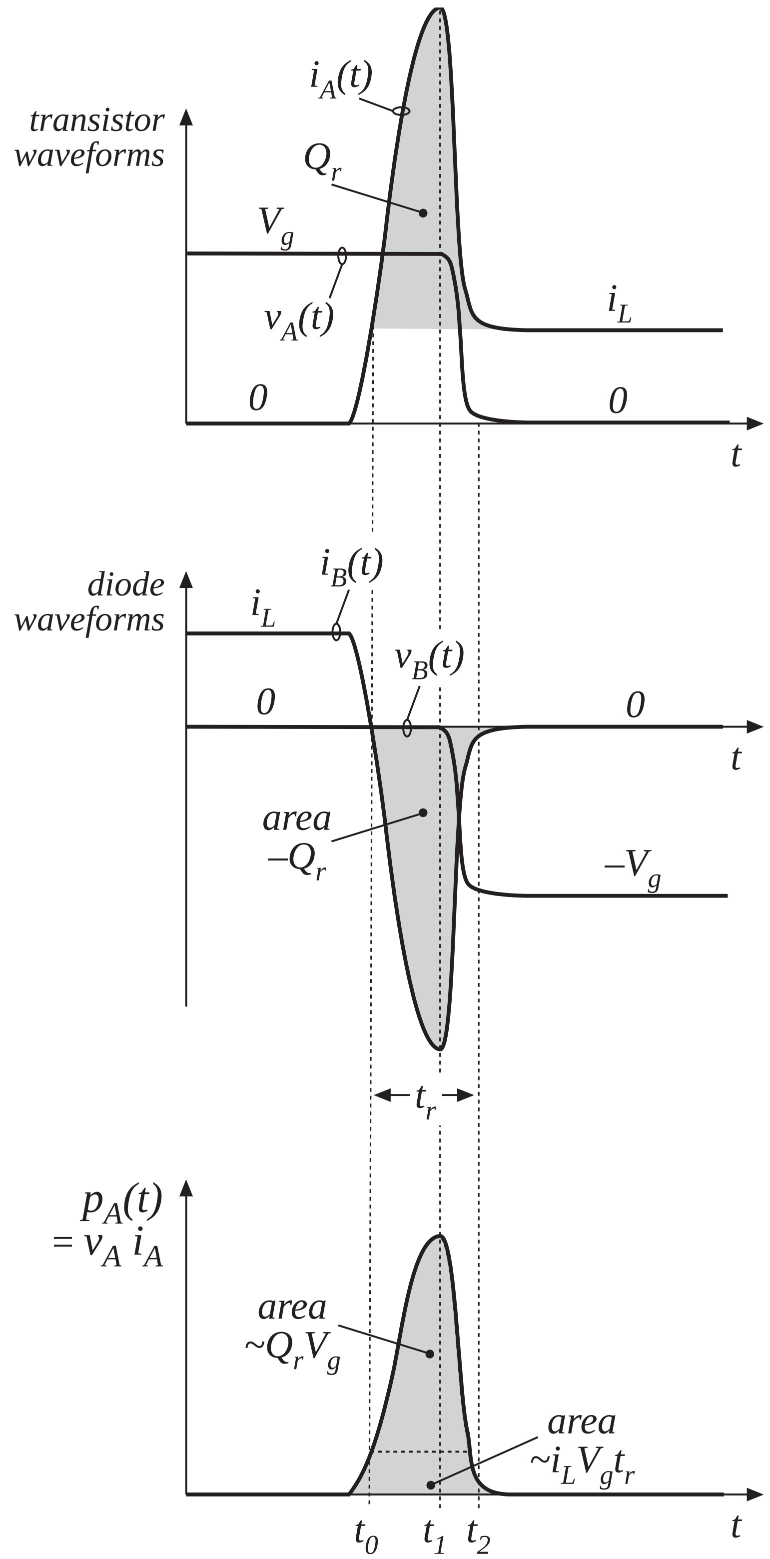
Switch realization: semiconductor devices

The IGBT





Switching loss

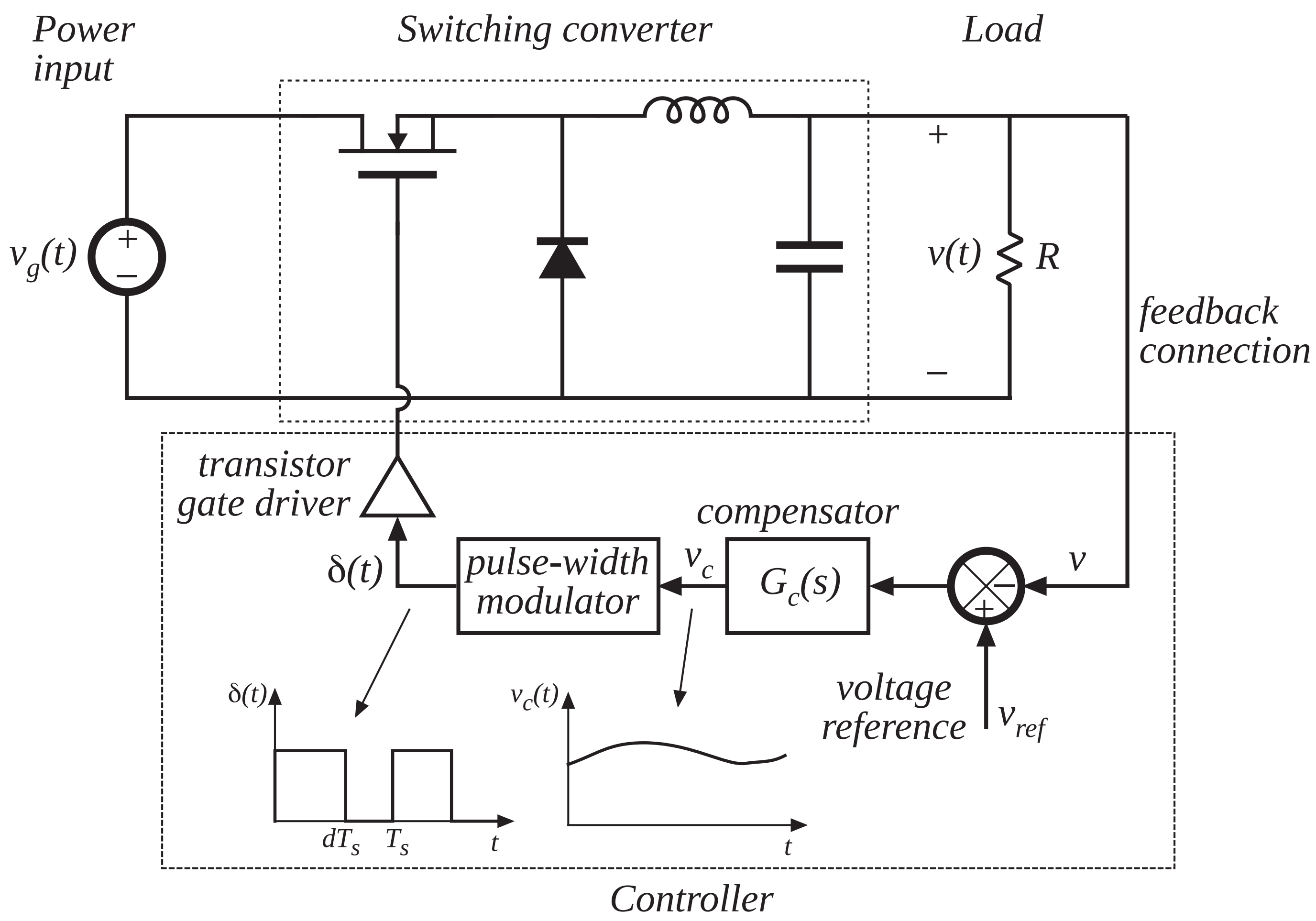


Part I. Converters in equilibrium

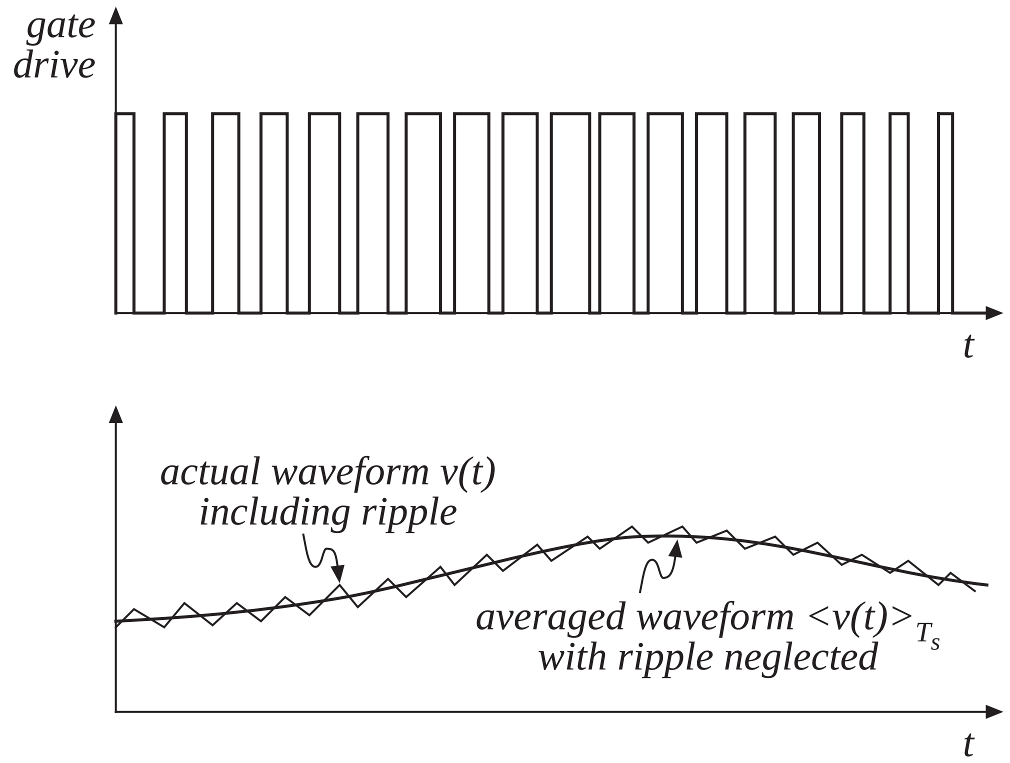
2. Principles of steady state converter analysis
3. Steady-state equivalent circuit modeling, losses, and efficiency
4. Switch realization
5. The discontinuous conduction mode
6. Converter circuits

Part II. Converter dynamics and control

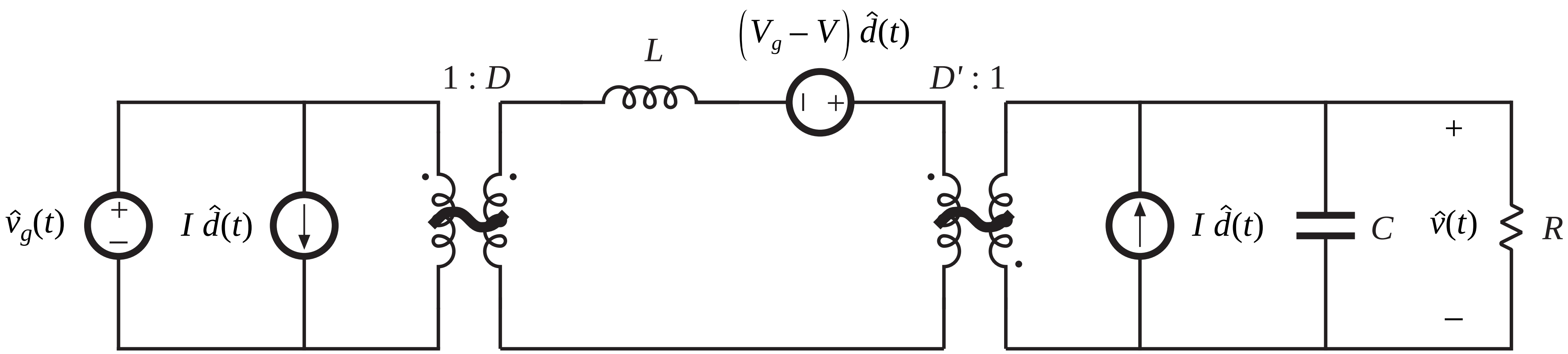
Closed-loop converter system



Averaging the waveforms



Small-signal averaged equivalent circuit

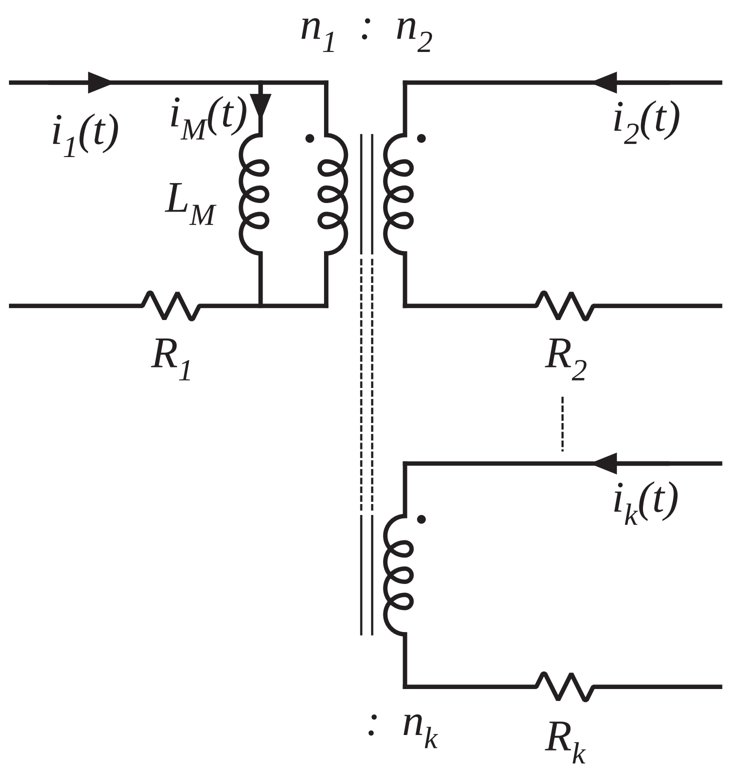


Part II. Converter dynamics and control

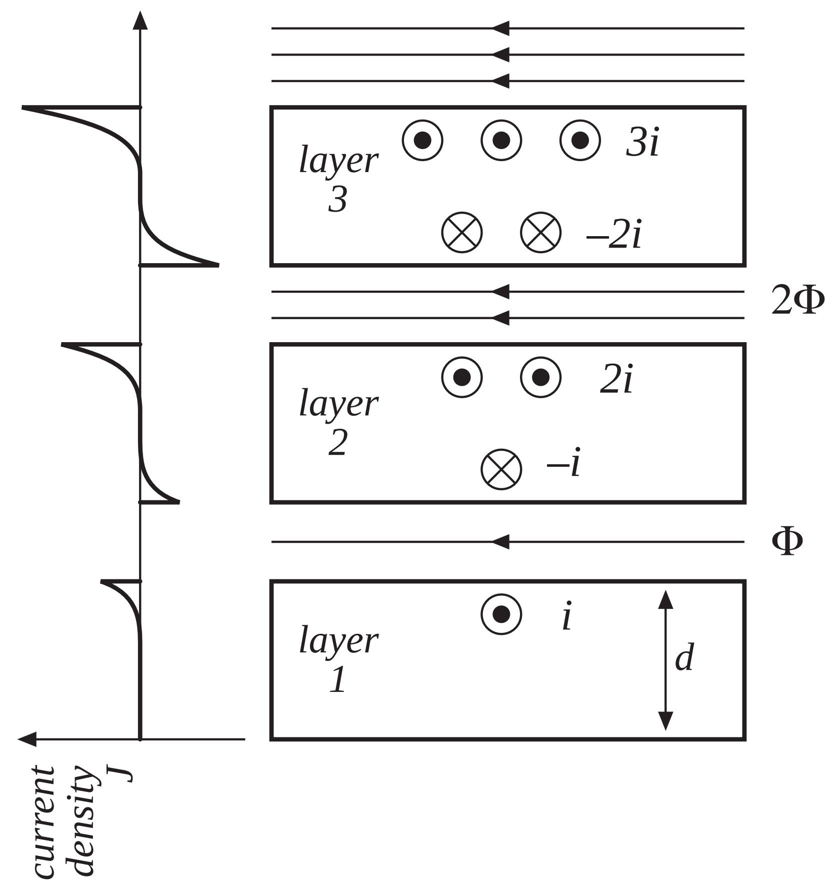
7. Ac modeling
8. Converter transfer functions
9. Controller design
10. Input filter design
11. Ac and dc equivalent circuit modeling of the discontinuous conduction mode
12. Current-programmed control

Part III. Magnetics

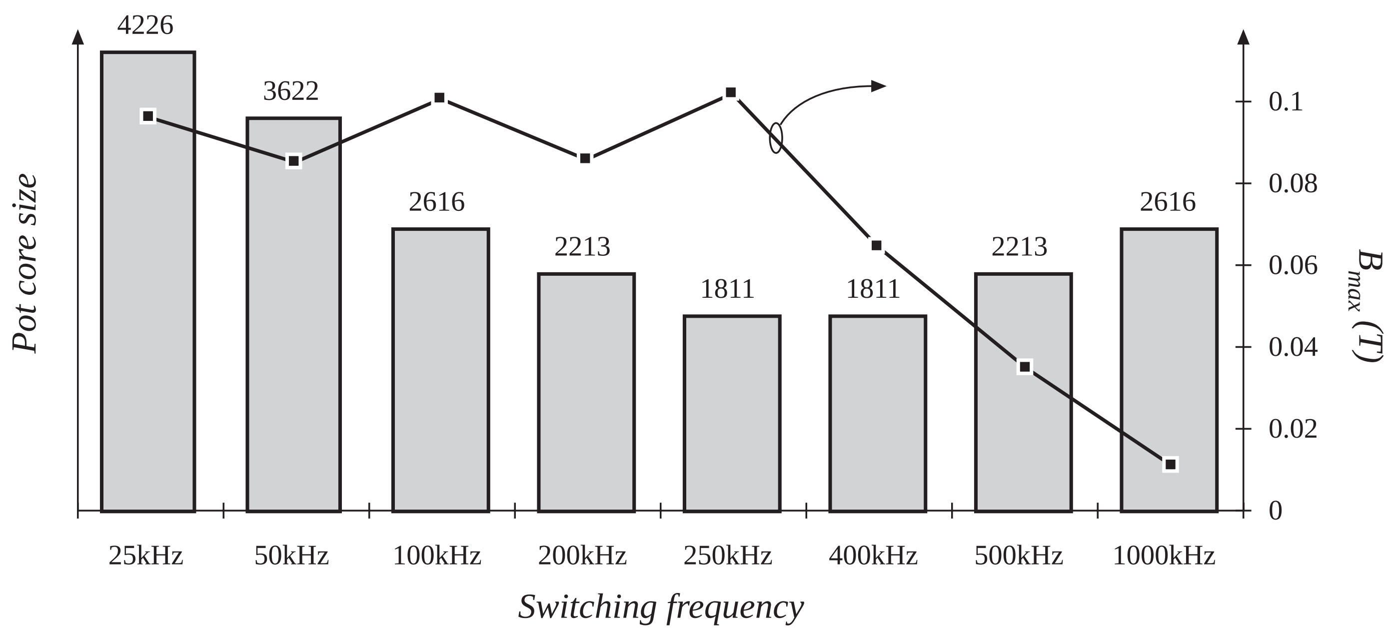
transformer design



the proximity effect



transformer size vs. switching frequency

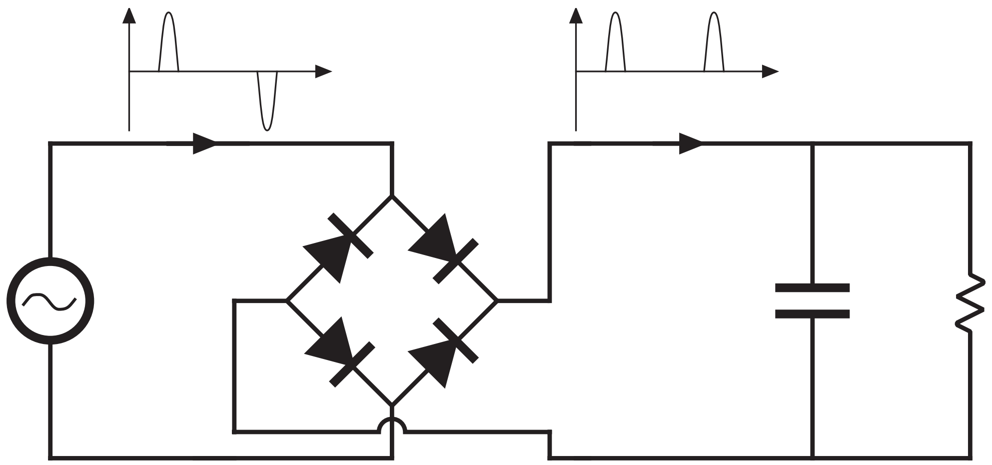


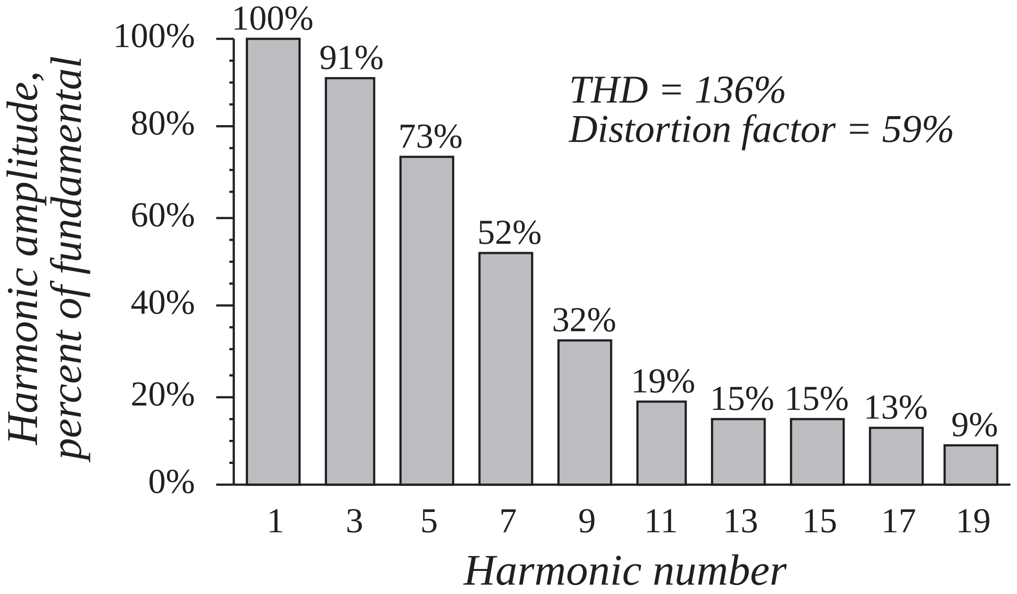
Part III. Magnetics

13. Basic magnetics theory
14. Inductor design
15. Transformer design

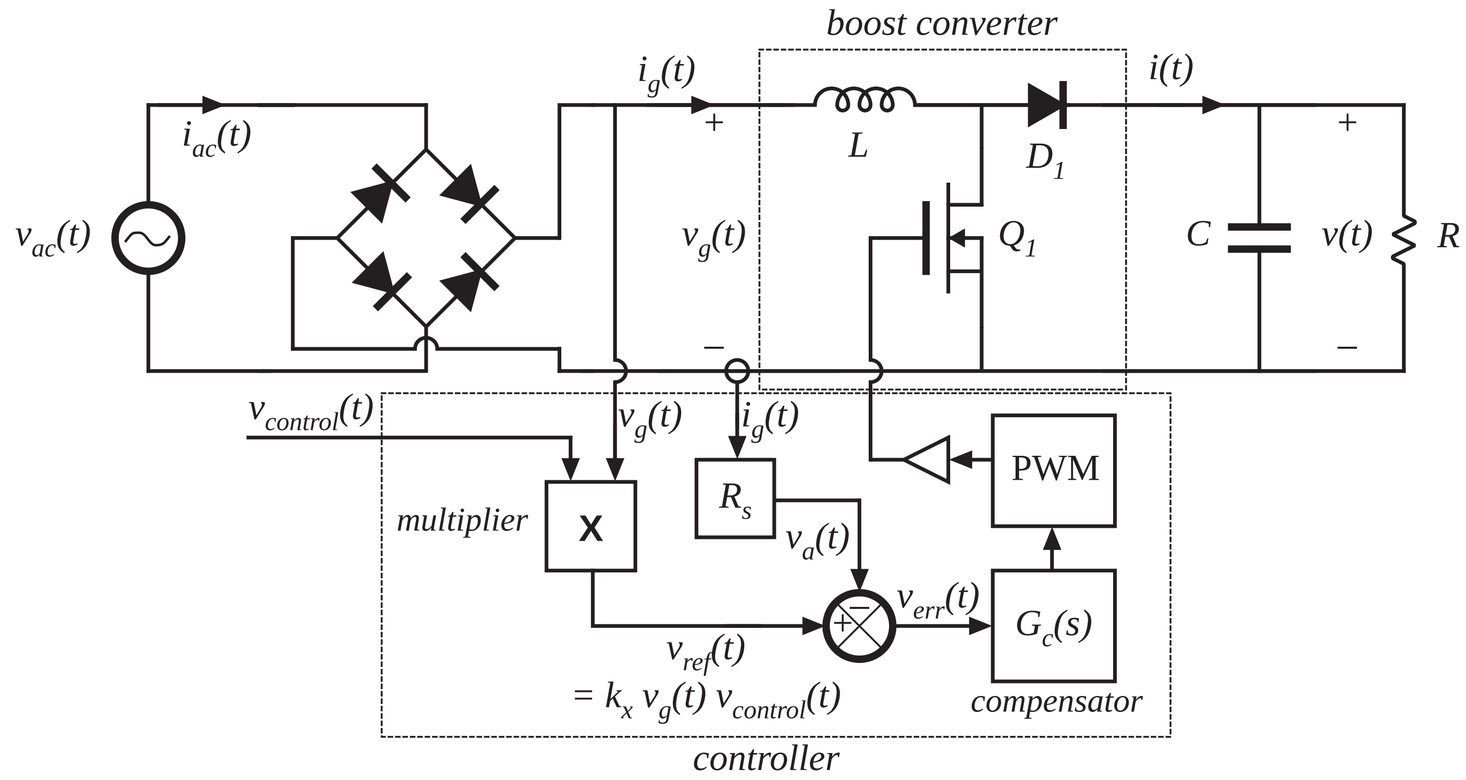
Part IV. Modern rectifiers,
and power system harmonics

Pollution of power system by rectifier current harmonics

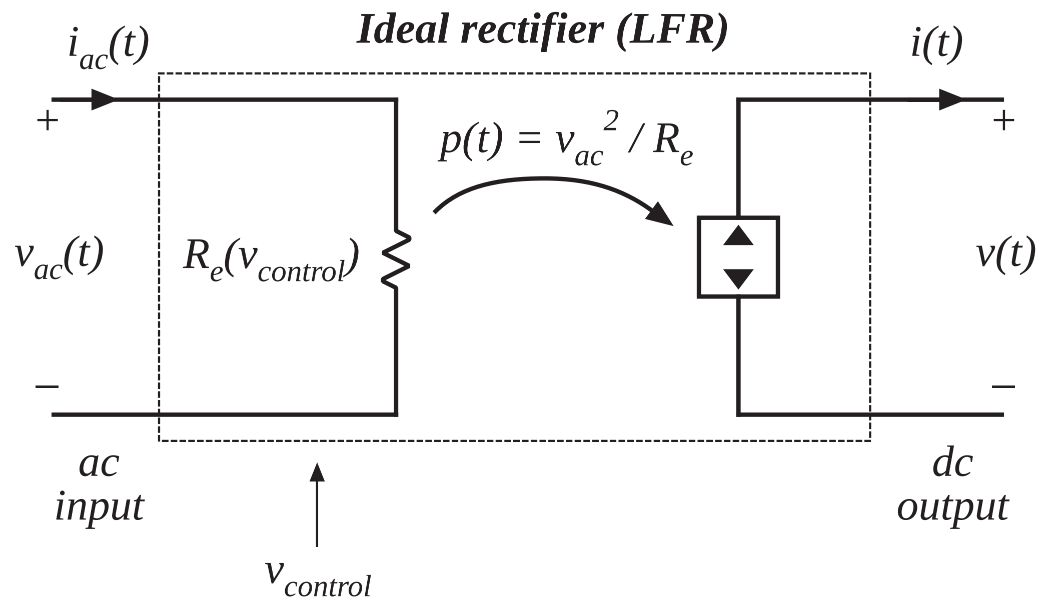




A low-harmonic rectifier system



Model of the ideal rectifier



Part IV. Modern rectifiers,
and power system harmonics

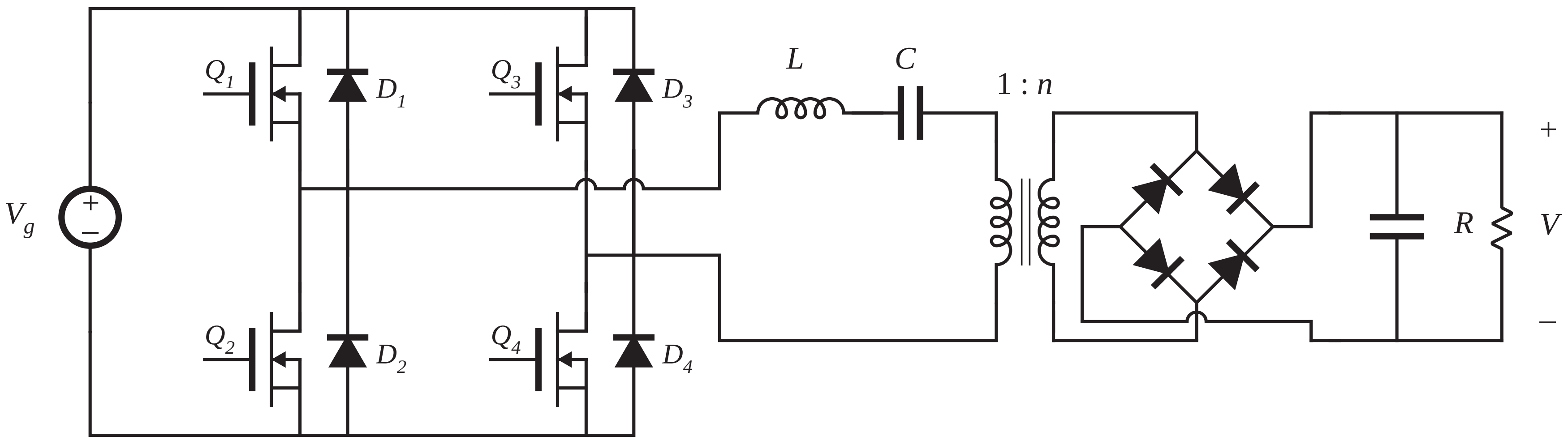
16. Power and harmonics in nonsinusoidal systems

17. Line-commutated rectifiers

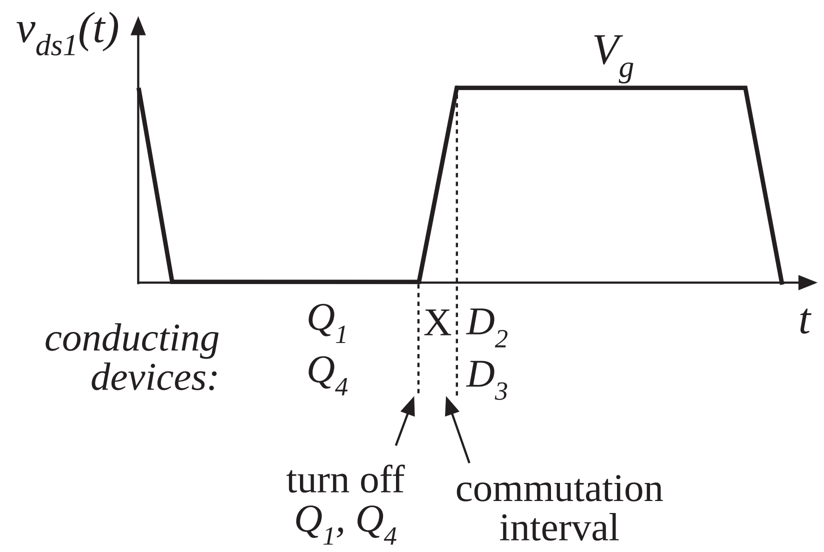
18. Pulse-width modulated rectifiers

Part V. Resonant converters

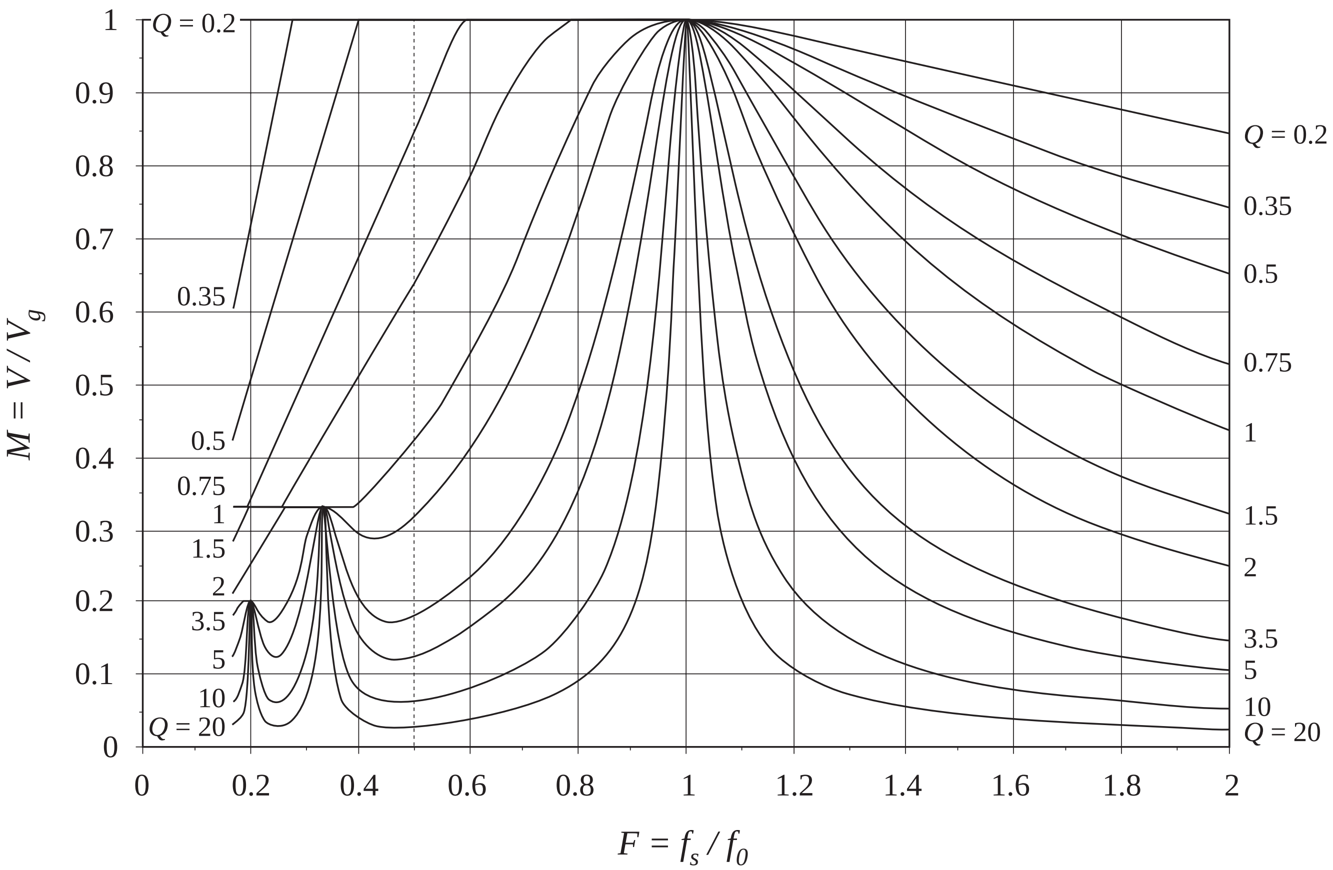
The series resonant converter



Zero voltage switching



Dc characteristics



Part V. Resonant converters

19. Resonant conversion
20. Soft switching

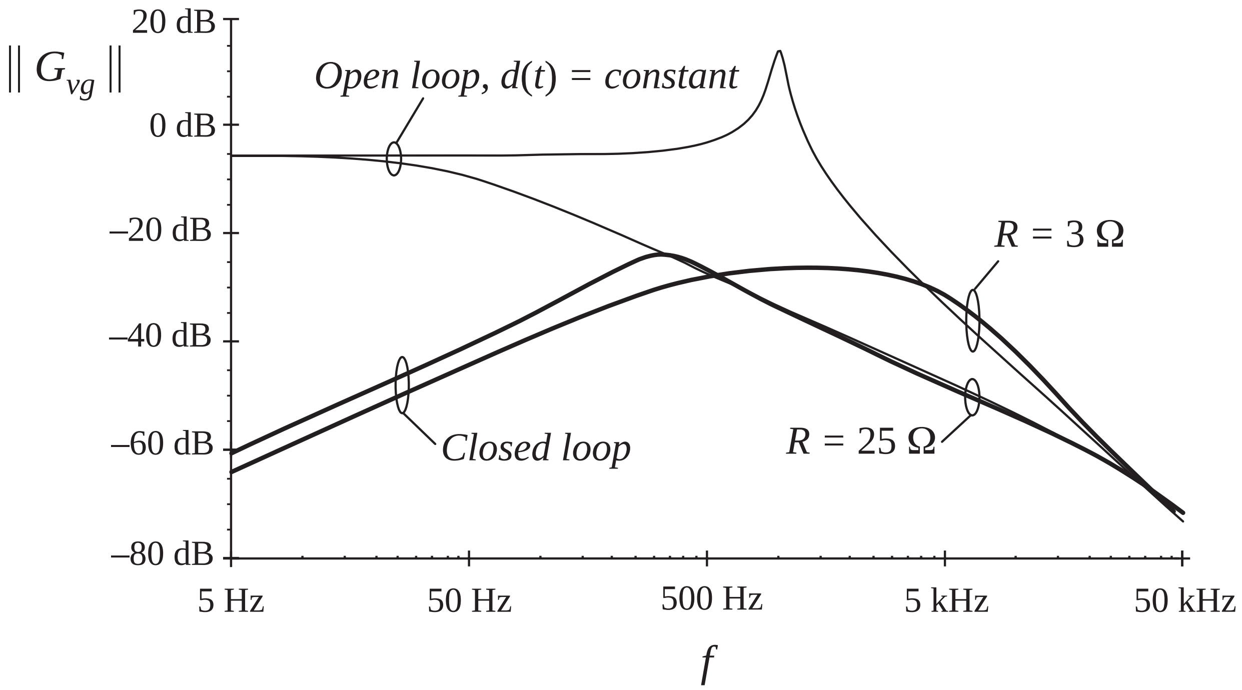
Appendices

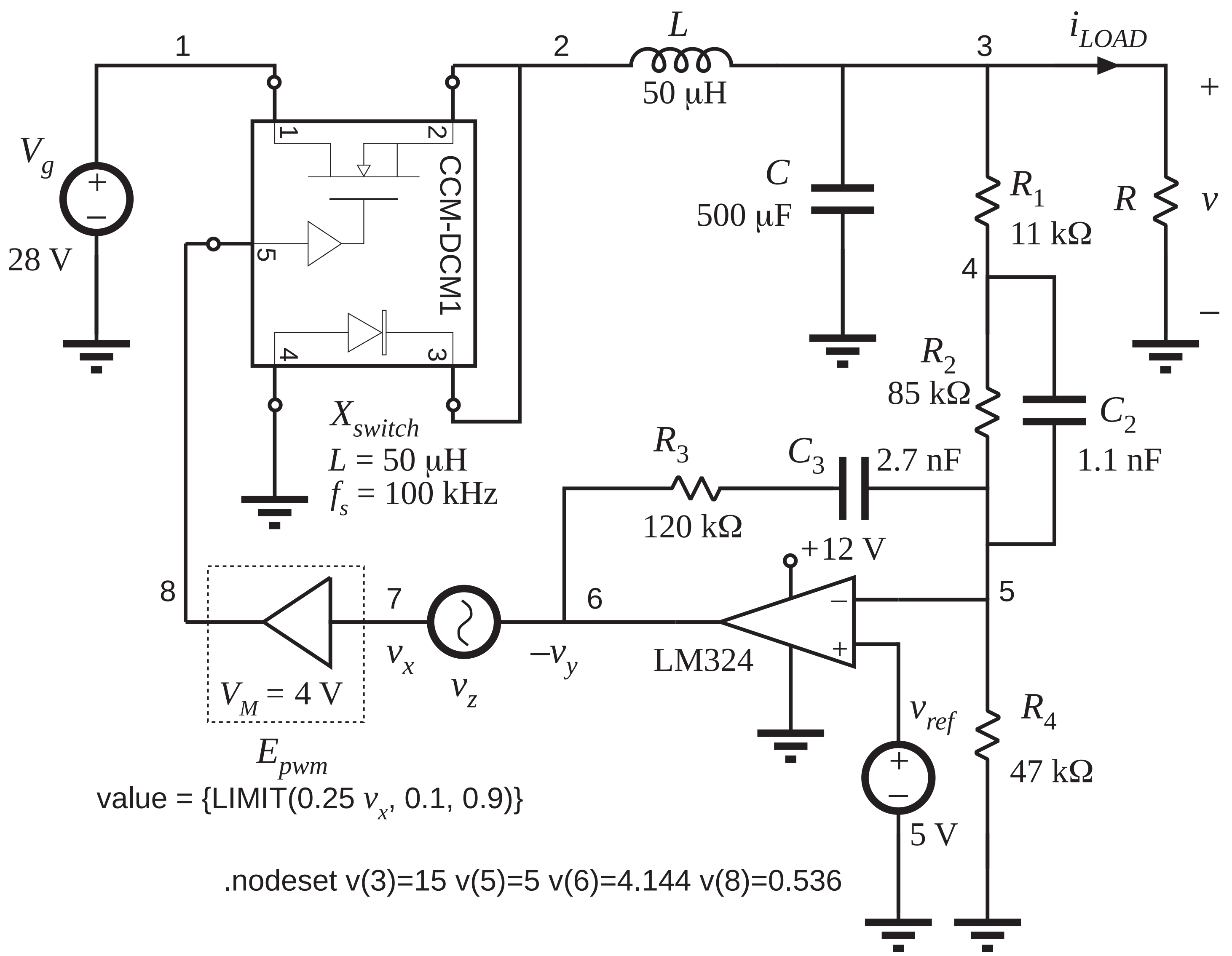
A. RMS values of commonly-observed converter waveforms

B. Simulation of converters

C. Middlebrook's extra element theorem

D. Magnetics design tables





Chapter 2
Principles of Steady-State Converter Analysis

2.1. Introduction

2.2. Inductor volt-second balance, capacitor charge balance, and the small ripple approximation

2.3. Boost converter example

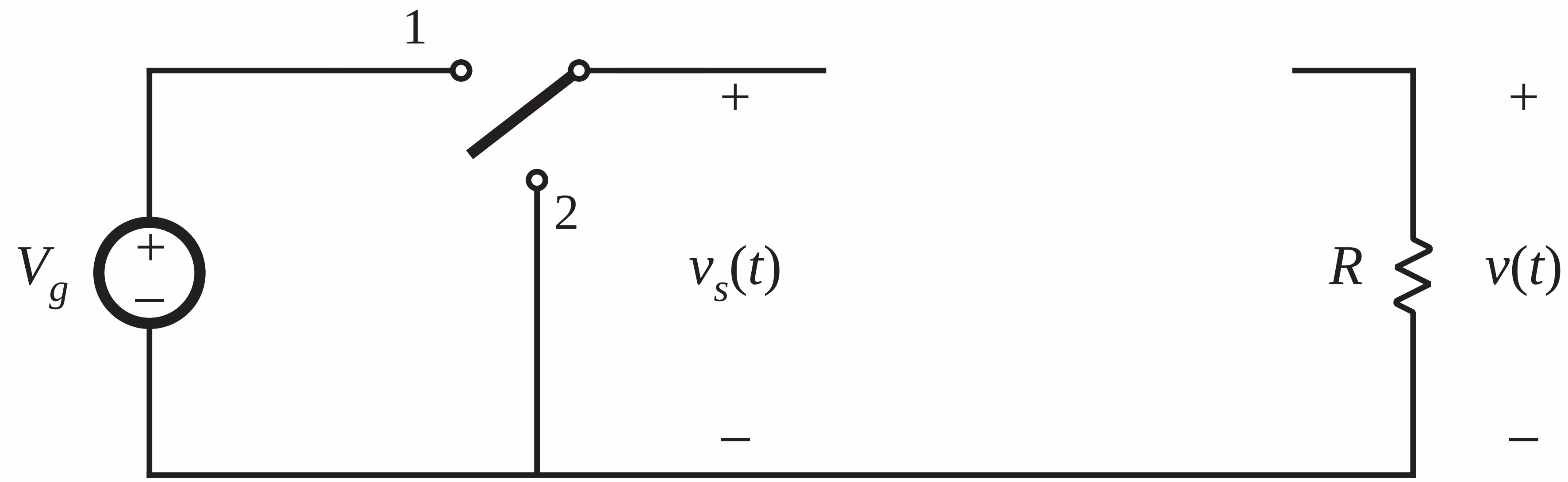
2.4. Cuk converter example

2.5. Estimating the ripple in converters containing two-pole low-pass filters

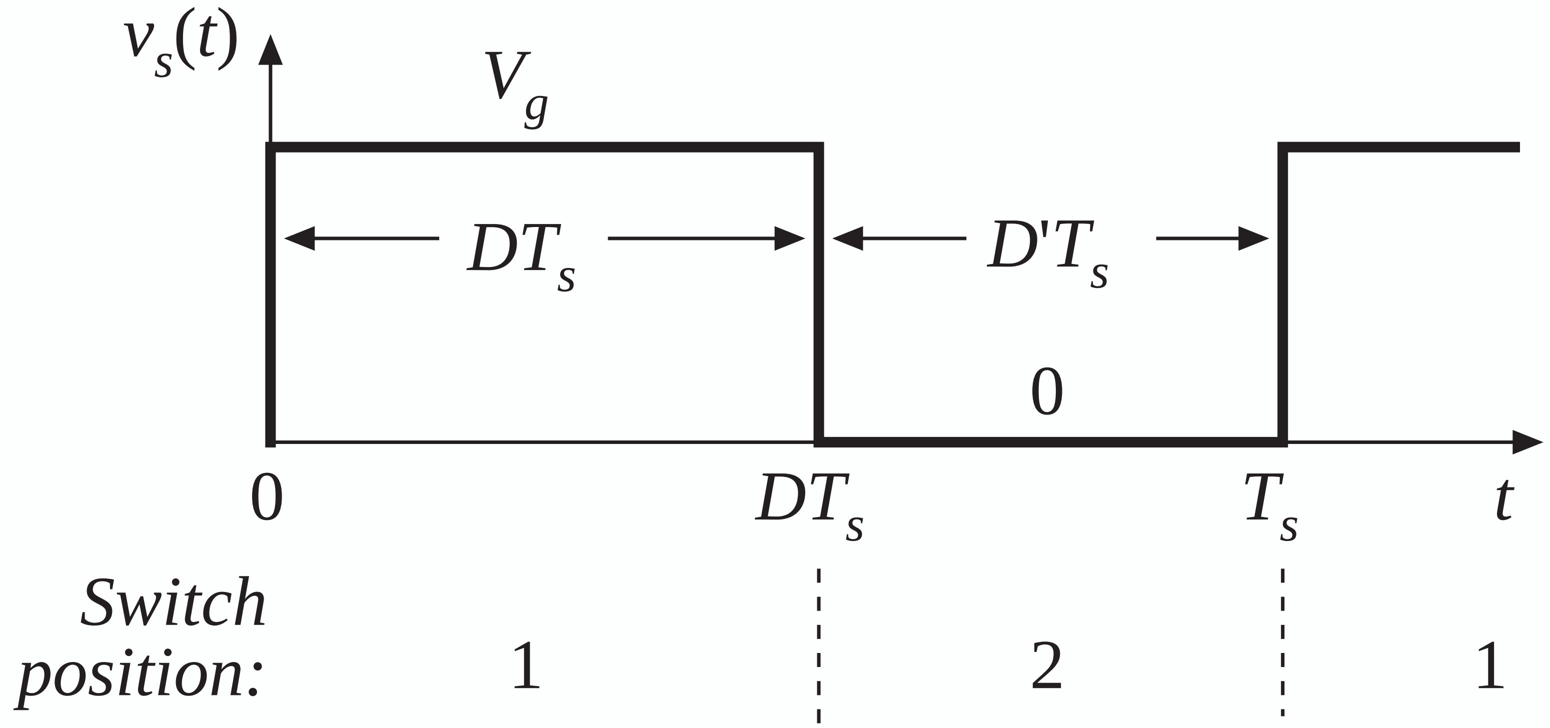
2.6. Summary of key points

2.1 Introduction
Buck converter

SPDT switch changes dc component



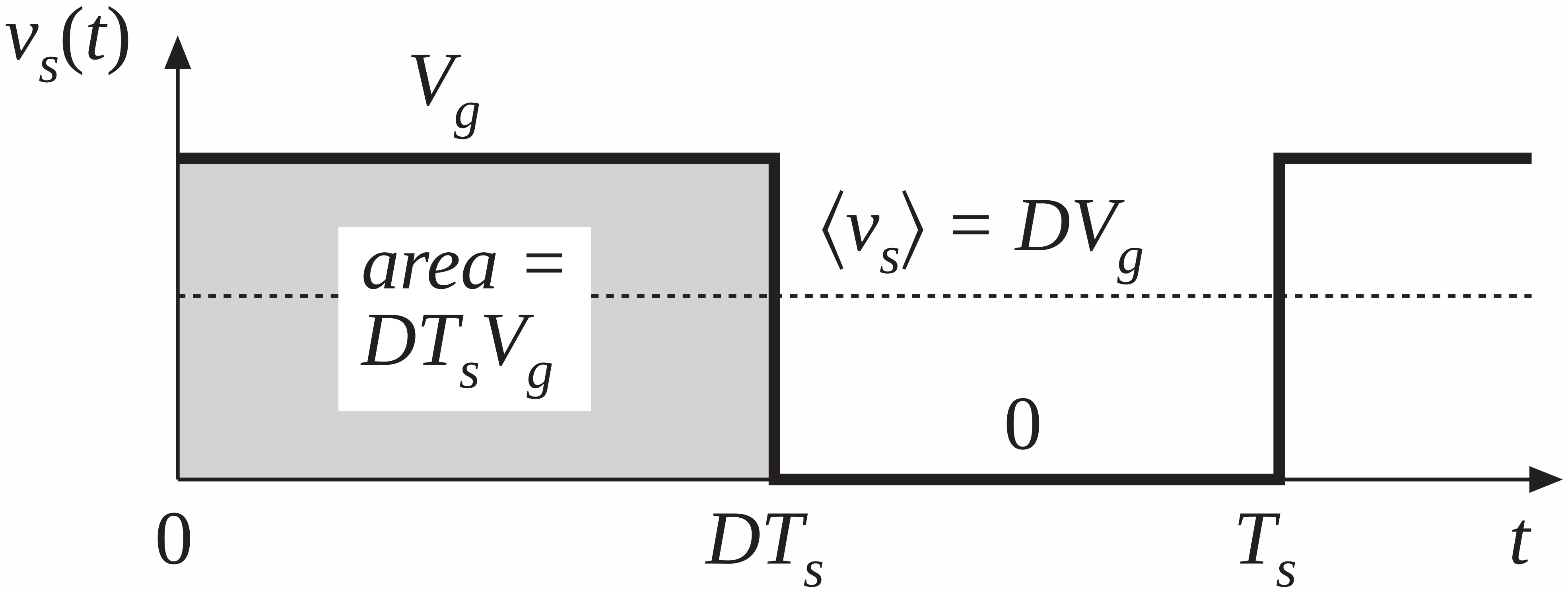
Switch output voltage waveform



Duty cycle *D:*
$0 \le D \le 1$

complement D′:
$D' = 1 - D$

Dc component of switch output voltage

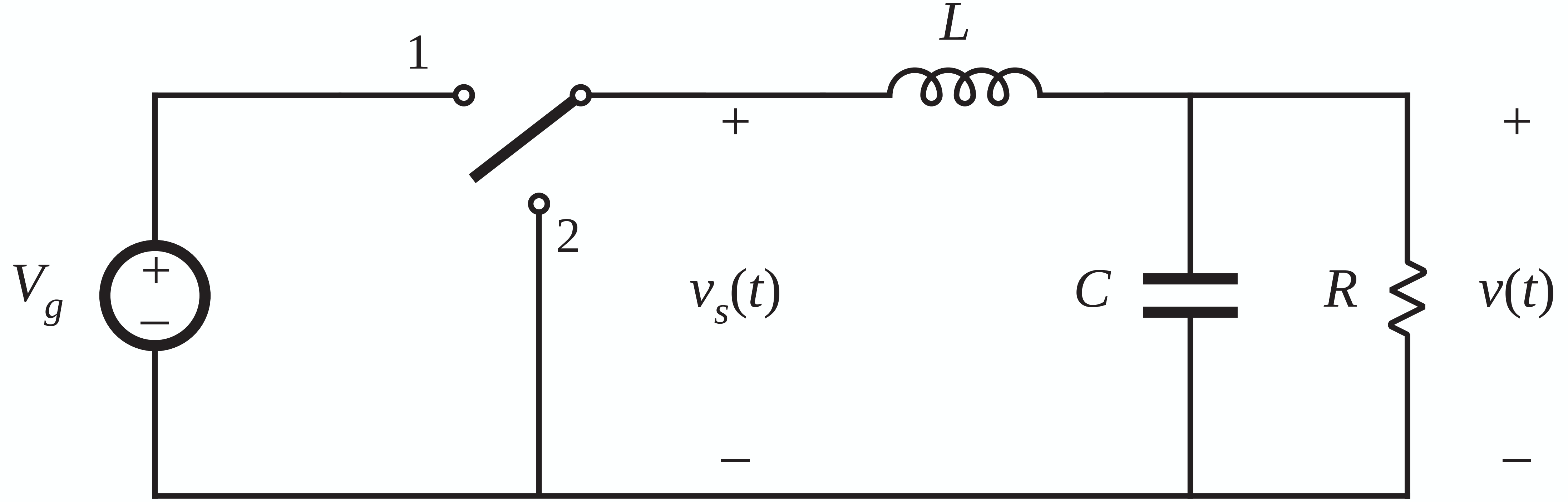


Fourier analysis: Dc component = average value

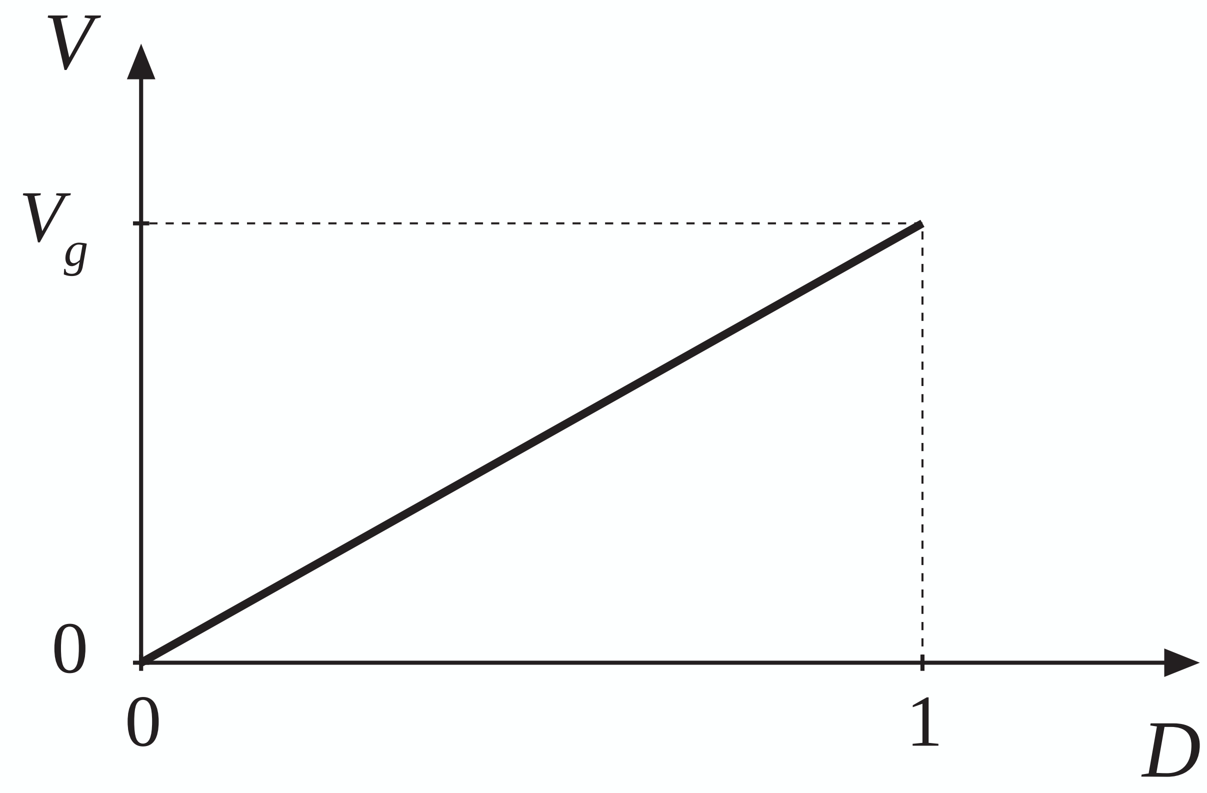
$$\langle v_s \rangle = \frac{1}{T_s}\int_0^{T_s} v_s(t)\, dt$$

$$\langle v_s \rangle = \frac{1}{T_s}\left(DT_sV_g\right) = DV_g$$

Insertion of low-pass filter to remove switching harmonics and pass only dc component

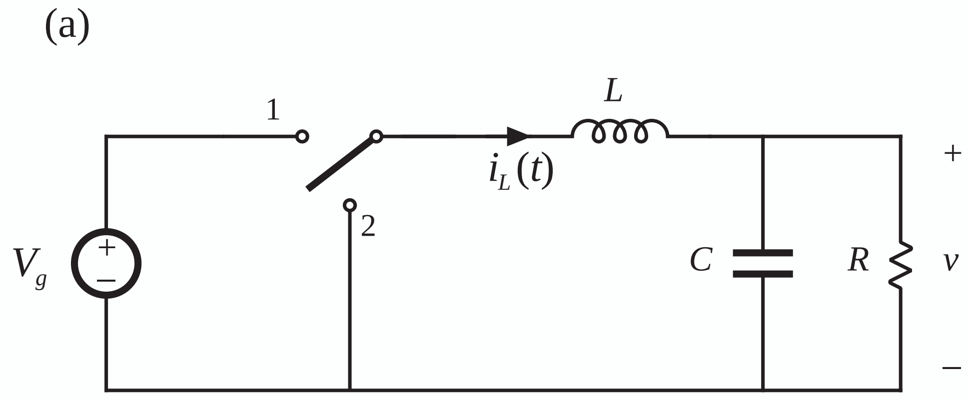


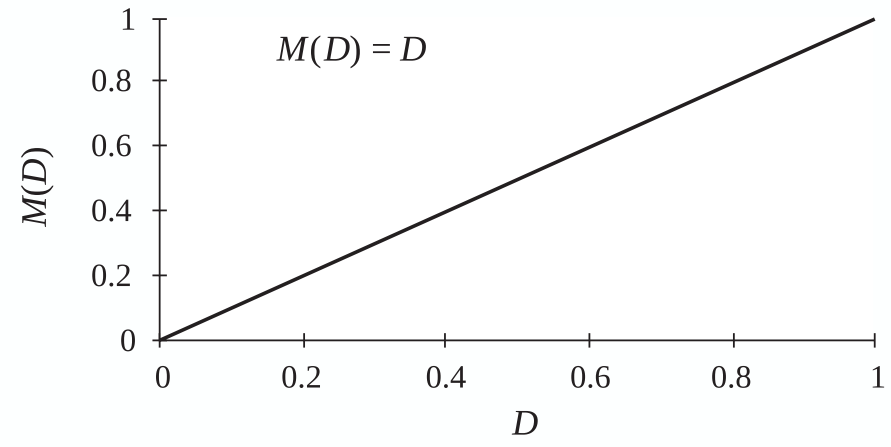
$$v \approx \langle v_s \rangle = DV_g$$



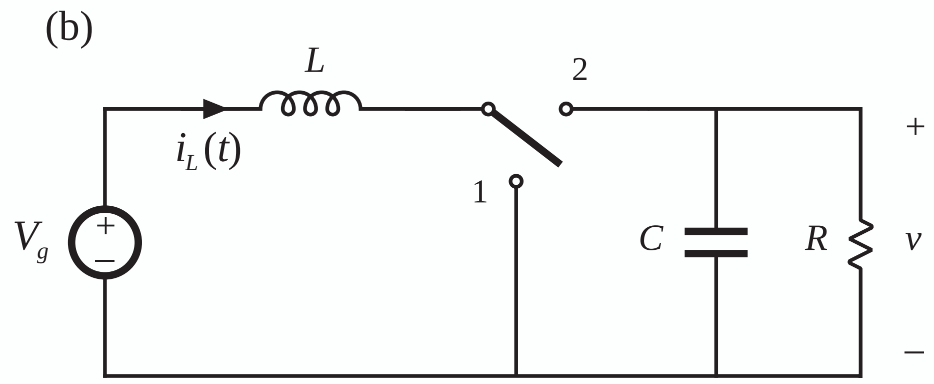
Three basic dc-dc converters

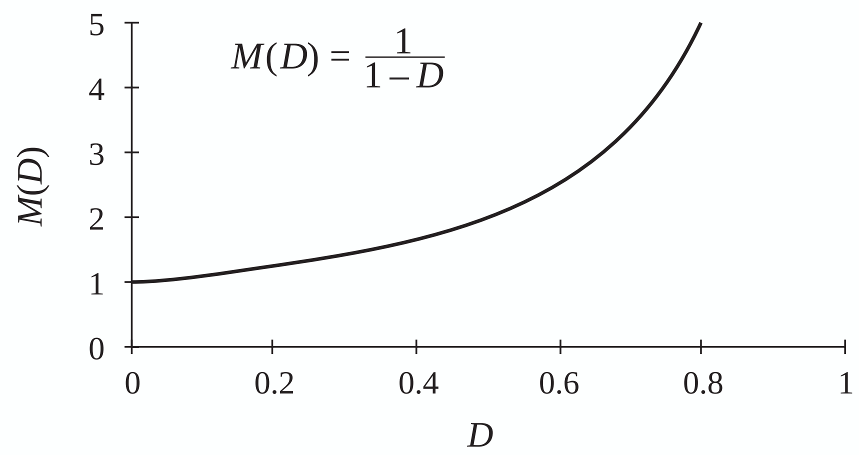
Buck



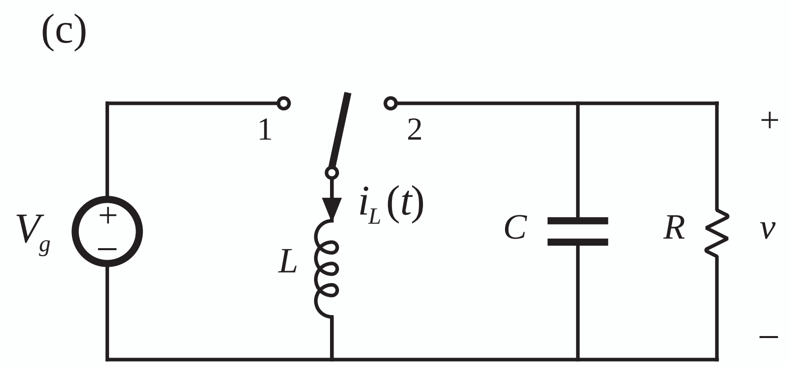


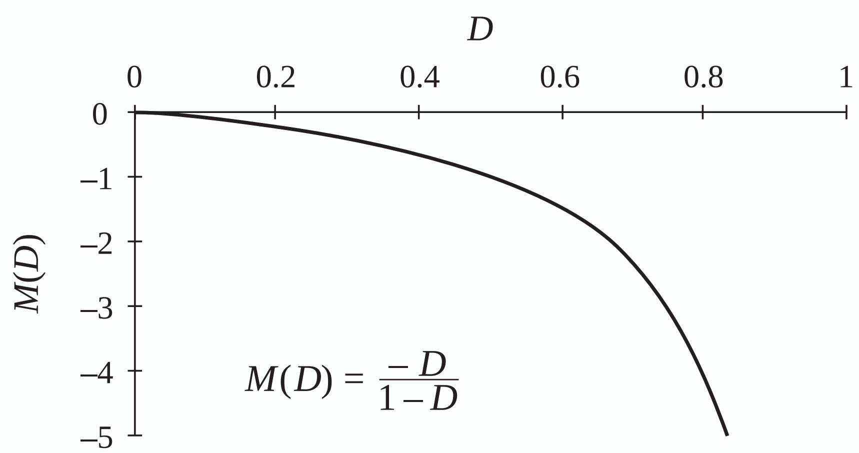
Boost





Buck-boost





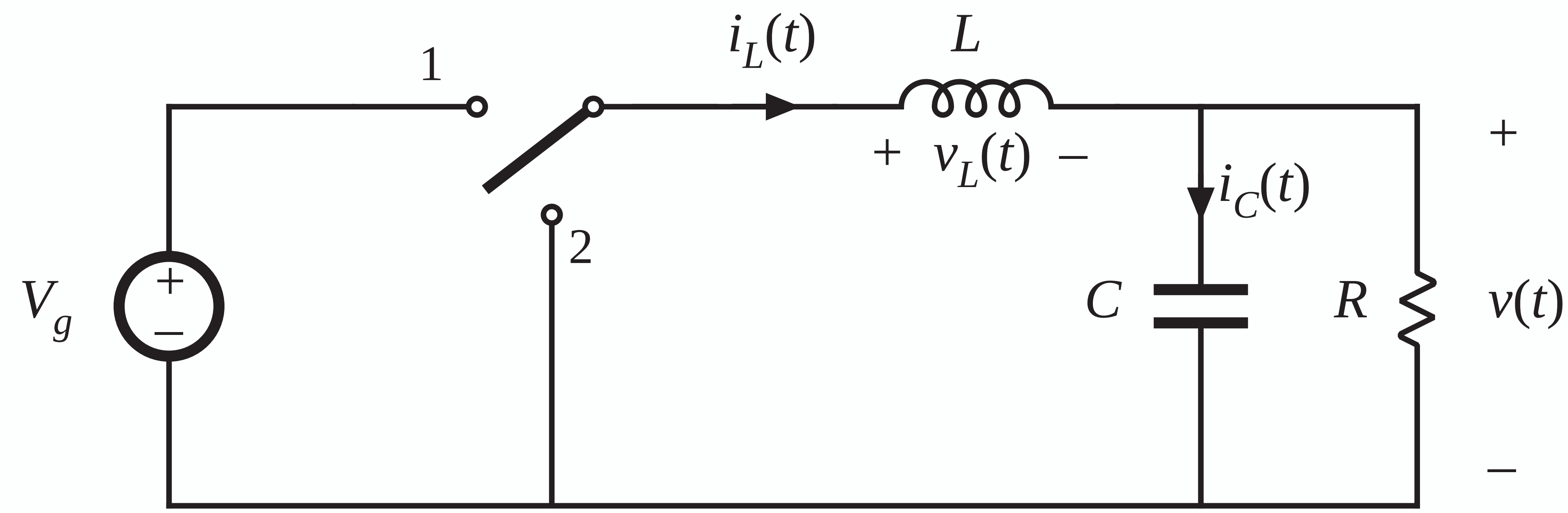
Objectives of this chapter

- Develop techniques for easily determining output voltage of an arbitrary converter circuit
- Derive the principles of *inductor volt-second balance* and *capacitor charge (amp-second) balance*
- Introduce the key *small ripple approximation*
- Develop simple methods for selecting filter element values
- Illustrate via examples

2.2. Inductor volt-second balance, capacitor charge balance, and the small ripple approximation

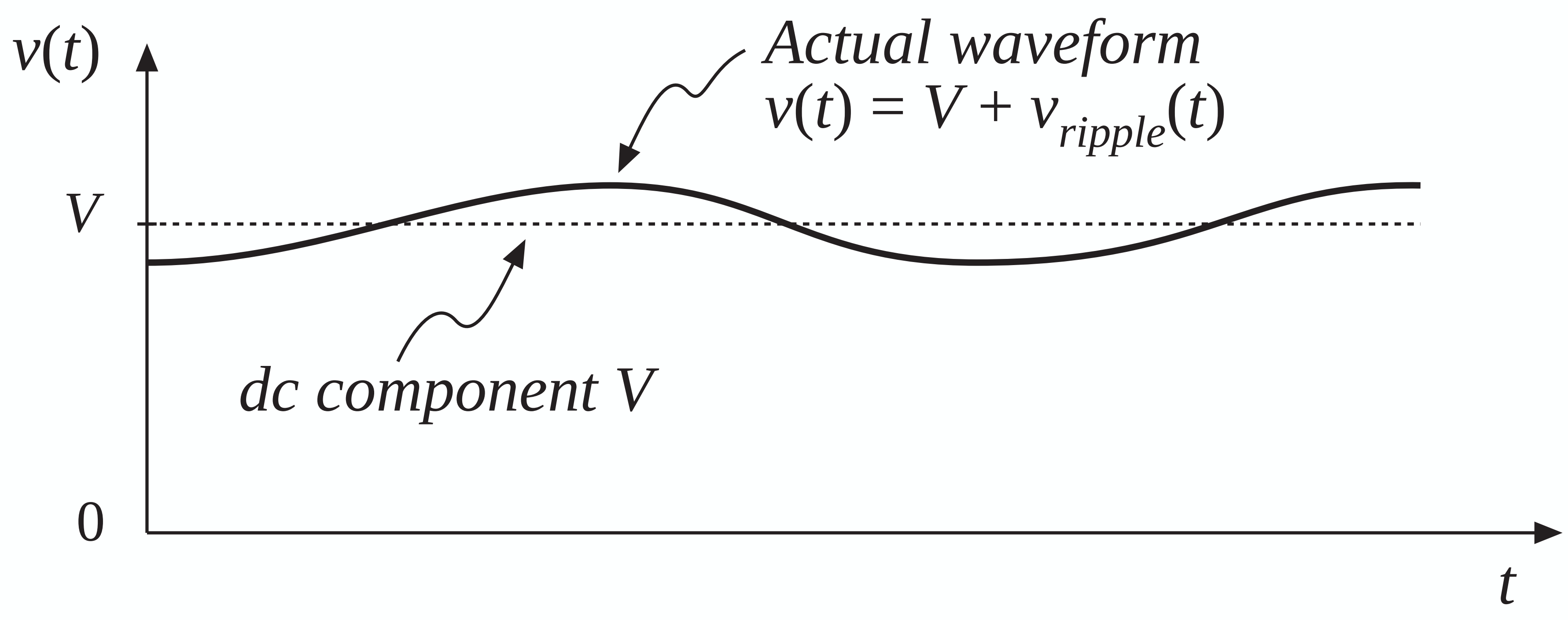
Actual output voltage waveform, buck converter

Buck converter containing practical low-pass filter



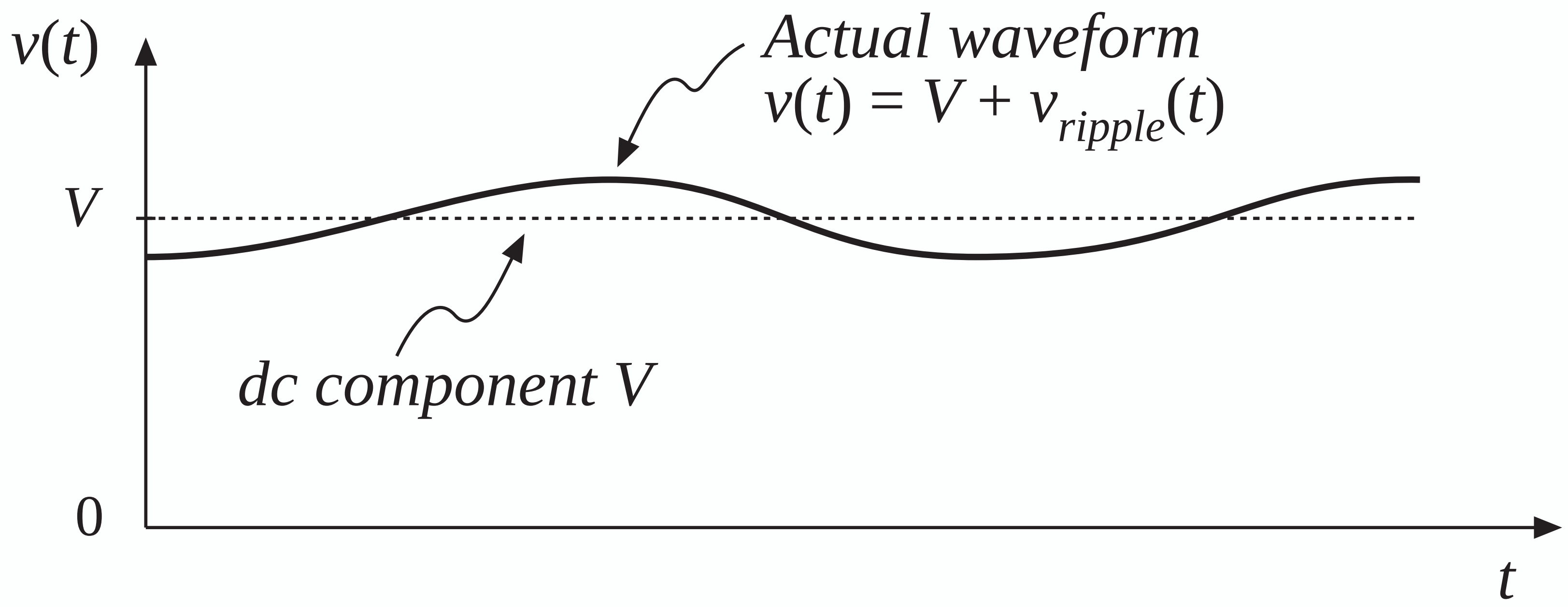
Actual output voltage waveform

$$v(t) = V + v_{ripple}(t)$$



The small ripple approximation

$$v(t) = V + v_{ripple}(t)$$



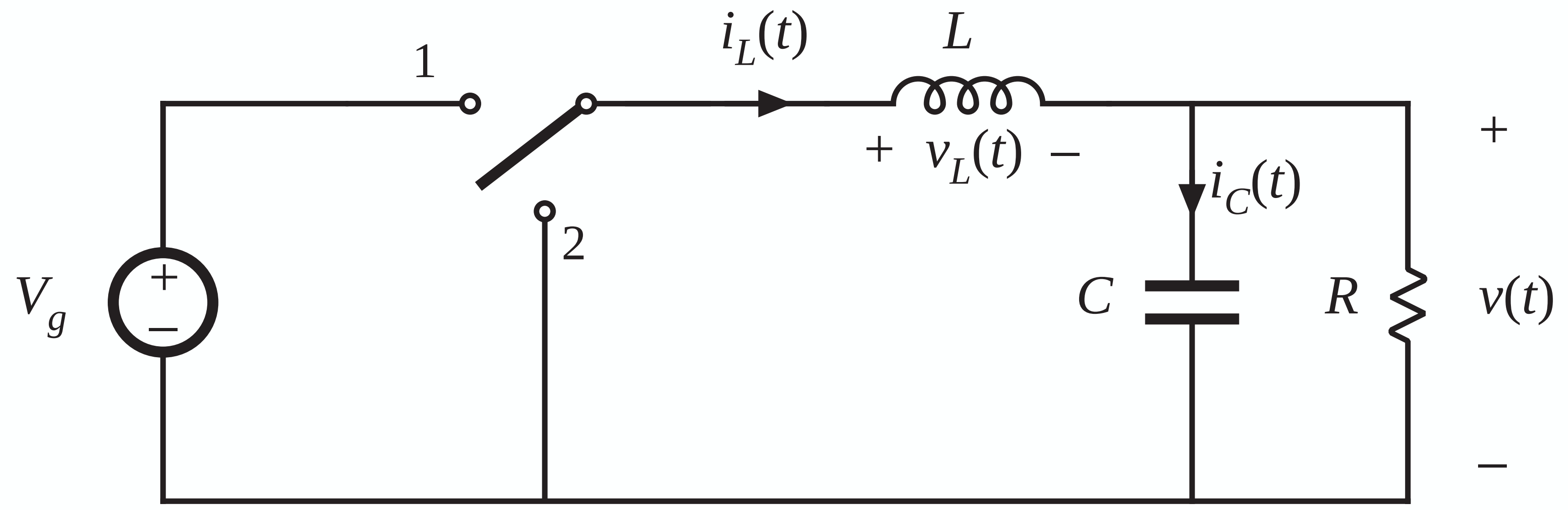
In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple:

$$\left\| v_{ripple} \right\| \ll V$$

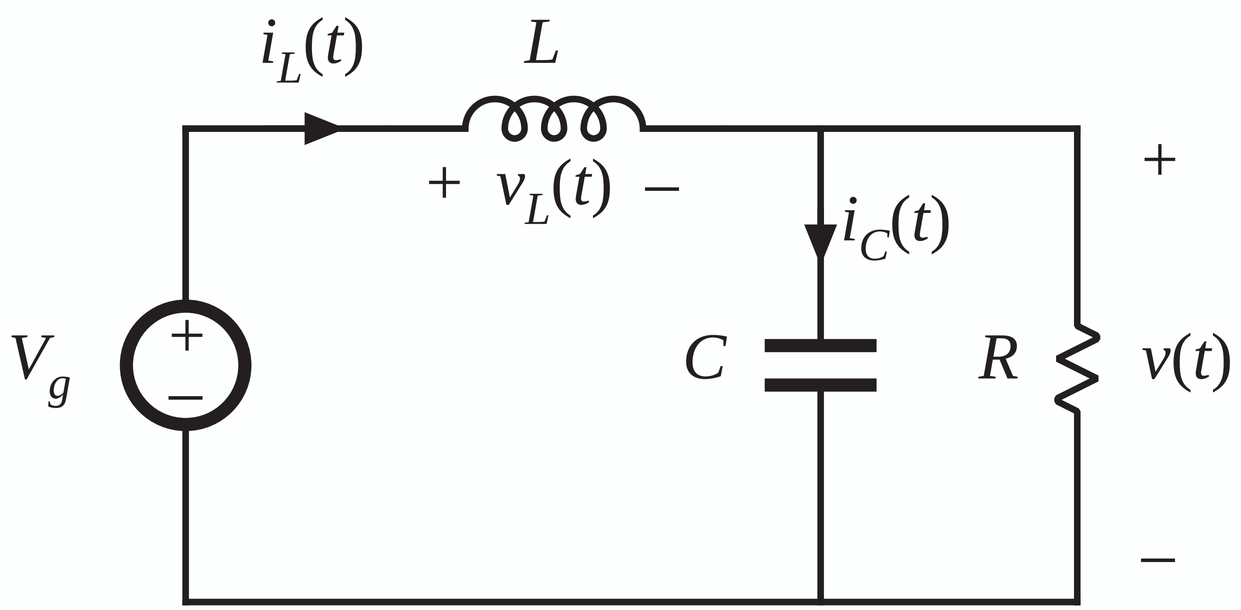
$$v(t) \approx V$$

Buck converter analysis: inductor current waveform

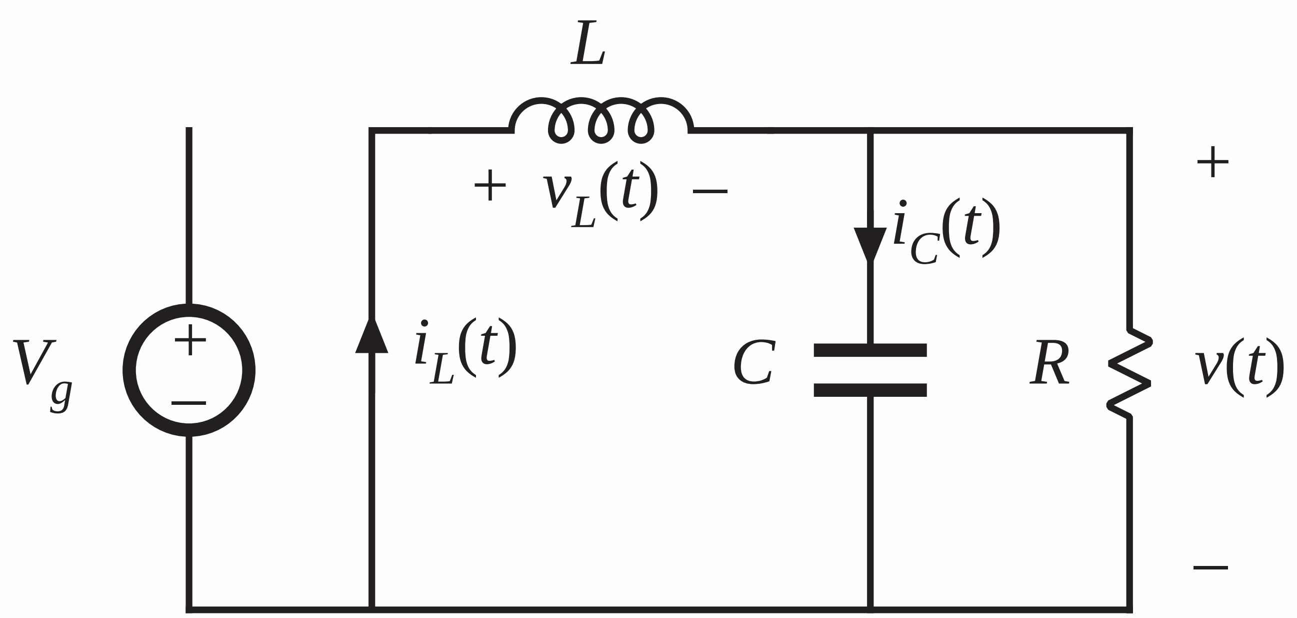
original converter



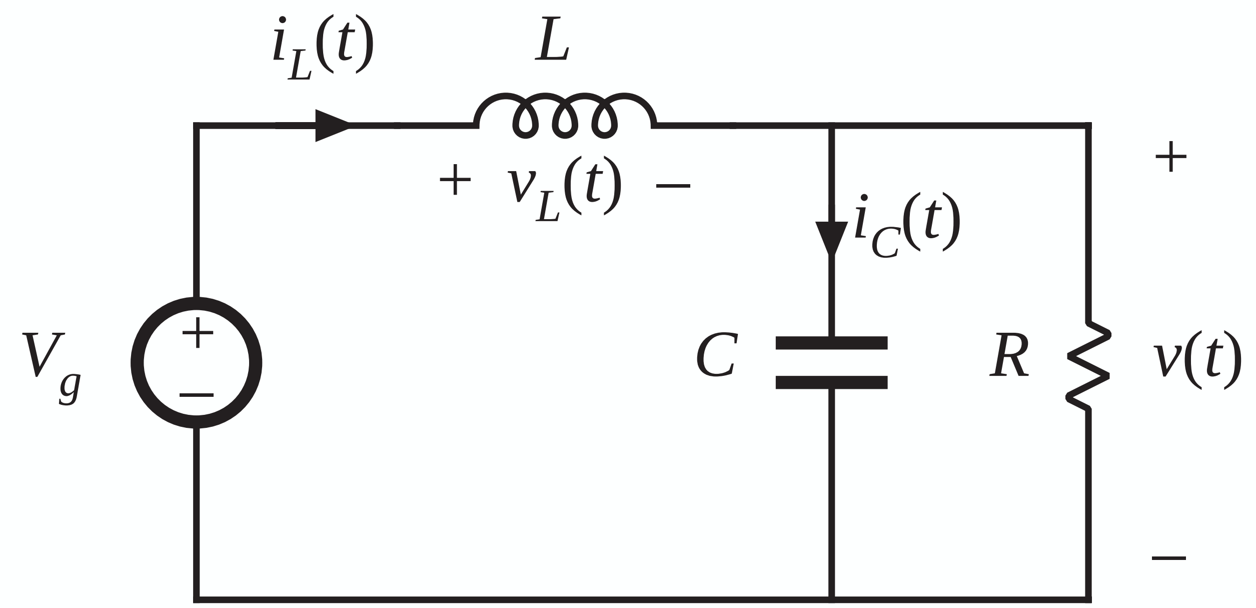
switch in position 1



switch in position 2



Inductor voltage and current
Subinterval 1: switch in position 1



Inductor voltage

$$v_L = V_g - v(t)$$

Small ripple approximation:

$$v_L \approx V_g - V$$

Knowing the inductor voltage, we can now find the inductor current via

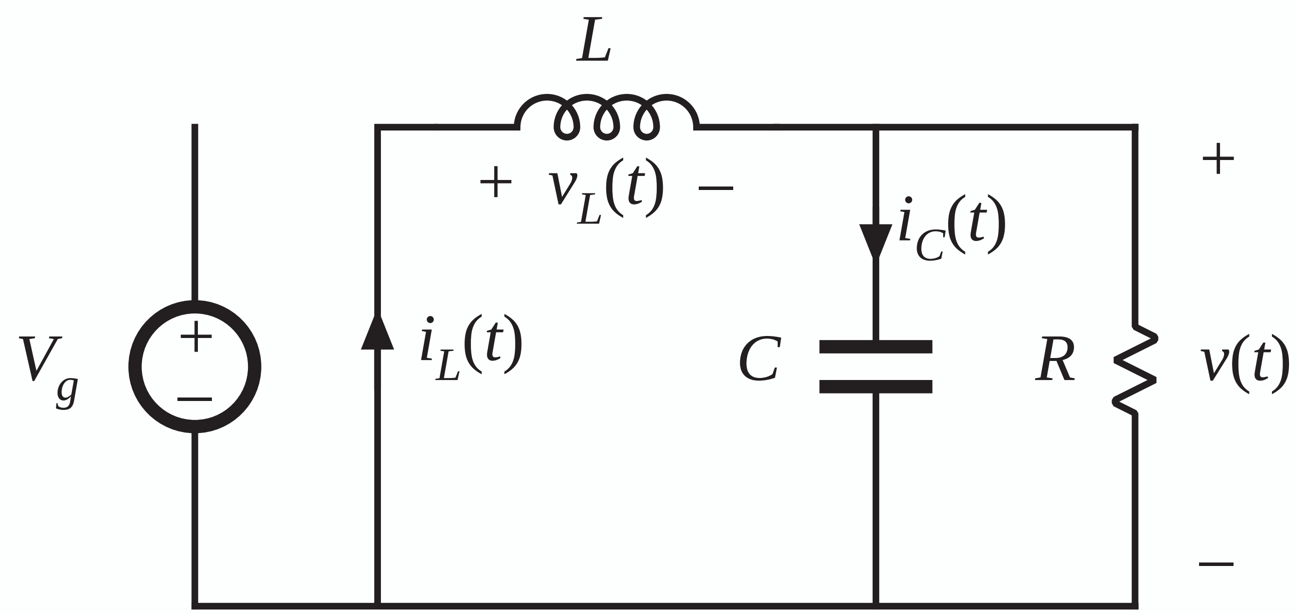
$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$

$\Rightarrow$ *The inductor current changes with an essentially constant slope*

Inductor voltage and current
Subinterval 2: switch in position 2



Inductor voltage

$$v_L(t) = -\,v(t)$$

Small ripple approximation:

$$v_L(t) \approx -\,V$$

Knowing the inductor voltage, we can again find the inductor current via

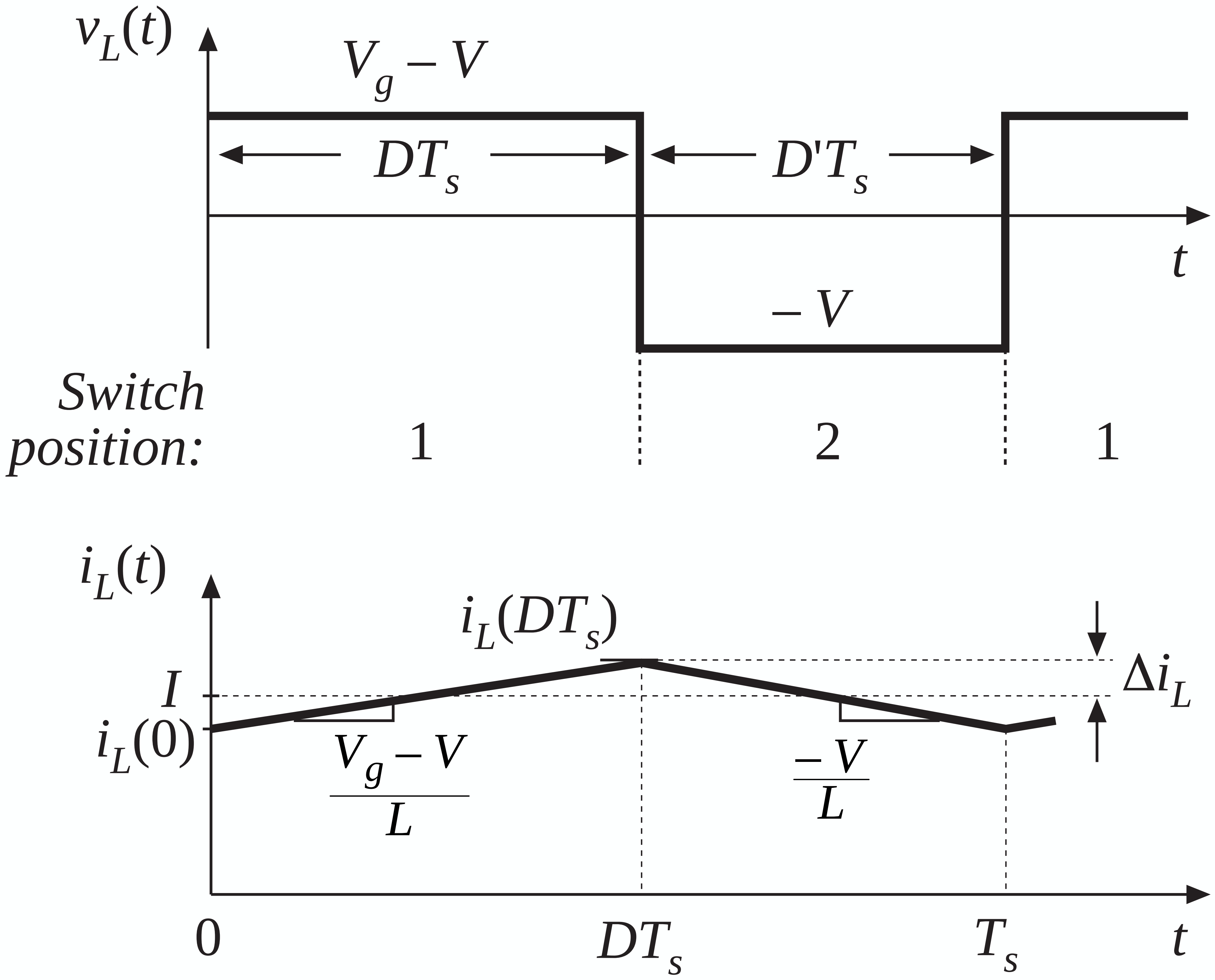
$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} \approx -\,\frac{V}{L}$$

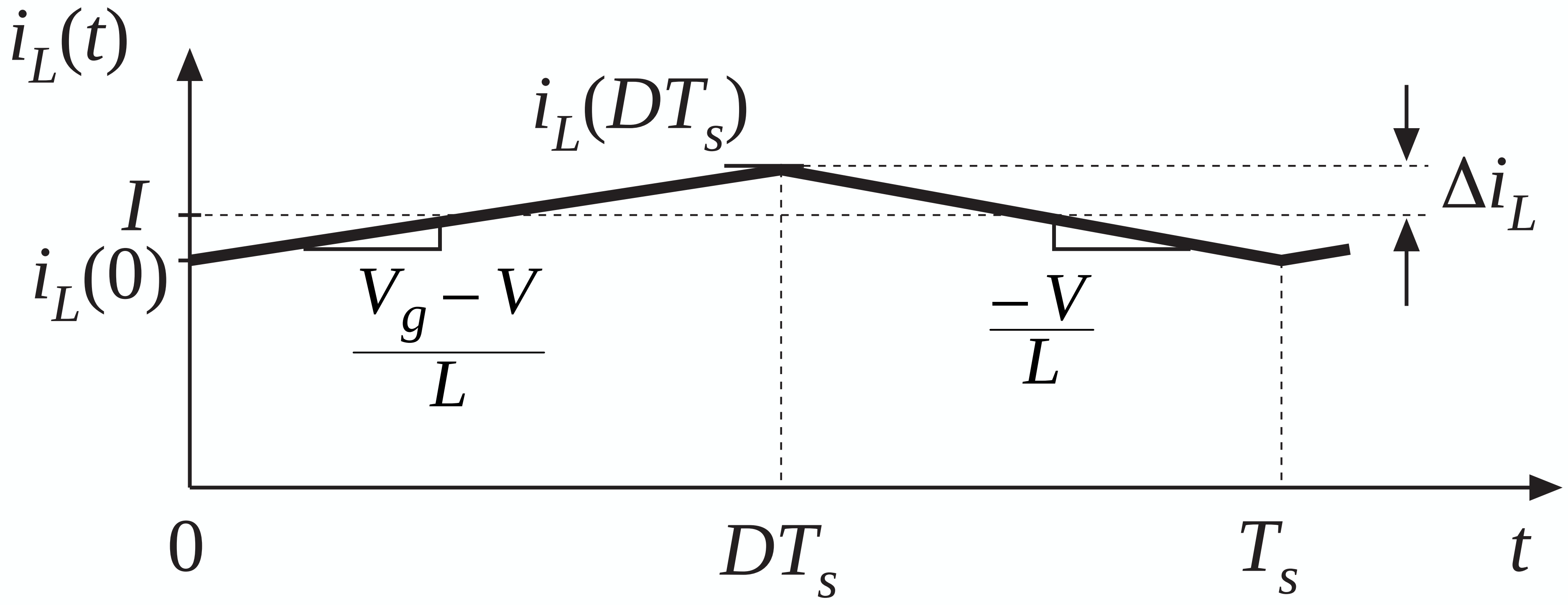
$\Rightarrow$ *The inductor current changes with an essentially constant slope*

Inductor voltage and current waveforms



$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Determination of inductor current ripple magnitude

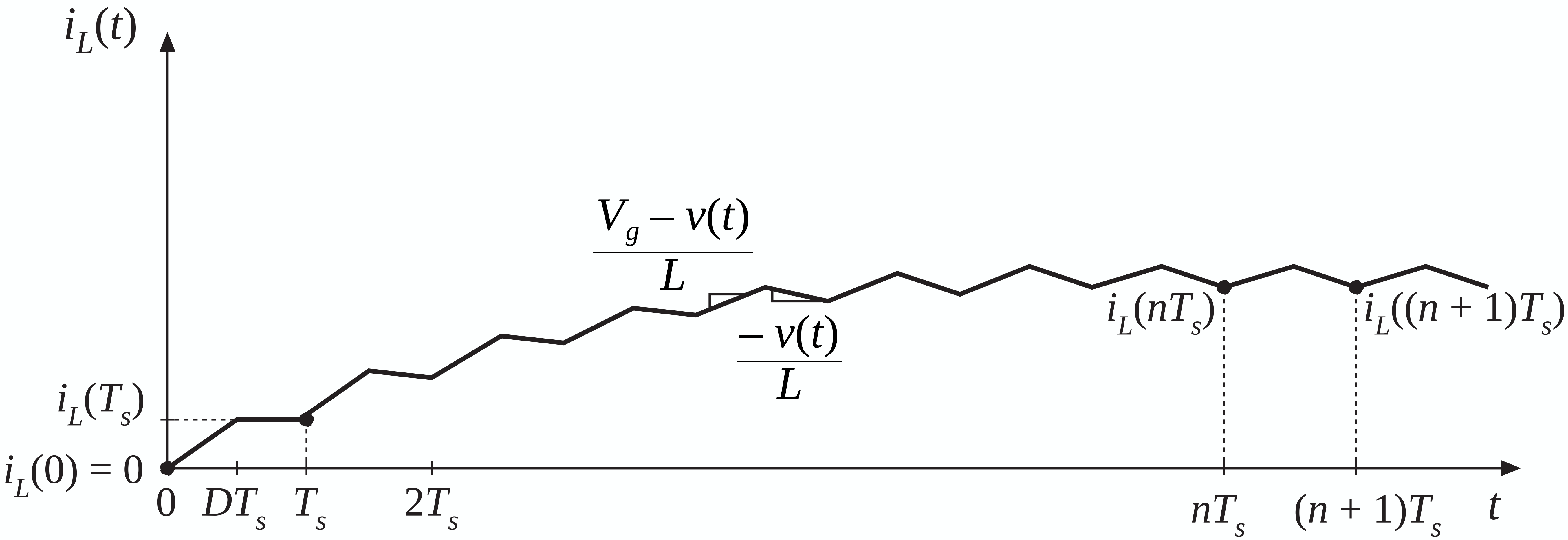


$$(\textit{change in } i_L) = (\textit{slope})(\textit{length of subinterval})$$

$$\left(2\Delta i_L\right) = \left(\frac{V_g - V}{L}\right)\left(DT_s\right)$$

$$\Rightarrow \qquad \Delta i_L = \frac{V_g - V}{2L} DT_s \qquad\qquad L = \frac{V_g - V}{2\Delta i_L} DT_s$$

Inductor current waveform during turn-on transient



When the converter operates in equilibrium:

$$i_L((n+1)T_s) = i_L(nT_s)$$

The principle of inductor volt-second balance: Derivation

Inductor defining relation:

$$v_L(t) = L \frac{di_L(t)}{dt}$$

Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L} \int_0^{T_s} v_L(t)\, dt$$

In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t)\, dt$$

Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

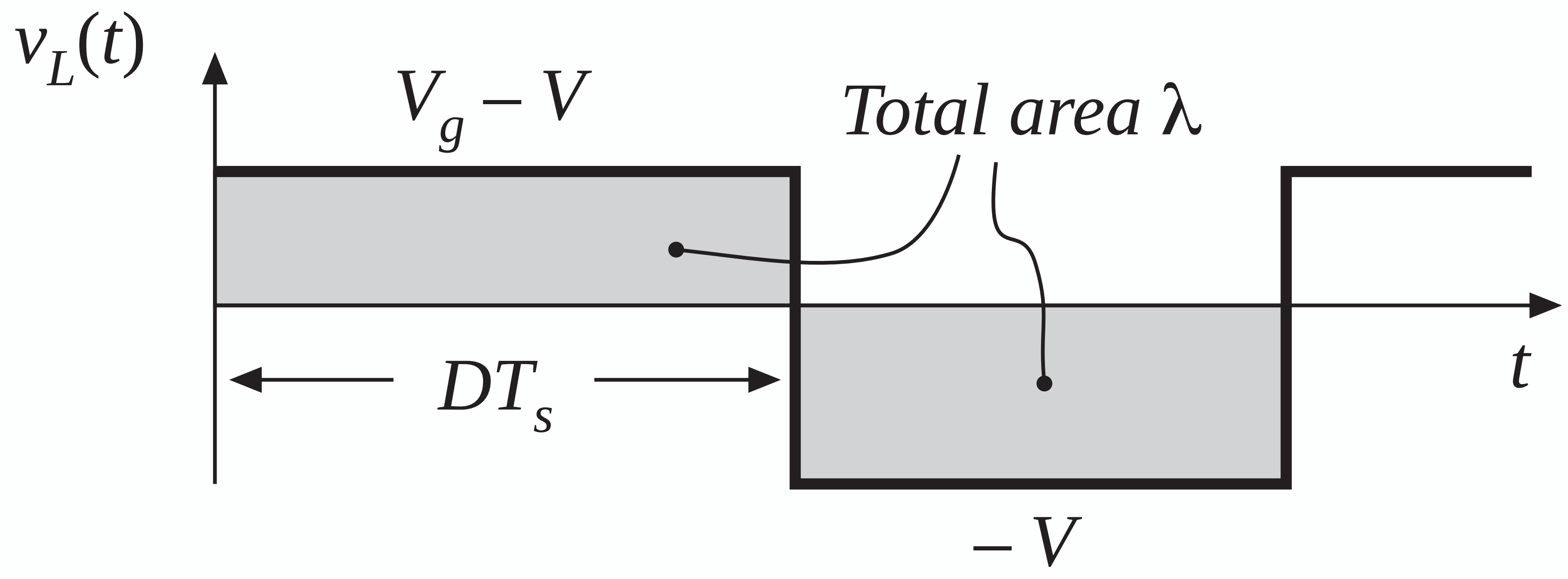
An equivalent form:

$$0 = \frac{1}{T_s} \int_0^{T_s} v_L(t)\, dt = \left\langle v_L \right\rangle$$

The average inductor voltage is zero in steady state.

Inductor volt-second balance: Buck converter example

Inductor voltage waveform, previously derived:



Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\, dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

Average voltage is

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for *V:*

$$0 = DV_g - (D + D')V = DV_g - V \qquad \Rightarrow \qquad V = DV_g$$

The principle of capacitor charge balance: Derivation

Capacitor defining relation:

$$i_C(t) = C\,\frac{dv_C(t)}{dt}$$

Integrate over one complete switching period:

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)\,dt$$

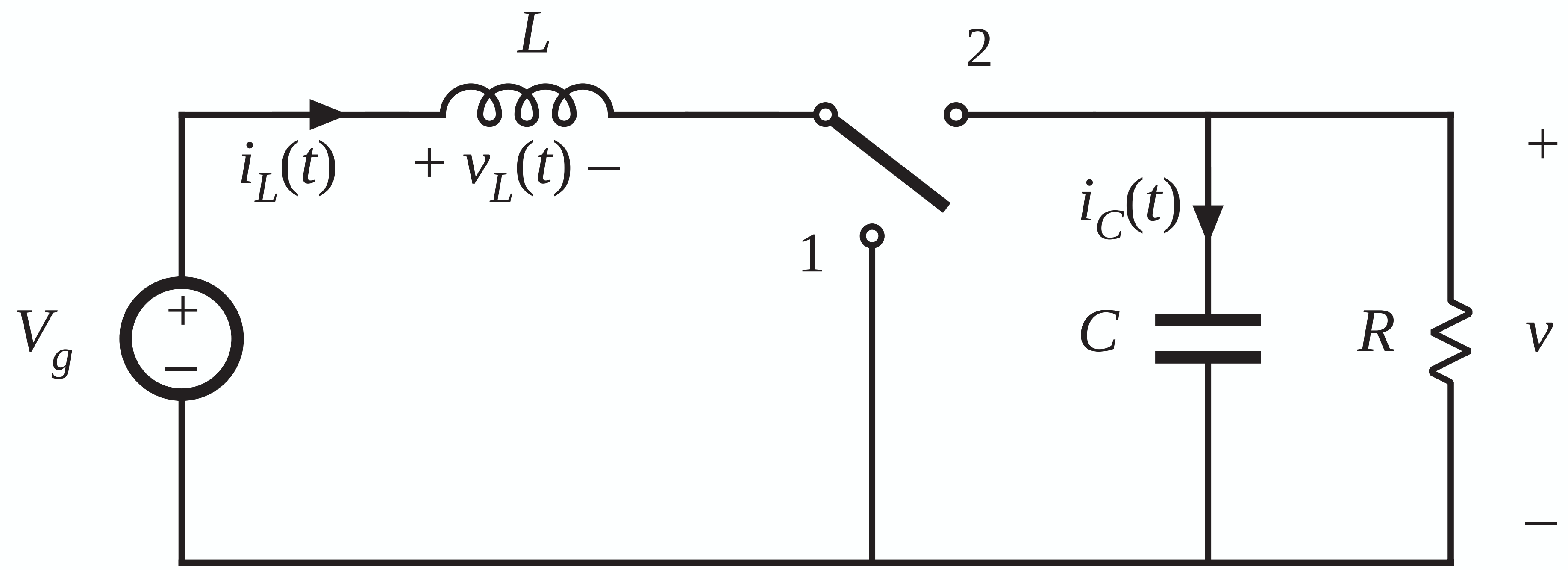
In periodic steady state, the net change in capacitor voltage is zero:

$$0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = \left\langle i_C \right\rangle$$

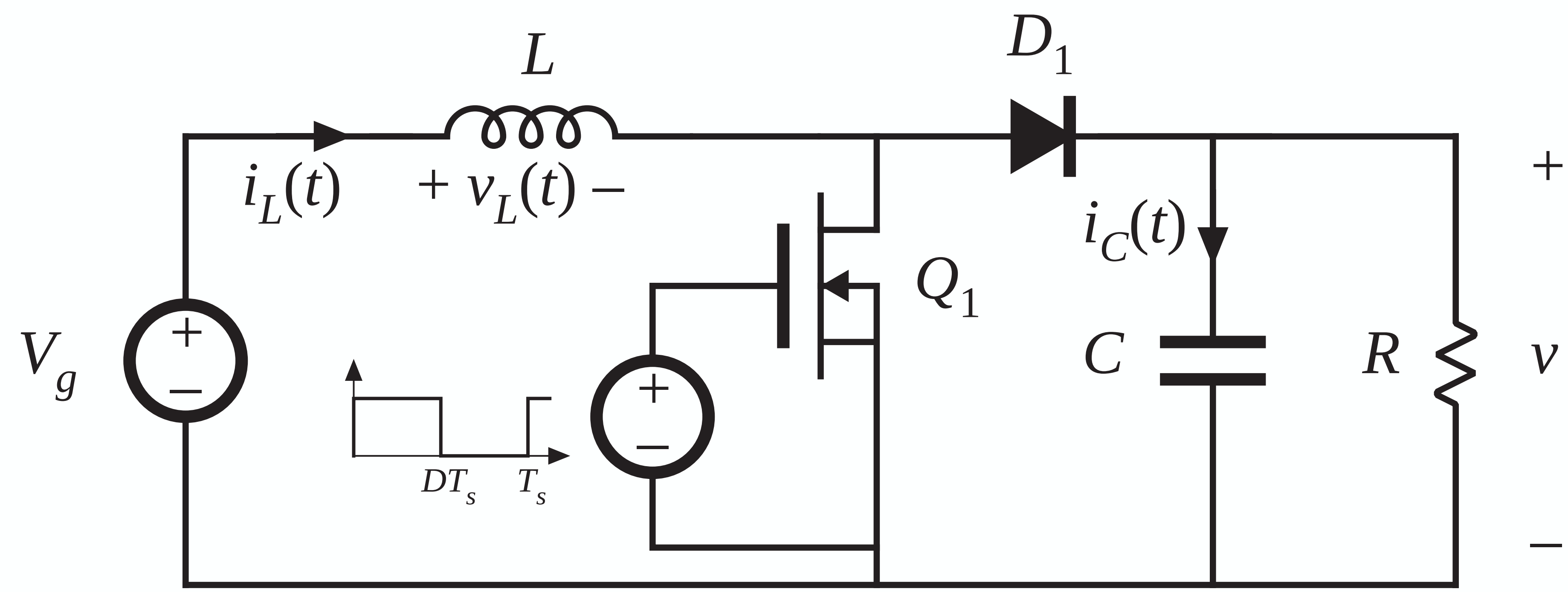
Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.

2.3 Boost converter example

Boost converter with ideal switch

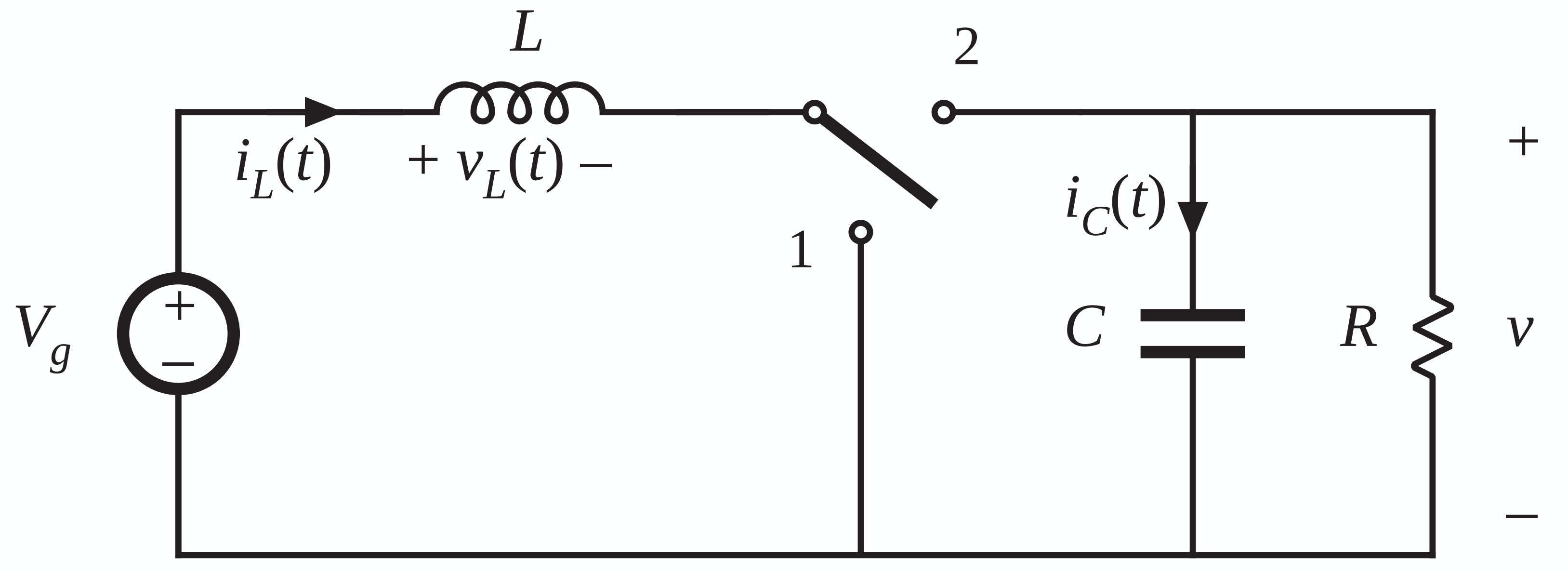


Realization using power MOSFET and diode

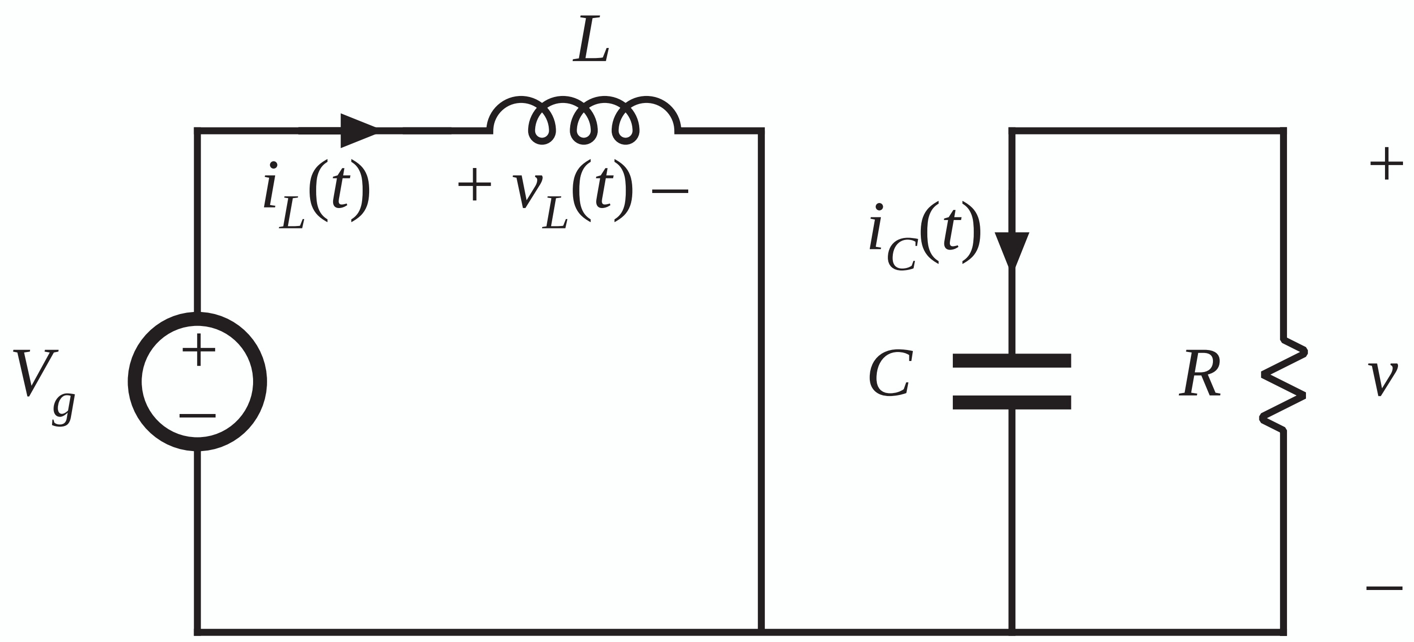


Boost converter analysis

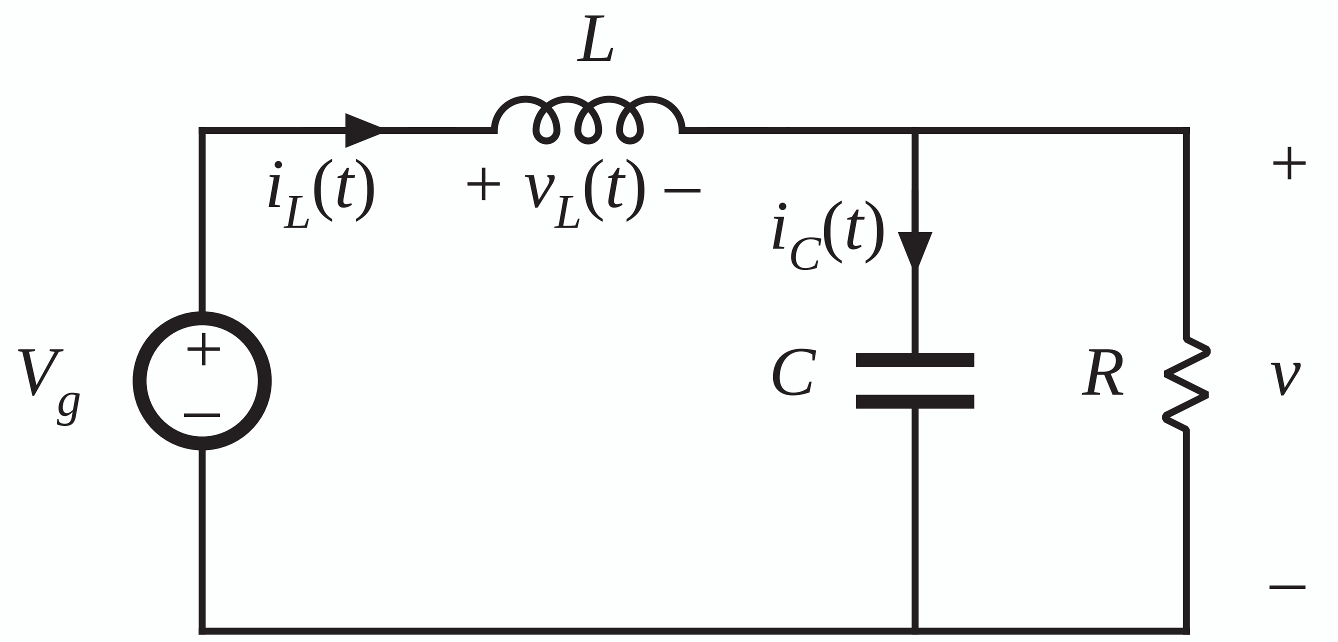
original converter



switch in position 1



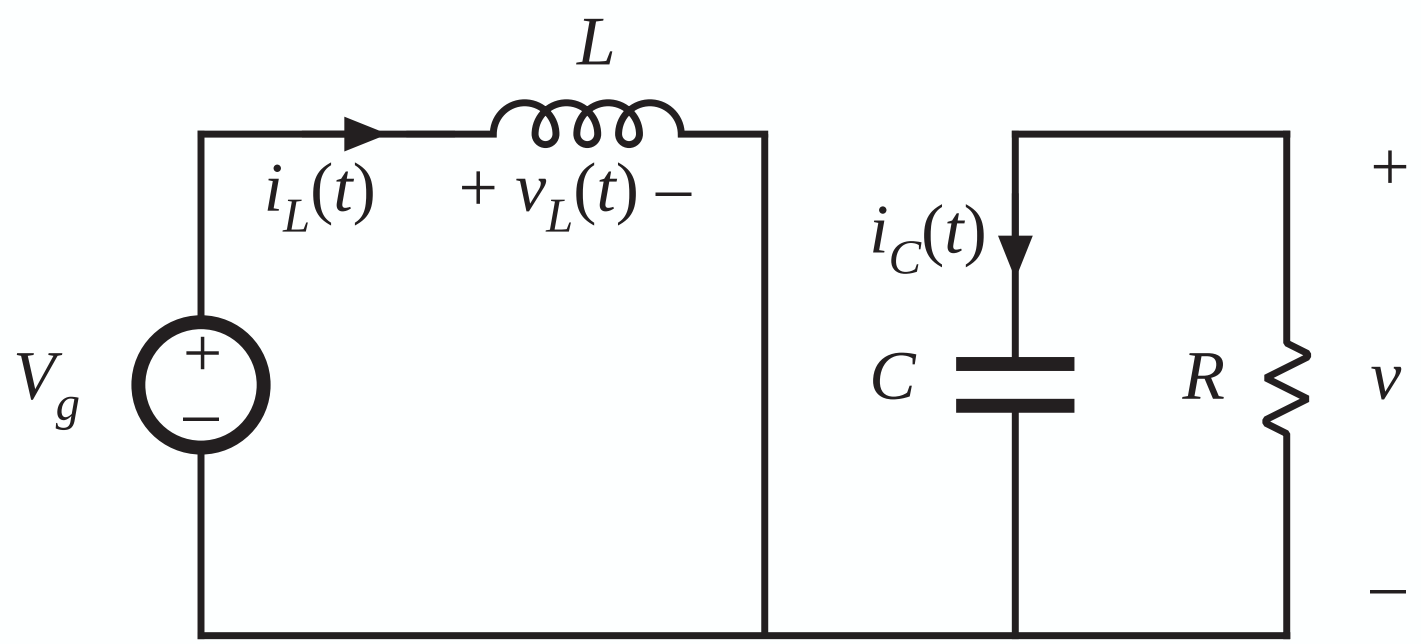
switch in position 2



Subinterval 1: switch in position 1

Inductor voltage and capacitor current

$$v_L = V_g$$
$$i_C = -v / R$$



Small ripple approximation:

$$v_L = V_g$$
$$i_C = -V / R$$

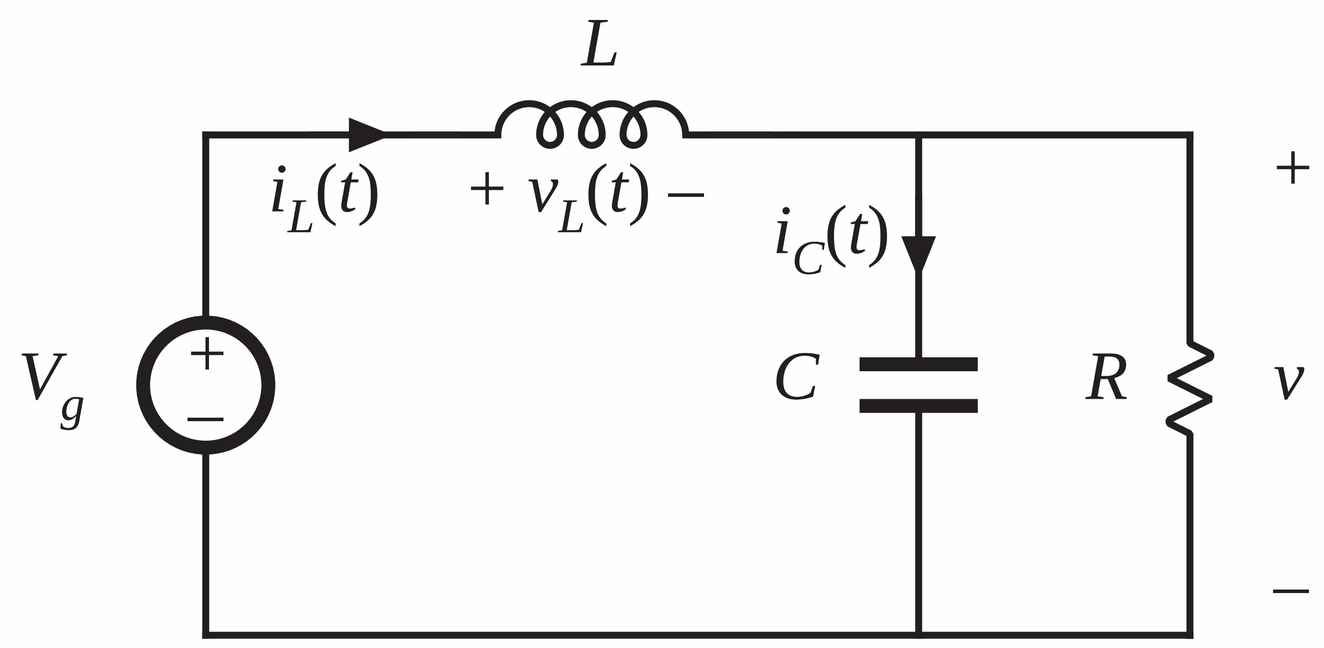
Subinterval 2: switch in position 2

Inductor voltage and capacitor current

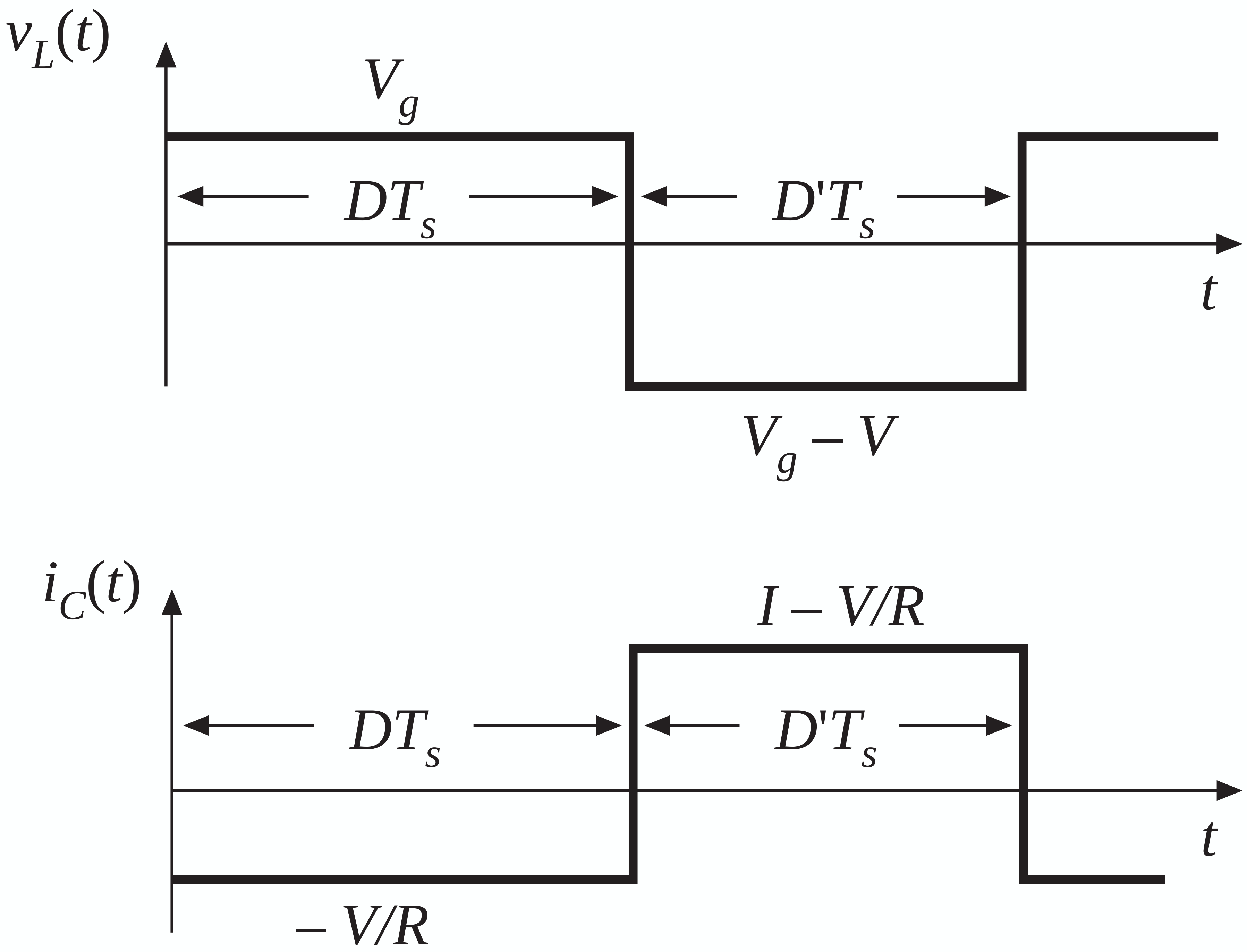
$$v_L = V_g - v$$
$$i_C = i_L - v / R$$

Small ripple approximation:

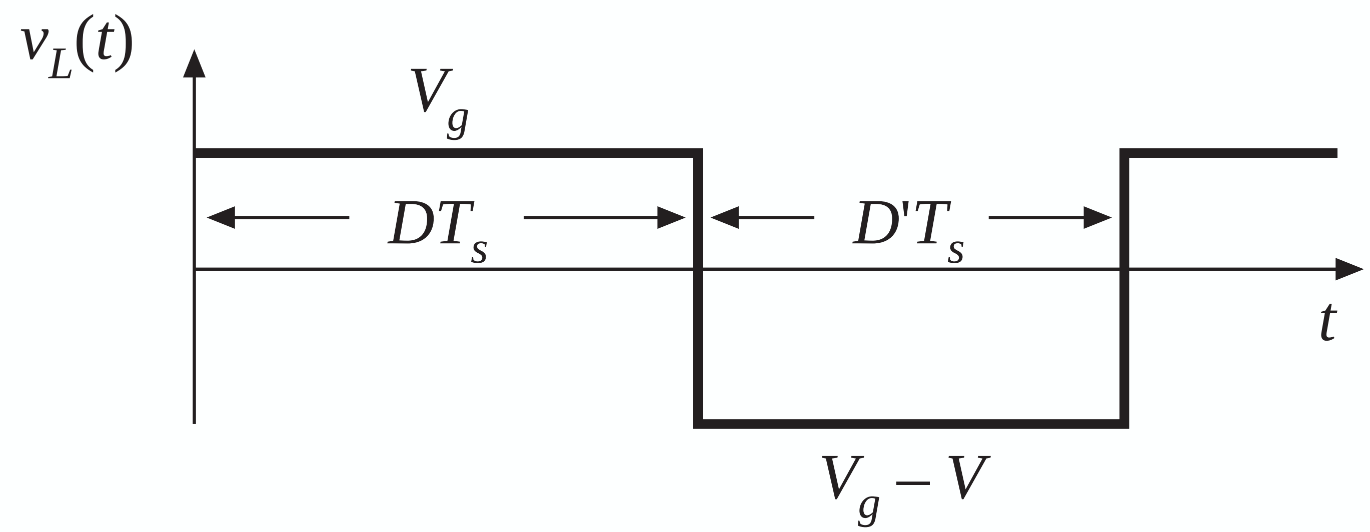
$$v_L = V_g - V$$
$$i_C = I - V / R$$



Inductor voltage and capacitor current waveforms



Inductor volt-second balance



Net volt-seconds applied to inductor over one switching period:

$$\int_0^{T_s} v_L(t)\, dt = (V_g)\, DT_s + (V_g - V)\, D'T_s$$

Equate to zero and collect terms:

$$V_g\, (D + D') - V\, D' = 0$$

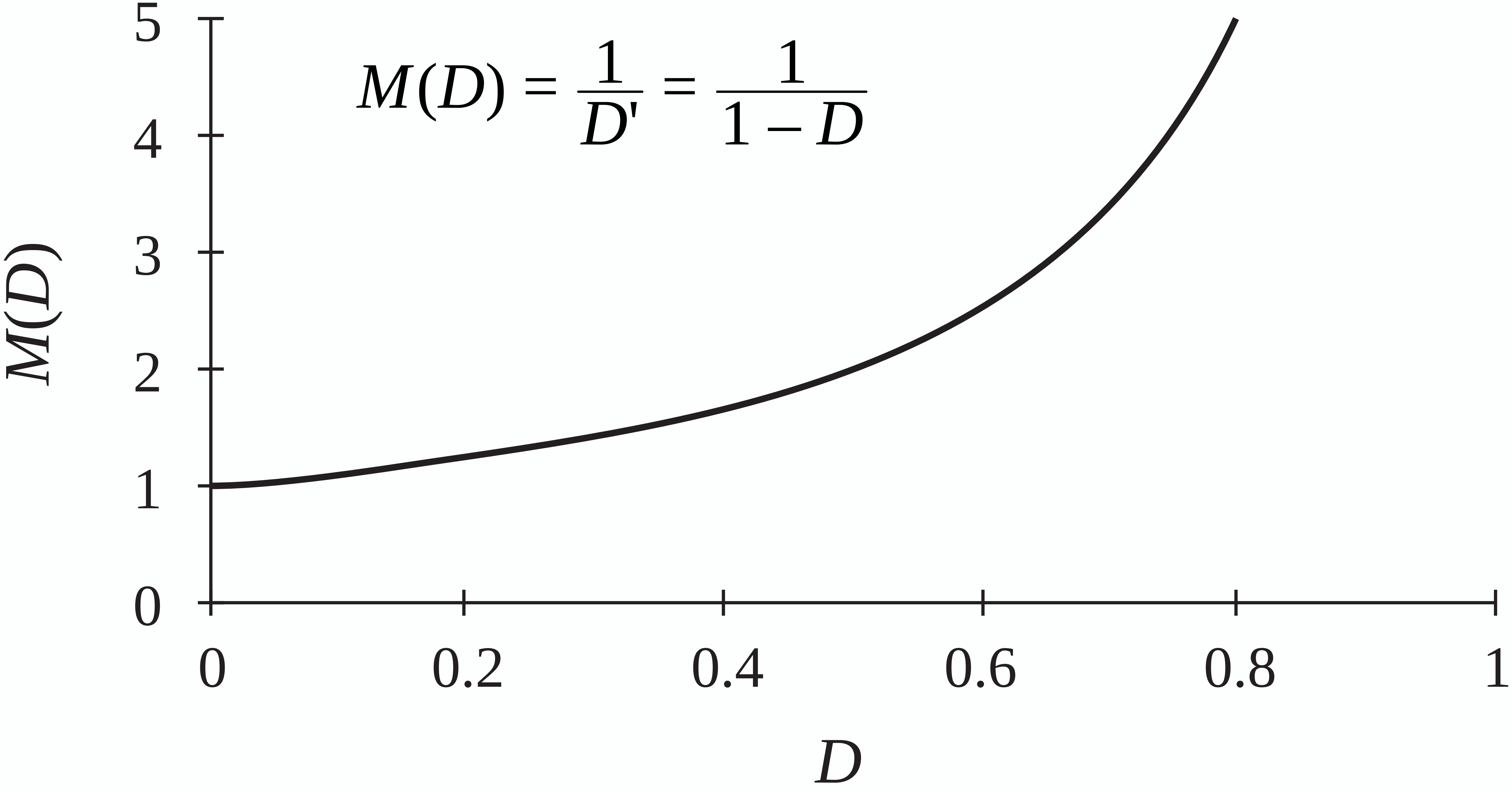
Solve for V:

$$V = \frac{V_g}{D'}$$

The voltage conversion ratio is therefore

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1 - D}$$

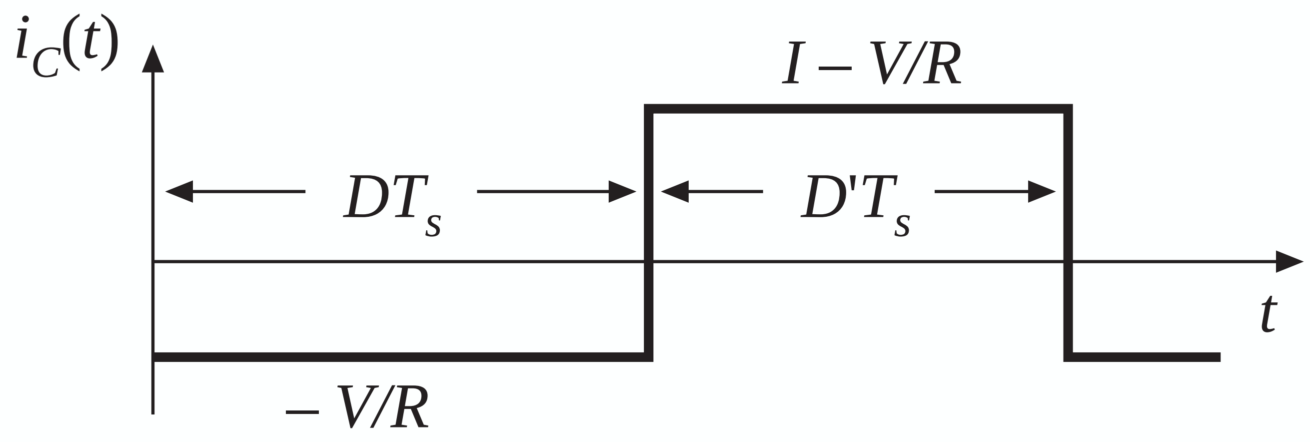
Conversion ratio *M(D)* of the boost converter



Determination of inductor current dc component

Capacitor charge balance:

$$\int_0^{T_s} i_C(t)\, dt = \left(-\frac{V}{R}\right) DT_s + \left(I - \frac{V}{R}\right) D'T_s$$



Collect terms and equate to zero:

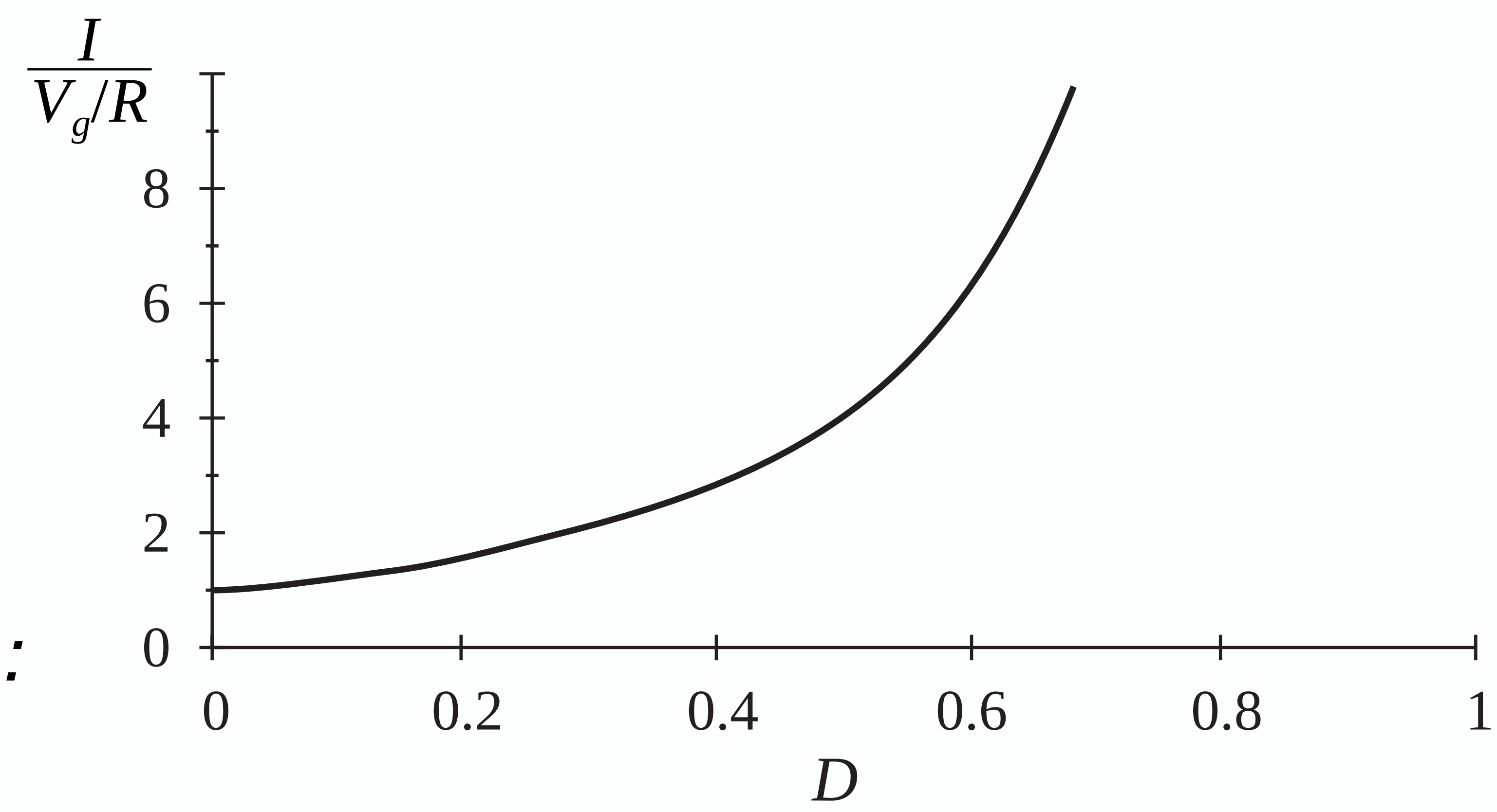
$$-\frac{V}{R}(D + D') + I\,D' = 0$$

Solve for *I*:

$$I = \frac{V}{D'\,R}$$

Eliminate *V* to express in terms of V_g:

$$I = \frac{V_g}{D'^2\,R}$$



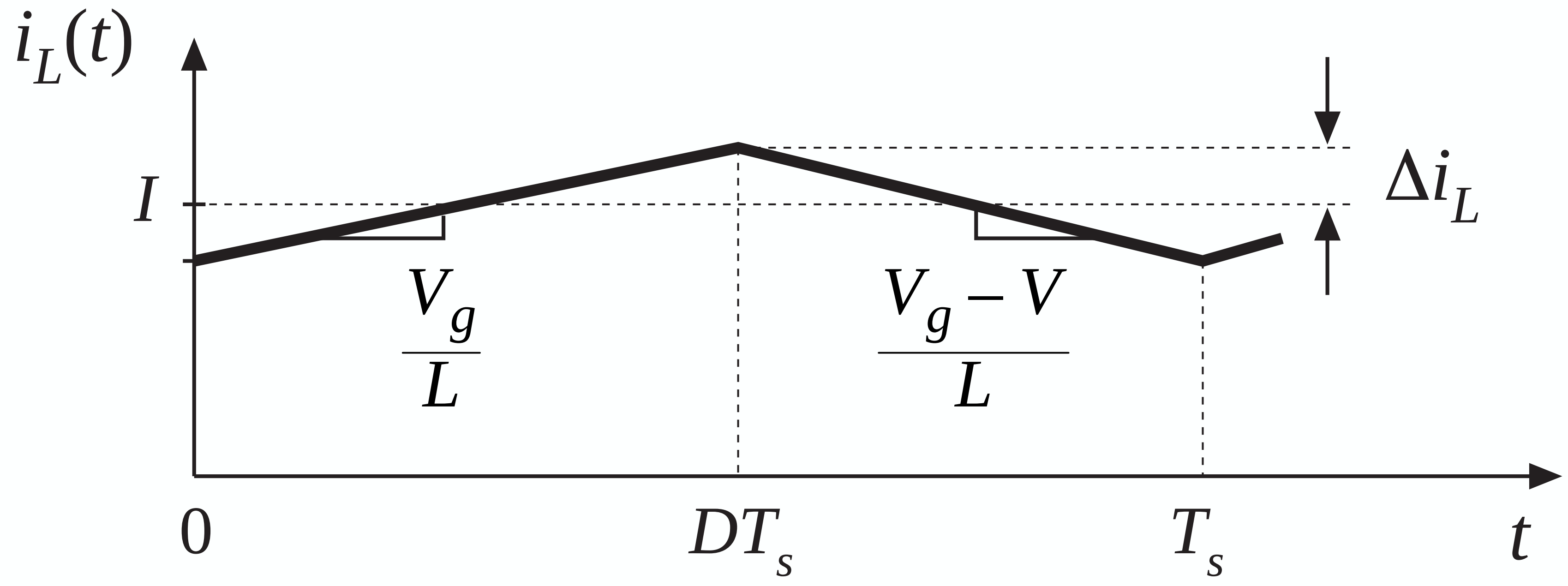
Determination of inductor current ripple

Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$



Change in inductor current during subinterval 1 is *(slope) (length of subinterval):*

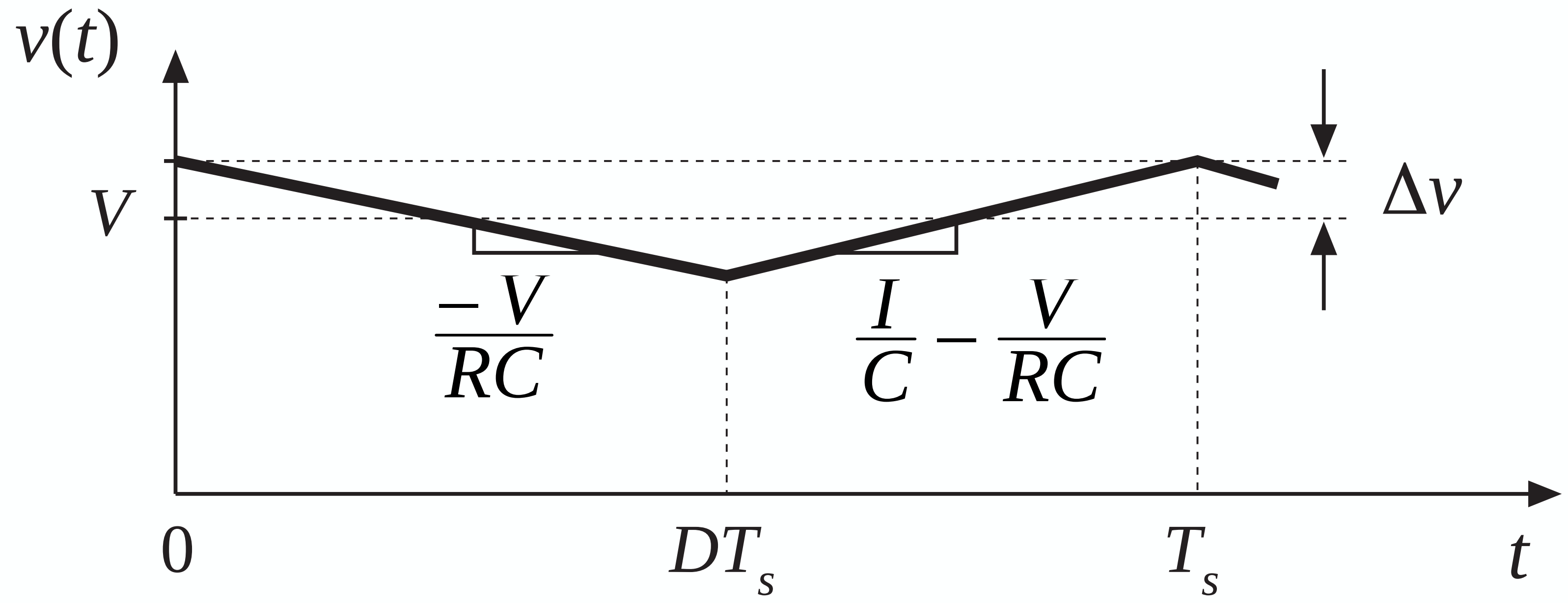
$$2\Delta i_L = \frac{V_g}{L} DT_s$$

Solve for peak ripple:

$$\Delta i_L = \frac{V_g}{2L} DT_s$$

- Choose *L* such that desired ripple magnitude is obtained

Determination of capacitor voltage ripple



Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$

Change in capacitor voltage during subinterval 1 is *(slope) (length of subinterval):*

$$-2\Delta v = \frac{-V}{RC} DT_s$$

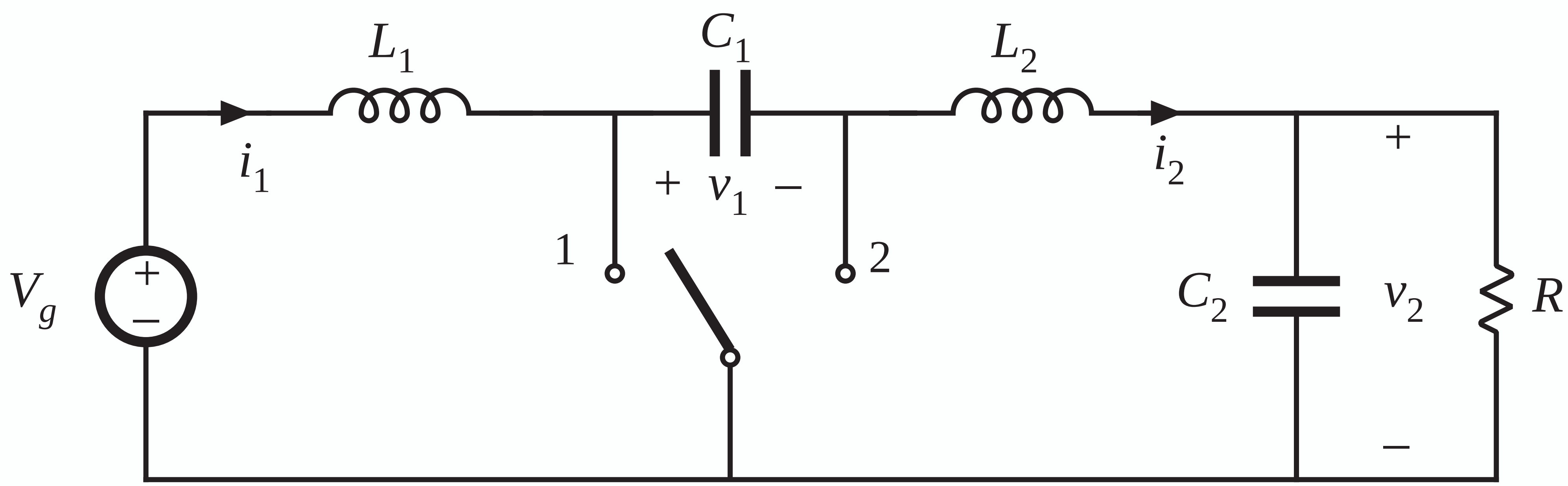
Solve for peak ripple:

$$\Delta v = \frac{V}{2RC} DT_s$$

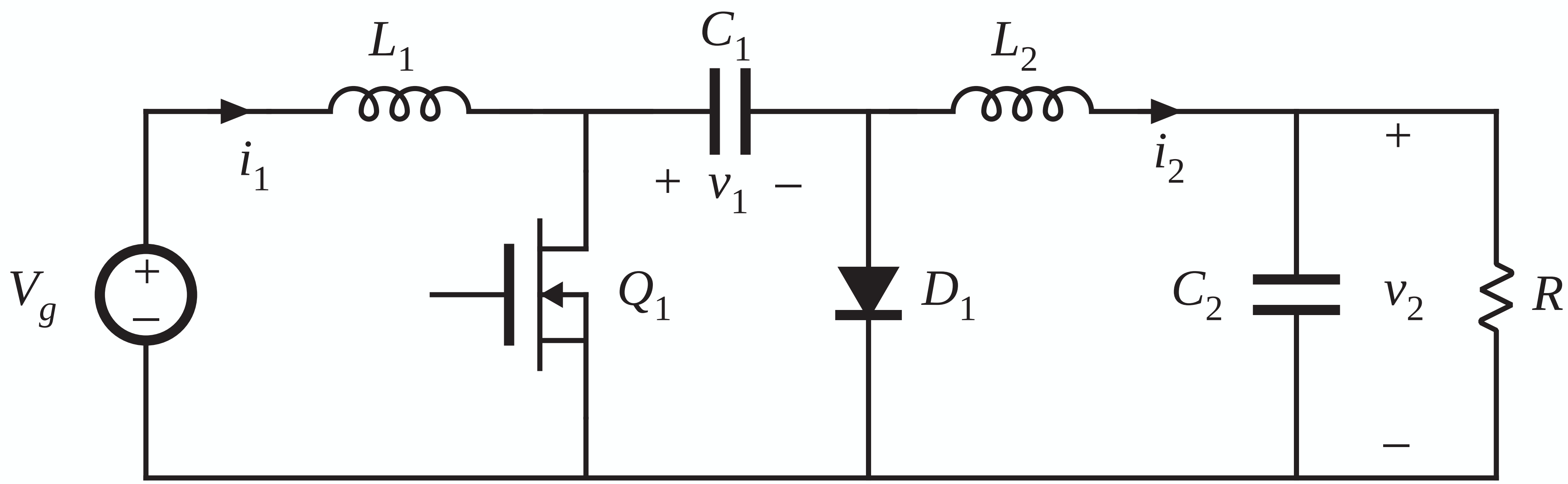
- Choose *C* such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple

2.4 Cuk converter example

Cuk converter, with ideal switch



Cuk converter: practical realization using MOSFET and diode

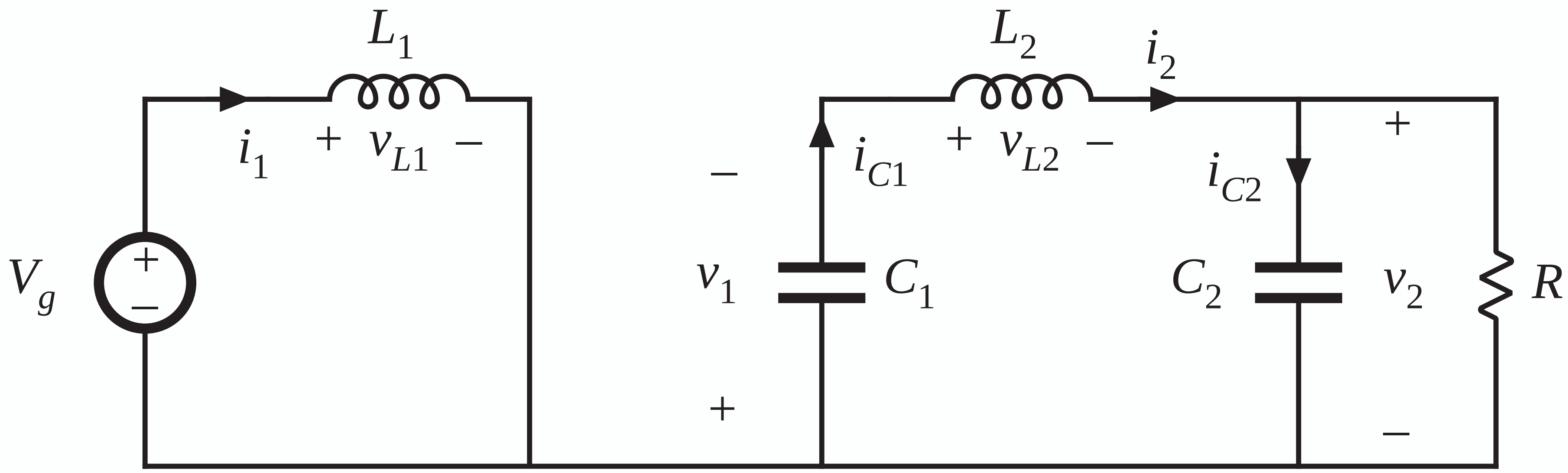


Cuk converter circuit

with switch in positions 1 and 2

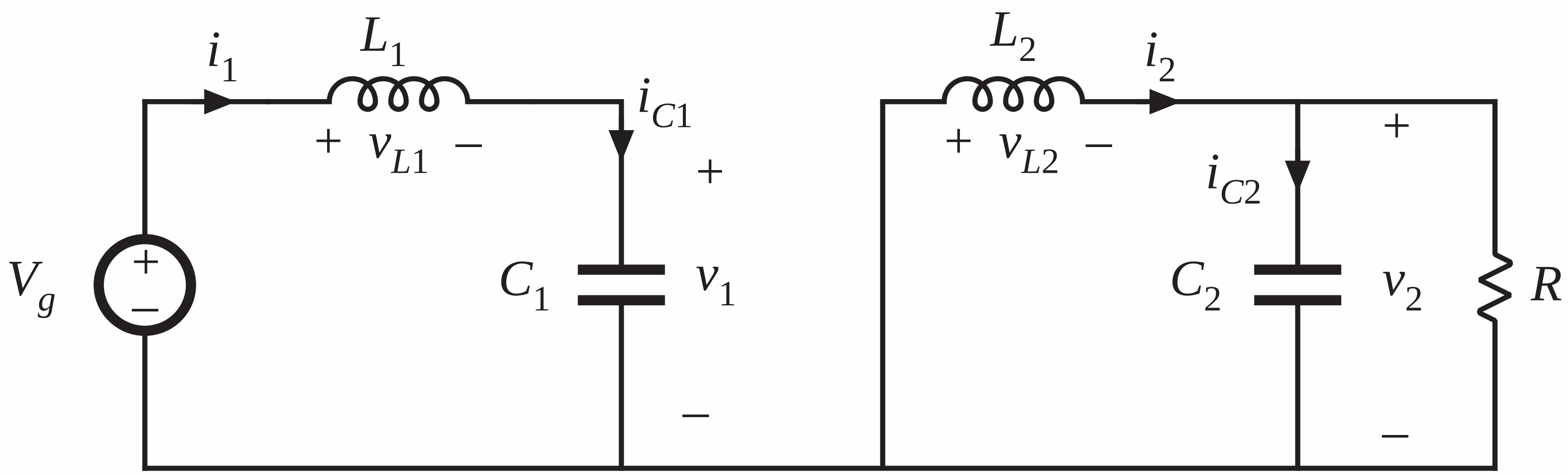
Switch in position 1:
MOSFET conducts

Capacitor C_1 releases energy to output

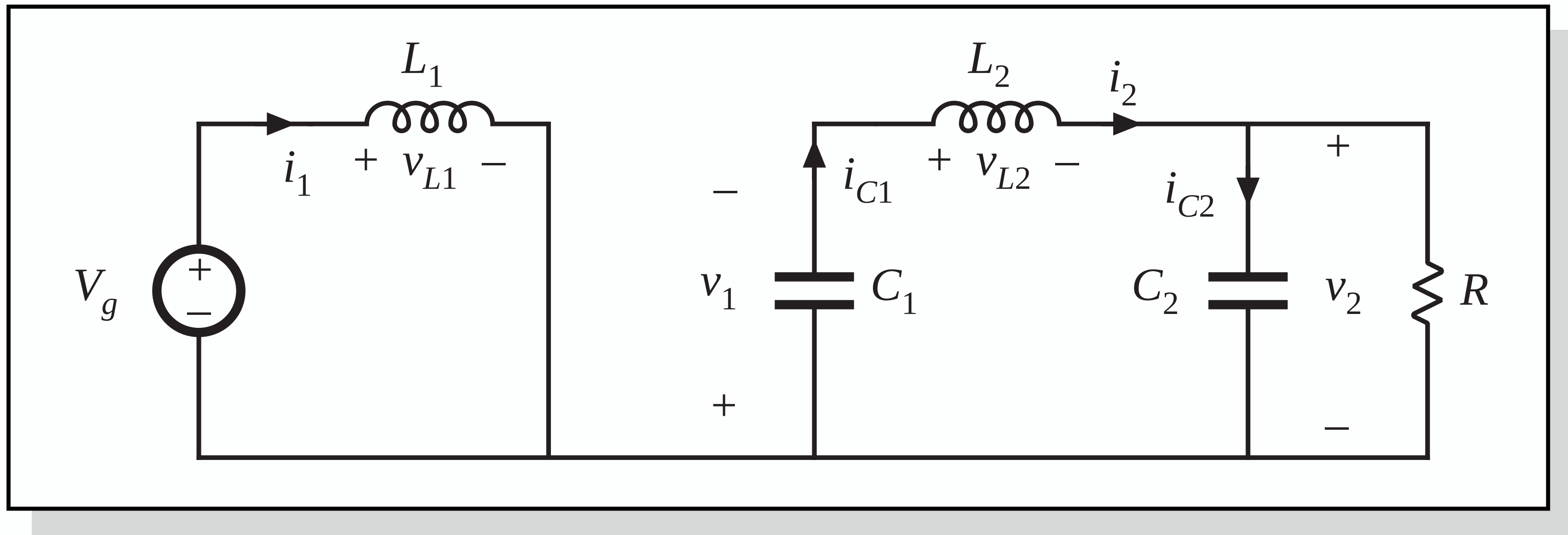


Switch in position 2:
diode conducts

Capacitor C_1 is charged from input



Waveforms during subinterval 1
MOSFET conduction interval



Inductor voltages and capacitor currents:

$$v_{L1} = V_g$$
$$v_{L2} = -v_1 - v_2$$
$$i_{C1} = i_2$$
$$i_{C2} = i_2 - \frac{v_2}{R}$$

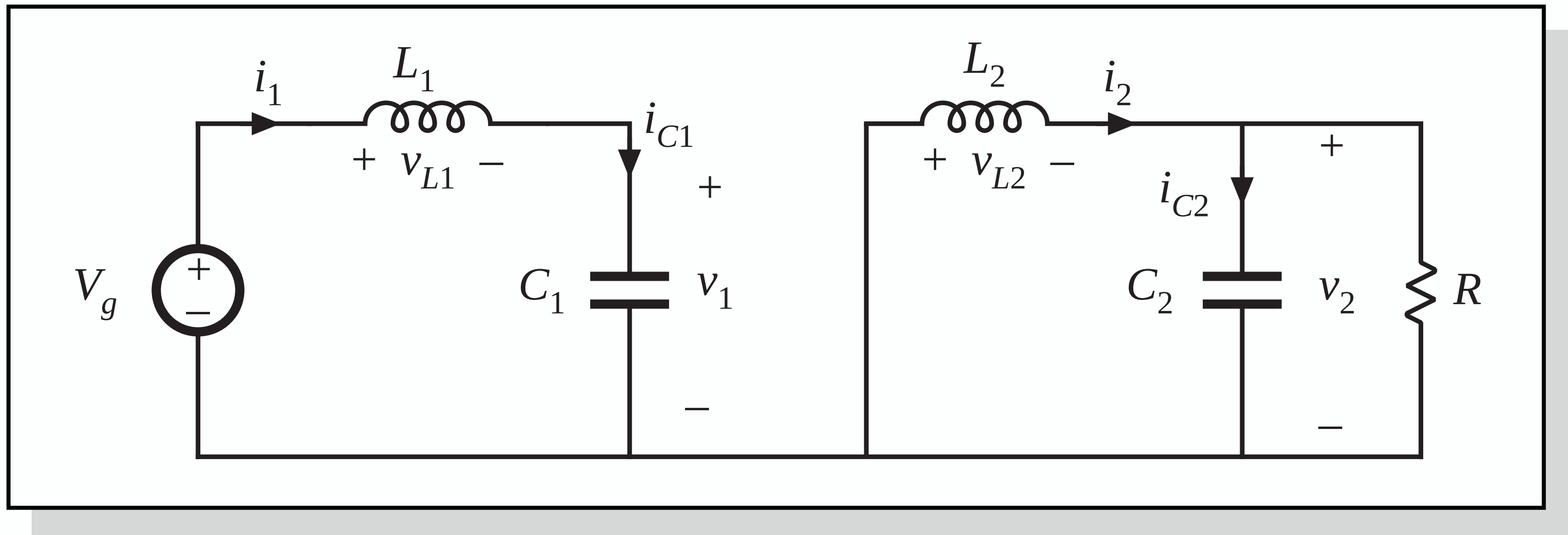
Small ripple approximation for subinterval 1:

$$v_{L1} = V_g$$
$$v_{L2} = -V_1 - V_2$$
$$i_{C1} = I_2$$
$$i_{C2} = I_2 - \frac{V_2}{R}$$

Waveforms during subinterval 2
Diode conduction interval

Inductor voltages and capacitor currents:

$$v_{L1} = V_g - v_1$$
$$v_{L2} = -v_2$$
$$i_{C1} = i_1$$
$$i_{C2} = i_2 - \frac{v_2}{R}$$



Small ripple approximation for subinterval 2:

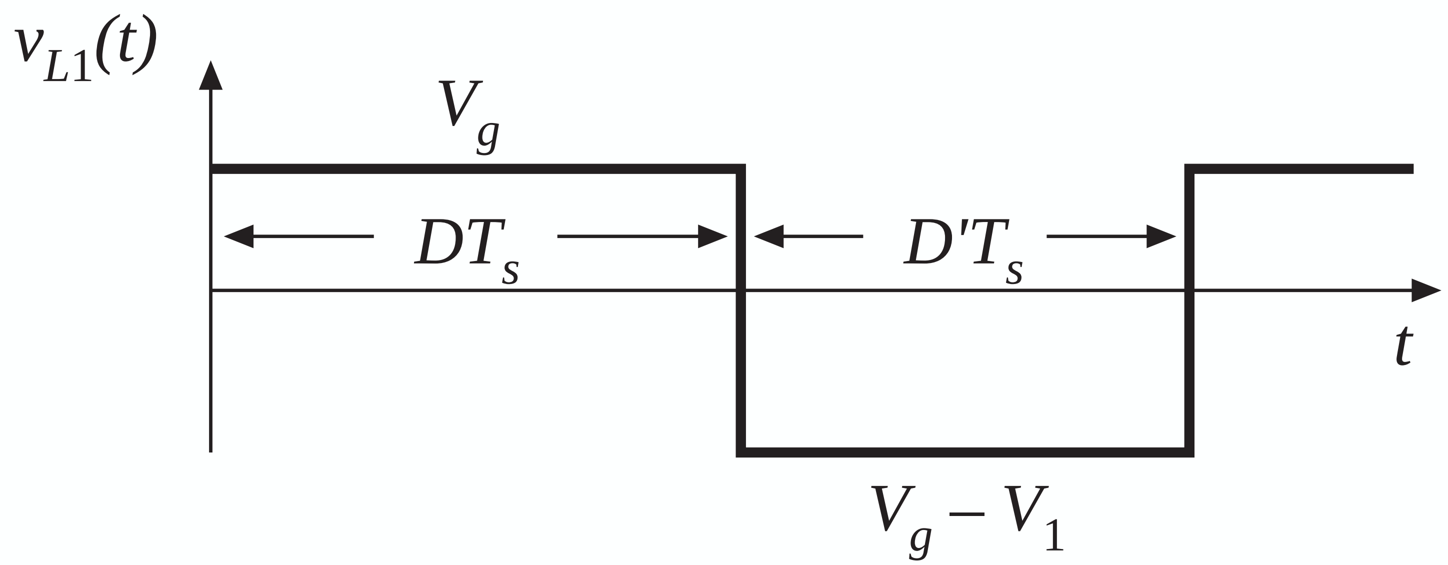
$$v_{L1} = V_g - V_1$$
$$v_{L2} = -V_2$$
$$i_{C1} = I_1$$
$$i_{C2} = I_2 - \frac{V_2}{R}$$

Equate average values to zero

The principles of inductor volt-second and capacitor charge balance state that the average values of the periodic inductor voltage and capacitor current waveforms are zero, when the converter operates in steady state. Hence, to determine the steady-state conditions in the converter, let us sketch the inductor voltage and capacitor current waveforms, and equate their average values to zero.

Waveforms:

Inductor voltage $v_{L1}(t)$

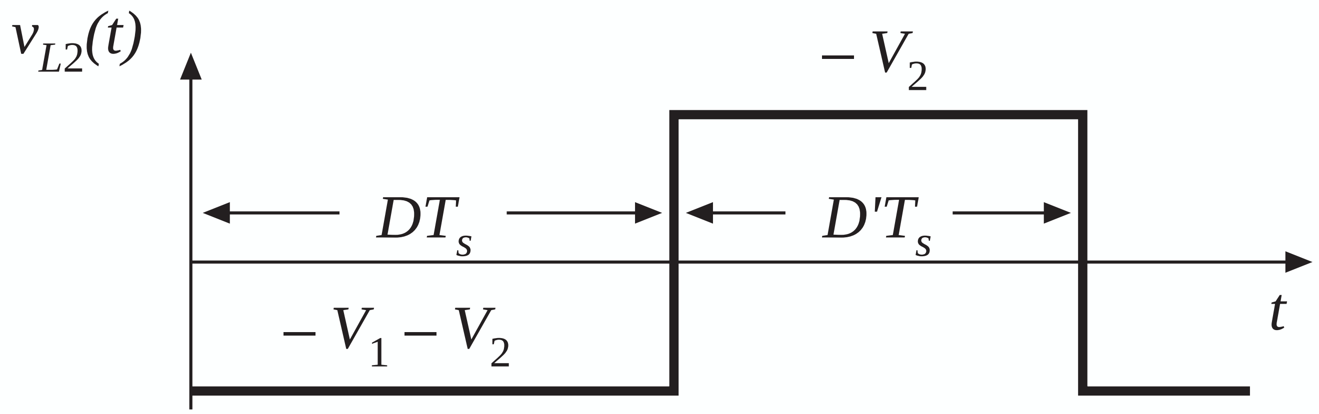


Volt-second balance on L_1:

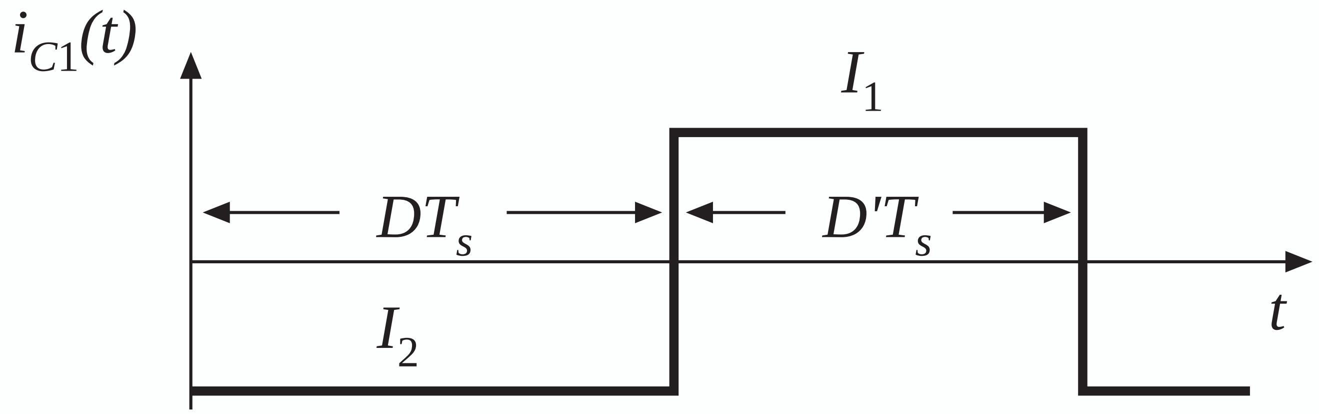
$$\langle v_{L1} \rangle = DV_g + D'(V_g - V_1) = 0$$

Equate average values to zero

Inductor L_2 voltage



Capacitor C_1 current



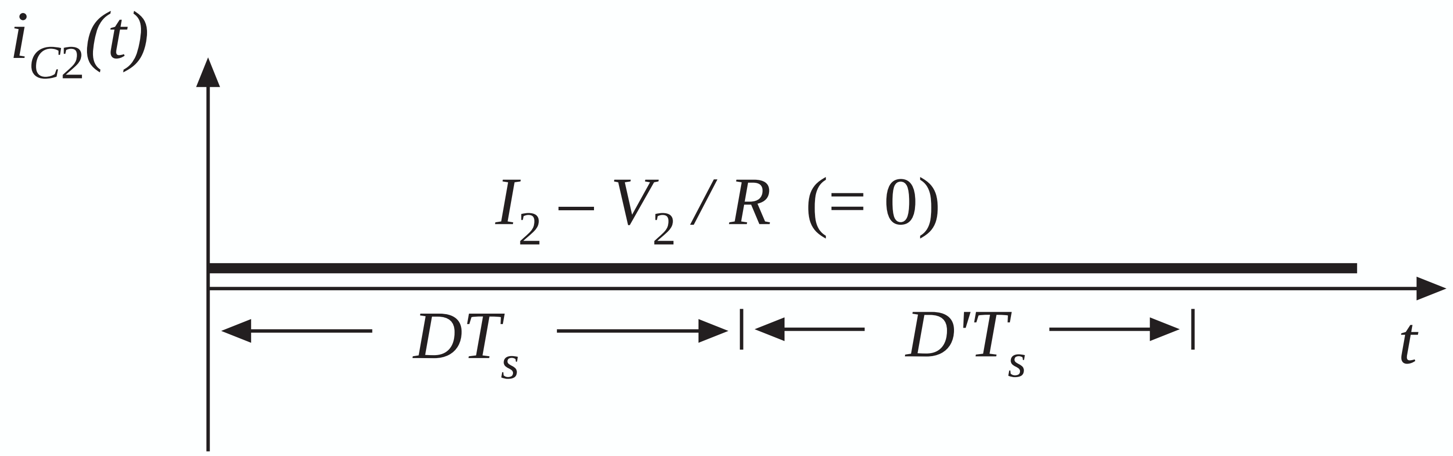
Average the waveforms:

$$\left\langle v_{L2} \right\rangle = D(-V_1 - V_2) + D'(-V_2) = 0$$

$$\left\langle i_{C1} \right\rangle = DI_2 + D'I_1 = 0$$

Equate average values to zero

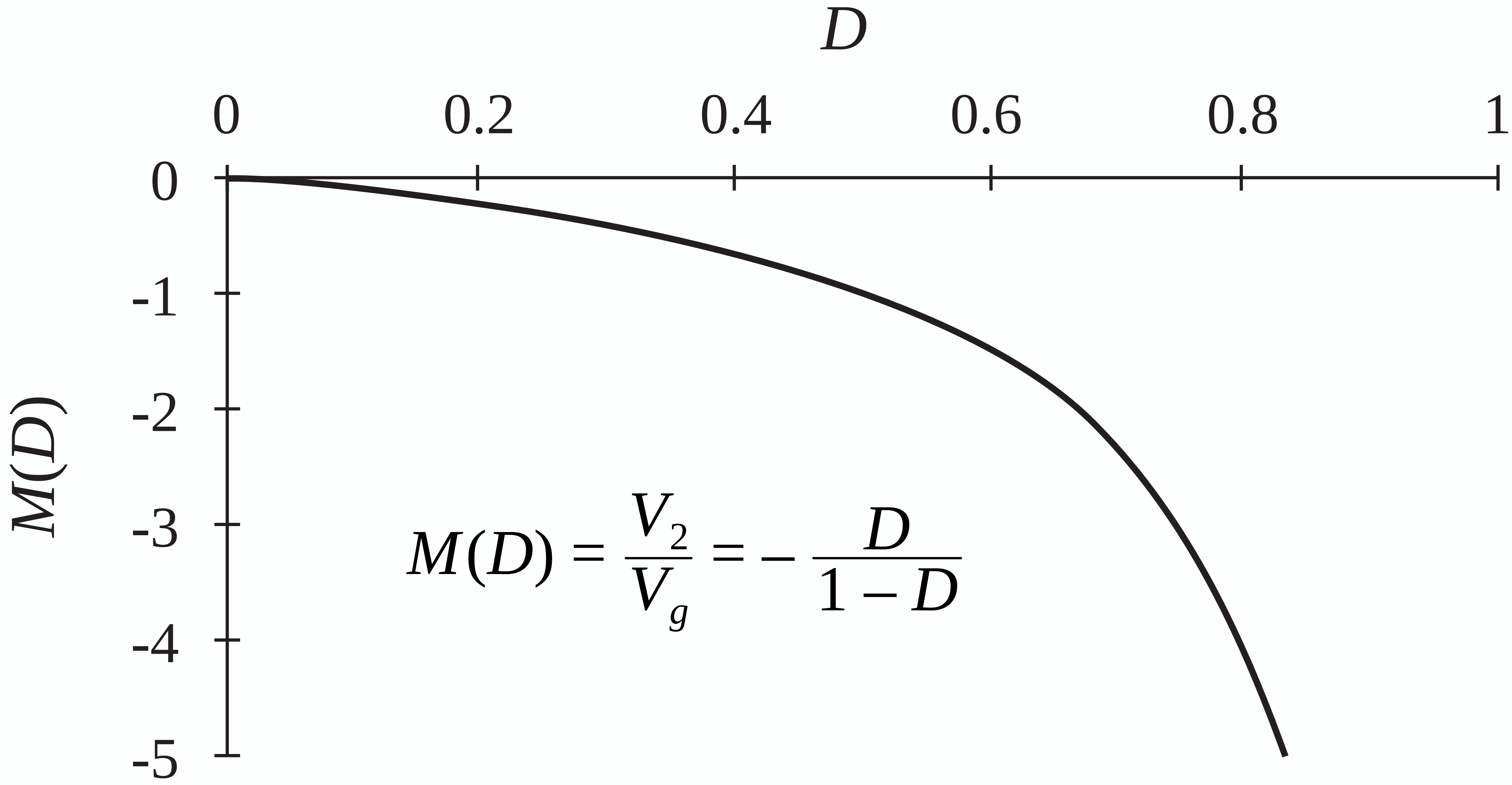
Capacitor current $i_{C2}(t)$ waveform



$$\left\langle i_{C2} \right\rangle = I_2 - \frac{V_2}{R} = 0$$

Note: during both subintervals, the capacitor current i_{C2} is equal to the difference between the inductor current i_2 and the load current V_2/R. When ripple is neglected, i_{C2} is constant and equal to zero.

Cuk converter conversion ratio $M = V / V_g$

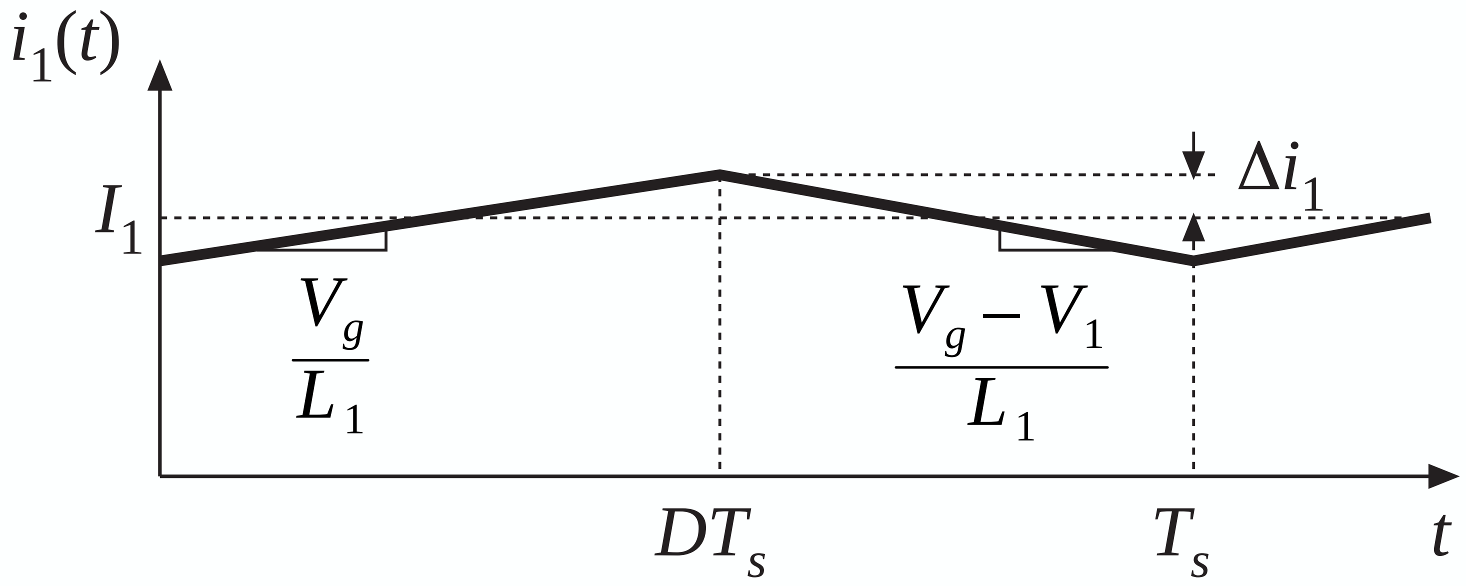


Inductor current waveforms

Interval 1 slopes, using small ripple approximation:

$$\frac{di_1(t)}{dt} = \frac{v_{L1}(t)}{L_1} = \frac{V_g}{L_1}$$

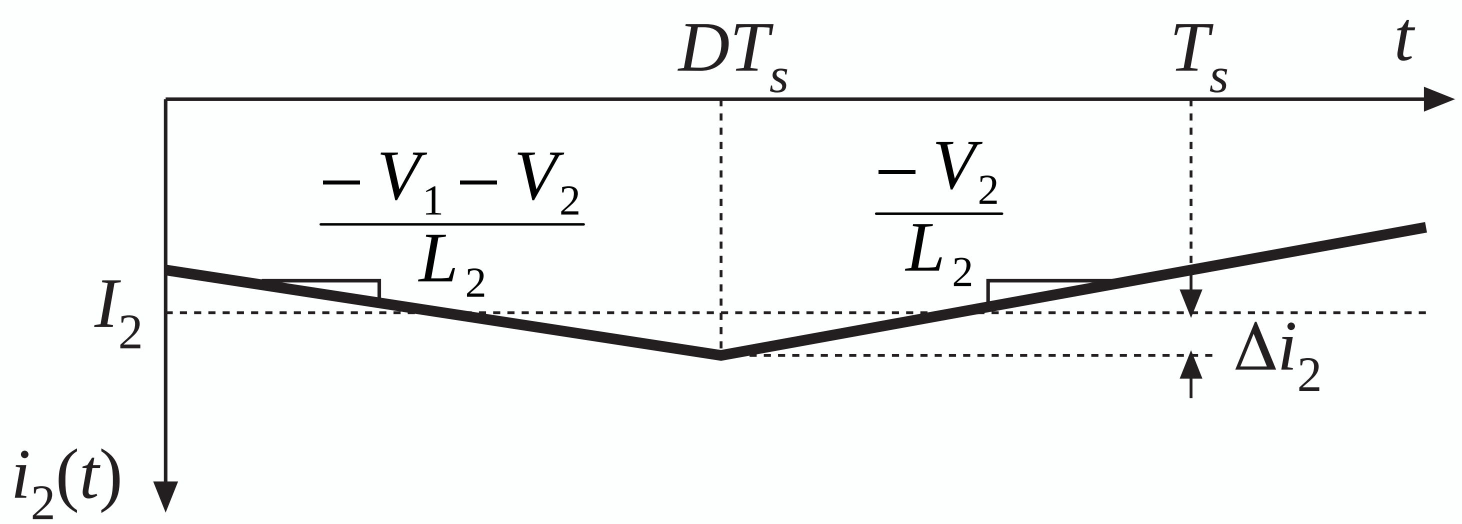
$$\frac{di_2(t)}{dt} = \frac{v_{L2}(t)}{L_2} = \frac{-V_1 - V_2}{L_2}$$



Interval 2 slopes:

$$\frac{di_1(t)}{dt} = \frac{v_{L1}(t)}{L_1} = \frac{V_g - V_1}{L_1}$$

$$\frac{di_2(t)}{dt} = \frac{v_{L2}(t)}{L_2} = \frac{-V_2}{L_2}$$



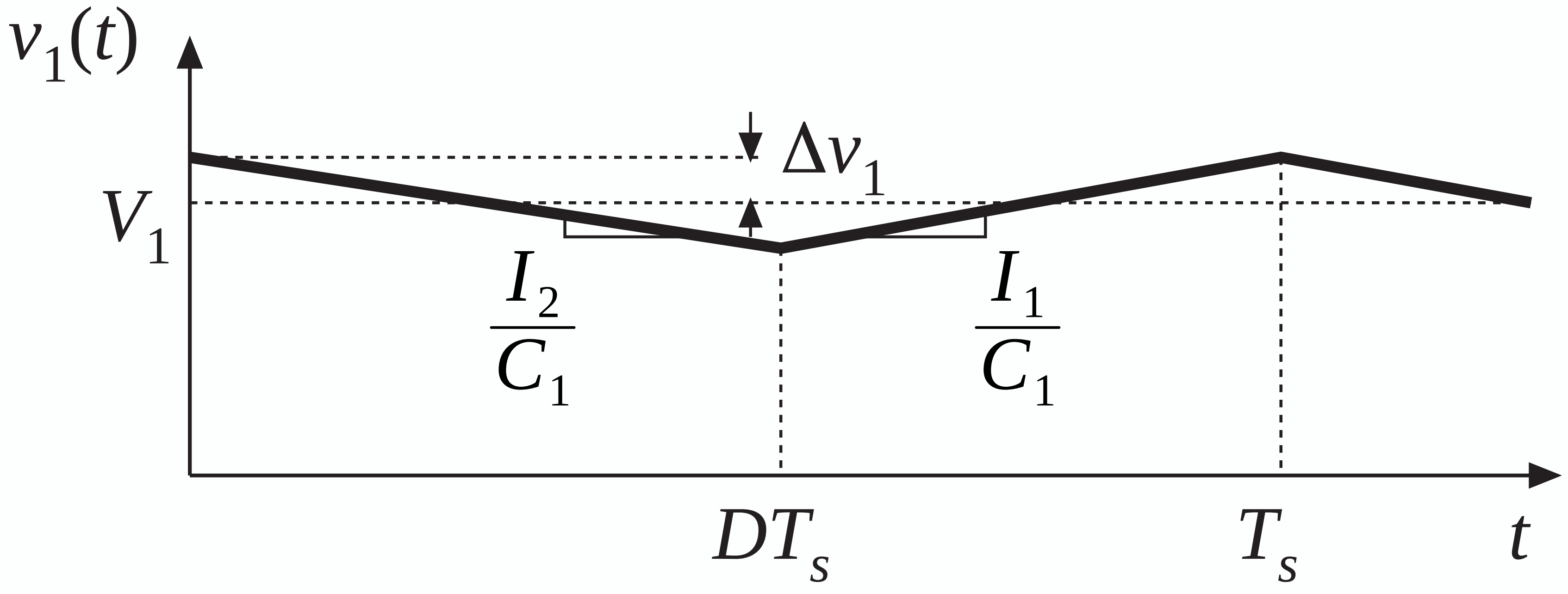
Capacitor C_1 waveform

Subinterval 1:

$$\frac{dv_1(t)}{dt} = \frac{i_{C1}(t)}{C_1} = \frac{I_2}{C_1}$$

Subinterval 2:

$$\frac{dv_1(t)}{dt} = \frac{i_{C1}(t)}{C_1} = \frac{I_1}{C_1}$$



Ripple magnitudes

Analysis results

$$\Delta i_1 = \frac{V_g D T_s}{2L_1}$$

$$\Delta i_2 = \frac{V_1 + V_2}{2L_2} D T_s$$

$$\Delta v_1 = \frac{-I_2 D T_s}{2C_1}$$

Use dc converter solution to simplify:

$$\Delta i_1 = \frac{V_g D T_s}{2L_1}$$

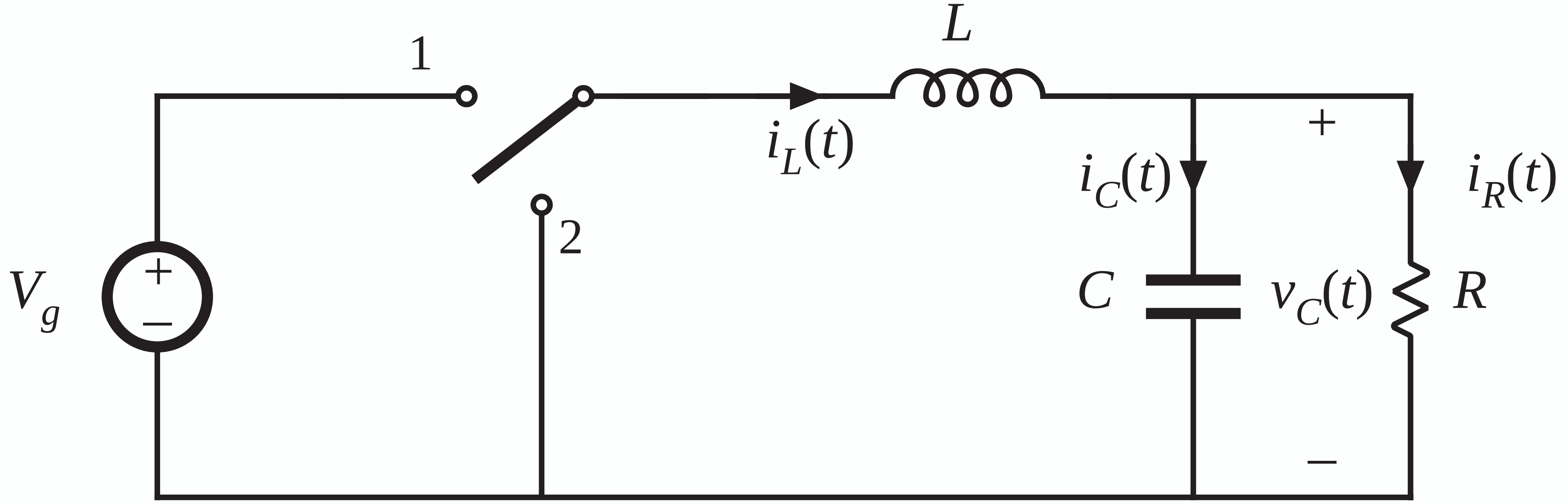
$$\Delta i_2 = \frac{V_g D T_s}{2L_2}$$

$$\Delta v_1 = \frac{V_g D^2 T_s}{2D'RC_1}$$

Q: How large is the output voltage ripple?

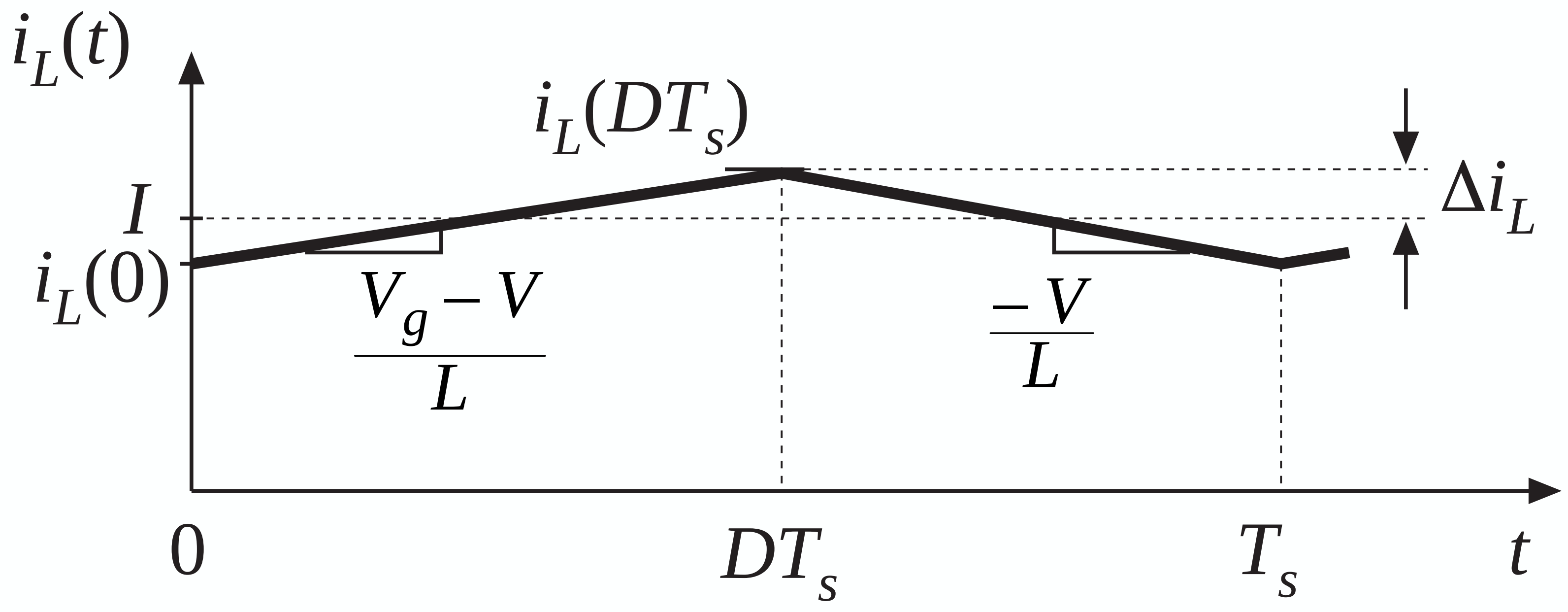
2.5 Estimating ripple in converters containing two-pole low-pass filters

Buck converter example: Determine output voltage ripple



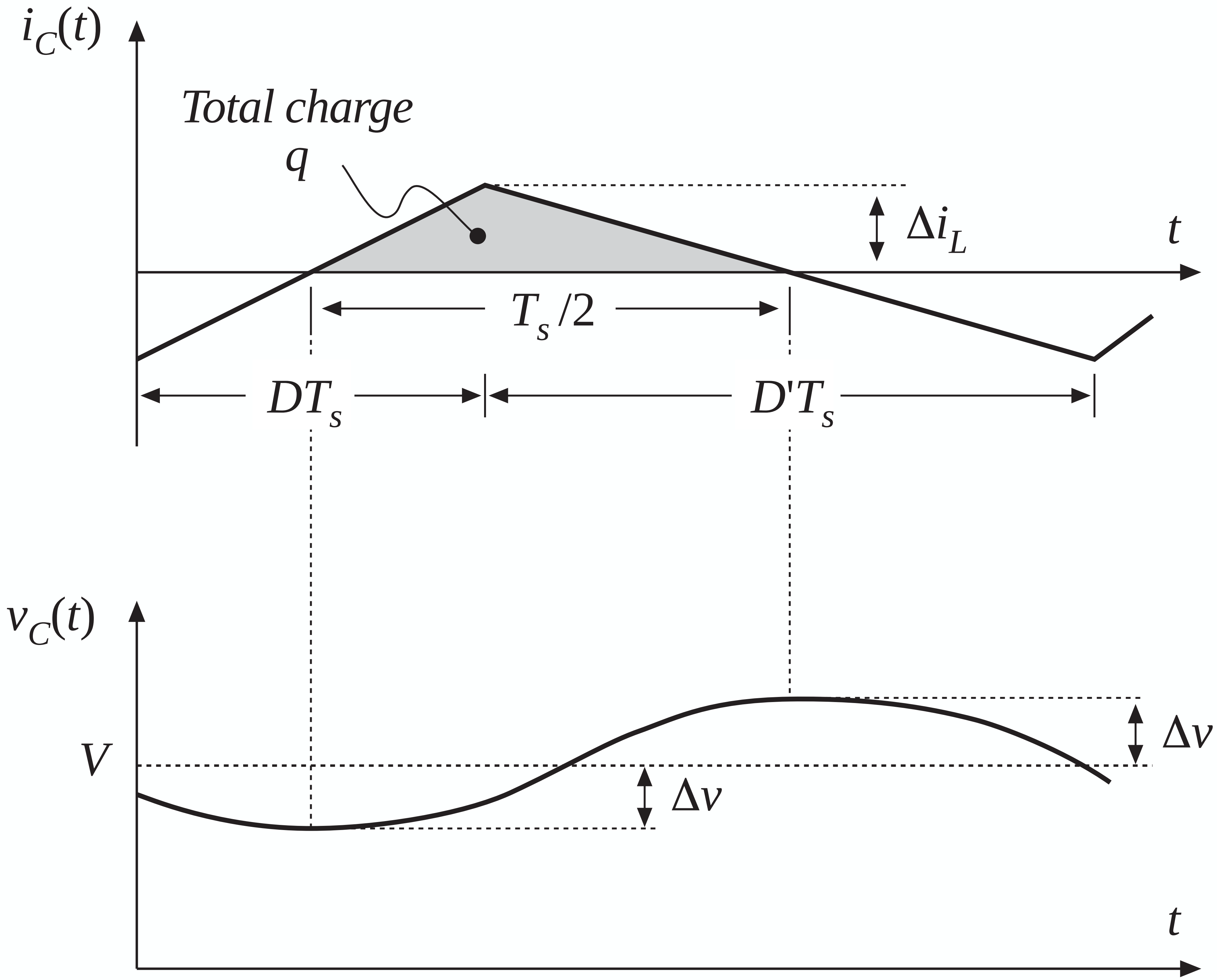
Inductor current waveform.

What is the capacitor current?



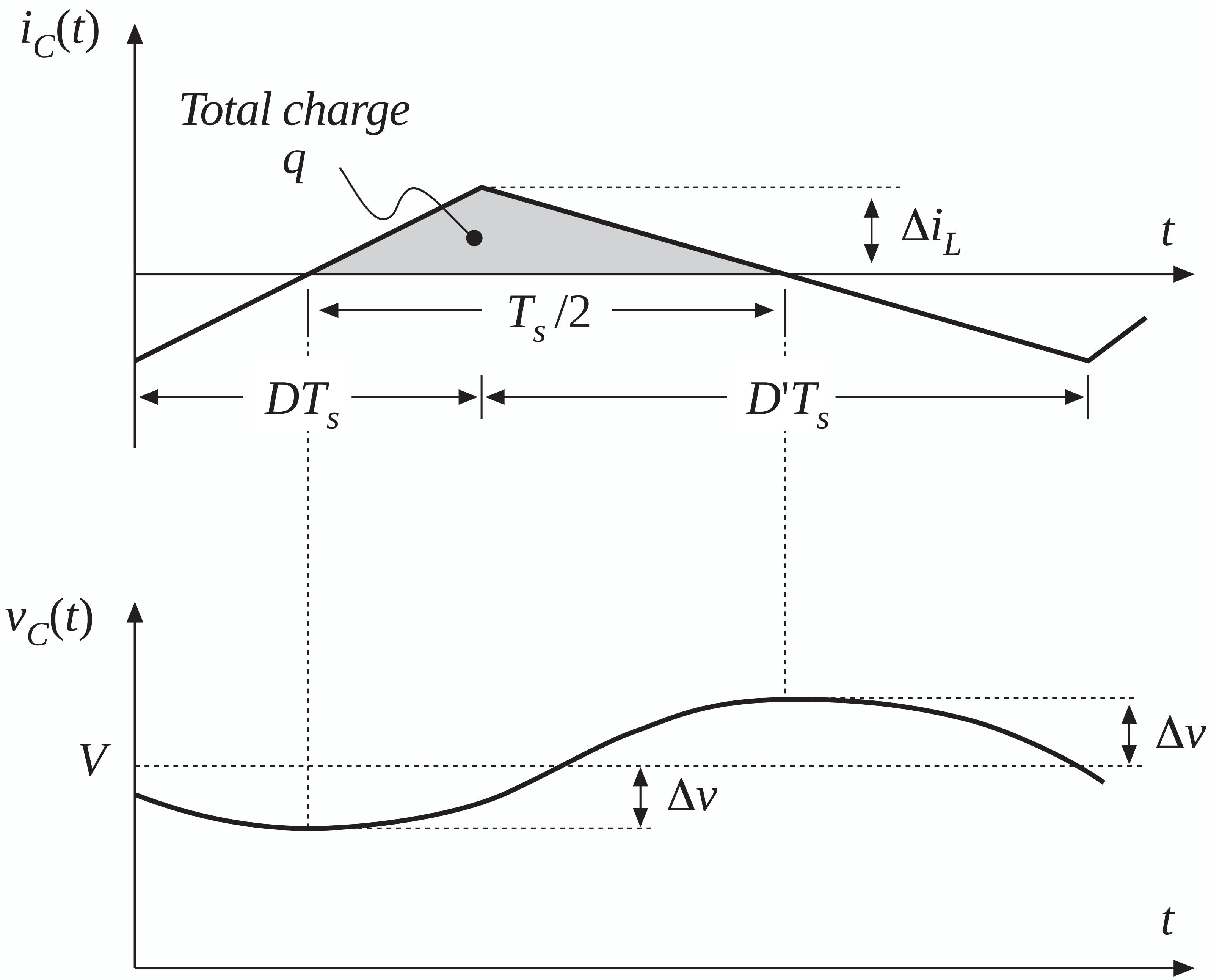
Capacitor current and voltage, buck example

Must not neglect inductor current ripple!



If the capacitor voltage ripple is small, then essentially all of the ac component of inductor current flows through the capacitor.

Estimating capacitor voltage ripple Δv

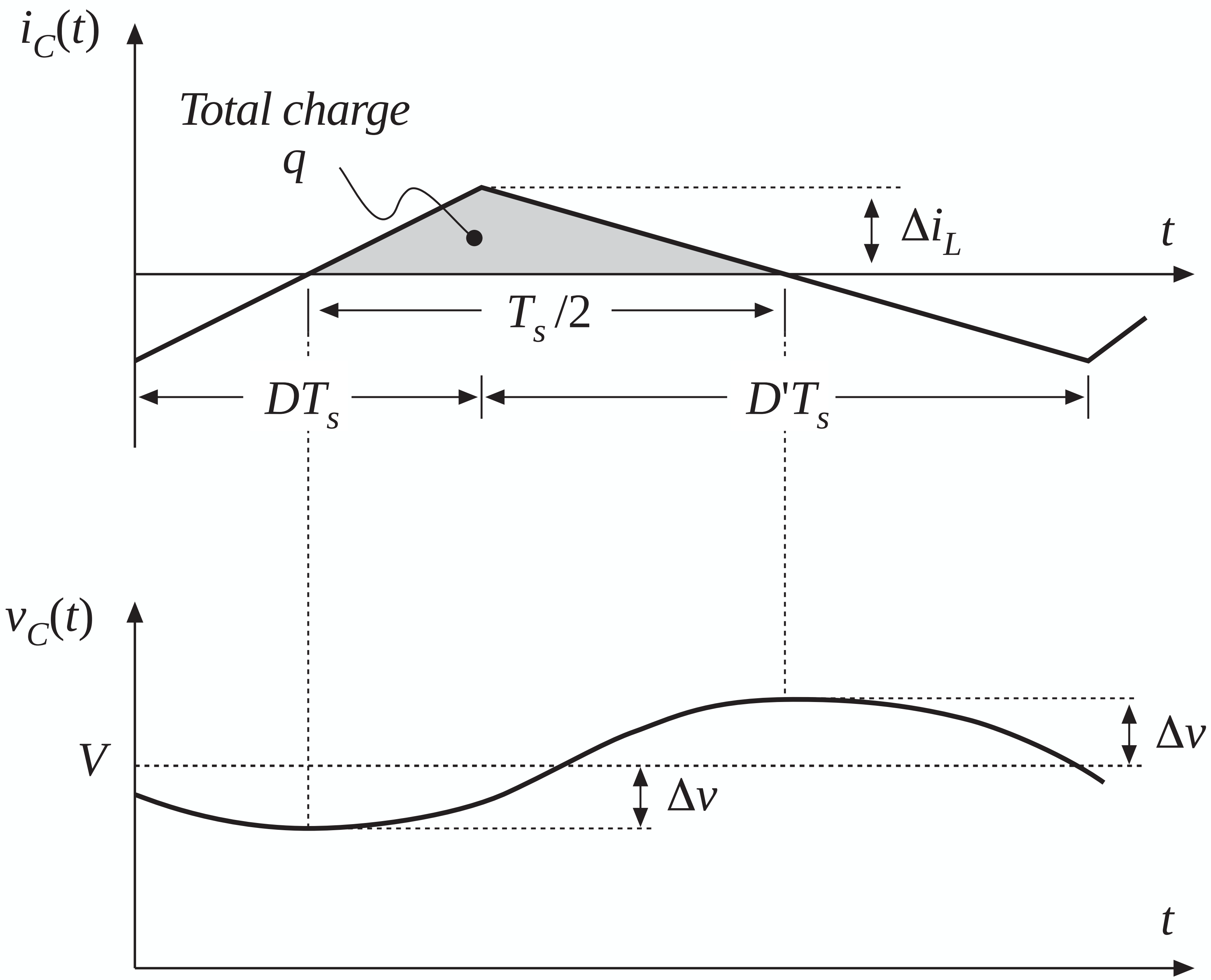


Current $i_C(t)$ is positive for half of the switching period. This positive current causes the capacitor voltage $v_C(t)$ to increase between its minimum and maximum extrema. During this time, the total charge q is deposited on the capacitor plates, where

$$q = C\,(2\Delta v)$$

$$(\textit{change in charge}) = C\,(\textit{change in voltage})$$

Estimating capacitor voltage ripple Δv



The total charge q is the area of the triangle, as shown:

$$q = \frac{1}{2}\,\Delta i_L\,\frac{T_s}{2}$$

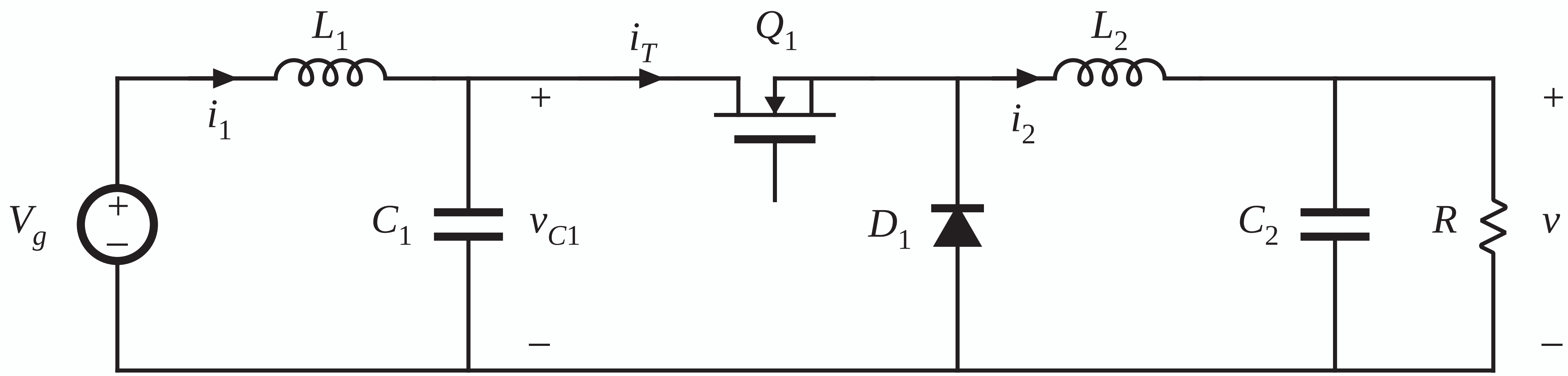
Eliminate q and solve for Δv:

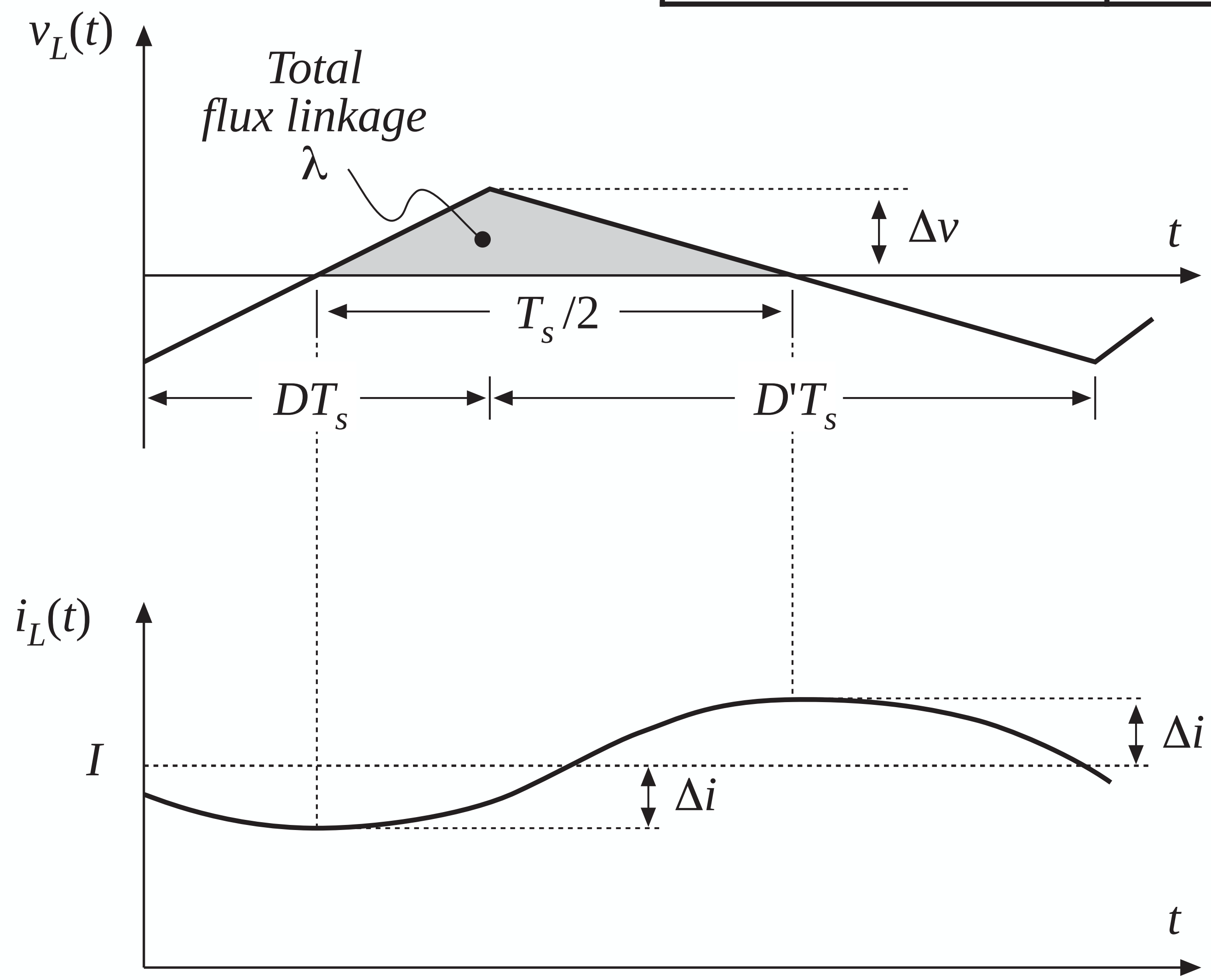
$$\Delta v = \frac{\Delta i_L\, T_s}{8\,C}$$

Note: in practice, capacitor equivalent series resistance (esr) further increases Δv.

Inductor current ripple in two-pole filters

Example:
problem 2.9





can use similar arguments, with

$$\lambda = L\,(2\Delta i)$$

λ = inductor flux linkages

= inductor volt-seconds

2.6 Summary of Key Points

1. The dc component of a converter waveform is given by its average value, or the integral over one switching period, divided by the switching period. Solution of a dc-dc converter to find its dc, or steady-state, voltages and currents therefore involves averaging the waveforms.
2. The linear ripple approximation greatly simplifies the analysis. In a well-designed converter, the switching ripples in the inductor currents and capacitor voltages are small compared to the respective dc components, and can be neglected.
3. The principle of inductor volt-second balance allows determination of the dc voltage components in any switching converter. In steady-state, the average voltage applied to an inductor must be zero.

Summary of Chapter 2

4. The principle of capacitor charge balance allows determination of the dc components of the inductor currents in a switching converter. In steady-state, the average current applied to a capacitor must be zero.
5. By knowledge of the slopes of the inductor current and capacitor voltage waveforms, the ac switching ripple magnitudes may be computed. Inductance and capacitance values can then be chosen to obtain desired ripple magnitudes.
6. In converters containing multiple-pole filters, continuous (nonpulsating) voltages and currents are applied to one or more of the inductors or capacitors. Computation of the ac switching ripple in these elements can be done using capacitor charge and/or inductor flux-linkage arguments, without use of the small-ripple approximation.
7. Converters capable of increasing (boost), decreasing (buck), and inverting the voltage polarity (buck-boost and Cuk) have been described. Converter circuits are explored more fully in a later chapter.