

# BLG527E Machine Learning Homework 1

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Anıl Öztürk

504181504

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## 1 BAYESIAN DECISION THEORY

We are needed to create a discriminant function of the form:

$$g_i(x) = \ln p(x|w_i) + \ln P(w_i) \quad (1.1)$$

Then we need to define a class which has 3 label probabilities (one is later). Each label has its own mean, standard deviation and PDE. These are what wanted from us to do;

- With given mean, standard deviations and prior probabilities, plot the pdf and the separating surface
- With given mean, standard deviations and changed priors, plot the pdf and the separating surface
- Generate random datasets with given element numbers for first two assignments
  - Compute the estimates for the mean, plot histograms and computed separating points
- Add another class label with its own mean, standard deviation and prior probability. Compute decision regions again.

In order to find the decision surface, we need to create an equation of two classes. In PDFs, when two class have the same probability value, in that point they have a decision surface. Following that information, we must equalize the function  $g(x)$  of each class.

$$g_1(x) = g_2(x) \quad (1.2)$$

$$\frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right) \quad (1.3)$$

$$\frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right) - \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right) = 0 \quad (1.4)$$

If we solve this equation for certain  $\mu$  and  $\sigma$  values, we will get proper  $x$  values. And these values will be our decision regions. For example, if we solve this equation for 1.a, the equation will be as follows.

$$\frac{1}{\sqrt{2\pi}4} \exp\left(-\frac{(x-(-5))^2}{32}\right) - \frac{1}{\sqrt{2\pi}4\sqrt{2}} \exp\left(-\frac{(x-10)^2}{64}\right) = 0 \quad (1.5)$$

$$x = 1.7297195052747578 \quad (1.6)$$

This  $x$  value will be our decision surface. Applying this formula for each section in the question, we can get the following values as decision surfaces.

$$1.a(x) = 1.7297195052747578 \quad (1.7)$$

$$1.b(x) = -1.7912565829379605 \quad (1.8)$$

$$1.c(x)(N10a) = 1.2563433341620744 \quad (1.9)$$

$$1.c(x)(N10b) = -3.252287590835177 \quad (1.10)$$

$$1.c(x)(N100a) = 1.6843889288822038 \quad (1.11)$$

$$1.c(x)(N100b) = -1.009899579735004 \quad (1.12)$$

$$1.d(x)(First) = 1.6033512225956448 \quad (1.13)$$

$$1.d(x)(Second) = 2.055393315932129 \quad (1.14)$$

### 1.1 PLOTTING THE PDFS AND THE SEPARATING SURFACE

$$P(w_j|x) = \frac{p(x|w_j)P(w_j)}{p(x)} \quad (1.15)$$

$$g_i(x) = \ln p(x|w_i) + \ln P(w_i) \quad (1.16)$$

In the first equation, we have the basic definition of the Bayes formula. In the second equation, we have the posterior probability in a log scale. Summing logs means multiplying parameters inside the log. We have the prior probability, mean and standard deviation as follows;

$$P(w_1), P(w_2) = 0.5 \quad (1.17)$$

$$\sigma = 4 \quad (1.18)$$

$$\mu_1 = -5, \mu_2 = 10 \quad (1.19)$$

With given mean and standard deviation values, the PDF (posterior) and separating surface plot will be like the following;

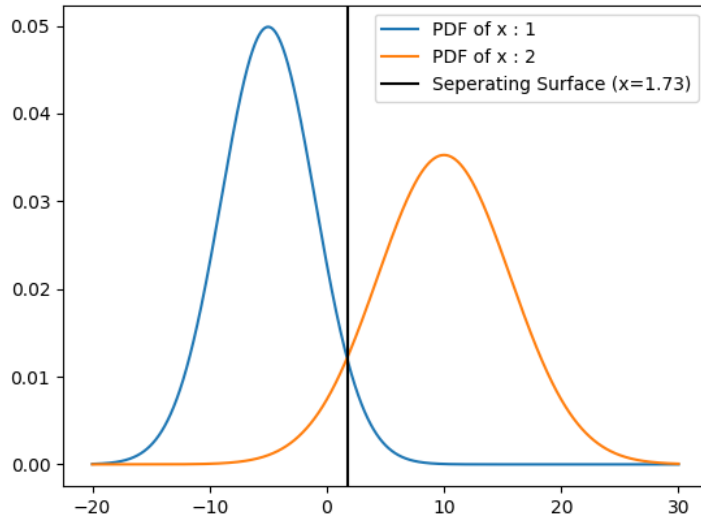


Figure 1.1: Plot of (Q1.a)

We can see that the probability of making decision in favor of the class 2 is higher.

## 1.2 PLOTTING THE PDFS AND THE SEPARATING SURFACE (WITH 0.1 PROBABILITY)

$$P(w_j|x) = \frac{p(x|w_j)P(w_j)}{p(x)} \quad (1.20)$$

$$g_i(x) = \ln p(x|w_i) + \ln P(w_i) \quad (1.21)$$

In the first equation, we have the basic definition of the Bayes formula. In the second equation, we have the posterior probability in a log scale. Summing logs means multiplying parameters inside the log. We have the prior probability, mean and standard deviation as follows;

$$P(w_1) = 0.1, P(w_2) = 0.9 \quad (1.22)$$

$$\sigma = 4 \quad (1.23)$$

$$\mu_1 = -5, \mu_2 = 10 \quad (1.24)$$

With given mean and standard deviation values, the PDF (posterior) and separating surface plot will be like the following;

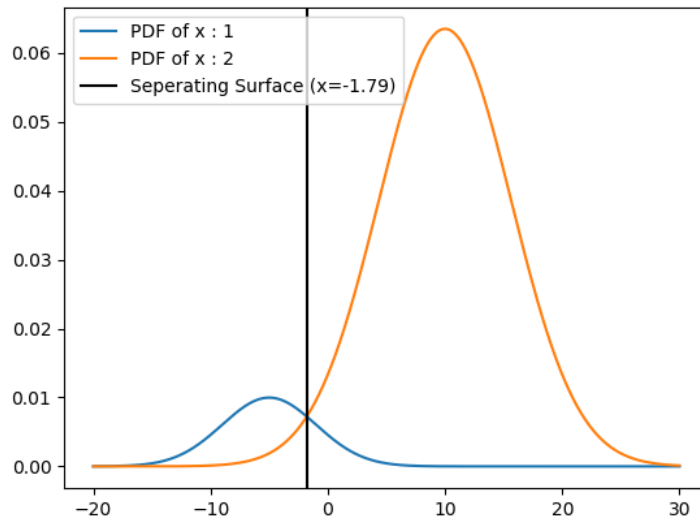


Figure 1.2: Plot of (Q1.b)

We can see that the probability of making decision in favor of the class 2 is higher than before.

### 1.3 PLOTTING THE HISTOGRAMS AND THE SEPARATING SURFACE (N=10)

We are needed to create random samples based on the given standard deviation and mean values. First, we created two 10 sample sized spaces and plotted their histogram with these probability and mean values.

$$P(w_1) = 0.5, P(w_2) = 0.5, P(w_3) = 0.1, P(w_4) = 0.9 \quad (1.25)$$

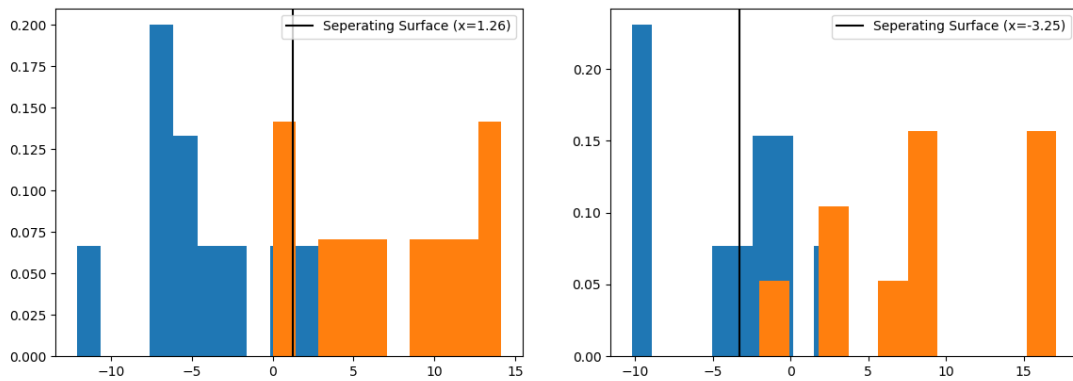
$$\sigma = 4 \quad (1.26)$$

$$\mu_1 = -5, \mu_2 = 10 \quad (1.27)$$

Calculated the new mean values from these histograms and run the separating surface function with these new mean values.

$$\mu_1 = -4.56, \mu_2 = 7.52, \mu_3 = -3.74, \mu_4 = 8.28 \quad (1.28)$$

With given mean and standard deviation values, the histograms and separating surface plot will be like the following;



(a) With probabilities in (1.a)

(b) With probabilities in (1.b)

Figure 1.3: Histograms and Separating Surface Values for N=10

We can see that the histogram based data verifies our previous assumptions.

### 1.4 PLOTTING THE HISTOGRAMS AND THE SEPARATING SURFACE (N=100)

We are needed to create random samples based on the given standard deviation and mean values. First, we created two 100 sample sized spaces and plotted their histogram with these probability and mean values.

$$P(w_1) = 0.5, P(w_2) = 0.5, P(w_3) = 0.1, P(w_4) = 0.9 \quad (1.29)$$

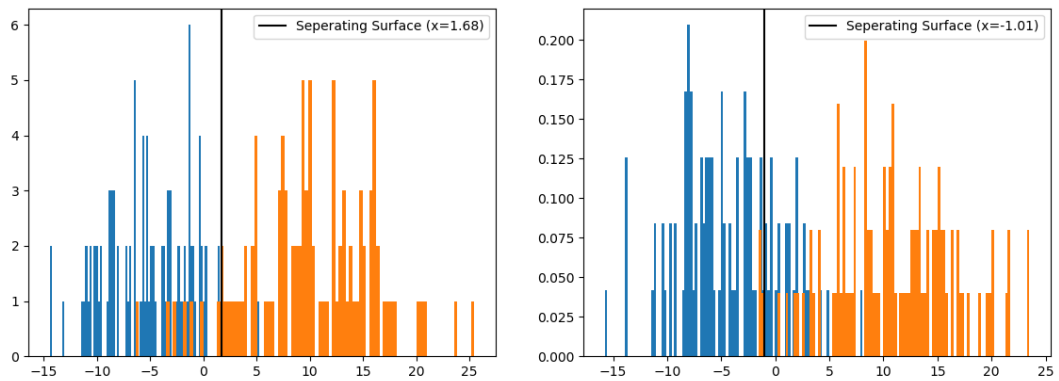
$$\sigma = 4 \quad (1.30)$$

$$\mu_1 = -5, \mu_2 = 10 \quad (1.31)$$

Calculated the new mean values from these histograms and run the separating surface function with these new mean values.

$$\mu_1 = -4.99, \mu_2 = 9.95, \mu_3 = -4.47, \mu_4 = 10.98 \quad (1.32)$$

With given mean and standard deviation values, the histograms and separating surface plot will be like the following;



(a) With probabilities in (1.a)

(b) With probabilities in (1.b)

Figure 1.4: Histograms and Separating Surface Values for N=100

We can see that the histogram based data verifies our previous assumptions.

### 1.5 PLOTTING THE PDFS AND THE SEPARATING SURFACE (WITH 3RD CLASS)

$$P(w_j|x) = \frac{p(x|w_j)P(w_j)}{p(x)} \quad (1.33)$$

$$g_i(x) = \ln p(x|w_i) + \ln P(w_i) \quad (1.34)$$

In the first equation, we have the basic definition of the Bayes formula. In the second equation, we have the posterior probability in a log scale. Summing logs means multiplying parameters inside the log. We have the prior probability, mean and standard deviation as follows;

$$P(w_1) = 0.4, P(w_2) = 0.4, P(w_3) = 0.2 \quad (1.35)$$

$$\sigma = 4 \quad (1.36)$$

$$\mu_1 = -5, \mu_2 = 10, \mu_3 = 5 \quad (1.37)$$

With given mean and standard deviation values, the PDF (posterior) and decision region plot will be like the following;

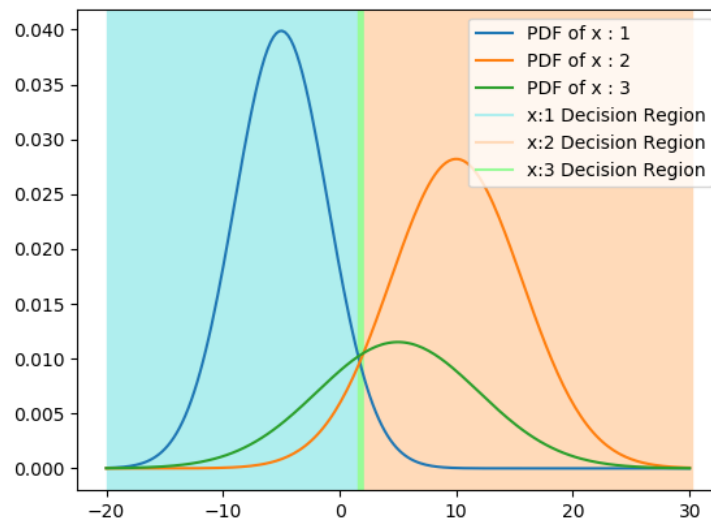


Figure 1.5: Plot of (Q1.d)

We can see that the decision region of class 3 is the smallest one.

## 2 RUNNING THE CODE

The assignment has been written in **Python 3.6.2** with **Sublime Text 3.1.1** IDE. In order to run the assignment code, the following Python libraries are required;

- math
- numpy
- matplotlib
- scipy

Simply running the code will give **Figure 1a,1b,1c(N10,1a),1c(N100,1a), 1c(N10,1b),1c(N100,1b), 1d**, respectively. Also in the console you can find the parameter outputs.

```
Function output example: The 1.a. seperator is on x(1.7297195052747578)
1.A Decision Surface at 1.7297195052747578
1.B Decision Surface at -1.7912565829379605
1.C.1 Decision Surface at 1.119241394364648
1.C.1 New Means mu1: -6.26, mu2: 10.78
1.C.1 New Std Devs sigma1: 3.59, sigma2: 5.16
1.C.2 Decision Surface at 1.9806857335358319
1.C.2 New Means mu1: -4.8, mu2: 9.82
1.C.2 New Std Devs sigma1: 4.21, sigma2: 5.44
1.C.3 Decision Surface at -1.5044024865649814
1.C.3 New Means mu1: -5.74, mu2: 7.75
1.C.3 New Std Devs sigma1: 3.0, sigma2: 3.81
1.C.4 Decision Surface at -1.980074505180342
1.C.4 New Means mu1: -4.84, mu2: 9.57
1.C.4 New Std Devs sigma1: 3.75, sigma2: 5.67
1.D First Decision Surface at 1.6033512225956448
1.D Second Decision Surface at 2.055393315932129
```

Figure 2.1: Parameter outputs in the console