ENGINEERING MATHEMATICS 2.EXAM JANUARY 2012

1. Find the solution of the initial value problem given below

$$(2xy - 1)dx + x^2dy = 0$$

$$for\ \ x = 1 \quad y = 2,$$

2. Find the most general solution of the system of linear algebraic equations given below

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \\ 2 & 0 & 5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix}$$

3. Find the most general solution of the differential equation given below

$$L(\mathbf{D})y = e^t + sin(t)\ where\ \ L(x) = (x-1)^3\ \ \mathbf{D} = \frac{d}{dt}$$

4. Find the linear independent eigenvectors of the matrix $\mathbf{A}$ given below.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 9 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 9 & 2 \end{bmatrix}$$

1) $M = 2xy - 1 \qquad N = x^2 \qquad \frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2x$ exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

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$\frac{\partial g}{\partial x} = M \qquad \frac{\partial g}{\partial y} = N \qquad \frac{\partial g}{\partial x} dx = \int M\,dx = \int (2xy-1)\,dx = x^2y - x + c(y) = g(x,y)$

$\frac{\partial g}{\partial y} = N(x,y) = x^2 \qquad c'(y) = 0 \qquad c(y) = c_1 \qquad g(x,y) = x^2y - x + c_1$

$x^2 + c'(y) = x^2 \qquad d(g(x,y)) = 0 \qquad g(x,y) = c_2$

$x^2y - x + c_1 = c_2$

$x^2y - x = c_3$

$1^2 \cdot 2 - 1 = c_3 \qquad c_3 = 1$

$x^2y - x - 1 = 0 \qquad y = \frac{x+1}{x^2} = \frac{1}{x} + \frac{1}{x^2}$

2) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 1 & -1 & 3 & 3 \\ 2 & 0 & 5 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & -2 & +1 & -1 \\ 0 & -2 & +1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{array}\right] \to$ inconsistent

$(x-1)^3 = 0 \qquad x_{1,2,3} = 1 \qquad y_h = c_1e^t + c_2te^t + c_3t^2e^t$

$y_{p_1} = A\sin t + B\cos t$

$y_{p_1}' = A\cos t - B\sin t \qquad (x-1)^3 = x^3 - 3x^2 + 3x - 1$

$y_{p_1}'' = -A\sin t - B\cos t \qquad -A\cos t + B\sin t + 3A\sin t + \underline{3B\cos t} + \underline{3A\cos t} - 3B\sin t - A\sin t - \underline{B\cos t} = \sin t$

$y_{p_1}''' = -A\cos t + B\sin t$

$2A\cos t + 2B\cos t = 0 \qquad A = -B$

$2(A+B) = 0 \qquad -2B + 2A = 1$

$A = \frac{1}{4} \qquad B = -\frac{1}{4}$

$y_{p_1} = \frac{1}{4}\sin t - \frac{1}{4}\cos t = \frac{1}{4}(\sin t - \cos t)$

$y_{p_2} \to \left|\begin{array}{ccc|c} e^t & te^t & t^2e^t & 0 \\ e^t & e^t(1+t) & (t^2+2t)e^t & 0 \\ e^t & e^t(2+t) & (t^2+4t+2)e^t & e^t \end{array}\right| \sim \left|\begin{array}{ccc|c} 1 & t & t^2 & 0 \\ 1 & 1+t & t^2+2t & 0 \\ 1 & 2+t & t^2+4t+2 & 1 \end{array}\right| \sim \left|\begin{array}{ccc|c} 1 & t & t^2 & 0 \\ 0 & 1 & 2t & 0 \\ 0 & 2 & 4t+2 & 1 \end{array}\right| \sim$

$\left|\begin{array}{ccc|c} 1 & t & t^2 & 0 \\ 0 & 1 & 2t & 0 \\ 0 & 0 & 2 & 1 \end{array}\right| \sim \left|\begin{array}{ccc|c} 1 & 0 & -t^2 & 0 \\ 0 & 1 & 2t & 0 \\ 0 & 0 & 2 & 1 \end{array}\right| \qquad 2u_3' = 1 \qquad u_3' = \frac{1}{2}$

$u_3 = \frac{1}{2}t + d_3$

$u_2' = -2tu_3'$

$u_2' = -t \qquad u_2 = -\frac{t^2}{2} + d_2$

$u_1' = t^2u_3' = t^2\frac{1}{2} \qquad u_1 = \frac{t^3}{6} + d_1$

$y_{p_2} = \left(\frac{t^3}{6} + d_1\right)e^t + \left(-\frac{t^2}{2} + d_2\right)te^t + \left(\frac{1}{2}t + d_3\right)t^2e^t$

$= e^t\left[\left(\frac{t^3}{6}\right) + d_3t^2 + d_2t + d_1\right]$

$y_p = e^t\left(\frac{t^3}{6} + d_3t^2 + d_2t + d_1\right) + \frac{1}{4}(\sin t - \cos t)$

4) $\det(\lambda I - A) = \begin{vmatrix} \lambda-2 & -1 \\ -9 & \lambda-2 \end{vmatrix} = \lambda^2 - 4\lambda - 5 = 0$

$\lambda_{1,2} = 5 \qquad \lambda_{2} = -1$