(4) The time arrows show that time is being used to carry out an event.

(5) The activity box shows an action either currently in progress or soon to be accomplished.

(6) The connection matrix box is a convenient way of showing how certain outputs are associated with activity boxes without cluttering up the chart with lines. The inputs go into either the right or left sides of the matrix rows, but only one input goes to any row. The outputs come out the bottom. The X shows the connection between the input and output.

(7) The ground is a symbol to denote the completion of a particular activity.

The sequence of elements (decision, events, activities, etc.) are shown on DELTA charts and convey the order in which it is planned to carry out the various operations and may indicate time durations for activities and time of occurrence for events and decisions.

The use of DELTA charts in project planning clearly illustrates the decision points and can stimulate more meaningful bidding practices. A precise syntax for the DELTA chart components is defined in order to make them capable of presenting a clear precise picture that is self-explanatory to a wide audience. This chart includes the decision and logic boxes which provide a method for clearly and realistically portraying project plans and promote innovative approaches to project planning. Because PERT does not conveniently allow for alternatives, decisions, and logic, it only tends to constrain thinking to a single narrow path. Hence, the DELTA chart was developed to satisfy a need for an improved method for depicting a planned flow of activities in research and development (R & D) projects.

15. GROUP DECISION MAKING UNDER MULTIPLE CRITERIA FOR EVALUATION/SELECTION OF ALTERNATIVES

15.1. INTRODUCTION

It has become increasingly more complicated for decision makers to make the right decision at the right time. For example, to select the winner for a Nobel peace prize or to find the right person to fill a certain position is difficult because there may be many qualified applicants. In these types of cases, they must compare, rank, rate, or score in order to make the best choice -- this task is definitely not easy.

Generally, the sequential procedures of decision making include: the preparatory phase, the screening phase, the evaluating phase, and the decision phase. The preparatory phase includes advertizing very specifically for what is desired in the applicant. The screening phase consists of using various methods to eliminate the unqualified candidates. The evaluating phase includes reviewing the application of the qualified candidates and interviewing them. Finally, in the decision phase, the committee members may either make a recommendation to the president or manager or they may make a list of pros and cons of each eligible applicant and let the president or manager decide.

This section discusses the evaluation and decision phases of group problem solving. It presents a possible mathematical and systematical approach to collective or group decision making under multiple criteria consideration. It also presents the ordinal and cardinal approaches (See Fig. 15.1).

In the mathematical and systematical approach, the criteria can either be classified as quantitative which is measurable or qualitative which is judgmental and difficult to measure. An example of quantitative would be job experience and an example of qualitative would be a personal characteristic such as dependability.

Probably the most commonly used evaluation techniques are ranking, rating, scoring and utility fuction, all of which indicate preferences in regard to a group of candidates under consideration. The ordinal approach, which involves the ranking of candidates, has been discussed by Souder [S68, S69], Bernardo [B19], Cook and Seiford [C37, C39], Franz, Lee, and Van Horn [F60], to mention just a few. The cardinal approach, which involves the scoring of candidates, has been discussed by Eckenrode [E1], Dean and Nishry [D13], Fishburn [F20], Souder [S67], Minnehan [M46], Keeney and Kirkwood [K19], Dyer and Miles [D29], Hwang and Yoon [H56], and many others.

If one candidate stands head and shoulders above all the rest in all respects, there is no problem and these approaches need not be applied. More often though, there are several candidates whose overall characteristics are fairly similar, then

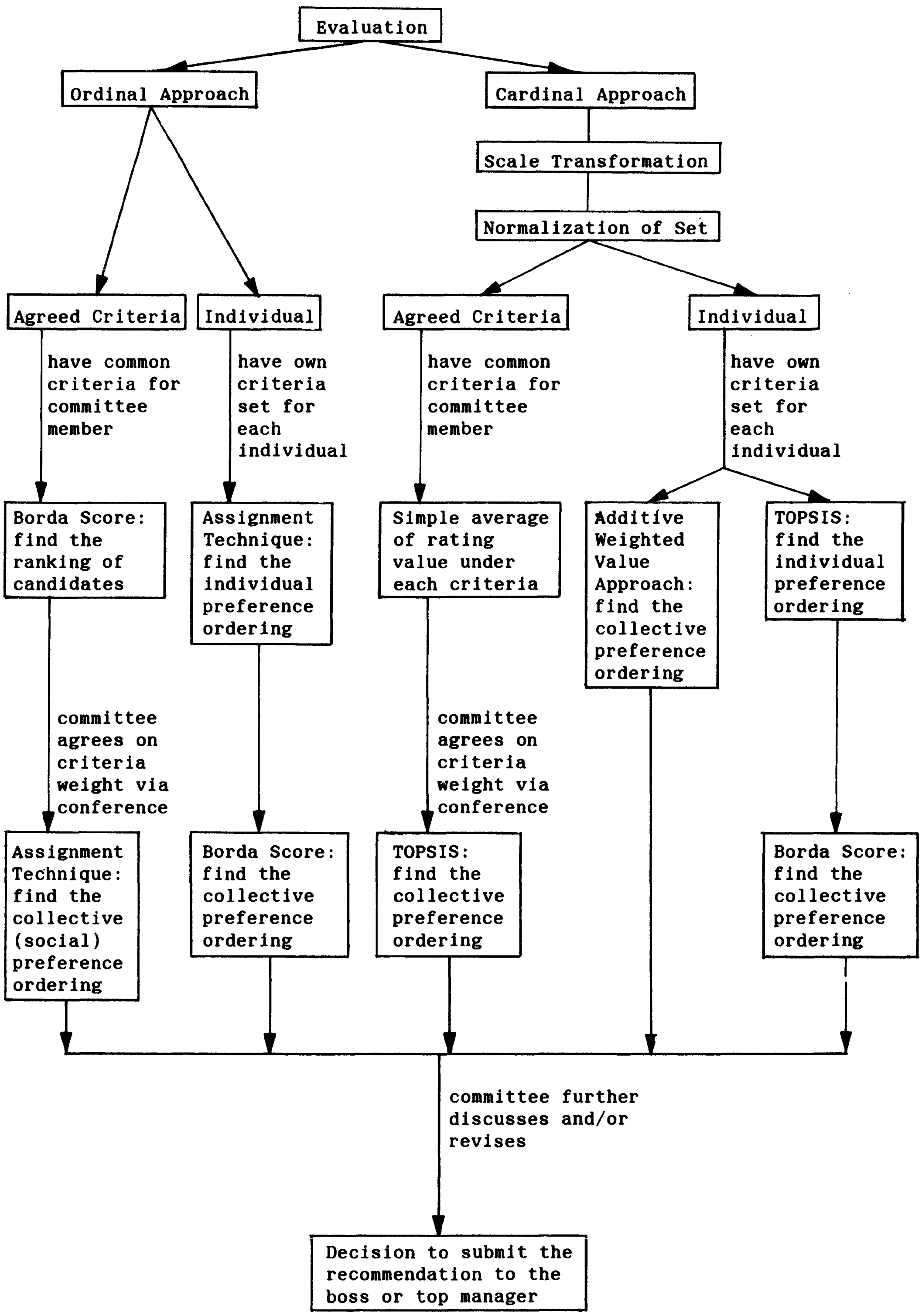

Figure 15.1 The Group Decision Process in the Phases of Evaluation and Selection

either one of the approaches may be applied.

15.2. A GENERAL FORMULATION

The process of evaluating candidates involves certain required criteria. For instance, suppose a school wants to find a faculty member. The committee evaluating the candidates for the position may consider each candidate's potential contribution to the school in reference to teaching, research development, university service, etc. It would be difficult for the committee to decide which of these contributions is most significant.

Let us assume that we have m candidates being evaluated by n committee members, who are using p criteria (each committee member may or may not be using the same criteria). The problem in matrix form would be the following:

$$A^k = [a_{ij}]^k = \begin{pmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1p} \\ a_{21} & \cdots & a_{2j} & \cdots & a_{2p} \\ \vdots & & & & \vdots \\ a_{m1} & \cdots & a_{mj} & \cdots & a_{mp} \end{pmatrix} \qquad (k = 1,\ldots,n)$$

The symbol $A^k_{i.} = [a_{i1}, \ldots, a_{ip}]^k$ means that candidates i are being evaluated by criteria from 1 to p by committee member k. The symbol $A^k_{.j} = [a_{1j}, \ldots, a_{mj}]^k$ means that criterion j is being used by committee member k to evaluate all candidates from 1 to m where m is the number of candidates.

The solution to this problem is to have each candidate be evaluated by n number of committee members, using various p criteria. This process can be summarized as the following mapping function.

$$\Psi : \{A^k \mid k=1,\ldots,n\} \rightarrow \{G\}$$

This mapping function could be obtained through ranking, rating, scoring, or voting. It is crucial that this mapping function represent all the various criteria that the committee members used in judging all the candidates.

The next step is to use either or both the ordinal function (such as ranking) and/or the cardinal function (such as rating, or scoring). The ordinal approach will be described first followed by a sketch of the cardinal approach.

15.3. THE ORDINAL APPROACH

The matrix contains all of the information pertinent to the problem. It includes all the criteria used in ranking all of the candidates by all of the committee members.

There are two approaches in ranking candidates -- the agreed criteria approach and the individual approach.

15.3.1 The Agreed Criteria Approach

The agreed criteria approach involves each committee member using the same criteria to find the matrices of all the candidates, the committee being in agreement on the type of criteria being used. For each criterion ℓ (ℓ =1, ..., p) we have a matrix

$$C = \begin{pmatrix} a^1_{1\ell} & a^2_{1\ell} & \cdots & a^n_{1\ell} \\ a^1_{2\ell} & a^2_{2\ell} & \cdots & a^n_{2\ell} \\ \vdots & & & \vdots \\ a^1_{m\ell} & a^2_{m\ell} & \cdots & a^n_{m\ell} \end{pmatrix} \qquad (\ell = 1, \ldots, p)$$

Borda's score is then determined for each candidate by each committee member. For example, since m is the total number of candidates, the first place candidate would receive a score of m-1, the second place, m-2, etc. Then the candidate with the highest Borda score, that is, the sum of all the committee members' Borda scores, would receive first place, the second highest, second place, etc.

Now, we have a collective ordered matrix which maps the form of $\{A^k ; K = 1, \ldots, n\}$ into $\{A'\}$. That is,

$$A' = [a'_{ij}] = \begin{bmatrix} a'_{11} & \cdots & a'_{1p} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a'_{m1} & \cdots & a'_{mp} \end{bmatrix}$$

where $[a'_{ij}]$ is the ordering of candidate i under criterion j.

However, it is possible that some criteria may be more important than others; therefore, a committee would want to place more weight on that criterion. To accomplish this, we need to have a vector of weights, $\underline{W} = \{w_1, \ldots, w_p\}$ where w_i is the weight assigned to the i^{th} criterion and $\sum_{i=1}^{p} w_i = 1$. These values or weights can be found by the eigenvector function which is when committee members compare all criteria on a one to one basis, or by other methods described in Hwang and Yoon [H56].

Then we formulate an agreement matrix, π ; this is a square mxm nonnegative matrix in which entries $\pi_{ij\ell}$ represent the number of orderings where the ith candidate is placed in the jth position for a given criterion ℓ. The set of weights for criteria should be used in the decision process, and we will have the collective weighted agreement matrix $G = [g_{ij} = \sum_{\ell=1}^{p} \pi_{ij\ell} w_\ell]$, where $\pi_{ij\ell} = 1$ if the ith candidate is placed in the jth position, otherwise it is zero.

We want to match candidate i with rank number j so that the sum of the corresponding assigned weight value is the largest possible. This task can be achieved by solving the so-called assignment problem of linear programming:

$$\text{Max} \quad \sum_{i=1}^{m} \sum_{j=1}^{m} g_{ij} \, x_{ij}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = 1 \quad , \; j = 1, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1 \quad , \; i = 1, \ldots, m$$

where $x_{ij} = 1$ if j has been assigned to i and $x_{ij} = 0$ otherwise.

15.3.2 The Individual Approach

The individual approach involves each committee member having his own criteria, which may or may not differ from other committee members, to determine each candidate's matrix. For example, committee member k has a set of criteria index by $\{1, \ldots, p\}^k$. The other committee members may or may not share some of committee member k's criteria. An individual would want to assign a set of importance weights to the criteria, $\underline{W} = (w_1, \ldots, w_p)^k$, k=1, ..., n and $\sum_{j=1}^{p} w_j^k = 1$, where w_j^k is the weight assigned to the j^{th} criterion by individual k. These weights can be obtained via the eigenvector function or other methods (see Hwang and Yoon [H56]). The committee member sets up his/her own agreement matrix π, a square mXm matrix in which entries $\pi_{ij\ell}$ represent the number of individual orderings where the ith candidate is placed in the jth position for the given criterion ℓ and the value $\pi_{ij\ell}$ is equal to one; otherwise, it is a zero. The inclusion of weight criteria allows the setting up of an assignment problem to find linear orderings of candidates for each individual $F^k = [f_{ij}] = [\sum_{\ell=1}^{p} \pi_{ij\ell} w_\ell]^k$, followed by the formulation of the assignment problem of linear programming for each committee member:

$$\text{Max} \quad \sum_{i=1}^{m} \sum_{j=1}^{m} f^k_{ij} x_{ij} \qquad , \; k=1, \ldots, n$$

subject to

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad , \quad j=1, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad , \quad i=1, \ldots, m$$

where $x_{ij} = 1$ if jth position has been assigned to ith candidate by committee member k and $x_{ij} = 0$ otherwise.

In each set of preference orderings of the candidates, scores of m-1, m-2, ..., 1, 0 to the first ranked, second ranked, ..., last ranked for each individual are assigned. Then the Borda score for each candidate (the sum of the individual scores for each candidate) is determined. The candidate with the highest Borda score is first place; the candidate with the next highest score is second; etc. In this way, a complete ordering of candidates is obtained.

15.3.3 Numerical Example (see section 2.12 of Part II)

Six experts in each area ranked each of the five feasible alternatives a_2 through a_6 according to each of the three criteria S_1, S_2, and S_3.

We have six ordinal rank matrices as follows:

$$A^1 = \begin{array}{c} \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} S_1 \;\; S_2 \;\; S_3 \\ \begin{bmatrix} 5 & 3 & 3 \\ 2 & 1 & 2 \\ 3 & 4 & 4 \\ 4 & 5 & 5 \\ 1 & 2 & 1 \end{bmatrix} \end{array}, \quad A^2 = \begin{array}{c} S_1 \;\; S_2 \;\; S_3 \\ \begin{bmatrix} 3 & 4 & 4 \\ 2 & 2 & 1 \\ 5 & 3 & 5 \\ 4 & 5 & 2 \\ 1 & 1 & 3 \end{bmatrix} \end{array}$$

$$A^3 = \begin{array}{c} a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{bmatrix} 3 & 4 & 4 \\ 1 & 1 & 2 \\ 5 & 3 & 5 \\ 4 & 5 & 1 \\ 2 & 2 & 3 \end{bmatrix}, \quad A^4 = \begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & 1 \\ 5 & 4 & 5 \\ 3 & 2 & 4 \\ 1 & 5 & 2 \end{bmatrix}$$

$$A^5 = \begin{array}{c} a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{bmatrix} 4 & 4 & 4 \\ 1 & 2 & 2 \\ 5 & 5 & 5 \\ 3 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}, \quad A^6 = \begin{bmatrix} 1 & 5 & 5 \\ 3 & 1 & 2 \\ 5 & 4 & 4 \\ 4 & 3 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

(A) The Agreed Criteria Approach

For each criterion there is matrix which includes all judges and alternatives. Scores of 4, 3, 2, 1, 0 to the first ranked, second ranked, ..., fifth ranked are then assigned. The results are as follows:

For criterion S_1, we have

	expert 1	expert 2	expert 3	expert 4	expert 5	expert 6
a_2	5	3	3	4	4	1
a_3	2	2	1	2	1	3
a_4	3	5	5	5	5	5
a_5	4	4	4	3	3	4
a_6	1	1	2	1	2	2

→ assign score of 4, 3, 2, 1, 0 to the first ranked, ..., last ranked

a_2	0	2	2	1	1	4
a_3	3	3	4	3	4	2
a_4	2	0	0	0	0	0
a_5	1	1	1	2	2	1
a_6	4	4	3	4	3	3

→ Borda scores which are sums across each row

a_2	10
a_3	19
a_4	2
a_5	8
a_6	21

The candidate with the highest Borda score is in the first place. Therefore, the preference ordering of the alternatives for criterion S_1 is: $\{a_6, a_3, a_2, a_5, a_4\}$. Similarly, the preference ordering of the alternatives for criterion S_2 is: $\{a_3, a_6, a_2, a_4 \sim a_5\}$ where ~ indicates that there is a tie between a_4 and a_5, the preference ordering of the alternatives for criterion S_3 is: $\{a_3, a_6, a_5, a_2, a_4\}$. The collective ordinal rank matrix is:

A' =

	S_1	S_2	S_3
a_2	3	3	4
a_3	2	1	1
a_4	5	4.5*	5
a_5	4	4.5*	3
a_6	1	2	2

The value of 4.5* means that the alternatives 4 and 5 share the position of fourth and fifth place. Then the collective weighted-agreement matrix which includes the agreement matrix π and takes account of the weight vector (w_1, w_2, w_3) can be formulated. For example, $g_{11} = \sum_{\ell=1}^{3} \pi_{11\ell} w_\ell = 0$, and $g_{13} = \sum_{\ell=1}^{3} \pi_{13\ell} w_\ell = (1)w_1 + (1)w_2 +$

$(0)w_3 = w_1 + w_2$, and so on. Then the a matrix is:

$$G = \begin{array}{c} \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccc} \text{1st} & \text{2nd} & \text{3rd} & \text{4th} & \text{5th} \end{array} \\ \left(\begin{array}{ccccc} 0 & 0 & w_1+w_2 & w_3 & 0 \\ w_2+w_3 & w_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2w_2 & w_1+w_2+1/2w_2 \\ 0 & 0 & w_3 & w_1+1/2w_2 & 1/2w_2 \\ w_1 & w_1+w_2 & 0 & 0 & 0 \end{array} \right) \end{array}$$

A vector of weights (w_1, w_2, w_3) should be agreed upon by all experts through some discussion, evaluation and compromise. The Eigenvector function method may be used in the process of determining the weights. For instance, let us assume that the committee members agreed with the weight of $\underline{W}$ = (0.2, 0.3, 0.5), then the collective weighted-agreement matrix is:

$$G = \begin{array}{c} \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccc} \text{1st} & \text{2nd} & \text{3rd} & \text{4th} & \text{5th} \end{array} \\ \left(\begin{array}{ccccc} 0 & 0 & 0.5 & 0.5 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \\ 0 & 0 & 0 & 0.35 & 0.15 \\ 0.2 & 0.8 & 0.5 & 0 & 0 \end{array} \right) \end{array}$$

The assignment problem technique can be used to solve the problem. This problem can also be solved by observation, because we are dealing with a maximization problem and can simply maximize the total weight of assigning alternative i to position j. For instance, we have

i \ j	1	2	3	4	5
a_2	0	0	0.5	(0.5)	0
a_3	(0.8)	0.2	0	0	0
a_4	0	0	0	0.15	(0.85)
a_5	0	0	(0.5)	0.35	0.15
a_6	0.2	(0.8)	0	0	0

The ranks are: $\{a_3, a_6, a_2, a_5, a_4\}$, and the corresponding maximized weight is 3.45 (=.8 + .8 + .5 + .5 + .85)

(B) The Individual Criteria Approach

Each individual has a personal set of criteria weights $\underline{W}^k = (w_1, w_2, w_3)^K$, k= 1, ..., n. Then he/she sets up an agreement matrix and also takes account of criteria weight. For expert 1, the agreement matrix is as follows:

Rank / Alternative	1st	2nd	3rd	4th	5th
a_2	0	0	w_2+w_3	0	w_1
a_3	w_2	w_1+w_3	0	0	0
a_4	0	0	w_1	w_2+w_3	0
a_5	0	0	0	w_1	w_2+w_3
a_6	w_1+w_3	w_2	0	0	0

If expert 1 sets up the weight of $\underline{W}$ = (0.2, 0.3, 0.5), then the F matrix becomes

$$F = \begin{array}{c} \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array} \begin{array}{c} \begin{array}{ccccc} \text{1st} & \text{2nd} & \text{3rd} & \text{4th} & \text{5th} \end{array} \\ \begin{pmatrix} 0 & 0 & \textcircled{0.8} & 0 & 0.2 \\ 0.3 & \textcircled{0.7} & 0 & 0 & 0 \\ 0 & 0 & 0.2 & \textcircled{0.8} & 0 \\ 0 & 0 & 0 & 0.2 & \textcircled{0.8} \\ \textcircled{0.7} & 0.3 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Then it can be solved by the assignment problem technique. The results of ranking are $\{a_6, a_3, a_2, a_4, a_5\}$ for the evaluation of expert 1. Similarly, if expert 2 gives the vector of weight $\underline{W}$ = (0.3, 0.3, 0.4), the ranking is $\{a_6, a_3, a_2, a_5, a_4\}$ or $\{a_6, a_3, a_4, a_2, a_5\}$; for expert 3, the vector of weight $\underline{W}$ = (0.2, 0.4, 0.4), the ranking is $\{a_3, a_6, a_4, a_2, a_5\}$; for expert 4, the vector of weight $\underline{W}$ = (0.3, 0.4, 0.3), the ranking is $\{a_2, a_5, a_3, a_4, a_6\}$; for expert 5, the equality weight for all of criteria, the ranking is $\{a_3, a_6, a_5, a_2, a_4\}$; and for expert 6, the vector of weight $\underline{W}$ = (0.3, 0.2, 0.5), the ranking is $\{a_6, a_3, a_5, a_4, a_2\}$.

To aggregate the preference orderings into a consensus ordering, the Borda score is used. Scores of 4, 3, 2, 1, 0 are assigned to the first ranked, second ranked..., last ranked candidates. In this case, Expert 2 has two different ranking orders which is evident in the cases of (a) and (b):

(a)

	expert 1	expert 2	expert 3	expert 4	expert 5	expert 6
a_2	3	3	4	1	4	5
a_3	2	2	1	3	1	2
a_4	4	5	3	4	5	4
a_5	5	4	5	2	3	3
a_6	1	1	2	5	2	1

→ assign score of 4, 3, 2, 1, 0 to first ranked,..., last ranked

a_2	2	2	1	4	1	0
a_3	3	3	4	2	4	3
a_4	1	0	2	1	0	1
a_5	0	1	0	3	2	2
a_6	4	4	3	0	3	4

→ find Borda score via sum of each row

a_2	10
a_3	19
a_4	5
a_5	8
a_6	18

The result of ranking is $\{a_3, a_6, a_2, a_5, a_4\}$.

(b)

	expert 1	expert 2	expert 3	expert 4	expert 5	expert 6
a_2	3	4	4	1	4	5
a_3	2	2	1	3	1	2
a_4	4	3	3	4	5	4
a_5	5	5	5	2	3	3
a_6	1	1	2	5	2	1

→ assign score for each ranked-order

a_2	2	1	1	4	1	0
a_3	3	3	4	2	4	3
a_4	1	2	2	1	0	1
a_5	0	0	0	3	2	2
a_6	4	4	3	0	3	4

→ find Borda score via sum of each row

a_2	9
a_3	19
a_4	7
a_5	7
a_6	18

Then the result of ranking is $\{a_3, a_6, a_2, a_4 \sim a_5\}$, where a_4 and a_5 are tied for last place.

15.4. THE CARDINAL APPROACH

Cost, time, speed, volume, and so on are expressed in numerical or quantitative terms, but they may be in different units. Performance, feeling, happiness, dependability, and so forth are expressed in nonnumerical or qualitative terms. The question that arises is how should these different types of criteria be compared? Furthermore, how can the nonhomogeneous units of measure such as \$, hrs, m/sec, m^3, lb be taken care of and so on? These are scaling problems, and they have been discussed in Hwang and Yoon [H56]. We need two stages to transform these criteria into a set of comparable scales. First, in the case of qualitative terms, we need to transfer the qualitative into an interval scale. For example, we may choose a 10-point scale and give 10 points to the maximum value and 0 points to the minimum value. Thus, the rating of "very high" may be assigned to the value 9.0, and the rating of "high" to the scores from 5.1 to 8.9. Therefore, 7.0 would be the scale value for the "high" group. On the low end of the scale, "very low" may be assigned to the value 1.0, and "low" to the value of 3.0. There are many other scales and points. For example, there is the scale of (very bad, bad, poor, fair, good, very good, excellent), and the point scale of 0-100. The committee members should agree on the scaling procedures they use. Secondly, how should values with different units of measurement be compared? There are different ways of normalizing values and scales which have been described in Hwang and Yoon [H56]. In this section, the vector normalization is used because all criteria are measured in dimensionless units. This procedure implies that each column vector of the individual decision matrix is divided by its norm, so that each normalized value d^k_{ij} of the individual normalized decision matrix D^k can be calculated as

$$d^k_{ij} = \frac{a^k_{ij}}{\sqrt{\sum_{i=1}^{m} (a^k_{ij})^2}} \quad , \; k= 1, \ldots, n \quad j= 1, \ldots, p$$

For a given set of criteria, the committee members should distinguish between "benefit" criteria and "cost" criteria. The larger the value (scale) outcomes, the greater the preference for the "benefit" criteria and the less the preference for the "cost" criteria. Therefore, each criterion in the individual decision matrix is either monotonically increasing or monotonically decreasing. The comparing of matrices includes all candidates, all criteria, and all evaluations by the committee members. Now the formulation of collective ordering, which is according to the agreed criteria or the individual approach, can be found.

15.4.1 The Agreed Criteria Approach

In this formulation of a committee choice problem, all members have equal power, and their evaluations have equal importance. Under a given criterion, we find a collective value which is an aggregation of the values of the committee members. We have the form of the collective matrix C as

$$C = [c_{ij}] = \left[\frac{\sum_{k=1}^{n} d^{k}_{ij}}{n} \right], \qquad i = 1, \ldots, m \quad j = 1, \ldots, p$$

Since all criteria may or may not be of equal importance, this method sets up a vector of weights from the committee. The methods of Delphi or NGT can be used to find this vector of weights.

A vector of weights is $\underline{W} = \{w_1, \ldots, w_p\}$, $\sum_{j=1}^{p} w_j = 1$. Now the weighted normalized collective matrix can be calculated by multiplying each column of the matrix C with its associated weight w_j. Therefore, the weighted normalized collective matrix, F, is

$$F = [\, f_{ij} \,] = [\, c_{ij} w_j \,] \quad , i = 1, \ldots, m \; ; \quad j = 1, \ldots, p$$

The technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which was developed by Hwang and Yoon [H56], is based upon the concept that the chosen candidate should be the shortest distance from the ideal solution and the farthest from the negative-ideal solution. An ordering of candidates which is based on the distance of relative closeness to the ideal solution is selected. The procedure of TOPSIS is presented as follows:

Step 1 Determining ideal and negative-ideal solutions

Let the two sets of artifical candidates A^* and A^- be defined as

$$A^* = \{(\max_i f_{ij} \mid j \in J), (\min_i f_{ij} \mid j \in J') \quad i = 1, \ldots, m\}$$

$$= \{f^*_1, \ldots, f^*_j, \ldots, f^*_p\}$$

$$A^- = \{(\min_i f_{ij} \mid j \in J), (\max_i f_{ij} \mid j \in J') \quad i = 1, \ldots, m\}$$

$$= \{f^-_1, \ldots, f^-_j, \ldots, f^-_p\}$$

where $J = \{j = 1, \ldots, p \mid j$ associated with benefit criteria$\}$ and $J' = \{j = 1, \ldots, p \mid j$ associated with cost criteria$\}$.

Then it is certain that the two new sets of candidates A^* and A^- indicate the most preferable candidate (ideal solution) and the least preferable candidate (negative-ideal solution), respectively.

Step 2 Calculating the separation measures

The separation between each candidate can be measured by the n-dimensional Euclidean distance. The separation of each candidate from the ideal one is given by

$$S_{i*} = \sqrt{\sum_{j=1}^{p} (f_{ij} - f^*_j)^2} \quad , \quad i = 1, \ldots, m$$

Similarly, the separation from the negative-ideal one is given by

$$S_{i-} = \sqrt{\sum_{j=1}^{p} (f_{ij} - f^-_j)^2} \quad , \quad i = 1, \ldots, m$$

Step 3 Calculating the relative closeness to the ideal solution

The relative closeness of A_i with respect to A^* is defined as

$$G_i = S_{i-} / (S_{i*} + S_{i-}) \quad , 0 < G_i < 1 \; , \quad i = 1, \ldots, m$$

It is clear that $G_i = 1$ if $A_i = A^*$ and $G_i = 0$ if $A_i = A^-$. When candidate A_i is closer to A^* then G_i is closer to 1.

Step 4 Ranking the collective (social) preference ordering

A set of candidates can now be ranked according to the descending order of G_i.

15.4.2 The Individual Approach

Committee member k has a personal set of criteria index $\{1, \ldots, p\}^k$ which may or may not share some of the other committee members' criteria. An individual has to assign a vector of criteria weights as $\underline{W}^k = (w^k_1, \ldots, w^k_p)$, k= 1, ..., n and $\sum_{j=1}^{p} w^k_j = 1$, where w^k_j is the weight assigned to the j criterion by individual k. In the following section, we discuss two different kinds of approaches.

15.4.2.1. The Additive Weighted Value Approach

When multiple criteria $j \in \{1, \ldots, p\}$ are involved, the simplest procedure to have is the summation process which is based on the criteria and the individual's transformation function. The formulation of the value function is

$$G_i = \{ \sum_{k=1}^{n} \sum_{j=1}^{p} w^k_j \, d^k_{ij} \mid j \in J\} - \{ \sum_{k=1}^{n} \sum_{j=1}^{p} w^k_j \, d^k_{ij} \mid j \in J'\},$$

$$i = 1, \ldots, m$$

where $J = \{j = 1, \ldots, p \mid j \text{ associated with benefit criteria}\}$ and

$J' = \{j = 1, \ldots, p \mid j \text{ associated with cost criteria}\}$

G_i measures the simple value of alternative i which is based on the difference value between the benefit criteria and the cost criteria which is calculated by all committee members. Then the collective preference orderings are ranked in the order of the value of G_i.

15.4.2.2. TOPSIS and Borda's Function Approach

The individual weighted normalized matrix ,F^k, can be calculated by multiplying each column of the matrix D^k with its associated weight w^k_j. Therefore, the individual weighted normalized matrix, F^k, is

$$F^k = [\, f^k_{ij} \,] = [\, w^k_j \, d^k_{ij} \,], \quad k= 1, \ldots, n; \quad i= 1, \ldots, m; \quad j= 1, \ldots p$$

Then TOPSIS can be used to find the individual preference ordering. The procedure is the same as described in section 15.4.2. Then Borda's function can be used to find the collective (social) preference orderings. With m candidates in A, scores of m-1, m-2, ..., 1, 0 can be assigned to the first ranked, second ranked, ..., last ranked candidate by each committee member. The Borda score (the sum of the committee members scores) can be determined for each candidate. Finally, the candidates are ranked according to their Borda scores.

15.4.3. **Example**: The Recreation Complex Problem

Suppose a company wants to build a new recreation complex. Company executives understand that the evaluation involves a large number of subjective, qualitative, and uncertain questions. Therefore, the company forms a committee to evaluate all the possible places where the complex may be built. Five committee members, who are experts, have been charged with the evaluation and selection task. Committee members must agree on what sites to evaluate, with respect to what criteria, and by use of what scale. Keeping this in mind, the committee decides to evaluate nine different places based on the eleven basic criteria (the committee members may or may not have the same criteria). The committee members then rate each place. In the next stage, these ratings will be calculated, discussed, and possibly revised. The committee must then reach a consensus and submit the overall committee results to the top manager.

Eleven basic criteria were finally agreed upon by the five committee members. The committee members also agreed on the use of the scale (very low, low, average, high, very high) to evaluate the criteria of x_1, x_2, x_4, x_5, x_6, x_{10}, and x_{11}. They also agreed on using the scale from 0 to 10 points to evaluate the criteria of x_3, x_7, x_8, and x_9. They classified them into two categories, one being benefit criteria, x_1, x_2, x_3, x_7, x_8, x_9, x_{10}, and the other being cost criteria, x_4, x_5, x_6, and x_{11}.

Each of the five members on the committee prepares a rating for each of the nine places. These ratings then serve as inputs into the committee's final decision. The ratings of the nine places by the five committee members using the eleven basic criteria are shown as follows:

For expert 1

$A^1 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	average	average	5	low	average	average	6	4	6	high	average
a_2	very low	low	7	high	average	average	3	2	4	average	low
a_3	average	low	5	average	average	average	5	8	1	high	average
a_4	average	low	10	average	average	average	3	6	6	low	average
a_5	low	very low	4	high	average	average	4	6	2	high	high
a_6	high	low	9	high	low	average	4	6	2	low	average
a_7	very high	very low	5	very high	low	average	5	7	6	high	average
a_8	average	very high	5	high	average	high	6	5	6	high	high
a_9	average	average	4	very high	low	average	5	8	5	high	high

For expert 2

$A^2 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	high	average	6	average	average	high	2	4	3	average	high
a_2	very low	low	5	low	low	high	5	5	5	high	high
a_3	high	average	7	average	average	average	3	3	2	average	average
a_4	average	low	2	high	average	average	7	4	2	very high	average
a_5	average	very low	7	average	average	average	9	3	3	average	average
a_6	average	very low	2	high	low	average	9	3	5	very high	average
a_7	very high	low	5	low	average	average	6	3	1	average	average
a_8	high	average	4	high	average	low	5	4	3	average	average
a_9	low	low	5	average	low	average	5	2	3	low	low

For expert 3

$A^3 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	high	very high	6	high	low	average	6	4	8	average	low
a_2	low	average	5	average	average	average	7	5	2	high	low
a_3	low	very high	7	high	low	low	5	3	4	high	average
a_4	average	low	2	high	average	low	6	4	4	very high	average
a_5	average	average	7	average	average	low	5	3	8	average	average
a_6	average	average	2	high	average	average	8	3	6	very high	average
a_7	high	average	5	average	low	high	5	3	3	average	average
a_8	high	average	8	high	high	low	4	3	7	average	high
a_9	very high	average	4	low	low	low	3	2	8	low	high

For expert 4

$A^4 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	average	very high	5	average	average	average	2	4	7	average	average
a_2	low	low	7	high	low	average	9	5	4	low	high
a_3	average	average	5	high	average	average	3	3	2	average	average
a_4	low	low	10	high	low	high	9	4	2	low	average
a_5	high	average	5	high	low	high	8	3	7	average	average
a_6	low	low	9	average	low	high	5	3	5	low	average
a_7	average	very high	5	very high	low	high	4	3	2	average	average
a_8	average	very low	4	average	low	low	3	3	5	average	average
a_9	high	average	4	very high	low	average	4	2	7	high	low

For expert 5

$A^5 =$

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
a_1	high	high	3	low	low	average	8	4	3	high	high
a_2	average	low	5	average	low	average	7	9	5	high	average
a_3	average	high	2	average	average	average	8	1	2	high	average
a_4	average	low	9	average	low	average	2	4	2	low	average
a_5	average	high	2	high	low	low	7	2	5	high	low
a_6	average	low	8	average	low	low	6	2	1	average	high
a_7	low	high	2	very high	low	average	8	2	3	high	low
a_8	high	high	1	average	average	low	8	3	3	high	low
a_9	high	high	1	very high	low	average	8	2	3	high	low

The committee members were able to agree on the scale of qualitative terms. For example, "very high", "high", "average", "low", and "very low" may be associated with the values of 9, 7, 5, 3, 1, respectively. Then the individual normalized decision matrix D^k, k= 1, ..., 5, is determined, which is calculated via

$$d^k_{ij} = \frac{a^k_{ij}}{\sqrt{\sum_{i=1}^{9} (a^k_{ij})^2}}, \quad j= 1, \ldots, 11,$$

as follows:

$D^1 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	.307	.385	.263	.147	.376	.317	.427	.220	.431	.381	.298
a_2	.061	.231	.386	.343	.376	.317	.214	.110	.287	.272	.179
a_3	.307	.231	.263	.245	.376	.317	.356	.440	.072	.381	.298
a_4	.307	.231	.525	.245	.376	.317	.214	.330	.431	.163	.298
a_5	.184	.077	.210	.343	.376	.317	.285	.330	.144	.381	.418
a_6	.430	.231	.473	.343	.226	.317	.285	.330	.144	.163	.298
a_7	.553	.077	.263	.441	.226	.317	.356	.385	.431	.381	.298
a_8	.307	.692	.263	.343	.376	.444	.427	.275	.431	.381	.418

$D^2 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	.396	.470	.393	.307	.376	.437	.109	.376	.308	.269	.437
a_2	.056	.282	.328	.184	.226	.437	.273	.470	.513	.377	.437
a_3	.396	.470	.459	.307	.376	.312	.164	.282	.205	.269	.312
a_4	.283	.282	.131	.430	.376	.312	.382	.376	.205	.485	.312
a_5	.283	.094	.459	.307	.376	.312	.492	.282	.308	.269	.312
a_6	.283	.094	.131	.430	.226	.312	.492	.282	.513	.485	.312
a_7	.509	.282	.328	.184	.376	.312	.328	.282	.103	.269	.312
a_8	.396	.470	.262	.430	.376	.187	.273	.376	.308	.269	.312
a_9	.170	.282	.328	.307	.226	.312	.273	.188	.308	.162	.187

$D^3 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	.391	.502	.364	.386	.221	.385	.355	.388	.446	.260	.193
a_2	.167	.279	.303	.276	.368	.385	.415	.485	.111	.364	.193
a_3	.167	.502	.424	.386	.221	.231	.296	.291	.223	.364	.322
a_4	.279	.167	.121	.386	.368	.231	.355	.388	.223	.469	.322
a_5	.279	.279	.424	.276	.368	.231	.296	.291	.446	.260	.322
a_6	.279	.279	.121	.386	.368	.385	.474	.291	.334	.469	.322
a_7	.391	.279	.303	.276	.221	.538	.296	.291	.167	.260	.322
a_8	.391	.279	.485	.386	.515	.231	.237	.291	.390	.260	.451
a_9	.502	.279	.243	.165	.221	.231	.178	.194	.446	.156	.451

$D^4 =$

a_1	.333	.553	.263	.240	.470	.286	.115	.388	.467	.353	.328
a_2	.200	.184	.368	.336	.282	.286	.515	.485	.267	.212	.459
a_3	.333	.307	.263	.336	.470	.286	.172	.291	.133	.353	.328
a_4	.200	.184	.525	.336	.282	.401	.515	.388	.133	.212	.328
a_5	.467	.307	.263	.336	.282	.401	.458	.291	.467	.353	.328
a_6	.200	.184	.473	.240	.282	.401	.286	.291	.333	.212	.328
a_7	.330	.553	.263	.432	.282	.401	.229	.291	.133	.353	.328
a_8	.330	.061	.210	.240	.282	.172	.172	.291	.333	.353	.328
a_9	.467	.307	.210	.432	.282	.286	.229	.194	.467	.494	.197

$D^5 =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	.418	.391	.216	.161	.282	.376	.374	.339	.308	.360	.484
a_2	.298	.167	.360	.269	.282	.376	.327	.763	.513	.360	.346
a_3	.298	.391	.144	.269	.470	.376	.374	.085	.205	.360	.346
a_4	.298	.167	.648	.269	.282	.376	.093	.339	.205	.154	.346
a_5	.298	.391	.144	.377	.282	.226	.327	.170	.513	.360	.207
a_6	.298	.167	.576	.269	.282	.226	.280	.170	.103	.257	.484
a_7	.179	.391	.144	.485	.282	.376	.374	.170	.308	.360	.207
a_8	.418	.391	.072	.269	.470	.226	.374	.254	.308	.360	.207
a_9	.418	.391	.072	.485	.282	.376	.374	.170	.308	.360	.207

(A) The Agreed Criteria Approach

For each criterion, we have the collective matrix, C, which is based on the aggregation of all the committee members. Then $C = [c_{ij}] = \left[\frac{\sum_{k=1}^{5} d_{ij}^{k}}{5}\right]$ and then the result is as follows:

$C =$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
a_1	.369	.460	.300	.248	.345	.360	.276	.342	.392	.253	.251
a_2	.156	.229	.345	.282	.307	.360	.349	.463	.338	.317	.323
a_3	.300	.380	.311	.309	.383	.304	.272	.278	.168	.345	.321
a_4	.273	.206	.390	.330	.337	.327	.312	.364	.239	.297	.321
a_5	.302	.230	.300	.328	.337	.297	.372	.273	.376	.325	.317
a_6	.298	.191	.355	.344	.277	.328	.356	.273	.285	.317	.349
a_7	.393	.316	.260	.364	.277	.389	.317	.284	.228	.325	.293
a_8	.369	.379	.258	.334	.404	.252	.297	.297	.354	.325	.343
a_9	.373	.329	.213	.366	.247	.304	.282	.237	.378	.311	.292

Let us assume that the relative importance for each of the eleven basic criteria is regarded equally by all the committee members. Therefore, the matrix F is the same as matrix C. Then TOPSIS is used to find the collective preference ordering. The procedures are shown as follows:

procedures are shown as follows:

Step 1 Determining the ideal and negative-ideal solutions

$$A^* = \{\max_i f_{i1}, \max_i f_{i2}, \max_i f_{i3}, \min_i f_{i4}, \min_i f_{i5}, \min_i f_{i6}, \max_i f_{i7}, \max_i f_{i8}, \max_i f_{i9}, \max_i f_{i10}, \min_i f_{i11}\}$$

$$= \{.393, .460, .390, .248, .247, .252, .372, .463, .392, .345, .251\}$$

$$A^- = \{\min_i f_{i1}, \min_i f_{i2}, \min_i f_{i3}, \max_i f_{i4}, \max_i f_{i5}, \max_i f_{i6}, \min_i f_{i7}, \min_i f_{i8}, \min_i f_{i9}, \min_i f_{i10}, \max_i f_{i11}\}$$

$$= \{.156, .191, .213, .366, .404, .389, .272, .237, .168, .253, .349\}$$

Step 2 Calculating the separation measure

$$S_{i*} = \sqrt{\sum_{j=1}^{11} (f_{ij} - f^*_j)^2}, \qquad i = 1, \ldots, 9$$

$S_{1*} = .250$, $S_{2*} = .371$, $S_{3*} = .381$,

$S_{4*} = .379$, $S_{5*} = .356$, $S_{6*} = .394$,

$S_{7*} = .367$, $S_{8*} = .316$, $S_{9*} = .357$

$$S_{i-} = \sqrt{\sum_{j=1}^{11} (f_{ij} - f^-_j)^2}, \qquad i = 1, \ldots, 9$$

$S_{1-} = .463$, $S_{2-} = .356$, $S_{3-} = .296$,

$S_{4-} = .283$, $S_{5-} = .350$, $S_{6-} = .296$,

$S_{7-} = .326$, $S_{8-} = .383$, $S_{9-} = .386$

Step 3 Calculating the relative closeness to the ideal solution

$$G_1 = \frac{S_{1-}}{S_{1-} + S_{1*}} = .649, \qquad G_2 = .490, \qquad G_3 = .482,$$

$G_4 = .427$, $G_5 = .496$, $G_6 = .429$,

$G_7 = .470$, $G_8 = .548$, $G_9 = .520$

Step 4 Ranking the collective preference orderings

According to the descending value of G_i, the collective preference orderings are: a_1, a_8, a_9, a_5, a_2, a_3, a_7, a_6, a_4.

(B) The Individual Approach

Each committee member may have his/her own criteria set which may or may not be the same as the other committee members. For simplicity, in the following analysis, let us assume that each committee member has the same criteria and uses equal weight for each criterion. Two kinds of analysis will be discussed:

(1) Additive weighted value approach

$$G_i = \{ \sum_{k=1}^{5} \sum_{j=1}^{11} w^k_j \, d^k_{ij} \mid j \in (x_1, x_2, x_3, x_7, x_8, x_9, x_{10})\} -$$

$$\{ \sum_{k=1}^{5} \sum_{j=1}^{11} w^k_j \, d^k_{ij} \mid j \in (x_4, x_5, x_6, x_{11})\}$$

$G_1 = 5.936$, $G_2 = 4.628$, $G_3 = 3.687$,

$G_4 = 3.815$, $G_5 = 4.485$, $G_6 = 3.941$,

$G_7 = 3.999$, $G_8 = 4.730$, $G_9 = 4.559$

According to the descending value of G_i, the collective preference orderings are: a_1, a_8, a_2, a_9, a_5, a_7, a_6, a_4, a_3.

(2) The TOPSIS and Borda's Approach

Each committee member has his own individual weighted normalized matrix. According to the procedure of TOPSIS, the preference ordering is:

for expert 1: a_8, a_1, a_9, a_7, a_4, a_6, a_3, a_2, a_5

for expert 2: a_8, a_6, a_3, a_7, a_2, a_5, a_1, a_4, a_9

for expert 3: a_1, a_3, a_5, a_9, a_8, a_2, a_6, a_4, a_7

for expert 4: a_5, a_1, a_9, a_2, a_7, a_4, a_6, a_8, a_3

for expert 5: a_2, a_4, a_1, a_5, a_8, a_6, a_9, a_7, a_3

8, 7, ..., 1, 0 scores are assigned to the first rank, second rank, ..., last rank

8, 7, ..., 1, 0 scores are assigned to the first rank, second rank, ..., last rank place made by each committee member. Then the Borda score can be determined for each place as the sum of the committee members' scores for that place. There are: $a_1 = 30$, $a_2 = 21$, $a_3 = 15$, $a_4 = 16$, $a_5 = 21$, $a_6 = 17$, $a_7 = 15$, $a_8 = 25$, and $a_9 = 19$. The candidates are then ranked in the order of the Borda score: a_1, a_8, $a_2 \sim a_5$, a_9, a_6, a_4, $a_3 \sim a_7$ where a_2 and a_5 are tied for third place, and a_3 and a_7 are tied for last place.

Note. The identical criteria approach and the different criteria approach resulted in two different rankings. However, a_1 was given a top rating by both. The committee then discusses the rankings and submits the overall recommendation to the boss or top manager.

15.5 NOTE

The ordinal (ranking) and cardinal (rating) approaches allow committee members to individually evaluate each candidate and to find the collective preference ordering. In addition, these approaches will show to what extent candidates are preferred over others. The committee members making the decision have certain characteristics. They may or may not share the same criteria. And even if they do share the same criteria, each still may weigh each part of the criteria differently.

Both approaches may be applied in real-life situations, are simple to use, and include all relevant factors and important intangible factors. The advantage of using the ordinal approach is that the assignment problem technique can be used quite easily. The Borda score, used in the ordinal approach, is very popular. An example of this is the weekly poll made by AP or UPI of the top 20 college basketball teams in the USA. In the cardinal approach, the TOPSIS method is used. TOPSIS takes into account the distance the candidates are from each other, and considers the relative closeness the top candidate is to the ideal candidate.

These approaches are useful to the committee members in the evaluating and selecting of candidates. However, many questions still remain unanswered. For example, how should committee members interpret the available information and arrive criteria is yet to be determined. This is a psychological and political problem. However, the methods proposed do aid in evaluating and selecting candidates. The methods can be applied to more complex cases such as the allocation of scarce resources, time dependent judgements, and the probabilistic or fuzzy presentation in the decision process.

16. A SYSTEMS APPROACH TO EXPERT JUDGMENTS AND/OR GROUP PARTICIPATION ANALYSIS

16.1 INTRODUCTION

Today's decision makers and problem solvers in government, military, business, industry, and education--in any area of our society--are confronted with a variety of problems. These problems are highly complex, often interdisciplinary or transdisciplinary, with social, economic, political, and emotional factors intertwined with more quantifiable factors of physical technology. Therefore, when attempting to solve a problem, all important factors of the problem should be considered.

In society, decisions often affect groups of people instead of isolated individuals. However, group decision making is usually understood to be the reduction of many different individual preferences (interests) to a single choice, either by conflict or by compromise. Frequently, social problems appear to be so complex as to be insolvable. High-quality decision making requires the decision maker to see through the problem and its complexities. This can be accomplished by focusing on the problem from a systems viewpoint and by adopting a systems philosophy and attitude toward problem solving. In the systems approach, solutions must succeed for all systems and for all people, regardless of their political, religious, geographical, or other affiliations.

Many methods and techniques have been proposed during the past 50 years. Unfortunately, each technique was designed to solve some specific problem. This section presents a proposed guide for selecting and using techniques in the complexity of society.

An excellent presentation of systems analysis and procedures for problem