

Lesson 10: Predator-Two Prey and ode45

10.1 Applied Problem.

In this lesson we will allow for more than two populations where they depend on each other. One population could be the predator such as a fox, and the second and third could be the prey such as a rabbits and turkeys. The populations will be modeled by three or more differential equations. The Matlab command ode45 can be used to solve such systems of differential equations.

10.2 Differential Equation Model.

We will consider last two continuous predator-prey models:

- one predator, and one prey with constant birth and death rate,
- one predator, and one prey with variable birth and death rates and
- one predator, and two preys.

We consider the predator to be a fox and a prey to be either rabbits or turkeys. One could also consider different species of fish such as sharks and bass.

One Predator, and One Prey with Variable Birth and Death Rates.

The equation for the fox population will remain the same. The rabbit population will now have a logistic model when there are no foxes. This means the birth and death rate for the rabbit population will vary and be set equal to $k(M - y)$. So, with no foxes the differential equation model for the rabbits is $y' = k(M - y)y$. If there is a nonzero fox population, then there will be an additional death rate for the rabbit population so that

birth rate minus death rate equals $k(M - y) - cx$. The constants k , M and c must be determined from careful observations of the populations. The new mathematical model is

$$x' = (-d + ey)x \quad \text{and} \quad x(0) = x_o, \quad \text{Fox Equation}$$

$$y' = (k(M - y) - cx)y \quad \text{and} \quad y(0) = y_o. \quad \text{Rabbit Equation}$$

One Predator, and Two Prey with Variable Birth and Death Rates.

Consider one fox and two prey: rabbits and turkeys. Let the turkey population be given by $z(t)$. Since the turkey population is also a prey, the birth rate minus the death rate of the turkeys must be similar to the foxes so that birth rate minus death rate equals $k_T(M_T - z) - c_Tx$. The birth rate minus the death rate for the fox population will increase to $-d + ey + e_Tz$. Therefore, the new system of differential equations will have three equations

$$x' = (-d + ey + e_Tz)x \quad \text{and} \quad x(0) = x_o, \quad \text{Fox Equation}$$

$$y' = (k(M - y) - cx)y \quad \text{and} \quad y(0) = y_o, \quad \text{Rabbit Equation}$$

$$z' = (k_T(M_T - z) - c_Tx)z \quad \text{and} \quad z(0) = z_o. \quad \text{Turkey Equation}$$

10.3 Method of Solution.

In this lesson we will use the Matlab command `ode45` to solve our systems of differential equation. This command is a robust implementation for systems of differential equations, which uses a variable step size method and the fourth and fifth order Runge-Kutta method.

10.4 Matlab Implementation.

In the following calculations we have attempted to solve the second predator-prey model

$$x' = -.5x + .01xy \text{ and}$$

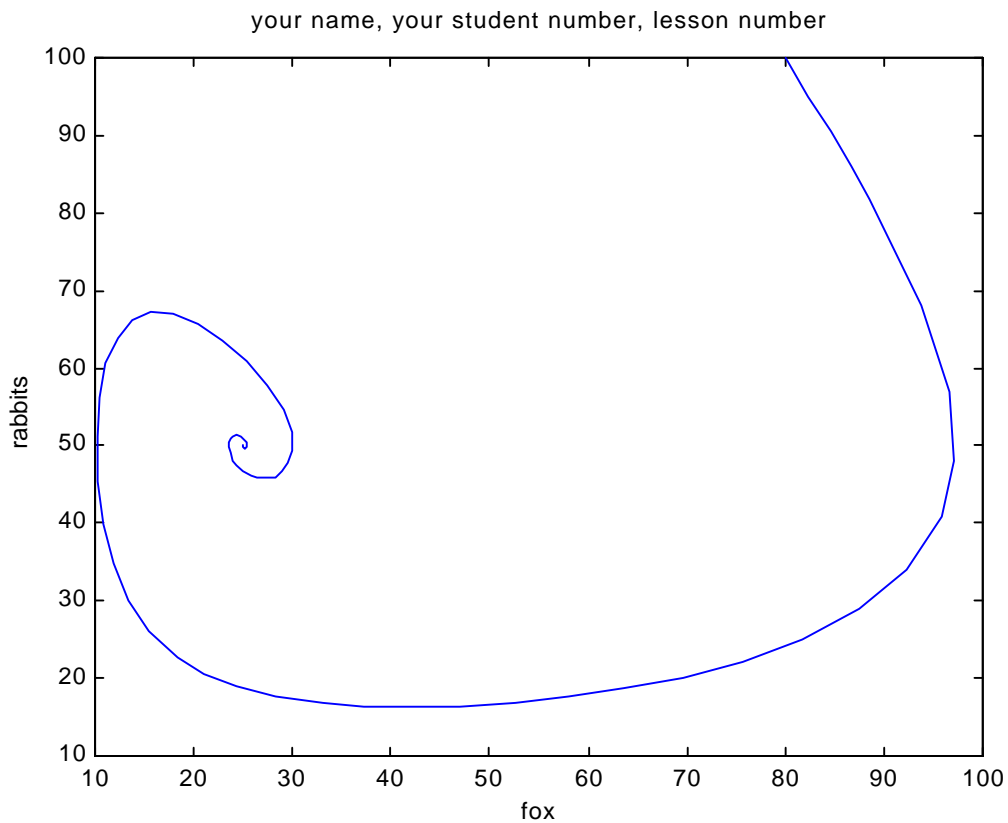
$$y' = .005(100 - y)y - .01xy.$$

The **steady state solutions** are given by

$$x' = -.5x + .01xy = 0 \text{ and}$$

$$y' = .005(100 - y)y - .01xy = 0.$$

The nonzero steady state solutions are $x = 25$ and $y = 50$. Our numerical solution spirals in towards the steady state solution.

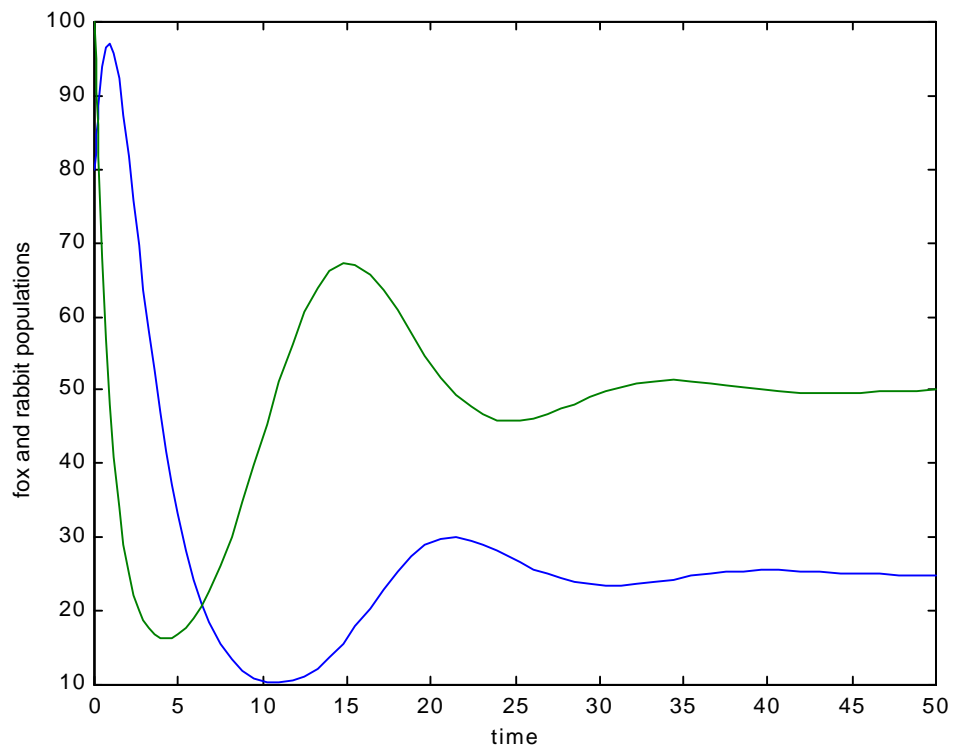


m-file yprf.m:

```
function yprf =yprf(t,y)
    yprf(1) =-.5*y(1) + .01*y(1)*y(2);
    yprf(2) = .005*(100 - y(2))*y(2) -.01*y(1)*y(2);
    yprf = [yprf(1) yprf(2)]';
```

m-file rf.m:

```
%your name, your student number, lesson number
clear;
to = 0;
tf =50;
yo = [80 100];
[t y] = ode45('yprf',[to tf],yo);
plot(y(:,1),y(:,2))
title('your name, your student number, lesson number')
ylabel('rabbits')
xlabel('fox')
%plot(t,y(:,1),t,y(:,2))
%xlabel('time')
%ylabel('rabbits and fox')
```



In the yprf.m file the $x(t)$, fox, is associated with the symbol $y(1)$, and $y(t)$, rabbits, is associated with the symbol $y(2)$. The right sides of the differential equations are given by yprf(1) and yprf(2). In rf.m the $y_0 = [80 \ 100]$ is a 2x1 array of initial populations where $x(0) = 80$ and $y(0) = 100$. The output $[t \ y]$ from the rf.m file has three column vectors where the first column is for all the time values and the second and third columns correspond to the values of $x(t)$ and $y(t)$. These values can be viewed by typing $[t \ y]$ at the Matlab prompt. Also the following graph of $x(t)$ versus t and $y(t)$ versus t was generated by typing $\text{plot}(t, y(:,1), t, y(:,2))$ at the Matlab prompt.

10.5 Numerical Experiments.

In the following calculations we have attempted to solve the third predator-prey model

$$x' = -.5x + .01xy + .02xz,$$

$$y' = .01(100 - y)y - .01xy \text{ and}$$

$$z' = .04(80 - z)z - .03xz.$$

In the following Matlab calculations with ode45 we used the initial conditions $x(0) = 100$, $y(0) = 90$ and $z(0) = 80$. From these one can determine which of the three curves in the graph below are fox, rabbits and turkeys.

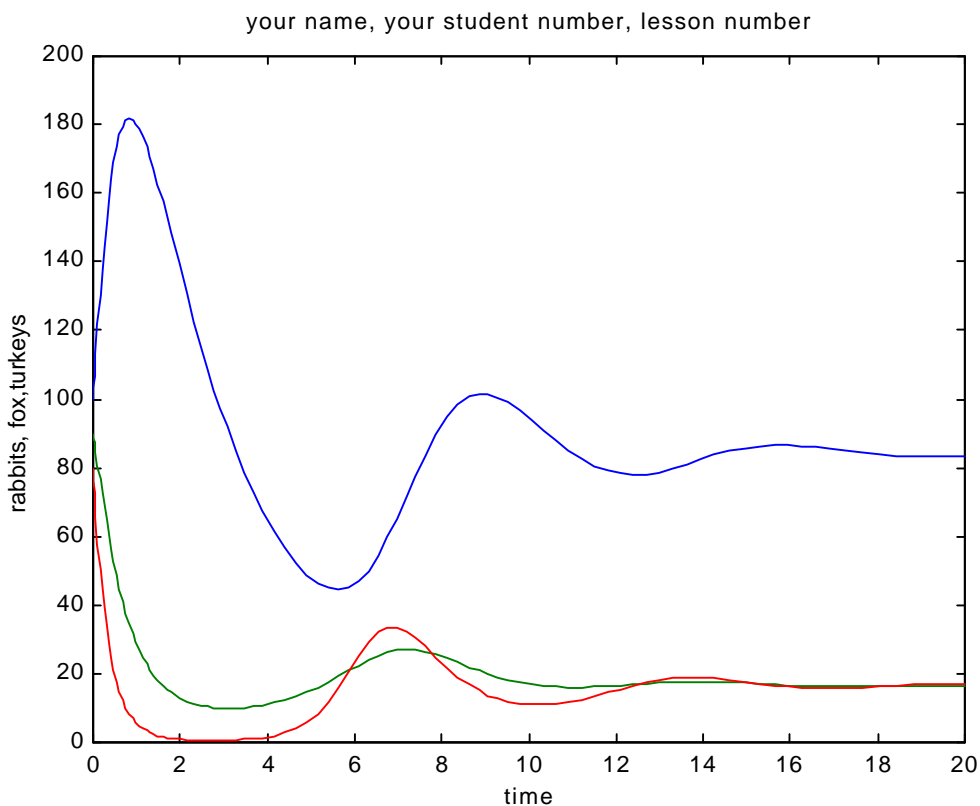
In the yprft.m file the symbol $y(1)$ represents fox, the symbol $y(2)$ represent rabbits and the symbol $y(3)$ represents turkeys. In the rft.m file the 1x3 array $y_0 = [100 \ 90 \ 80]$ has the initial conditions. The output of the rft.m file has four columns of vectors where the first column has the times and the next three columns have the associated population sizes.

m-file yprft.m:

```
function yprft =yprft(t,y)
    yprft(1) =-.5*y(1) + .01*y(1)*y(2) + .02*y(1)*y(3);
    yprft(2) = .01*(100 - y(2))*y(2) - .01*y(1)*y(2);
    yprft(3) = .04*(80 - y(3))*y(3) - .03*y(1)*y(3);
    yprft = [yprft(1) yprft(2) yprft(3)]';
```

m-file rft.m:

```
%your name, your student number, lesson number
clear;
to = 0;
tf =20;
yo = [100 90 80];
[t y] = ode45('yprft',[to tf],yo);
plot(t,y(:,1),t,y(:,2),t,y(:,3))
title('your name, your student number, lesson number')
xlabel('time')
ylabel('rabbits, fox,turkeys')
```



10.6 Additional Calculations.

Consider the above fox, rabbit, turkey problem with variable initial conditions

$$[x(0) \ y(0) \ z(0)] = [100 + 10*(1+.S) \ 90 \ 80], [100 + 10*(1+.S) \ 1 \ 80] \text{ and}$$

$$[100 + 10*(1+.S) \ 90 \ 1]$$

- (a). Find a steady state solution where all three populations are positive.
- (b). Modify the rft.m file.
- (c). Insert your name and student number into the rft.m file.

Execute rft.m for each set of initial conditions.

- (d). Print the yprft.m and rft.m files and the graphs.