DISCRETE MATHEMATICS 1st MIDTERM EXAM

90 minutes March 19, 2014

Id	Fullname	Signature

Q1	Q2	Q3	Q4	Q5	Q6	Total
/15	/15	/20	/15	/15	/20	/100

No questions are allowed. Answer the questions to the best of your understanding. If you need to make extra assumptions, state them clearly. Make sure that all your answers are sufficiently (and mathematically) explained.

1. Show that $\neg p \to (q \to r) \Leftrightarrow q \to (p \vee r)$ using the laws of logic. Be sure to cite each law whenever used.

2. Determine whether the following proof is valid. If it is valid, provide a valid proof of the result. If it is not valid, provide specific truth values of $p, q,$ and r as a counterexample.

$$\begin{array}{l} p \to r \\ q \to r \\ \neg(p \vee q) \\ \hline \therefore \neg r \end{array}$$

[ ] VALID

PROOF:

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[ ] NOT VALID

COUNTEREXAMPLE:

p :
q :
r :

Explanation:

3. Evaluate each of the following for the universe $\mathbb{Z}$.

(a) $\exists x \forall y \; x < y$ where $x, y \in \mathbb{Z}$

(b) $\forall y \exists x \; x < y$ where $x, y \in \mathbb{Z}$

(c) $\exists x \exists y \; x + y = 0 \vee x \cdot y = 0$ where $x, y \in \mathbb{Z}^+$

(d) $\forall x \forall y \; x \cdot y \geq x + y$ where $x, y \in \mathbb{Z}^+$

ANYTHING WRITTEN BELOW THIS LINE WILL NOT BE GRADED.

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4. 11 men and 8 women are to be seated such that no two women sit together. In how many ways, can this be done

   (a) if they are seated in a straight row?

   (b) if they are seated around a circular table?

5. Prove that $A \cap B = B \Rightarrow \overline{A} \subseteq \overline{B}$. Do not use Venn diagrams.

6. Consider the equations below:

$$\begin{aligned} 1 \cdot 1! &= 2! - 1 \\ 1 \cdot 1! + 2 \cdot 2! &= 3! - 1 \\ 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! &= 4! - 1 \end{aligned}$$

(a) If the equations above correspond to $k = 1, 2,$ and 3, what is the nth equation?

(b) Prove by mathematical induction that the nth equation is true for all $n \geq 1$.

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