## DISCRETE MATHEMATICS 2nd MIDTERM EXAM

90 minutes
May 7, 2014

| Id | Fullname | Signature |
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| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Total |
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| $/ 20$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 20$ | $/ 100$ |

No questions are allowed. Answer the questions to the best of your understanding. If you need to make extra assumptions, state them clearly. Make sure that all your answers are sufficiently (and mathematically) explained.

1. The relation $\alpha$ is cyclic if it satisfies this property: $a \alpha b \wedge b \alpha c \Rightarrow c \alpha a$.

Prove that if a relation is cyclic and symmetric then it is an equivalence relation.
2. Let $f: A \rightarrow B$ where $|A|=m$ and $|B|=n$. What should be the relationship between $m$ and $n$ for $f$ to be (a) injective?
(b) surjective?
(c) bijective?
3. In an exam with 30 students, 1 student has made 13 mistakes and all other students have made fewer mistakes. Prove that at least 3 students have made the same number of mistakes.

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|  |  |  | $/ 15$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 20$ | $/ 100$ |  |

4. Draw all non-isomorphic plain graphs on 5 vertices and exactly 3 edges.
5. For what values of $n$
(a) does $K_{n}$ have an Euler path?
(b) does $K_{n}$ have a Hamiltonian path?
6. Consider the expression:

$$
\frac{1}{\sqrt{2 \pi}} e^{-\frac{x^{2}}{2}}
$$

(a) Draw the expression tree for this expression. Use the $\uparrow$ symbol for exponentiation and represent $\sqrt{a}$ as $a \uparrow 0.5$.
(b) Write the postorder traversal for this expression tree.

